Online Appendix to: *Is Government Spending at the Zero Lower Bound Desirable?*

Full DSGE Model

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1 Optimal pricing: recursive structure

We assume a typical environment with a continuum of monopolistic producers, each of measure zero. We begin by illustrating the problem in the absence of capital accumulation and of price indexation. Our final description of the equilibrium, however, incorporates both features. The production function of each monopolistic producer is:

\[ Y_t(i) = N_t(i)^{1-\alpha} \]  

where \( N_t(i) \) is total labor demand by individual producer \( i \).

The optimal demand for the individual variety \( i \) reads:

\[ Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} Y_t \]  

where \( Y_t \) is total demand for variety \( i \).

In equilibrium, the following relationship between individual and average nominal marginal cost, \( MC_{t+k} \), holds

\[ MC_{t+k|t} = \frac{W_{t+k}}{1 - \alpha} N_{t+k}^{-\alpha} \left( \frac{N_{t+k}}{N_{t+k|t}} \right)^{-\alpha} \]  

\[ = MC_{t+k} \left( \frac{N_{t+k}}{N_{t+k|t}} \right)^{-\alpha} \]  

\[ = MC_{t+k} \left( \frac{P_{t+k}}{P_t} \right)^{\frac{\varepsilon_p \alpha}{1 - \alpha}} \]  

where \( MC_{t+k|t} \) is the nominal marginal cost at \( t + k \) of a firm that last reset its price at time \( t \). Notice that the last equality follows from (2), and \( MC_{t+k} \) is the average nominal marginal cost. Similarly, notice that:

\[ Y_{t+k|t} = \left( \frac{P_t}{P_{t+k}} \right)^{-\varepsilon_p} Y_{t+k} \]  


Optimal Pricing  The first order condition with respect to $\overline{P}_t$ is (abstracting from index i):

$$
\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k_p \{ Q_{t,t+k} \left( \prod_{s=1}^k \Pi_{t+s} \right) \}^{-1} Y_{t+k|t} \overline{P}_t \} = \mathcal{M}_p(1 - S) \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k_p \{ Q_{t,t+k} \left( \prod_{s=1}^k \Pi_{t+s} \right) \}^{-1} Y_{t+k|t} MC_{t+k|t} \}
$$

where $Q_{t,t+k} = \beta^k \frac{\gamma_{c,t+k}}{\gamma_{c,t}}$, $S = 1 - \frac{1}{\mathcal{M}_p \mathcal{M}_w}$ is an employment subsidy financed via lump sum taxes, and $MC_{t+k|t}$ is the nominal marginal cost at $t + k$ of a firm that last reset its price at time $t$.

Dividing through by $P_t$ we can write the LHS of the above equation as follows (using 3 and 4):

$$
LHS \equiv \left( \frac{\overline{P}_t}{P_t} \right)^{1 - \epsilon_p} \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k_p \{ Q_{t,t+k} \left( \prod_{s=1}^k \Pi_{t+s} \right) \}^{-1} Y_{t+k|t} \left( \frac{P_{t+k}}{P_t} \right)^{\alpha \epsilon_p} \frac{1}{1 - \alpha}
$$

where $\Pi_{t+s} = P_{t+s}/P_{t+s-1}$.

Consider next the RHS of (5):

$$
RHS = \mathcal{M}_p(1 - S) \left( \frac{\overline{P}_t}{P_t} \right)^{1 - \epsilon_p} \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k_p \{ Q_{t,t+k} \left( \prod_{s=1}^k \Pi_{t+s} \right) \}^{-1} Y_{t+k|t} MC_{t+k|t} \left( \frac{P_{t+k}}{P_t} \right)^{\alpha \epsilon_p} \frac{1}{1 - \alpha}
$$

where $mc_{t+k} \equiv MC_{t+k|t} / P_{t+k}$ is the average real marginal cost.

Equating LHS and RHS and rearranging we finally obtain:

$$
\left( \frac{\overline{P}_t}{P_t} \right)^{1 - \alpha + \epsilon_p} \frac{1}{1 - \alpha} \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k_p Q_{t,t+k} Y_{t+k} \left( \prod_{s=1}^k \Pi_{t+s} \right)^{\epsilon_p - 1} = \mathcal{M}_p(1 - S) \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k_p Q_{t,t+k} Y_{t+k} mc_{t+k} \left( \prod_{s=1}^k \Pi_{t+s} \right)^{\epsilon_p} \frac{1}{1 - \alpha}
$$

Recursive representation  Define

$$
\mathcal{K}_t^p \equiv \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k_p Q_{t,t+k} Y_{t+k} \left( \prod_{s=1}^k \Pi_{t+s} \right)^{\epsilon_p - 1}
$$

$$
\mathcal{Z}_t^p \equiv \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k_p Q_{t,t+k} Y_{t+k} mc_{t+k} \left( \prod_{s=1}^k \Pi_{t+s} \right)^{\epsilon_p} \frac{1}{1 - \alpha}
$$

Express recursively as:

$$
\mathcal{K}_t^p = Y_t + \theta_p \left( \frac{\gamma_{c,t+1}}{\gamma_{c,t}} \right) \Pi_{t+1}^{\epsilon_p - 1} \mathcal{K}_{t+1}^p
$$
Similarly:

\[ Z_t^p = Y_t mc_t + \theta_p \left( \beta \frac{U_{c,t+1}}{U_{ct}} \right) \Pi_{t+1}^{\epsilon_p} Z_{t+1}^p. \]

We also have:

\[ 1 = \theta_p \Pi_t^{\epsilon_p-1} + (1 - \theta_p) \left( \frac{P_t}{P_t} \right)^{1-\epsilon_p}. \quad (6) \]

Summarizing, the pricing block comprises the following set of equilibrium conditions:

\[ K_t^p = Y_t + \theta_p \left( \beta \frac{U_{c,t+1}}{U_{ct}} \right) \Pi_{t+1}^{\epsilon_p} K_{t+1}^p. \]

\[ Z_t^p = Y_t mc_t + \theta_p \left( \beta \frac{U_{c,t+1}}{U_{ct}} \right) \Pi_{t+1}^{\epsilon_p} Z_{t+1}^p. \]

\[ \frac{1-\alpha + \epsilon_p \alpha}{P_t^{1-\alpha}} K_t^p = M_p (1 - S) Z_t^p \]

\[ 1 = \theta_p \Pi_t^{\epsilon_p-1} + (1 - \theta_p)P_t^{1-\epsilon_p}. \]

where \( P_t \equiv P_t/P_t \).

2 Optimal wage setting: recursive form

In this section we introduce nominal wage rigidity along the lines of Erceg et al. (2000). The economy is populated by a continuum of households, each supplying a differentiated labor type \( j \), and by a continuum of firms. .

Deriving total demand for each labor type Each firm \( i \) employs all differentiated labor types. Hence total labor demand by firm \( i \) can be written:

\[ N_t(i) = \left( \int_0^1 N_t(i, j) \frac{\epsilon_w}{1-\epsilon_w} \, dj \right)^{-\epsilon_w} \]

where \( N_t(i, j) \) is demand by firm \( i \) of labor type \( j \).

Optimal demand for labor type \( j \) by firm \( i \) reads:

\[ N_t(i, j) = \left( \frac{W_t(j)}{W_t(i)} \right)^{-\epsilon_w} N_t(i) \quad (7) \]

Integrating across firms, we can derive the equilibrium total demand for each labor type \( j \) (using (7) above):
Optimal wage setting problem  Next, consider the optimal wage setting problem for household $j$:

$$\max \mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_w)^k U(\tilde{C}_{t+k|t}(j), N_{t+k|t}(j))$$

where $N_{t+k|t}(j)$ is time $t+k$ labor supply by household type $j$ who last reset her wage in time $t$.

At the chosen wage $\overline{W}_t(j)$, household type $j$ is assumed to supply enough labor to satisfy demand. The constraint reads, using (8):

$$N_{t+k|t}(j) = (\overline{W}_t(j))^{-\varepsilon_w} \frac{N_t}{W_t}$$

Notice that $N_{t+k}$ bears the index $t+k$ (and not $t+k|t$) because it corresponds to aggregate (or average) labor demand.

The additional household’s constraint is the budget constraint:

$$P_{t+k}C_{t+k|t}(j) + \mathbb{E}_t \left\{ Q_{t+k, t+k+1} B_{t+k+1|t} \right\} \leq B_{t+k|t} + \overline{W}_t(j) N_{t+k|t}(j) - T_{t+k}$$

Each household $j$ reoptimizing the wage at a given time $t$ will choose the same optimal wage. It is therefore convenient to abstract from index $j$.

Household problem  The (relevant portion of the) Lagrangian of the household’s problem is

$$\mathcal{L}^w = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_w)^k \left\{ U\left( \tilde{C}_{t+k|t}, N_{t+k|t} \right) - \lambda_{t+k|t} \left[ P_{t+k}C_{t+k|t} - \overline{W}_t N_{t+k|t} \right] \right\} .$$

(9)

The FOC of the problem with respect to $\overline{W}_t$ is:

$$\sum_{k=0}^{\infty} (\beta \theta_w)^k \mathbb{E}_t \left\{ U_{N_{t+k|t}} \frac{\partial N_{t+k|t}}{\partial \overline{W}_t} + \lambda_{t+k|t} \left( N_{t+k|t} + \overline{W}_t \frac{\partial N_{t+k|t}}{\partial \overline{W}_t} \right) \right\} = 0$$
Notice:

\[
\frac{\partial N_{t+k|t}}{\partial W_t} = -\varepsilon_w \left( \frac{W_t}{W_{t+k}} \right)^{-\varepsilon_w-1} \frac{N_{t+k}}{W_{t+k}}
\]

Hence we can write:

\[
-\sum_{k=0}^{\infty} (\beta \theta_w)^k E_t \left\{ U_{N',t+k|t} \varepsilon_w N_{t+k|t} \frac{1}{W_t} + \lambda_{t+k|t} N_{t+k|t} (\varepsilon_w - 1) \right\} = 0
\]

Under complete markets and separable utility we have \( U_{c,t+k}(C_{t+k|t}, N_{t+k|t}) = U_{c,t+k}(C_{t+k}) \).
In addition, equilibrium implies \( U_{c,t+k} = \lambda_{t+k} P_{t+k} \) (since \( \lambda_{t+k} \) is the shadow value of one unit of nominal income at \( t + k \)).

Hence we have:

\[
-\sum_{k=0}^{\infty} (\beta \theta_w)^k E_t \left\{ U_{N,t+k|t} \varepsilon_w N_{t+k|t} \mathcal{M}_w + U_{c,t+k} N_{t+k|t} \frac{W_t}{P_{t+k}} \right\} = 0
\]

where \( \mathcal{M}_w \equiv \varepsilon_w / (\varepsilon_w - 1) \).

The above expression can be rewritten:

\[
\sum_{k=0}^{\infty} (\beta \theta_w)^k E_t \left\{ U_{c,t+k} N_{t+k|t} \mathcal{M}_w + U_{N,t+k|t} \frac{W_t}{P_{t+k}} \right\} = 0
\]

Recursive representation  Condition (10) reads:

\[
E_t \sum_{k=0}^{\infty} (\beta \theta_w)^k N_{t+k|t} U_{c,t+k} \frac{W_t}{P_{t+k}} = E_t \sum_{k=0}^{\infty} (\beta \theta_w)^k N_{t+k|t} \mathcal{M}_w (U_{N,t+k|t})
\]

Using the optimal labor demand condition

\[
N_{t+k|t} = \left( \frac{W_t}{W_{t+k}} \right)^{-\varepsilon_w} N_{t+k},
\]

we can write the LHS as follows:

\[
LHS \equiv \left( \frac{W_t}{P_t} \right)^{1-\varepsilon_w} \left\{ \left( \frac{W_t}{P_t} \right)^{\varepsilon_w} N_t U_{c,t} + \beta \theta_w \left( \frac{W_{t+1}}{P_{t+1}} \right)^{\varepsilon_w} \Pi_{t+1} N_{t+1} U_{c,t+1} + \right. \]
\[
+ \left( \beta \theta_w \right)^2 \left( \frac{W_{t+2}}{P_{t+2}} \right)^{\varepsilon_w} \Pi_{t+1} \Pi_{t+2} \left( N_{t+2} U_{c,t+2} + \ldots \right) \right\}
\]

\[
= \overline{w}_t^{1-\varepsilon_w} E_t \sum_{k=0}^{\infty} (\beta \theta_w)^k w_{t+k}^{\varepsilon_w} \left( \prod_{s=1}^{k} \Pi_{t+s} \right)^{\varepsilon_w-1} N_{t+k} U_{c,t+k},
\]

where \( \overline{w}_t \equiv W_t / P_t \).

Next consider RHS:

\[
RHS \equiv - \left( \frac{W_t}{P_t} \right)^{-\varepsilon_w} \left\{ \left( \frac{W_t}{P_t} \right)^{\varepsilon_w} N_t \mathcal{M}_w U_{N,t|t} + \beta \theta_w \left( \frac{W_{t+1}}{P_{t+1}} \right)^{\varepsilon_w} N_{t+1} \Pi_{t+1} \mathcal{M}_w U_{N,t+1|t} + \ldots \right\}
\]
This can be written

\[
RHS \equiv \pi_t^{-\varepsilon_w} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_w)^k w_{t+k}^{\varepsilon_w} \left( \prod_{s=1}^{k} \Pi_{t+s} \right) \varepsilon_w N_{t+k} \mathcal{M}_w (-U_{N,t+k|t})
\]

Under the assumption that \( U_N(\bullet) \) is homogenous of degree \( \varphi \) in \( N \) we have (using (11)):

\[
-U_{N,t+k|t} = \left( \frac{W_t}{W_{t+k}} \right) \varepsilon_{w\varphi} (-U_{N,t+k}(N_{t+k}))
\]

Substituting:

\[
RHS \equiv \pi_t^{-\varepsilon_w(1+\varphi)} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_w)^k w_{t+k}^{\varepsilon_w(1+\varphi)} N_{t+k} \mathcal{M}_w \left( \prod_{s=1}^{k} \Pi_{t+s} \right) \varepsilon_w(1+\varphi) (-U_{N',t+k}(N_{t+k}))
\]

Combining LHS and RHS we obtain:

\[
\pi_t^{1+\varepsilon_w\varphi} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_w)^k w_{t+k}^{\varepsilon_w(1+\varphi)} N_{t+k} \mathcal{M}_w \left( \prod_{s=1}^{k} \Pi_{t+s} \right) \varepsilon_w(1+\varphi) (-U_{N',t+k}(N_{t+k}))
\]

We can rewrite recursively:

\[
k^w_t = w_t^{\varepsilon_w} N_t U_{c,t} + \theta w_\varepsilon w \Pi_{t+1}^{-1} k_{t+1}^w
\]

\[
z^w_t = w_t^{\varepsilon_w(1+\varphi)} N_t (-U_{N',t}(N_t)) + \theta w_\varepsilon w \Pi_{t+1}^{\varepsilon_w(1+\varphi)} z_{t+1}^w
\]

Hence the first order condition can be written in compact form:

\[
\pi_t^{1+\varepsilon_w\varphi} k^w_t = \mathcal{M}_w z_t^w
\]

**Summary of wage setting equilibrium conditions**

\[
w_t^{1-\varepsilon_w} = \theta_w (w_{t-1} \Pi_t)^{1-\varepsilon_w} + (1 - \theta_w)w_t^{1-\varepsilon_w}
\]

\[
k^w_t = w_t^{\varepsilon_w} N_t U_{c,t} + \theta w_\varepsilon w \Pi_{t+1}^{-1} k_{t+1}^w
\]

\[
z^w_t = w_t^{\varepsilon_w(1+\varphi)} N_t (-U_{N',t}(N_t)) + \theta w_\varepsilon w \Pi_{t+1}^{\varepsilon_w(1+\varphi)} z_{t+1}^w
\]

\[
\pi_t^{1+\varepsilon_w\varphi} k^w_t = \mathcal{M}_w z_t^w
\]

6
2.1 Price dispersion, wage dispersion, and equilibrium

Market clearing for each individual variety implies:

\[
\frac{N_t(i)}{P_t} = \left( \frac{P_i}{P_t} \right)^{-\varepsilon} Y_t \quad \text{(12)}
\]

where \(N_t(i)\) denotes the total amount of labor employed by firm i. Rearranging:

\[
N_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t^{\frac{1}{1-\alpha}}
\]

Integrating across all producers:

\[
\int_0^1 N_t(i) \, di = \int_0^1 \left[ \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} Y_t \right]^{\frac{1}{1-\alpha}} \, di = Y_t^{\frac{1}{1-\alpha}} \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} \, di = Y_t^{\frac{1}{1-\alpha}} \Delta_{p,t} \quad \text{(13)}
\]

where \(\Delta_{p,t} \equiv \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} \, di\) measures the dispersion of relative prices across producers. In a more compact form:

\[
N_t = Y_t^{\frac{1}{1-\alpha}} \Delta_{p,t} \quad \text{(15)}
\]

where \(N_t = \int_0^1 N_t(i) \, di\).

Equilibrium in the market for the final good requires:

\[
Y_t = C_t + G_t \quad \text{(16)}
\]

Hence conditions (15) and (16) describe aggregate market clearing.\(^1\)

Expressing \(\Delta_{p,t}\) in recursive form:

\[
\Delta_{p,t} = \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} \, di
\]

\[
= \int_{1-\theta_p}^{1} \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} \, di + \int_{\theta_p}^{1} \left( \frac{P_{t-1}(i)}{P_{t-1}} \right)^{-\varepsilon_p} \, di
\]

\[
= (1-\theta_p) \left( \frac{P_t}{P_t} \right)^{-\varepsilon_p} + \theta_p \Pi_t^{\varepsilon_p} \Delta_{p,t-1}
\]

\(^1\)Equivalently, let \(y^*_t(i) \equiv A_t N_t^\beta(i)^{1-\alpha}\) denote the supply of variety i. In equilibrium:

\[
y^*_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t
\]

Integrating across i:

\[
Y^*_t = \int_0^1 y^*_t(i) \, di = \Delta_{p,t} Y_t
\]

From this notation it is clear the interpretation of \(Y_t = C_t\) as an index of aggregate demand.
Let \( N_t(j) \) denote labor supply by each differentiated household. Since each household is assumed to satisfy labor demand at the given posted wage, equilibrium in the labor market requires:

\[
N_t(j) = N_t(j)
\]

Aggregating across each household \( j \) one obtains, using (8):

\[
N_t = \int_0^1 N_t(j) dj = \int_0^1 N_t(j) = \int_0^1 \left( \frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} dj N_t
\]

where \( N_t \) is an index of aggregate labor supply. By defining \( \Delta_{w,t} \equiv \int_0^1 \left( \frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} \) as wage dispersion, the above equation becomes.

\[
N_t = \Delta_{w,t} N_t
\]

Notice that by substituting (17) into (15) one obtains:

\[
N_t = \frac{\Delta_{w,t} N_t}{Y_t^{1-\alpha} \Delta_{p,t}}
\]

which shows that the relationship between aggregate employment \( N_t \) and aggregate output \( Y_t \) depends on both price and wage dispersion.

3 Capital accumulation

Suppose each monopolistic firm \( i \) produces a homogenous good according to the production function:

\[
Y_t(i) = \left[ N_t(i)^{1-\alpha} K_t^\alpha(i) \right]^\xi
\]

where is a labor productivity shifter (common across firms). Notice that parameter \( \xi \geq 1 \) measures the degree of returns to scale in production.

The cost minimizing choice of labor and capital input implies:

\[
\frac{W_t}{P_t(i)} = \frac{MC_t}{P_t(i)} (1 - \alpha) \left( \frac{K_t(i)}{N_t(i)} \right)^{\alpha \xi}
\]

\[
\frac{Z_t}{P_t(i)} = \frac{MC_t}{P_t(i)} \alpha \left( \frac{N_t(i)}{K_t(i)} \right)^{(1-\alpha)\xi}
\]

where \( Z_t \) is the nominal rental cost of capital.

Notice that the above conditions imply:

\[
MC_t(i) = \frac{W_t^{(1-\alpha)\xi} Z_t^{\alpha \xi}}{(\alpha \xi)^{\alpha \xi} (\xi(1 - \alpha))^{(1-\alpha)\xi} Y_t(i)^{1-\xi}}.
\]
**Constant returns to scale.** We assume $\xi = 1$. Hence we have $MC_t(i) = MC_t$ for all $i$, i.e., the nominal marginal cost is identical across firms. Notice also that we can write:

$$MC_t(i) = \frac{W_t}{\xi(1-\alpha)} \left( \frac{K_t(i)}{N_t(i)} \right)^{\xi \alpha}$$

and

$$MC_t(i) = \frac{Z_t K_t(i)}{\xi \alpha N_t(i)}$$

In the case $\xi = 1$, since $MC_t(i) = MC_t$ for all $i$, we also have $K_t(i)/N_t(i) = K_t/N_t$ for all $i$. In other words, under constant returns to scale, the capital labor ratio is equalized across firms.

**Market clearing** Henceforth we assume $\xi = 1$. Market clearing for each individual variety implies:

$$N_t(i)^{1-\alpha} K_t^\alpha(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t$$

Equilibrium in the market for the final good requires:

$$Y_t = C_t + I_t + G_t$$

Integrating (25) across $i$, and combining with (26):

$$\left( \frac{K_t}{N_t} \right)^\alpha \int_0^1 N_t(i) di = \Delta_{p,t} Y_t$$

or alternatively:

$$K_t^\alpha N_t^{1-\alpha} = \Delta_{p,t} Y_t$$

**4 Equilibrium**

Let $\Delta_{p,t} \equiv \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon p} di$ and $\Delta_{w,t} \equiv \int_0^1 \left( \frac{W_t(j)}{W_t} \right)^{-\varepsilon w} dj$ denote price and wage dispersion respectively. Let $\bar{p}_t \equiv \bar{P}_t/P_t$, $\bar{w}_t \equiv \bar{W}_t/W_t$, $z_t \equiv Z_t/P_t$ and $mc_t$ be the real marginal cost of production (equal for all firms). Let $K_t^p$, $Z_t^p$, $K_t^w$, $Z_t^w$ be recursive objects in the optimal pricing and wage setting problems. For any given exogenous processes $\{\theta_t, G_t\}$, an equilibrium is a set of endogenous variables $\{\lambda_t, C_t, Y_t, N_t, \bar{Y}_t, mc_t, i_t, \Pi_t, \bar{P}_t, I_t, K_t, \bar{K}_t, u_t, q_t, z_t, w_t, \bar{w}_t, \Delta_{w,t}, \Delta_{p,t}, K_t^p, Z_t^p, K_t^w, Z_t^w\}$ solving the following set of conditions:

1. Marginal utility of consumption:

$$\lambda_t = (C_t - hC_{t-1})^{-\sigma}$$

2. Euler equation:

$$\lambda_t = \beta_t(1 + i_t)(1 + \rho_t)E_t \left[ \frac{\lambda_{t+1}}{\pi_{t+1}} \right]$$

3. Production:

$$\Delta_{p,t} Y_t = K_t^\alpha N_t^{1-\alpha}$$
4. Optimal labour demand:
\[ w_t = mc_t(1 - \alpha) \frac{Y_t}{N_t} \]

5. Optimal demand for capital:
\[ z_t = mc_t \alpha \frac{Y_t}{K_t} \]

6. Price of capital:
\[ q_t = \beta_t E_t \left( \frac{\lambda_{t+1}}{\lambda_t} (z_{t+1}u_{t+1} - a(u_{t+1}) + (1 - \delta)q_{t+1}) \right) \]

7. Optimal investment:
\[ q_t \left[ 1 - \Omega(\cdot) - \Omega'(\frac{I_t}{I_{t-1}}) \frac{I_t}{I_{t-1}} \right] = 1 - \beta_t E_t \left[ q_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{I_{t+1}}{I_t} \right)^2 \Omega' \left( \frac{I_{t+1}}{I_t} \right) \right] \]

8. Capital accumulation:
\[ \tilde{K}_t = (1 - \delta)\tilde{K}_{t-1} + I_t \left[ 1 - \Omega \left( \frac{I_t}{I_{t-1}} \right) \right] \]

9. Utilization transformation
\[ K_t = u_t \tilde{K}_{t-1} \]

10. Optimal utilization
\[ z_t = a'(u_t) \]

11. Equilibrium in the final good market:
\[ Y_t = C_t + I_t + a(u_t) \tilde{K}_{t-1} + G_t \]

12. Monetary policy rule:
\[ 1 + i_t = \max \left\{ 1, (1 + i_{t-1})^{\phi_i} \left[ \frac{\pi}{\beta(1 + \rho_t)} \cdot \left( \frac{\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}}{\pi} \right)^{\phi_p} \cdot \left( \frac{Y_t}{Y_{t-1}} \right)^{\phi_y} \right]^{1-\phi_i} \right\} \]

13. Recursive representation for \( K_t^p \):
\[ K_t^p = Y_t + \theta_p \beta_t E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{e_p-1} \pi_t^{1-e_p} K_{t+1} \right] \]

14. Recursive representation for \( Z_t^p \):
\[ Z_t^p = Y_t mc_t + \theta_p \beta_t E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{e_p} \pi_t^{-e_p} Z_{t+1} \right] \]

15. Optimal pricing:
\[ \bar{p}K_t^p = \frac{\epsilon_p}{\epsilon_p - 1} Z_t^p \]
16. Inflation:

\[ 1 = \theta_p \left( \frac{\pi_t}{\pi_{t-1}} \right)^{\epsilon_p - 1} + (1 - \theta_p) \tilde{p}_t^{1-\epsilon_p} \]

17. Price dispersion:

\[ \Delta_{p,t} = (1 - \theta_p) \tilde{p}_t^{-\epsilon_p} + \theta_p \left( \frac{\pi_t}{\pi_{t-1}} \right)^{\epsilon_p} \Delta_{p,t-1} \]

18. Recursive representation for \( K^w_t \):

\[ K^w_t = w_t^{\epsilon_w} N_t \lambda_t + \theta_w \beta \mathbb{E}_{t-1} \left[ \frac{w_t}{w_{t-1}} \pi^{\epsilon_w - 1}_{t+1} \left( \frac{w_t}{w_{t-1}} \pi_t \right)^{-\epsilon_w} K^w_{t+1} \right] \]

19. Recursive representation for \( Z^w_t \):

\[ Z^w_t = w_t^{\epsilon_w (1+\varphi)} \chi_N N_t^{1+\varphi} + \theta_w \beta \mathbb{E}_{t-1} \left[ \frac{w_t}{w_{t-1}}^{\epsilon_w (1+\varphi)} \left( \frac{w_t}{w_{t-1}} \pi_t \right)^{-\epsilon_w (1+\varphi)} Z^w_{t+1} \right] \]

20. Optimal wage:

\[ \bar{w}^{1+\epsilon_w \varphi} K^w_t = \frac{\epsilon_w}{\epsilon_w - 1} Z^w_t \]

21. Wage level:

\[ w_t^{1-\epsilon_w} = \theta_w \left( \frac{w_{t-1}}{w_t} \right)^{1-\epsilon_w} + (1 - \theta_w) \bar{w}_t^{1-\epsilon_w} \]

22. Wage dispersion:

\[ \Delta_{w,t} = (1 - \theta_w) \left( \frac{\bar{w}_t}{w_t} \right)^{-\epsilon_w} + \theta_w \left( \frac{w_t}{w_{t-1}} \pi_t \pi_{t-1} \right)^{\epsilon_w} \Delta_{w,t-1} \]

23. Optimal labour supply:

\[ N_t = \Delta_{w,t} N_t \]