ONLINE APPENDIX: Andrea Lanteri, “The Market for Used Capital: Endogenous Irreversibility and Reallocation over the Business Cycle”[1]

Appendix A: Additional empirical evidence and data sources

This appendix contains additional empirical evidence on capital reallocation and secondary markets for investment goods. It also includes details on data sources and construction.

Evidence on capital reallocation

Figure 1 and Table 1 break down capital reallocation into its two components, namely Sales of Plants, Property and Equipment and Acquisitions and shows the cyclical properties of each of these two series. While Acquisitions are more volatile, both series, as well as total capital reallocation, display positive correlation with GDP. Figure 2 plots the time series of the cyclical components of global secondary-market sales of commercial ships. Figure 3 plots the shares of used capital expenditures from the Annual Capital Expenditures Survey.

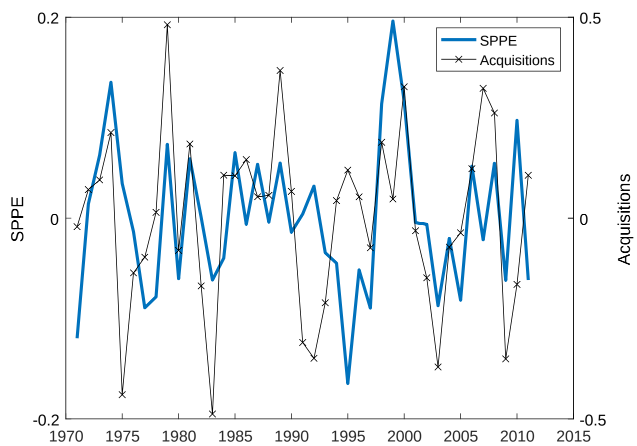


Figure 1: Components of capital reallocation (cyclical components)

Log-deviations from HP trend ($\lambda = 6.25$) of (i) Sales of Property, Plants and Equipment, (ii) Acquisitions. Variables deflated using the US GDP deflator, yearly frequency.

[1]Email: andrea.lanteri@duke.edu

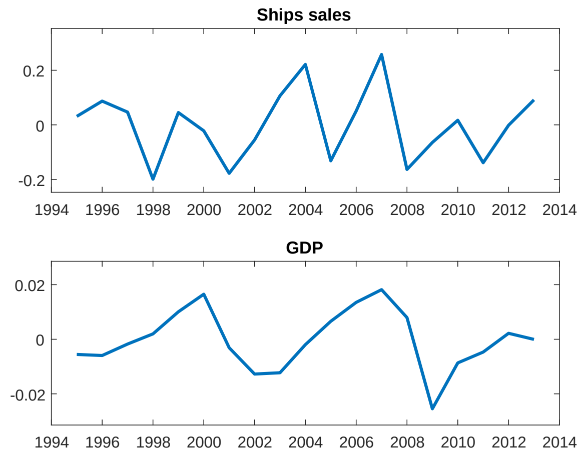


Figure 2: Sales of used ships over the business-cycle (cyclical components)

Top panel: log-deviations from trend (HP-filtered, $\lambda = 6.25$) of the number of sales of used commercial ships. Bottom panel: log-deviations from trend of real US GDP. Yearly frequency.

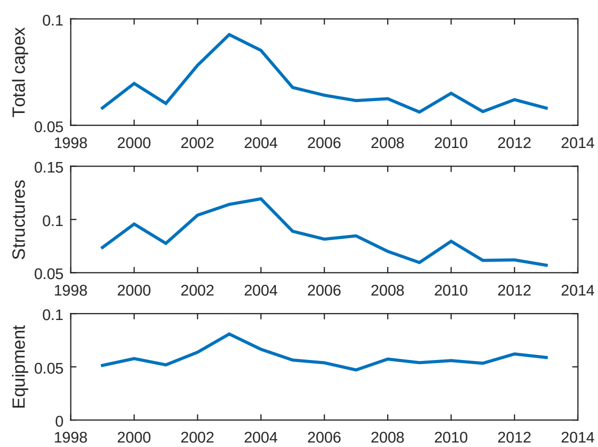


Figure 3: Share of used capital in capital expenditures

Annual Capital Expenditures Survey data on the share of used capital in total capital expenditures (first panel) and its two components: structures (second panel) and equipment (third panel). These data do not include Acquisitions.

Statistic	Reall	SPPE	Acq
$\sigma(.)$	0.159 (0.013)	0.075 (0.009)	0.221 (0.019)
corr(.,GDP)	0.712 (0.059)	0.305 (0.096)	0.765 (0.050)
autocorr	0.199 (0.110)	0.192 (0.166)	0.182 (0.102)

Table 1: Business-cycle properties of capital reallocation

Standard deviations, correlation coefficients with US GDP and coefficients of autocorrelation of the cyclical components of capital reallocation and its two components (SPPE and Acquisitions). Annual frequency 1971-2011. All variables in logs, deflated using GDP deflator and HP-filtered ($\lambda = 6.25$). Standard errors, in parentheses, are computed using a GMM Newey-West (1987) procedure. Source: Compustat, aggregate measures kindly made available by Andrea Eisfeldt on her webpage, author's calculations.

Evidence on prices in secondary markets

Aircraft. Starting with a dataset on the value of all Western-built commercial aircraft from 1967 to 2009, I construct a price index of used aircraft. This dataset was compiled by Aircraft Values (The Aircraft Value Analysis Company), a UK-based specialized consulting company that evaluates aircraft based on transactions prices for which the seller was not bankrupt. The dataset includes prices of all the different vintages of 38 types of aircraft, from their first production year onward. I restrict the attention to the values of aircraft starting one year after production (the notion of used capital consistent with my model). The observation unit is an aircraft of type j, vintage v in year t, with price p_{jvt}. To construct the index, I first deflate all prices using the US GDP deflator. Then, I create dummy variables for year, age and type (and interaction terms) and run a regression of $log(p_{jvt})$ on these dummies. In each subsample (new and used), the coefficients on the time dummies are the quality-age-adjusted price index of aircraft. Finally, I detrend the series using an HP filter, with a smoothing coefficient $\lambda = 6.25$. As a measure of new aircraft prices I use PPI Aircraft and Aircraft Equipment. To consider an alternative price index of new aircraft, I also repeat the same exercise described above for age-0 aircraft in my dataset. This leads to a procyclical index, with smaller volatility than the used price index, consistent with a procyclical relative price of used capital. However, this alternative measure is based on substantially fewer observations (relative to the used price index), particularly in the later part of the sample, resulting in a difficulty to reliably estimate the cyclical moments: alternative specifications of the controls (aircraft model types) lead to relatively large differences in business-cycle statistics for this series, inducing me to rely on the PPI index as the baseline measure of new aircraft prices in Section 2.

Ships. I gather price indices for new and used ships for the period 1996-2013 from Clarksons and VesselsValue, two specialized companies that collect transaction prices and assess ships' resale market values on behalf of financial institutions. It is interesting to observe that prices and quantities traded fell contemporaneously in 2008, and that the price index of used ships is more volatile than the price index of new ships (Figures 2 - in the paper- and 2 in this Appendix). Similar to the points made about aircraft in Section 2, in the case of ships, the resale price of more specific models (e.g. the very heavy and large Capesize bulk carrier) in terms of possible routes also grew more strongly in the period 2006-2008 and then fell by a larger fraction towards the end of 2008 than the price of less specific ones (e.g. the more flexible and small Handysize bulk carrier). This is shown in Figure 4. As an alternative price index of new ships, I also consider the PPI Ships index. This measure is also less volatile than the Clarksons price index of used ships (the standard deviation of its cyclical component is 0.008. Moreover, it is negatively correlated with GDP (at yearly frequency, the correlation between the cyclical components of PPI Ships and GDP is -0.55). Hence, this alternative measure also leads to a highly procyclical relative price of used ships.

Vehicles. In the case of vehicles and trucks, I compare two separate separate CPI series, one for new (CPI new vehicles) and one for used (CPI used cars and trucks) in the sample

1953-2013. It emerges that the price of used vehicles is more volatile and more procyclical than that of new ones, which is actually acyclical. The volatility of prices of used vehicles is smaller than that of other goods considered, possibly because vehicles are a less specific type of asset. The procyclicality of the relative price of used vehicles is more pronounced in the second half of the sample, 1985-2013, as illustrated in Figure 5. As an alternative price index of new vehicles, I also consider PPI Motor Vehicles. This measure is also less volatile than my price index of used vehicles (the standard deviation of its cyclical component is 0.012). Moreover, it is negatively correlated with GDP (at quarterly frequency, the correlation between the cyclical components of PPI Motor Vehicles and GDP is -0.41). Hence, this alternative measure also leads to a procyclical relative price of used vehicles.

Construction equipment. Edgerton (2011) constructs an index of the price of used construction machinery by collecting data on auctions in which this equipment is reallocated across US construction firms in the time period 1994-2011. The index is constructed following the same procedure described for aircraft above (regressing prices on observable characteristics and time fixed effects). Different from the other price indices, however, this index is available as not deflated and linearly detrended instead of HP-filtered. To be consistent with this feature, when I compute the relative price of used construction (relative to new) and study its cyclical properties I use a linear trend also for the price of new (PPI Construction Machinery and Equipment) and for GDP. The price of used construction equipment fell by more than the corresponding PPI (price index of new construction machinery) both in the 2001 and in the 2009 recession and was, in general, significantly more volatile. In particular in 2009, the index of used construction equipment was approximately 20% below trend, while the corresponding PPI of new construction machinery was slightly above trend.

Standard errors for the business-cycle statistics of reallocation and prices in Section 2 are constructed using Constantino Hevia's Matlab routines available on the World Bank website. For each series, I use a Newey-West procedure with a number of lags equal to the integer part of the power 1/4 of the sample size.

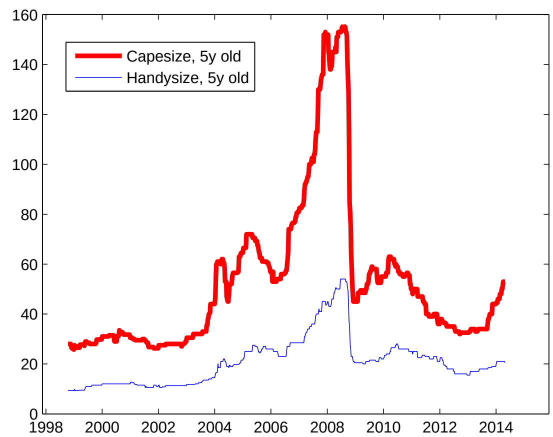


Figure 4: Ships: price of used Capesize and used Handysize

Prices in million $ of second-hand 5 year-old Capesize (more specific) and Handysize (less specific). Weekly frequency: estimated values based on actual transactions and shipping market information.

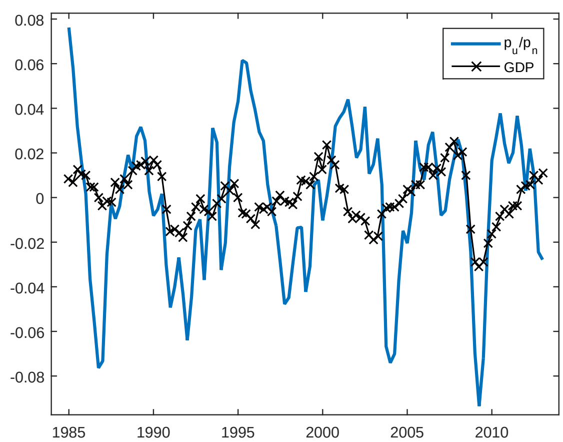


Figure 5: The price of used vehicles relative to new, 1985-2013

Cyclical components of the relative price of used vehicles (blue line) and US real GDP (black line with crosses). Subsample 1984-2013. Quarterly frequency (HP filtered, λ=1600).

VAR evidence

To provide further evidence on the cyclical behavior of the relative price of used capital as well as its response to business cycle shocks, I consider the two types of assets for which I have a long and unfiltered time series of prices and run bivariate VARs with real GDP. Figures 6 and 7 show the Impulse Response Function (IRF) of the relative price of used aircraft and vehicles respectively, to a shock to real GDP. Both relative prices show a statistically significant positive response to the shock, as well as a slowly mean-reverting dynamics of prices, which is qualitatively consistent with the implications of the quantitative model.

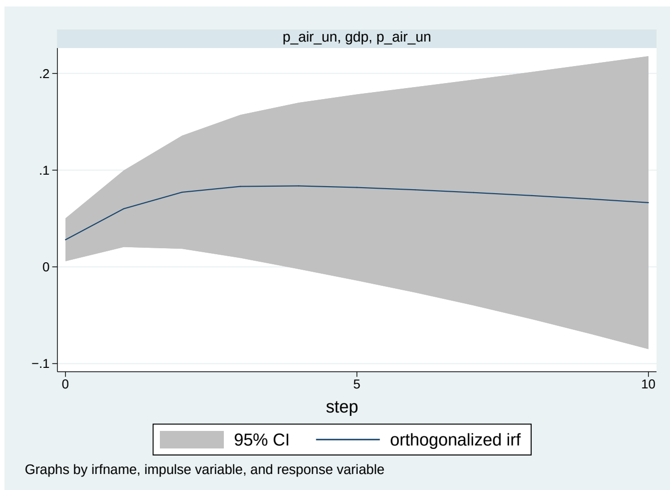


Figure 6: IRF of relative price of used aircraft to a positive shock to GDP

Bivariate VAR: real GDP and price index of used aircraft (relative to new). Yearly frequency, VAR estimated in levels. Cholesky identification with output ordered first. 95% confidence bands in grey.

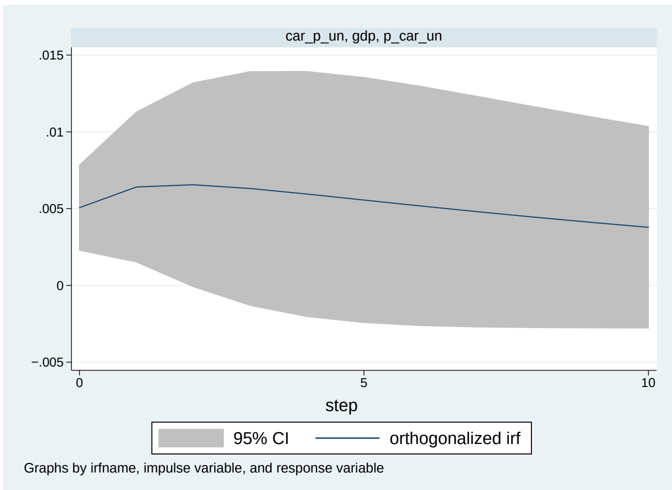


Figure 7: IRF of relative price of used vehicles to a positive shock to GDP

Bivariate VAR: real GDP and CPI price index of used vehicles (relative to new). Quarterly frequency, VAR estimated in levels. Cholesky identification with output ordered first. 95% confidence bands in grey.

Appendix B: Static Model

This Appendix contains the Proof of Proposition 1, the parameter values used in Figure 5 in the paper (Table 2 below) and presents an extension of the static model of Section 3 with positive new investment $\widetilde{i}_{new}$.

Proof of Proposition 1

(i) By equating demand and supply for used capital - equations (5) and (6) in the paper -, the market-clearing condition for used capital can be written as follows:

$$G(q,z,\epsilon) \equiv \theta(q) \int_{s^I} \left[\left(\frac{\alpha z s}{Q(q)} \right)^{\frac{1}{1-\alpha}} - k_0 \right] dF(s) - \int^{s^D} \left[k_0 - \left(\frac{\alpha z s}{q} \right)^{\frac{1}{1-\alpha}} \right] dF(s) = 0. \quad (1)$$

where $s^I = \frac{Q(q)}{\alpha z k_0^{\alpha-1}}$, $s^D = \frac{q}{\alpha z k_0^{\alpha-1}}$, $Q(q) = [\eta + (1-\eta)(q+\gamma)^{1-\epsilon}]^{\frac{1}{1-\epsilon}}$, $\theta(q) = (\frac{q+\gamma}{Q(q)})^{-\epsilon}(1-\eta)$ is the ratio of used investment to total investment for investing firms, and I have left implicit the dependence of θ, q and Q on ϵ. Equation (1) defines the market-clearing price q as an implicit function of the aggregate productivity parameter z and the elasticity of substitution between new and used capital ϵ. We can obtain the derivative of q with respect to z by applying the Implicit Function Theorem to function G, and we get[2]

$$\frac{dq}{dz} = -\frac{G_z}{G_q} \quad (2)$$

with

$$G_z = \frac{\theta}{(1-\alpha)z} \int_{s^I} \left(\frac{\alpha z s}{Q} \right)^{\frac{1}{1-\alpha}} dF(s) + \frac{1}{(1-\alpha)z} \int^{s^D} \left(\frac{\alpha z s}{q} \right)^{\frac{1}{1-\alpha}} dF(s)$$

and

$$G_q = \theta_q \int_{s^I} \left[\left(\frac{\alpha z s}{Q(q)} \right)^{\frac{1}{1-\alpha}} - k_0 \right] dF(s) - \frac{\theta Q_q}{(1-\alpha)Q} \int_{s^I} \left(\frac{\alpha z s}{Q} \right)^{\frac{1}{1-\alpha}} dF(s) - \frac{1}{(1-\alpha)q} \int^{s^D} \left(\frac{\alpha z s}{q} \right)^{\frac{1}{1-\alpha}} dF(s)$$

In applying Leibniz rule to derive these expressions, we do not need to worry about the derivatives of the end points s^D and s^I because, by their definition, the respective integrands are equal to zero when evaluated at these points. Note that G_z is positive and G_q is negative (as $\theta_q < 0$), hence q is increasing in z.

(ii) The elasticity of q with respect to z, call it $\phi_{q,z}(\epsilon) \equiv \frac{dq}{dz}\frac{z}{q}$, is given by

$$\phi_{q,z}(\epsilon) = \frac{\frac{\theta}{(1-\alpha)} \int_{s^I} \left(\frac{\alpha z s}{Q} \right)^{\frac{1}{1-\alpha}} dF(s) + \frac{1}{(1-\alpha)} \int^{s^D} \left(\frac{\alpha z s}{q} \right)^{\frac{1}{1-\alpha}} dF(s)}{-\theta_q q \int_{s^I} \left[\left(\frac{\alpha z s}{Q(q)} \right)^{\frac{1}{1-\alpha}} - k_0 \right] dF(s) + \frac{\theta Q_q q}{(1-\alpha)Q} \int_{s^I} \left(\frac{\alpha z s}{Q} \right)^{\frac{1}{1-\alpha}} dF(s) + \frac{1}{(1-\alpha)} \int^{s^D} \left(\frac{\alpha z s}{q} \right)^{\frac{1}{1-\alpha}} dF(s)} \quad (3)$$

[2]Notation: Call f_x be the partial derivative of function f with respect to argument x.

Now, note that when $\epsilon = 0$ (Leontief investment technology), the share of used capital to total investment becomes $\theta = 1 - \eta$, so that $\theta_q = 0$, while the price index becomes $Q = \eta + (1-\eta)(q+\gamma)$, so that we get $Q_q = 1 - \eta$. Hence, we can write

$$\phi_{q,z}(0) = \frac{(1-\eta)\int_{s^I}\left(\frac{\alpha zs}{Q}\right)^{\frac{1}{1-\alpha}} dF(s) + \int^{s^D}\left(\frac{\alpha zs}{q}\right)^{\frac{1}{1-\alpha}} dF(s)}{(1-\eta)^2 \frac{q}{\eta+(1-\eta)(q+\gamma)}\int_{s^I}\left(\frac{\alpha zs}{Q}\right)^{\frac{1}{1-\alpha}} dF(s) + \int^{s^D}\left(\frac{\alpha zs}{q}\right)^{\frac{1}{1-\alpha}} dF(s)} \tag{4}$$

and this establishes that $\phi_{q,z}(0) > 1$ as $q < 1 \Rightarrow (1-\eta)\frac{q}{\eta+(1-\eta)(q+\gamma)} < 1$. Standard arguments can be used to show that $\phi_{q,z}$ is continuous.

To show that reallocation is increasing in z, it suffices to observe that the equilibrium supply of used capital S^*_{used} - i.e. total reallocation - is a decreasing function of $\frac{z}{q}$ (as above, we can disregard the derivative of s^D as the integrand is zero when evaluated at s^D):

$$S^*_{used} = \int^{s^D}\left[k_0 - \left(\frac{\alpha zs}{q}\right)^{\frac{1}{1-\alpha}}\right] dF(s) \tag{5}$$

Thus, the sign of its derivative with respect to z is the sign of $\phi_{q,z} - 1$. This establishes that in the limit for sufficiently low elasticity of substitution between new and used capital, reallocation is increasing in z - i.e. "procyclical".

Parameter	Value
α	1/3
k_0	0.2
$F(s)$	uniform(0.85,1.15)
η	0.75
γ	0.1

Table 2: Parameter values used in Figure 5 in the paper

Extension: a model with $\widetilde{i}_{new} > 0$

In this extension, I present a model in which expanding firms employ a positive amount of new goods both in the bundle with used goods and as fully new investment, unrelated to reallocation of used capital. I show that the key insights of Section 3 are robust to this extension. This clarifies that the fact that used investment affects the marginal product of all new investment in the model of Section 3 is not important to determine the results.

Consider this more general firm-level investment technology for expanding firms, i.e., choosing $k' \geq (1-\delta)k$:

$$k' - (1-\delta)k = \widetilde{i}_{new} + g(i_{new}, i_{used})$$

where g is a CES aggregator with elasticity ϵ and, additionally,

$$\widetilde{i}_{new} \geq \omega\,[k' - (1-\delta)k]$$

where both $\widetilde{i}_{new}$ and i_{new} are new investment goods, i_{used} are used investment goods and $\omega \in [0,1]$. In this modified model, firms need to use new investment goods at least for a fraction ω of their desired expansion.

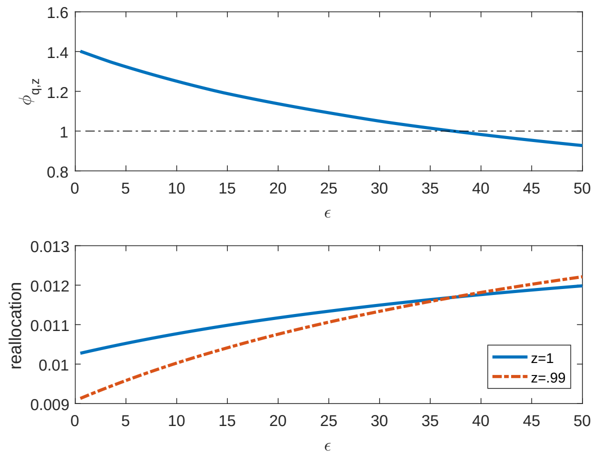
Notice that a standard one-sector model is nested in this formulation for $\omega = 0$ and $\epsilon = \infty$. The model of Section 3 is nested for $\omega = 0$ and $\epsilon < \infty$.

Consider the case $\omega > 0$ and $\epsilon < \infty$. Under this assumption, at least a fraction ω of the investment is always made using new goods. This formulation separately accommodates both positive investment directly employed in the refurbishment of used capital (i_{new}) and other, unrelated new investment ($\widetilde{i}_{new}$).

Firm optimization implies $\widetilde{i}_{new} = \omega\,[k' - (1-\delta)k]$ whenever the cost of a unit of used capital is less than the cost of a unit of new capital, which is the empirically relevant case. As the price of new capital in the model is 1, the overall cost of a unit of future capital k' for expanding firms is $Q = \omega + (1-\omega)Q_g$ where Q_g is the price index associated with the CES bundle g, given by equation (4) in the paper. Notice that the marginal product of $\widetilde{i}_{new}$, which is now positive, is independent of i_{used}.

In order to illustrate the result and obtain a numerical range for ϵ with procyclical reallocation, I repeat the exercise illustrated in Figure 5 in the paper using this alternative specification. The value of the parameter η has to be reduced in the model with positive ω because part of the new goods are now employed in the component $\widetilde{i}_{new}$. To allow for a fair comparison between my assumptions (ω=0) and this version of the model, I set $\omega = .5$ and $\eta = .5$ in order to (approximately) match the share of used goods in investment in the two cases. Figure 8 illustrates the outcome. As in Figure 5 in the paper, the top panel shows the elasticity of the price of used capital with respect to the aggregate productivity parameter z, as a function of the elasticity ϵ. The bottom panel illustrates capital reallocation for two values of z, also as a function of ϵ. First, notice that the results are qualitatively similar to those obtained in the baseline model with $\omega = 0$. The elasticity of the price of used capital with respect to the aggregate productivity parameter z is larger than 1 and reallocation is still

Figure 8: Elasticity of substitution ϵ, price adjustment and capital reallocation with $\omega > 0$



Top panel: $\phi_{q,z}$, the elasticity of the price of used capital with respect to aggregate productivity z, as a function of ϵ, the elasticity of substitution between new and used capital. Bottom panel: Capital reallocation (as a fraction of aggregate capital k_0), as a function of ϵ, for $z = 1$ (blue solid line) and $z = .99$ (red dashed-dotted line).

procyclical for sufficiently low elasticity of substitution ϵ. Furthermore, the threshold for ϵ under which reallocation is procyclical actually increases substantially relative to the version of the model in the paper (from 20 to 37) and for any given ϵ reallocation is more procyclical under the alternative version of the model that satisfies the three desired properties. For instance, when $\epsilon = 10$, reallocation is 5% higher in the high z state in my baseline model and 7.5% higher in the alternative model with positive ω.

It is also easy to verify that Proposition 1 holds in the extended model with ω, following the same steps as in the proof presented above. Furthermore, while in the model of Section 3 I abstract from convex adjustment cost, similar results can be obtained obtained by adding standard quadratic adjustment costs on new investment $\widetilde{i}_{new}$ in this modified model.

Appendix C: Algorithm for DSGE model, more aggregate results and robustness exercises

This appendix contains details of the solution method for the DSGE model of Sections 4-5 and discusses accuracy of the solution. It also displays additional results and illustrates the robustness of the quantitative results to different parameter values.

Algorithm

I solve the model using an extension of the method of Krusell and Smith (1998) and Khan and Thomas (2008, 2013), which takes care of market clearing in the market for used capital. In Khan and Thomas (2008, 2013), there is one endogenous price to solve for (the wage, or, equivalently, the marginal utility valuation of the output good). In my model, there are two equilibrium prices to solve for: the wage and the price of used capital. The price index Q_t is then obtained analytically given the CES price index formula.

Key steps of the algorithm:

- I approximate the distribution m with its first moment, aggregate capital K, and the covariance between firm-level capital and idiosyncratic productivity, cov_{ks}. Agents perceive laws of motion:

$$\log(K') = \hat{\phi}_0^K + \hat{\phi}_1^K \log(K) + \hat{\phi}_2^K I(z^H) + \hat{\phi}_3^K \log(K) I(z^H)$$

$$\log(cov'_{ks}) = \hat{\phi}_0^{cov} + \hat{\phi}_1^{cov} \log(K) + \hat{\phi}_2^{cov} I(z^H) + \hat{\phi}_3^{cov} \log(K) I(z^H) + \hat{\phi}_4^{cov} \log(cov_{ks}) + \hat{\phi}_5^{cov} \log(cov_{ks}) I(z^H)$$

  where $I(z^H)$ is an indicator function for the high-productivity aggregate state.

- Agents perceive price functions:

$$\log(w) = \hat{\phi}_0^w + \hat{\phi}_1^w \log(K) + \hat{\phi}_2^w I(z^H) + \hat{\phi}_3^w \log(K) I(z^H)$$

$$\log(q) = \hat{\phi}_0^q + \hat{\phi}_1^q \log(K) + \hat{\phi}_2^q I(z^H) + \hat{\phi}_3^q \log(K) I(z^H) + \hat{\phi}_4^q \log(cov_{ks}) + \hat{\phi}_5^q \log(cov_{ks}) \log(K)$$

  While for aggregate capital and wage a rule based only on aggregate capital achieves very high accuracy, the covariance term helps achieve high accuracy in the prediction of the price of used capital. I also experimented with different interaction terms in the regressions, with similar results.

- Given these perceived laws of motion, I solve the individual firm's problem by value function iteration and obtain the policy functions (inner loop).

- I simulate a continuum of firms for 5000 periods using the simulation method of Young (2010) and update the price functions by explicitly imposing market clearing in the used capital market along the simulation.

- To impose market clearing, in each period, I interpolate the continuation value function at the current approximate aggregate state (K, cov_{ks}, z), allow current prices to be free parameters, re-solve the individual firm problem (using interpolation on the capital grid) at each candidate vector of prices and solve for the values of the prices that clear markets.
- I update the laws of motion for aggregate capital, covariance and prices (outer loop) using standard regression methods up to convergence.

The state vector has overall 84000 nodes (1,000 for k, 7 for s, 2 for z, 3 for K and 2 for cov_{ks}). A single iteration of the outer loop requires approximately thirty minutes on a twelve-workers Matlab cluster. Hence, I first solve the model with a shorter simulation (2000 periods), and without explicitly clearing markets, instead gradually updating prices in the direction implied by the sign of excess demand in each market in every period. When both the average excess demand and the forecast errors are sufficiently small, markets are cleared as a system using a non-linear solver and I proceed with the full simulation of 5000 periods.

Furthermore, the long simulation is divided into ten smaller simulations of equal size that I run in parallel. In every new iteration of the outer loop, I start each of this sub-simulations at the initial distribution implied by the end of the previous sub-simulation. I thank Aubhik Khan for kindly sharing this insight to gain efficiency in simulation-based solution algorithms.

Accuracy

The R^2 (maximum and mean error in parenthesis) of the regressions for future aggregate capital, future covariance, wage and price of used capital are 0.9999 (0.0019, 0.0002), 0.9998 (0.0028, 0.0004), 0.9999 (0.0030, 0.0003) and 0.9938 (0.0025, 0.0004) respectively.

The lower variance of q relative to the other approximated variables explains its somewhat lower R^2 even for a similar size of mean and maximum errors. This point is consistent with the analysis of den Haan (2010), who proposes the following accuracy check. To formally test for accuracy, I perform a long simulation of the model (5000 periods) and compare the paths of the approximated variables with an alternative simulation obtained iterating only on the estimated laws of motion. I compute the maximum and mean distance of each approximated variable from the value taken in the actual simulation. These values are reported in Table 3. As all variables are in logs, the errors can be interpreted as percentage deviations. These results show that the solution is not only accurate in the sense of producing accurate one-step ahead forecasts, but also that the forecast errors do not accumulate over time.

Error	K	cov_{ks}	w	q
max	0.0031	0.0041	0.0034	0.0026
mean	0.0007	0.0009	0.0004	0.0004

Table 3: den Haan (2010) accuracy test. Maximum and mean errors in long simulation based on subjective laws of motion

I also compare the solution with an alternative solution that only uses aggregate capital to the price of used capital, abstracting from dynamics in the covariance term. While the main quantitative business-cycle properties of the model display only very small differences, the accuracy of the solution is lower: in this case, the R^2 of the regression for q decreases to .9830 and the maximum and mean forecast errors during the simulation are twice as large relative to the baseline solution with covariance: they are .0053 and .0008 respectively. I verify that including second and third order powers of capital leads to similar results. This shows that the covariance term cov_{ks} is a valuable moment to forecast the equilibrium dynamics in the market for used capital.

Additional model figures

I present three additional figures: Figure 9 shows a 3-D plot of the joint distribution of firm-level capital and idiosyncratic productiviy shocks in the stationary equilibrium of the model; Figure 10 plots the distribution of firm-level investment rates in stationary equilibrium; Figure 11 plots the response of standard business-cycle variables (output, consumption, investment, capital and employment) to a negative aggregate TFP shock.

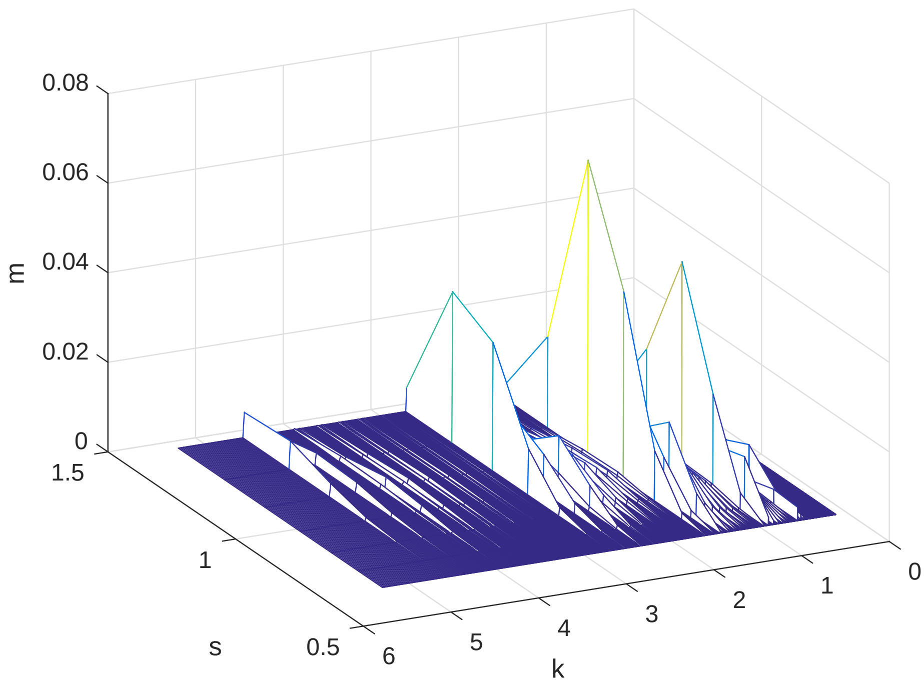


Figure 9: Stationary joint distribution of k and s

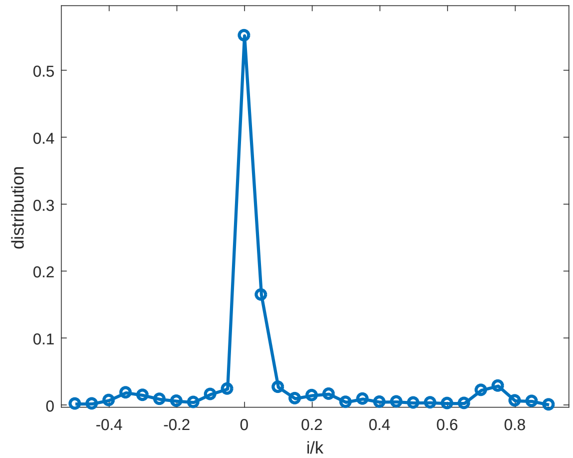


Figure 10: Stationary distribution of investment rates

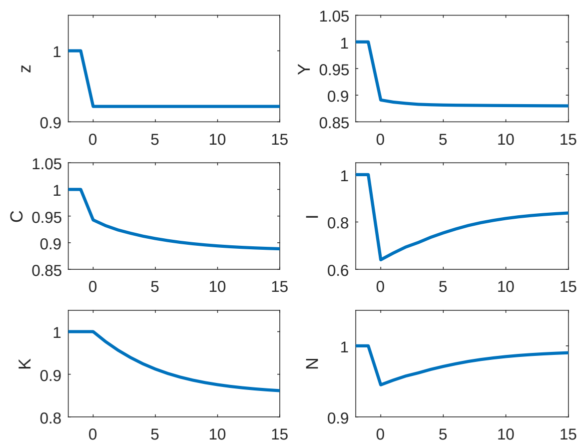


Figure 11: Aggregate shock: response of standard RBC variables

Transition from long sequence $z_t = z^H$ to long sequence $z_t = z^L$. Response of output (Y), consumption (C), investment (I), capital stock (K) and employment (N). Unfiltered data. Series normalized to one before the shock hits.

Robustness checks

The following tables report business-cycle statistics for different values of model parameters and HP-filtering smoothing parameter.

Specifically, Table 4 refers to the version of the model in which I set $\eta = 0.92$ to exclude Acquisitions from the notion of reallocation. Tables 5 and 6 reports the results of robustness checks with respect to ϵ, by setting this parameter equal to 1 and 10 respectively. In all these three cases, no other parameter values are changed relatively to the baseline calibration. Procyclical reallocation is robust to all these modifications of the parameter values.

Table 7 refers to a model in which I change the process of idiosyncratic productivity and reallocation cost to obtain a higher fraction of lumps (investment rate above 20%), specifically equal to 0.18 (Cooper and Haltiwanger, 2006). The parameter values that differ from the baseline calibration are as follows: $(\sigma_{inn,s} = 0.106, \rho_s = 0.63, \gamma = 0)$; besides a fraction of lumps of 0.18, these parameter values induce a standard deviation of investment rates of 0.27 (my model does not appear to allow to exactly match the standard deviation of investment rates jointly with the fraction of lumps, partly because of the non-negativity constaint on γ), autocorrelation of investment rates equal to 0.057 and fraction of firms doing negative investment equal to 0.116. The idiosyncratic shock is then discretized with a Tauchen (1986) procedure. The business-cycle properties of the model are very similar to the ones obtained under the baseline calibration. With the caveat of a smaller standard deviation of investment rates, this exercise suggests that the key results are robust with respect to empirically plausible changes in the fraction of firms doing large adjustments.

Tables 8, 9 and 10 show the business-cycle statistics of the baseline model, constant-q model and US data for a HP smoothing parameter equal to 100, in order to facilitate the comparison of the results with Khan and Thomas (2013) .

Statistic	Y	C	I	K	N	r	q	q/Q	reall
mean	0.612	0.508	0.104	1.59	0.336	0.041	0.888	0.896	0.012
$\sigma(.)/\sigma(Y)$	(1.54)	0.469	3.758	0.248	0.549	0.08	0.168	0.15	3.136
corr(.,Y)	1	0.979	0.987	-0.332	0.984	0.872	0.977	0.977	0.981
autocorr	0.08	0.152	0.056	0.495	0.053	-0.023	0.246	0.246	0.017

Table 4: Business-cycle statistics: excluding Acquisitions ($\eta = 0.92$, HP filter $\lambda = 6.25$)

Rows: mean, standard deviation relative to standard deviation of output, autocorrelation. Columns: output, consumption, investment, capital, hours, real interest rate, price of used capital, degree of irreversibility (price of used capital relative to marginal cost of expanding a firm), SPPE (value in terms of output good).

Statistic	Y	C	I	K	N	r	q	q/Q	reall
$\sigma(.)/\sigma(Y)$	(1.51)	0.493	3.607	0.238	0.524	0.076	0.203	0.153	3.61
corr(.,Y)	1	0.982	0.988	-0.334	0.984	0.839	0.978	0.978	0.988
autocorr	0.081	0.15	0.051	0.495	0.049	-0.048	0.214	0.214	0.052

Table 5: Business-cycle statistics: $\epsilon = 1$, HP filtered, $\lambda = 6.25$

Rows: standard deviation relative to standard deviation of output, autocorrelation. Columns: output, consumption, investment, capital, hours, real interest rate, price of used capital, degree of irreversibility (price of used capital relative to marginal cost of expanding a firm), capital reallocation (value in terms of output good).

Statistic	Y	C	I	K	N	r	q	q/Q	reall
$\sigma(.)/\sigma(Y)$	(1.52)	0.486	3.654	0.245	0.53	0.071	0.157	0.105	2.31
corr(.,Y)	1	0.983	0.989	-0.336	0.986	0.87	0.986	0.986	0.981
autocorr	0.083	0.141	0.064	0.503	0.06	-0.048	0.176	0.177	-0.004

Table 6: Business-cycle statistics: $\epsilon = 10$, HP filtered, $\lambda = 6.25$

Rows: standard deviation relative to standard deviation of output, autocorrelation. Columns: output, consumption, investment, capital, hours, real interest rate, price of used capital, degree of irreversibility (price of used capital relative to marginal cost of expanding a firm), capital reallocation (value in terms of output good).

Statistic	Y	C	I	K	N	r	q	q/Q	reall
$\sigma(.)/\sigma(Y)$	(1.56)	0.449	3.822	0.251	0.568	0.07	0.138	0.097	3.283
corr(.,Y)	1	0.979	0.989	-0.348	0.987	0.867	0.981	0.982	0.987
autocorr	0.07	0.157	0.035	0.488	0.025	-0.149	0.16	0.158	0.014

Table 7: Business-cycle statistics: model with fraction of lumps = 18%, HP filtered, $\lambda = 6.25$

Rows: standard deviation relative to standard deviation of output, autocorrelation. Columns: output, consumption, investment, capital, hours, real interest rate, price of used capital, degree of irreversibility (price of used capital relative to marginal cost of expanding a firm), capital reallocation (value in terms of output good).

Statistic	Y	C	I	K	N	r	q	q/Q	reall
$\sigma(.)/\sigma(Y)$	(2.28)	0.533	3.542	0.415	0.511	0.064	0.192	0.136	2.814
corr(.,Y)	1	0.96	0.974	0.041	0.956	0.704	0.955	0.956	0.974
autocorr	0.499	0.598	0.454	0.839	0.449	0.292	0.556	0.555	0.426

Table 8: Business-cycle statistics: baseline model, HP filtered, $\lambda = 100$

Rows: standard deviation relative to standard deviation of output, autocorrelation. Columns: output, consumption, investment, capital, hours, real interest rate, price of used capital, degree of irreversibility (price of used capital relative to marginal cost of expanding a firm), capital reallocation (value in terms of output good).

Statistic	Y	C	I	K	N	r	reall
$\sigma(.)/\sigma(Y)$	(2.37)	0.495	3.869	0.449	0.57	0.076	1.316
corr(.,Y)	1	0.93	0.963	0.057	0.947	0.818	-0.864
autocorr	0.486	0.624	0.441	0.831	0.438	0.397	0.554

Table 9: Business-cycle statistics: constant q, HP filtered, $\lambda = 100$

Rows: mean, standard deviation relative to standard deviation of output, autocorrelation. Columns: output, consumption, investment, capital, hours, real interest rate, price of used capital, capital reallocation.

Statistic	Y	C	I	K	N	w	r	TFP	reall	SPPE only
$\sigma(.)/\sigma(Y)$	(2.16)	0.567	2.821	1.129	1.14	0.575	0.711	0.522	12.031	6.981
corr(.,Y)	1	0.828	0.791	0.585	0.857	0.312	-0.024	0.452	0.671	0.370
autocorr	0.534	0.608	0.546	0.696	0.517	0.537	0.325	0.571	0.558	0.655

Table 10: Business-cycle statistics: US annual data, HP filtered with $\lambda = 100$

US business-cycle statistics 1947-2015. Rows: standard deviation relative to standard deviation of GDP, correlation with GDP, autocorrelation. Columns: real GDP, consumption (personal consumption expenditures on non-durables and services, deflated with GDP deflator), investment (fixed private investment and personal consumption expenditures on durables, deflated with GDP deflator), capital (fixed private assets and stock of consumer durables, deflated with GDP deflator), hours (all persons, nonfarm business sector), real wage (real compensation per hour, nonfarm business sector), real interest rate (three-month T-bill, net of ex-post GDP-deflator inflation), aggregate TFP (constructed as in the model, i.e. $\log(GDP) - \alpha \log(K) - \nu \log(N)$), capital reallocation (SPPE + Acquisitions) and SPPE (1971-2011), deflated with GDP deflator. Sources: BEA, BLS, Board of Governors of the Federal Reserve System, Compustat, author's calculations.

Appendix D: The constant-q model

In this appendix, I provide some more details on the comparison model with constant partial irreversibility. The household's problem is the same as for the model with endogenous irreversibility, hence I do not repeat its description. Firms' dynamic program is also the same, except for the fact that q_t and Q_t are constant. This can be rationalized under the following technological assumptions:

- one unit of output good can be transformed into Q^{-1} units of capital that can be freely specialized and installed by any firm;
- one unit of capital can be re-transformed into q units of output good.

Hence, investing firms buy each unit of extra capital at price Q, while disinvesting firms sell capital at price q. The aggregate resource constraint reads

$$C_t + QI_t^+ - qI_t^- = Y_t \tag{6}$$

where I_t^+ is total positive investment of expanding firms and I_t^- is total disinvestment of downsizing firms.

Notice that if $q < Q = 1$ this model coincides with a standard RBC model with production heterogeneity and partial irreversibility (e.g. Veracierto, 2002). However, in parametrizing the model, I choose not to set $Q = 1$. This is because I want this comparison model to only differ from my model in terms of the dynamic properties of capital prices. Hence, I exogenously set both q and Q to be equal to their steady-state equilibrium values in the baseline model with endogenous q. This way, the stationary equilibria of the two economies coincide exactly. This implies that all calibrated parameters also hit the same targets. When they are hit by aggregate shocks, however, the two economies differ in that capital prices only adjust in my model.

The solution method is analogous to the one described for the baseline model, with the simplification that the wage is the only endogenous price.

References

[1] den Haan, W. (2010), Assessing the Accuracy of the Aggregate Law of Motion in Models with Heterogeneous Agents, *Journal of Economic Dynamics and Control*, 34(1), 79-99.

[2] Tauchen, G. (1986), Finite State Markov-Chain Prroximation to Univariate and Vector Autoregressions, *Economic Letters*, 20, 177-181.

[3] Young, E.R. (2010), Solving the Incomplete Markets Model with Aggregate Uncertainty Using the Krusell-Smith Algorithm and Non-stochastic Simulations, *Journal of Economic Dynamics and Control*, 34(1), 36-41.