The Gains from Input Trade with Heterogeneous Importers

by Joaquin Blaum, Claire Lelarge and Michael Peters

Online Appendix

This Online Appendix contains the following additional results and material:

1) Generalizations of equation (7),
2) Details about the identification of the Input-Output Matrix $\Xi$,
3) Empirical evidence on the correlates of the firm-level gains from input trade,
4) The change in consumer prices with heterogeneous export participation,
5) A description of the bootstrap procedure,
6) Empirical results on the Elasticity Bias at the sector level,
7) An extension of the welfare equation (32) to a multi-sector environment,
8) Detailed theoretical derivations of the models of Section III.D,
9) The estimation of $\eta$,
10) Details about the algorithm used to calibrate the model of Section III.D.
In this section, we consider three generalizations of equation (7), which states that the firm’s unit costs is given by

\[
\hat{u}_i = \frac{1}{\hat{\varphi}_i} \times (s_{Di})^{\frac{\gamma}{1-\gamma}} \times \left(\frac{p_{Di}}{q_{Di}}\right)^\gamma w^{1-\gamma}.
\]

Equation (O27) was derived under the restrictions: (i) the production function has a constant elasticity of materials \(\gamma\), (ii) domestic and foreign inputs are combined in a CES fashion with elasticity of substitution \(\epsilon\) and (iii) foreign inputs are differentiated at the country, but not at the product level. We now relax these assumptions and derive expressions akin to (O27).

**Extension 1: CES Upper Tier.** — Suppose that the production function between materials \(x\) and primary factors \(l\) is CES instead of Cobb-Douglas, i.e.

\[
y = \varphi \left((1-\gamma) \frac{\zeta+1}{\zeta} + \gamma x \frac{x-1}{x}\right)^{\frac{1}{\zeta}}.
\]

The rest of the environment is exactly as in Section I. Let \(Q\) denote again the price index of materials \(x\) and \(w\) denote the price of primary factors \(l\). In this case, the firm’s unit cost is given by

\[
u = \frac{1}{\varphi} \left(\gamma^\zeta Q^{1-\zeta} + (1-\gamma)^\zeta w^{1-\zeta}\right)^{\frac{1}{1-\zeta}}.
\]

Noting that the optimal expenditure share on materials is given by

\[
s_M = \frac{\gamma^\zeta Q^{1-\zeta}}{\gamma^\zeta Q^{1-\zeta} + (1-\gamma)^\zeta w^{1-\zeta}},
\]

we can write the firm’s unit cost as

\[
u = \frac{1}{\varphi} s_M^{\frac{1}{\zeta-1}} \left(1\right)^{\frac{\zeta}{\zeta-1}} s_D^{\frac{1}{\zeta-1}} \left(\frac{1}{\gamma}\right)^{\frac{1}{\zeta-1}} \left(\frac{p_{Di}}{q_{Di}}\right)^\gamma \times s_M^{\frac{1}{\zeta-1}} s_D^{\frac{1}{\zeta-1}},
\]

where we have substituted for \(Q\) using (4). (O29) shows that measuring the effect of input trade on the unit cost requires knowledge of the counterfactual material share in the autarky equilibrium, \(s_M^{Aut}\). Because this object is not observed in

\[1\] The Cobb-Douglas assumption in (1) in the main text bypasses this issue because it implies that the material share is constant and given by \(\gamma\). In the non-Cobb-Douglas case, the material share endogenously reacts to changes in the import environment. A move to autarky, for example, makes materials relatively more expensive and should induce firms to substitute towards primary inputs.
the data, we can use (4) and (O28) to compute it:

\[ s_{M}^{\text{Aut}} = \left( \frac{\gamma}{1-\gamma} \right) \beta^{-\frac{1}{\varepsilon}} (1-\gamma) \left( \frac{p_{D}/q_{D}}{w} \right)^{1-\gamma} \frac{1-\varepsilon}{1+\left( \frac{\gamma}{1-\gamma} \right) \beta^{-\frac{1}{\varepsilon}} (1-\gamma) \left( \frac{p_{D}/q_{D}}{w} \right)^{1-\gamma}}. \]

The firm-level gains from input trade are therefore given by

\[ \ln \left( \left. \left( \frac{u}{u} \right) \right|_{p_{D},w} \right) = \ln \left( 1 + \left( \frac{\gamma}{1-\gamma} \right) \beta^{-\frac{1}{\varepsilon}} (1-\gamma) \left( \frac{p_{D}/q_{D}}{w} \right)^{1-\gamma} \frac{1-\varepsilon}{1+\left( \frac{\gamma}{1-\gamma} \right) \beta^{-\frac{1}{\varepsilon}} (1-\gamma) \left( \frac{p_{D}/q_{D}}{w} \right)^{1-\gamma}} \right). \]

(O30) is the generalization of (7) for the case where the aggregator between materials and primary factors is CES. We see that, in this case, quantifying the change in the unit cost relative to autarky requires knowledge of additional parameters \([\beta, \zeta, p_{D}/q_{D}]\) to predict the material share in autarky. Under the additional assumption that there is no variation in \(\beta\) and \(p_{D}/q_{D}\) across firms, we can bypass the estimation of some of these additional parameters. In this case, all firms would feature the same material share in autarky, which is given by the material share of a domestic firm in the observed trade equilibrium, \(s_{D}^{M}\). In this case, (O30) reduces to

\[ \ln \left( \left. \left( \frac{u}{u} \right) \right|_{p_{D},w} \right) = \ln \left( 1 - s_{D}^{M} + s_{D}^{M} \times s_{M}^{D} \right)^{\frac{1}{\varepsilon}}, \]

so that only micro-data on domestic expenditure shares \(s_{D}\) and the two elasticities of substitution \(\varepsilon\) and \(\zeta\) are required.\(^2\)

**Extension 2: General Production Function for Materials.** — In Section (I), we assumed that material services were a CES aggregator of a domestic variety \(z_{D}\) and a foreign input bundle \(x_{I}\). Suppose now that the aggregator for materials is given by a general function

\[ x = g(q_{D}z_{D}, x_{I}). \]

We continue to assume that materials \(x\) and primary factors \(l\) are combined with a Cobb-Douglas production function given in (1). Again let \(A(\mathscr{S})\) be the price index of the import bundle and \(Q(\mathscr{S})\) be the price index of materials. Consider any shock to the trading environment that affects \(A(\mathscr{S})\). Then

\[ d \ln (u) \bigg|_{p_{D},w} = \gamma \times d \ln (Q) \bigg|_{p_{D}} = \gamma \frac{z_{I}A}{u} \frac{dA}{A} = \gamma (1 - s_{D}) d \ln (A). \]

\(^2\) Note that, when \(\zeta \to 1\), (O31) reduces to the expression in (7):

\[ \lim_{\zeta \to 1} \ln \left( \left. \left( \frac{u}{u} \right) \right|_{p_{D},w} \right) = \frac{\gamma}{1-\varepsilon} \ln (s_{D}). \]
The optimality conditions from the cost-minimization problem imply that
\[
d \ln (A) = -\left(\frac{\frac{1}{\varepsilon_L}}{1 - \frac{1}{\varepsilon_L}}\right) \frac{1}{1 - s_D} d \ln (s_D),
\]
where
\[
- \frac{1}{\varepsilon_L} \equiv \frac{\partial \ln \left(\frac{\partial g(q_D z_D, x_I) / \partial x_I}{\partial g(q_D z_D, x_I) / \partial x_D}\right)}{\partial \ln \left(\frac{\partial g(z_D, x) / \partial x_D}{\partial g(z_D, x) / \partial x_I}\right)}
\]
is the local elasticity of substitution. Substituting this into (O33) yields
\[
(O34) \quad d \ln (u)|_{p_D, w} = \gamma \frac{\frac{1}{\varepsilon_L}}{1 - \frac{1}{\varepsilon_L}} d \ln (s_D) = - \frac{\gamma}{1 - \varepsilon_L} d \ln (s_D).
\]
In case the elasticity of substitution is constant, i.e. \(\varepsilon_L = \varepsilon\), (O34) can be integrated to yield (7).

Extension 3: Multiple Foreign Products. — In the main analysis, we assumed that firms source a single product from each sourcing country. In the data, firms often import multiple products from a given country. We now explore how (O27) would change in a multi-product environment. Consider first the case where the product aggregator is nested in the country aggregator, i.e. the production structure is given by (1)-(3), where

\[
q_{ci} z_c \equiv \psi_{ci} \left(\left[q_{kci} z_{kc}\right]_{k \in K_{ci}}\right),
\]
\(k\) is a product index, \(K_{ci}\) denotes the set of products that firm \(i\) sources from country \(c\), \(\psi_{ci}\) is a constant-returns-to-scale production function and (O35) applies also to the domestic variety. As long as the number of products sourced domestically does not change when firms are forced into input-autarky, the analysis in the main text remains entirely unchanged and the firm-level gains are still given by (7).

Consider next the case where the country aggregator is nested in the product aggregator. Suppose for example that the production structure for intermediates \(x\) is given by

\[
(O36) \quad x = \left(\sum_{k=1}^{K} \left(\eta_k x_k\right)^{\frac{1}{\varepsilon_k}}\right)^{\frac{\varepsilon_k}{\varepsilon_k - 1}}
\]
\[
x_k = \left(\beta_{ki} \left(q_{kD} z_{kD}\right)^{\frac{\varepsilon_k - 1}{\varepsilon_k}} + \left(1 - \beta_{ki}\right) x_{kI}\right)^{\frac{\varepsilon_k}{\varepsilon_k - 1}}
\]
\[
x_{kI} = h_{ki} \left(\left[q_{kcI} z_{kc}\right]_{c \in S_{ki}}\right).
\]
Note that the sourcing strategy is now a list of countries for each product. Letting
$Q_i$ and $Q_{ki}$ denote the price indices for materials $x$ and product-specific material services $x_k$ respectively, it can be easily shown that

$$Q_i = \left( \sum_{k=1}^{K} \left( \frac{Q_{ki}}{\eta_k} \right)^{1-\iota} \right)^{\frac{1}{1-\iota}},$$

$$Q_{ki} = (s_{kD_i})^{\frac{\epsilon_k}{\iota-\epsilon_k}} \beta_{ki}^{-\frac{s_k}{\iota-\epsilon_k}} p_{kD}/q_{kD},$$

where $s_{kD_i}$ is firm $i$’s domestic expenditure share for product $k$. The firm-level gains are therefore given by

$$\ln \left( \frac{u^{Aut}}{u} \right)_{pD,w} = \frac{\gamma}{\iota-1} \times \ln \left( \sum_{k=1}^{K} \chi_{ki} (s_{kD_i})^{\frac{\iota-1}{\iota-\epsilon_k}} \right),$$

where

$$\chi_{ki} \equiv \frac{\left( \beta_{ki}^{-\frac{s_k}{\iota-\epsilon_k}} p_{kD}/q_{kD} \right)^{1-\iota}}{\sum_{k=1}^{K} \left( \beta_{ki}^{-\frac{s_k}{\iota-\epsilon_k}} p_{kD}/q_{kD} \right)^{1-\iota}}.$$

We see that the producer gains are akin to a weighted average of the product-specific producer gains $(s_{kD_i})^{\frac{\iota-1}{\iota-\epsilon_k}}$. In our empirical application, we cannot implement (O37) because we do not observe domestic shares at the product level $s_{kD_i}$ in the French data. Note that implementing (O37) also requires measuring the weights $\chi_{ki}$. In the case where (O36) takes the Cobb-Douglas form, i.e. $\iota = 1$ as in Halpern, Koren and Szeidl (2015), (O37) simplifies to

$$\ln \left( \frac{u^{Aut}}{u} \right)_{pD,w} = \sum_{k=1}^{K} \frac{\gamma}{\iota-\epsilon_k} \ln \left( s_{D_i} \right).$$

Thus, in the Cobb-Douglas case, the producer gains are a weighted average of the product-specific producer gains.
We use the French input-output tables from the OECD to discipline the demand parameters $[\alpha_s]$ and the matrix of input-output linkages $\Xi$. To determine $\Xi$, we focus on the intermediate supply from each industry $j$ to each industry $s$. We abstract from any taxes and subsidies. As $\Xi$ can be identified from expenditure shares by sourcing sector, see (19), we set

$$\zeta^s_j = \frac{\text{Intermediate supply from industry } j \text{ to industry } s}{\text{Intermediate consumption at final prices from industry } s}.$$  

That is, $\zeta^s_j$ measures the importance of sector $j$ in the production process of sector $s$. By construction, this ensures that $\sum_{j=1}^{S} \zeta^s_j = 1$ for all $s$. We arrange the input-output matrix so that the columns contain the distribution of expenditure for the different sectors:

$$\Xi = \begin{bmatrix} \zeta^1_1 & \zeta^2_1 & \cdots & \zeta^S_1 \\ \zeta^1_2 & \zeta^2_2 & \cdots & \zeta^S_2 \\ \vdots & \vdots & \ddots & \vdots \\ \zeta^1_S & \zeta^2_S & \cdots & \zeta^S_S \end{bmatrix}.$$  

To determine $[\alpha_s]$, we also use the input-output tables as they contain information on the composition of final demand. Since there is no trade in final goods in the theory, we exclude any exports and imports in final goods in the data. More specifically, the input-output tables report final consumption expenditure by households on sector $j$, denoted by $HHFC_j$. Following (19), we hence set

$$\alpha_s = \frac{HHFC_s}{\sum_{j=1}^{S} HHFC_j}.$$  

The OECD input-output tables report their data at the 2-digit level of ISIC Rev. 3, which gives 37 manufacturing industries. To deal with the non-manufacturing industries, we group them into a “residual” sector which we denote by $S$. To incorporate this sector in the analysis, we set

$$(O38) \quad \alpha_S = 1 - \sum_{j \in M} \alpha_j,$$  

where $M$ is the set of manufacturing sectors. Because in our theory this sector is not engaged in foreign sourcingootnote{Note that this sector may nevertheless benefit from input trade if it sources output from the manufacturing industries.}, we set

$$\Lambda_S = 0.$$  

The input-output structure of sector $S$ can be recovered from the input-output
tables. In particular, we set

$$\zeta_j^S \equiv \frac{\sum_{n=1}^{NM} \text{Intermediate supply from industry } j \text{ to industry } n}{\sum_{n=1}^{NM} \text{Intermediate consumption at final prices to industry } n},$$

where $NM$ is the number (set) of non-manufacturing sectors. To measure the materials coefficient in the production of sector $S$, we employ the Input-Output Matrix for the non-manufacturing sectors. As we observe value added and intermediary spending for each sector, we set

(O39) \[
\gamma_S = \frac{\sum_{n=1}^{NM} X_n}{\sum_{n=1}^{NM} (X_n + VA_n)},
\]

where $X_n$ denotes total intermediary spending by sector $n$.

Table O3 summarizes how $[\alpha_s]$ and $[\gamma_s]$ are computed. The input-output matrix $\Xi$ used in our empirical analysis is contained in Table O4.

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### Table O4—Input-Output Linkages: \( \Xi \)

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\( S \) 43.64 68.48 34.03 55.82 41.4 44.54 31.56 47.37 35.35

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<td>5.66</td>
<td>8.44</td>
<td>2.28</td>
<td>9.25</td>
<td>0.35</td>
</tr>
<tr>
<td>28</td>
<td>28.04</td>
<td>15.87</td>
<td>0.75</td>
<td>13.11</td>
<td>5.07</td>
<td>8.31</td>
<td>8.02</td>
<td>4.84</td>
<td>4.72</td>
<td>1.38</td>
</tr>
<tr>
<td>29</td>
<td>4.04</td>
<td>19.27</td>
<td>0.64</td>
<td>2.28</td>
<td>1.99</td>
<td>3.37</td>
<td>4.21</td>
<td>3.33</td>
<td>3.01</td>
<td>1.51</td>
</tr>
<tr>
<td>30</td>
<td>0.24</td>
<td>0.48</td>
<td>37.61</td>
<td>0.35</td>
<td>1.27</td>
<td>2.02</td>
<td>0</td>
<td>0.07</td>
<td>0.17</td>
<td>0.34</td>
</tr>
<tr>
<td>31</td>
<td>2.24</td>
<td>4.43</td>
<td>3.93</td>
<td>16.03</td>
<td>6.83</td>
<td>2.84</td>
<td>3.07</td>
<td>1.05</td>
<td>1.39</td>
<td>1.12</td>
</tr>
<tr>
<td>32</td>
<td>0.59</td>
<td>3.81</td>
<td>12.59</td>
<td>11.00</td>
<td>30.3</td>
<td>11.86</td>
<td>1.98</td>
<td>4.26</td>
<td>1.50</td>
<td>0.70</td>
</tr>
<tr>
<td>33</td>
<td>0.61</td>
<td>2.29</td>
<td>2.59</td>
<td>2.72</td>
<td>8.55</td>
<td>19.31</td>
<td>1.52</td>
<td>8.74</td>
<td>0.42</td>
<td>0.60</td>
</tr>
<tr>
<td>34</td>
<td>0.25</td>
<td>0.69</td>
<td>0.13</td>
<td>0.27</td>
<td>0.22</td>
<td>0.24</td>
<td>35.3</td>
<td>0.14</td>
<td>0.42</td>
<td>0.82</td>
</tr>
<tr>
<td>35</td>
<td>0.11</td>
<td>1.31</td>
<td>0.02</td>
<td>0.03</td>
<td>0.03</td>
<td>0.05</td>
<td>0.11</td>
<td>46.37</td>
<td>0.05</td>
<td>1.12</td>
</tr>
<tr>
<td>36-37</td>
<td>0.98</td>
<td>0.76</td>
<td>0.31</td>
<td>0.61</td>
<td>0.34</td>
<td>0.52</td>
<td>1.94</td>
<td>0.37</td>
<td>6.85</td>
<td>0.45</td>
</tr>
</tbody>
</table>

\( S \) 30.04 34.13 37.63 28.21 32.02 35.73 23.46 24.09 35.37 79.14

**Note:** The table contains the French input-output matrix used in our empirical work. We report numbers in percentage terms. Sectors are classified at the 2-digit-level according to ISIC Rev. 3. The non-manufacturing sector \( S \) is constructed as explained in the main text and Table O3.
O3. Firm-Level Heterogeneity in the Gains from Input Trade

We can measure the firm-level gains from input trade from \( \frac{\gamma_1}{1-\varepsilon} \ln (s_{Dist}) \) (see (7)). One can then use the micro-data to learn about the firm-level correlates of this heterogeneity. In particular, consider a regression of the firm-level gains \( \frac{\gamma_1}{1-\varepsilon} \ln (s_{Dist}) \) on different firm-characteristics, which according to (5) could reflect firm-variation in exogenous “import capabilities” (such as prices \( \frac{p_{ci}}{q_{ci}} \) or the home bias \( \beta_i \)) and firms’ endogenous sourcing strategies \( S_i \). Consider Table O5, which contains the results of regressions of the form \( \frac{\gamma_1}{1-\varepsilon} \ln (s_{Dist}) = \delta_s + \delta_t + \psi_i + u_{ist} \), where \( \delta_s \) and \( \delta_t \) are industry and time fixed effects and \( \psi_i \) is a vector of firm characteristics. We find that bigger firms and exporters see higher gains. When we restrict the analysis to the sample of importers, the positive relation between firm size and the firm-level gains becomes substantially weaker. This is consistent with the pattern documented in Figure 2. When we consider the role of firms’ sourcing strategies, we find a strong positive relation between firms’ extensive margin (which we measure by the average number of countries that the firm sources its products from) and the firm-level gains. This is consistent with theories where import participation in foreign markets reduces firms’ unit costs. The importance of other firm characteristics is substantially diminished once this extensive margin is controlled for.

O4. Accounting for Export Participation

To derive Proposition 1, we had to express firms’ unobserved productivity \( \varphi \) in terms of value added. We did so using equation (11). This equation, however, is only correct if firms’ international sales are proportional to their domestic sales. In case export participation is limited and productive firms are more likely to export, we would overestimate \( \varphi \) for large firms. This could be important, because export participation is correlated with both firm size and import intensity.

Fortunately, it is straight-forward to account for this effect. In particular, we directly observed domestic value added \( va^D_i \) in the micro-data. Because (11) is valid for domestic value added, we can simply evaluate Proposition 1, by using domestic value added instead of total value added. Hence, accounting for firms’ export intensity reduces to a re-weighting of firms’ domestic expenditure shares. In our context, we find that the gains from input trade are given by 24.4% for the manufacturing sector and 8.1% for the whole economy. These numbers are smaller than our baseline results, reflecting the negative correlation between export intensity and domestic expenditure shares.
Table O5—Correlates of the Producer Gains

<table>
<thead>
<tr>
<th>Sample</th>
<th>Full Sample</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td></td>
<td>633,240</td>
</tr>
<tr>
<td>(2)</td>
<td></td>
<td>640,610</td>
</tr>
<tr>
<td>(3)</td>
<td></td>
<td>633,240</td>
</tr>
<tr>
<td>(4)</td>
<td></td>
<td>118,799</td>
</tr>
<tr>
<td>(5)</td>
<td></td>
<td>120,344</td>
</tr>
<tr>
<td>(6)</td>
<td></td>
<td>118,799</td>
</tr>
<tr>
<td>(7)</td>
<td></td>
<td>120,344</td>
</tr>
<tr>
<td>(8)</td>
<td></td>
<td>118,799</td>
</tr>
</tbody>
</table>

**Firm-level gains**

\[
\gamma_1 - \varepsilon \ln(s_{Di})
\]

Sample: Full sample Importers Only

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value Added</td>
<td>0.028***</td>
<td>0.013***</td>
<td>0.005***</td>
</tr>
<tr>
<td>(ln)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Employment</td>
<td>0.028***</td>
<td>-0.000</td>
<td></td>
</tr>
<tr>
<td>(ln)</td>
<td>(0.000)</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>Exporter</td>
<td>0.085***</td>
<td>0.040***</td>
<td>0.024***</td>
</tr>
<tr>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>Nb. varieties</td>
<td>0.148***</td>
<td>0.138***</td>
<td>0.113***</td>
</tr>
<tr>
<td>(ln)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
</tbody>
</table>

Notes:
- Robust standard errors in parentheses with ∗∗∗, ∗∗ and ∗ respectively denoting significance at the 1%, 5% and 10% levels.
- All regressions include year fixed effects.
- The number of varieties is the number of countries the firm sources from (averaged across products).
- A firm is a member of an international group if at least one affiliate or the headquarter is located outside of France.
- The data corresponds to the full sample of firms between 2002 and 2006.
O5. Bootstrap Procedure

We sample firms from the empirical distribution with replacement to construct 200 replicates of our micro-data. For each of these samples, we re-calculate $\sigma_s$ and re-estimate $\varepsilon$ and $[\gamma_s]$ following the factor shares approach explained in Section III.B and then re-calculate $[\Lambda_s]$ and $[s^{Agg}_D]$. Figure O1 depicts the bootstrap distributions of these variables. For the three sector-level variables, we report the distribution of the sectoral averages, e.g. the upper right panel displays the distribution of $\frac{1}{S} \sum_{s=1}^{S} \gamma_s$. While the variation in $\gamma$ and $s^{Agg}_D$ is relatively modest, there is a quite a bit of uncertainty regarding $\varepsilon$. This is consistent with the non-negligible standard errors reported in Table 2. We conclude that it is the variation in $\varepsilon$ which induces most of the variation in $\Lambda$ and therefore in the consumer price gains from input trade reported in Table 4.

![Bootstrap Distribution of Structural Parameters and Direct Price Reductions](image)

**Figure O1. Bootstrap Distribution of Structural Parameters and Direct Price Reductions**

*Note:* The upper left panel contains the bootstrap distribution of $\varepsilon$. The remaining panels depict the bootstrap distributions of $\frac{1}{S} \sum_{s=1}^{S} \gamma_s$, $\frac{1}{S} \sum_{s=1}^{S} \Lambda_s$ and $\frac{1}{S} \sum_{s=1}^{S} s^{Agg}_D$. The point estimates used in the main analysis are reported as vertical lines.

In Figure O2 we also show the entire distribution of the consumer price gains and the bias with respect to an aggregate approach.
Figure O2. Sampling Variation in the Consumer Price Gains and the Bias

Note: The top panels of the figure depict the bootstrap distribution of the consumer price gains from input trade for the manufacturing sector \((P_{Aut}^M/P_M - 1) \times 100\) (left panel) and the entire economy \((P_{Aut}/P - 1) \times 100\) (right panel). These are computed according to Proposition 1. We display the gains based on the micro data, i.e. using \(\Lambda_{Aut}\), and aggregate data, i.e. using \(\Lambda_{Agg,s}\). The bottom panels depict the bootstrap distribution of the bias from using aggregate data, which is computed according to (17). The bootstrap procedure is described in the Online Appendix. We use 200 iterations.
At the end of Section III.D we argued that aggregate models could be subject to an "elasticity bias". Because the mapping between the model’s structural parameter $\varepsilon$ and the implied aggregate trade elasticity depends on the particular model and the implied $\varepsilon$ for a given trade elasticity is higher in the aggregate model (compared to the firm-based model with heterogeneous fixed costs), the implied gains from trade are downward biased. Importantly, we showed that - quantitatively - this bias can be substantial. To see that, Table O6 reports the consumer price gains from input trade for different values of the elasticity of substitution $\varepsilon$. Columns one and two replicate the results for our baseline estimate $\varepsilon = 2.38$. While column one reports the results based on the micro-data, column two reports the gains based on aggregate data, $\Lambda_{Aut,Agg,s}$. These results correspond to the ones reported in Table 4 above. In the remaining columns, we report the gains for a range of values of $\varepsilon$. The tables show that the gains are very sensitive to the value of $\varepsilon$. Table O6 shows that the economy-wide gains predicted by an aggregate approach under $\varepsilon = 5$ are about 65% lower than the gains predicted by the approach that relies on micro-data.

Table O6—The Consumer Price Gains for Different Values of $\varepsilon$

<table>
<thead>
<tr>
<th>Value for $\varepsilon$:</th>
<th>Micro Data</th>
<th>Aggregate Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.38</td>
<td>2.38</td>
</tr>
<tr>
<td>Entire Economy</td>
<td>9.04</td>
<td>9.9</td>
</tr>
<tr>
<td>Manufacturing Sector</td>
<td>27.52</td>
<td>30.8</td>
</tr>
</tbody>
</table>

Note: The table reports the reduction in consumer prices for the entire economy ($P^{Aut}/P - 1$) $\times 100$ (first row) and the manufacturing sector ($P^{M}_{Aut}/P_{M} - 1$) $\times 100$ (second row) for different values of the elasticity of substitution $\varepsilon$. In the first two columns, we report the baseline results under $\varepsilon = 2.38$ for comparison. Column one is based on Proposition 1 where $\Lambda_{s}$ are computed with micro data as reported in Table 4. The remaining columns contain results based on an aggregate model, i.e. they are based on Proposition 1 where the sectoral gains are measured by $\Lambda_{Aut,Agg,s}$ as per (16) instead of $\Lambda_{Aut,s}$. The values for $\Xi, \gamma_s, \sigma_s$ and $\alpha_s$ employed for all calculations are given in Table 1.

Table O7 reports the consumer price gains from input trade for different values of the elasticity of substitution $\varepsilon$ at the sectoral level. Columns one and two replicate the results for our baseline estimate $\varepsilon = 2.38$. While column one reports the results based on the micro-data, column two reports the gains based on aggregate data, $\Lambda_{Aut,Agg,s}$. As in Table O6, we find the gains from input trade are very sensitive to the value of $\varepsilon$. 

13
<table>
<thead>
<tr>
<th>Value for $\varepsilon$:</th>
<th>Micro Data</th>
<th>Aggregate Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.38</td>
<td>2.38</td>
</tr>
<tr>
<td>Mining</td>
<td>2.96</td>
<td>2.50</td>
</tr>
<tr>
<td>Food, tobacco, beverages</td>
<td>11.06</td>
<td>12.62</td>
</tr>
<tr>
<td>Textiles and leather</td>
<td>31.14</td>
<td>31.87</td>
</tr>
<tr>
<td>Wood and wood products</td>
<td>8.23</td>
<td>9.58</td>
</tr>
<tr>
<td>Paper, printing, publishing</td>
<td>12.15</td>
<td>10.96</td>
</tr>
<tr>
<td>Chemicals</td>
<td>27.23</td>
<td>28.14</td>
</tr>
<tr>
<td>Rubber and plastic products</td>
<td>20.12</td>
<td>21.53</td>
</tr>
<tr>
<td>Non-metallic mineral products</td>
<td>13.42</td>
<td>13.29</td>
</tr>
<tr>
<td>Basic metals</td>
<td>21.80</td>
<td>28.83</td>
</tr>
<tr>
<td>Metal products</td>
<td>8.17</td>
<td>7.70</td>
</tr>
<tr>
<td>Machinery and equipment</td>
<td>17.64</td>
<td>18.23</td>
</tr>
<tr>
<td>Office and computing machinery</td>
<td>20.42</td>
<td>37.00</td>
</tr>
<tr>
<td>Electrical machinery</td>
<td>19.77</td>
<td>21.64</td>
</tr>
<tr>
<td>Radio and communication</td>
<td>21.55</td>
<td>22.15</td>
</tr>
<tr>
<td>Medical and optical instruments</td>
<td>17.90</td>
<td>15.90</td>
</tr>
<tr>
<td>Motor vehicles, trailers</td>
<td>6.24</td>
<td>11.23</td>
</tr>
<tr>
<td>Transport equipment</td>
<td>15.32</td>
<td>11.83</td>
</tr>
<tr>
<td>Recycling, nec.</td>
<td>12.87</td>
<td>14.06</td>
</tr>
</tbody>
</table>

Non-manufacturing 0 0 0 0 0 0

Note: The table reports the reduction in consumer prices at the sectoral level $(P^\text{Aut}_{s}/P_{s} - 1) \times 100$ for different values of the elasticity of substitution $\varepsilon$. In the first two columns, we report the baseline results under $\varepsilon = 2.38$ for comparison. Column one is based on Proposition 1 where $\Lambda_{s}$ are computed with micro data as reported in Table 4. The remaining columns contain results based on an aggregate model, i.e. they are based on Proposition 1 where the sectoral gains are measured by $\Lambda^\text{Aut}_{s,Agg}$ as per (16) instead of $\Lambda^\text{Aut}_{s}$. The values for $\Xi, \gamma_{s}, \sigma_{s}$ and $\alpha_{s}$ employed for all calculations are given in Table 1.
Consider the setup of Section III.D. We now consider the aggregate allocations in this economy. An equilibrium has the usual definition:

**Definition 1.** An equilibrium is a set of prices $w, \{p_i\}$, labor demands for production and fixed costs $\{l_i, l_i^F\}$, differentiated product quantities, consumption levels and foreign demands $\{y_i, c_i, y_i^{ROW}\}$, domestic and international input demands by local firms $\{y_{i|\nu}, [z_{ci}]\}$ and sourcing strategies $\{n_i\}$ such that:

1) Firms maximize profits given by (28)-(29),

2) Consumers maximize utility given by (8) subject to their budget constraint

$$\int_i p_i c_i di = wL + \int_i \pi_i di,$$

3) Trade is balanced (31),

4) Labor and good markets clear

$$L = \int_i (l_i + l_i^F) di$$

$$y_i = c_i + y_i^{ROW} + \int_{i} y_{i|\nu} dv.$$

We fist characterize the general equilibrium in a multi-sector version of the economy of Section III.D. In particular, we consider the multi-sector structure of Section I. We derive a generalization of (32). We do not impose any assumptions on how firms' determine their extensive margin. That is, we allow for an arbitrary mapping $l_{s,i}$ which gives the labor resources that firm $i$ needs to spend in order to attain the sourcing strategy $S_{s,i}$. We assume that trade is balanced and that the value of exports in sector $s$ is given by $\alpha_s^{ROW} \times IM$, where $IM$ denotes the value of total spending on imported inputs.

**Proposition 2.** Let $W, I$ and $S$ denote welfare, consumer income and total spending, respectively. Then, the change in welfare relative to input autarky is given by

$$\frac{W}{W^{Aut}} = \frac{I}{I^{Aut}} \times \frac{P^{Aut}}{P},$$

where $I$ and $I^{Aut}$ are given by

$$I = L + \sum_{s=1}^{S} S_s/\sigma_s - \sum_{s=1}^{S} \left( \int_{0}^{N_s} l_{s,i} di \right),$$

$$I^{Aut} = L + \sum_{s=1}^{S} S^{Aut}_s/\sigma_s,$$
and \([S_j] \text{ and } [S_j^{Aut}]\) solve

\[
S_s = \alpha_s \left( L - \sum_{j=1}^{S} \left( \int_0^{N_j} l_{j, \mathcal{J}} \, dl \right) + \sum_{j=1}^{S} \frac{1 + \zeta_j^{\mathcal{J}} \gamma_j (\sigma_j - 1)}{\sigma_j} S_j \right) + \sum_{j=1}^{S} \left[ \alpha_s^{\mathcal{R}ow} - \zeta_j^{\mathcal{J}} \right] \gamma_j \frac{\sigma_j - 1}{\sigma_j} \int_0^{N_j} (1 - s_Dl) \omega_l \, dl
\]

and

\[
S_s^{Aut} = \alpha_s \left( L + \sum_{j=1}^{S} 1 + \zeta_j^{\mathcal{J}} \gamma_j (\sigma_j - 1) / \alpha_s S_j^{Aut} \right).
\]

Furthermore, \(G = \frac{p_s}{p_s^{Aut}}\) is given in Proposition 1.

**Proof.** As labor is the only factor of production, consumer welfare is given by real income \(W = I/P\), consumer income is given by

\[
I = L + \sum_{s=1}^{S} \left( \int_0^{N_s} \pi_i \, dl \right).
\]

Note that \(L\) represents total labor income and \(\pi_i\) denotes firm \(i\)'s profits. To derive \(\pi_i\), recall that firms in sector \(s\) have a mark-up of \(\alpha_s/\sigma_s - 1\) so that variable profits gross of any extensive margin resource loss are given by

\[
\pi_i^V = (p_i - u_i) y_i = p_i y_i / \sigma_s.
\]

Total revenue for firm \(i\) is given by

\[
p_i y_i = \left( \frac{p_i}{P_s} \right)^{1-\alpha_s} S_s,
\]

where \(P_s\) is the consumer price index for sector \(s\) and \(S_s\) denotes total spending for sector \(s\) goods. Hence,

\[
\pi_i = p_i y_i / \sigma_s - l_{j, \mathcal{J}} = \frac{1}{\sigma_s} \left( \frac{p_i}{P_s} \right)^{1-\alpha_s} S_s - l_{j, \mathcal{J}},
\]

so that

\[
(O40) \quad I = L + \sum_{s=1}^{S} \frac{1}{\sigma_s} S_s - \sum_{s=1}^{S} \left( \int_0^{N_s} l_{j, \mathcal{J}} \, dl \right).
\]

Hence, given \([S_s]\) and \([l_{j, \mathcal{J}}]\), total income \(I\) is fully determined. Now consider \([S_s]_s\). Note that

\[
S_s = S_s^C + S_s^X + S_s^{\mathcal{R}ow},
\]

16
where \( S^C_s, S^X_s \) and \( S_{ROW}s \) denote total spending by consumers, intermediary producers and the rest of the world, respectively. For our economy, we have that
\[
S^C_c = \alpha_s I
\]
and
\[
S_{ROW}s = \alpha_{ROW}s Im
\]
as consumers spend a fraction \( \alpha_s \) of their income \( I \) on sector \( s \) products and balanced trade requires that total spending by the rest of the world is equal to the value of imports \( Im \), a fraction \( \alpha_{ROW}s \) of which is spent on sector \( s \) products. To derive \( S^X_s \), let total domestic intermediary purchases in sector \( j \) be given by \( X_j \). Then

\[
(O41) \quad S^X_s = \sum_{j=1}^{S} \zeta_s^j X_j.
\]

Letting \( m_i \) be total material spending by firm \( i \) and \( s_i \) be total spending by firm \( i \), we know that

\[
X_j = \int_0^{N_j} s_{Di} m_i \, di = \int_0^{N_j} s_{Di} \gamma_j s_i \, di = \int_0^{N_j} s_{Di} \gamma_j \frac{\sigma_j - 1}{\sigma_j} p_i y_i \, di
\]

\[
(O42) \quad X_j = \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_0^{N_j} s_{Di} \left( \frac{p_i}{P_j} \right)^{1-\sigma_j} \, di,
\]

where we used that firms in sector \( j \) spend a fraction \( \gamma_j \) of their total input spending \( s_i \) on materials and that total spending \( s_i \) accounts for a fraction \( (\sigma_j - 1)/\sigma_j \) of revenue. Hence, (O41) and (O42) imply that

\[
(O43) \quad S^X_s = \sum_{j=1}^{S} \zeta_s^j \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_0^{N_j} s_{Di} \left( \frac{p_i}{P_s} \right)^{1-\sigma_s} \, di.
\]

Similarly, total import spending is equal to

\[
Im = \sum_{j=1}^{S} Im_j = \sum_{j=1}^{S} \int_0^{N_j} (1 - s_{Di}) m_i \, di
\]

\[
(O44) \quad Im = \sum_{j=1}^{S} \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_0^{N_j} (1 - s_{Di}) \left( \frac{p_i}{P_s} \right)^{1-\sigma_s} \, di.
\]
Hence (O43) and (O44) imply that

\[ S_s = \alpha_s I + \alpha_s^{ROW} \left( \sum_{j=1}^{S} \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_{0}^{N_j} (1 - s_{Di}) \left( \frac{p_i}{P_j} \right)^{1-\sigma_j} di \right) + \sum_{j=1}^{S} \zeta_j \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_{0}^{N_j} s_{Di} \left( \frac{p_i}{P_j} \right)^{1-\sigma_j} di \]

\[ = \alpha_s I + \sum_{j=1}^{S} \zeta_j \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_{0}^{N_j} s_{Di} \left( \frac{p_i}{P_j} \right)^{1-\sigma_j} di \]

Using (O40), we get that

\[ S_s = \alpha_s \left( L - \sum_{j=1}^{S} \left( \int_{0}^{N_j} l_{\gamma_j} di \right) + \sum_{j=1}^{S} \frac{1 + \frac{\zeta_j}{\alpha_s} \gamma_j (\sigma_j - 1)}{\sigma_j} S_j \right) + \sum_{j=1}^{S} \left[ \alpha_s^{ROW} - \zeta_j \right] \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_{0}^{N_j} (1 - s_{Di}) \left( \frac{p_i}{P_j} \right)^{1-\sigma_j} di. \]

Now note that

\[ \frac{va_i}{\int_{0}^{N_s} va_i di} = \frac{p_i y_i}{\int_{0}^{N_s} p_i y_i di} = \frac{(p_i/P_s)^{1-\sigma_s} S_s}{\int_{0}^{N_s} (p_i/P_s)^{1-\sigma_s} S_s di} = \left( \frac{p_i}{P_s} \right)^{1-\sigma_s}. \]

Hence,

\[ S_s = \alpha_s \left( L - \sum_{j=1}^{S} \left( \int_{0}^{N_j} l_{\gamma_j} di \right) + \sum_{j=1}^{S} \frac{1 + \frac{\zeta_j}{\alpha_s} \gamma_j (\sigma_j - 1)}{\sigma_j} S_j \right) + \sum_{j=1}^{S} \left[ \alpha_s^{ROW} - \zeta_j \right] \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_{0}^{N_j} (1 - s_{Di}) \omega_i di, \]

(O45)

where \( \omega_i = \frac{va_i}{\int_{0}^{N_s} va_i di} \). Given \( L^{NET} = L - \sum_{j=1}^{S} \left( \int_{0}^{N_j} l_{\gamma_j} di \right) \), (O45) are \( S \) equations in \( S \) unknowns \( S_s \), which we can easily solve. Now consider the case of autarky. There we have \( l_{\gamma_j} = 0 \) and \( s_{Di} = 1 \). Hence, (O45) yields

\[ S_s^{Aut} = \alpha_s \left( L + \sum_{j=1}^{S} \frac{1 + \frac{\zeta_j}{\alpha_s} \gamma_j (\sigma_j - 1)}{\sigma_j} S_j^{Aut} \right). \]
In the case of a single sector (i.e. $S = 1$) it has to be the case that 
\[
\alpha_S = \alpha^\text{ROW}_S = \zeta^S_S = 1.
\]

Hence, 
\[
S^{\text{Aut}} = L + \frac{1 + \gamma (\sigma - 1)}{\sigma} S^{\text{Aut}} = \frac{\sigma}{(1 - \gamma)(\sigma - 1)} L.
\]
Substituting this in (O40) yields 
\[
I^{\text{Aut}} = L + \frac{1}{\sigma} S = \frac{1 + (1 - \gamma)(\sigma - 1)}{(1 - \gamma)(\sigma - 1)} L.
\]
Similarly, we get from (O45) that 
\[
\sum_{j=1}^{S} \left[ \alpha^\text{ROW}_S - \zeta^j_S \right] \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_0^{N_j} (1 - s_{Di}) \omega_i di = 0
\]
so that 
\[
S = \frac{\sigma}{(1 - \gamma)(\sigma - 1)} \left( L - \left( \int_i^{N_i} l_{\omega_i} di \right) \right)
\]
\[
I = \frac{1 + (1 - \gamma)(\sigma - 1)}{(1 - \gamma)(\sigma - 1)} \left( L - \left( \int_i^{N_i} l_{\omega_i} di \right) \right).
\]
This implies directly (32). This concludes the proof of Proposition 2. \qed
Because of our assumption that fixed costs do not vary by country, countries can be indexed by their quality \( q \). We first show that the price index of the import bundle takes the power form in (27). The import price index is given by

\[
A(S) = \left( \int_{q \in S} \frac{p(q)}{q}^{1-\kappa} dG(q) \right)^{\frac{1}{1-\kappa}}.
\]

(O46)

As quality is Pareto distributed, (O46) becomes

\[
A(S)^{1-\kappa} = \theta q^{\theta} q^{-1}\min q^{-\theta-1} dq.
\]

Because fixed costs are constant across countries, the sourcing set \( S \) can be parametrized by a quality cutoff \( q \). In particular, the firm selects countries with high enough quality, i.e. \( q \in S \) as long as \( q \geq \overline{q} \). It follows that

(O47)

\[
A(\overline{q})^{1-\kappa} = q^{\theta} q^{-1}\min q^{-\theta-1} dq.
\]

We can rewrite this expression in terms of the mass of countries sourced, \( n \), which is given by

(O48)

\[
n = P(q \in S) = P(q \geq \overline{q}) = q^{\theta} q^{-\theta-1}.
\]

Substituting (O48) into (O47) yields

\[
A(n) = q^{-1}\min \left( \frac{\theta}{\theta - (\kappa - 1)} \right)^{\frac{1}{1-\kappa}} n^{-\left(\frac{1}{\kappa-1}\right)},
\]

which is (27) in the main text where

\[
z \equiv q^{-1}\min \left( \frac{\theta}{\theta - (\kappa - 1)} \right)^{\frac{1}{1-\kappa}}
\]

\[
\eta \equiv \frac{1}{\kappa - 1}.
\]

This completes the characterization of (27). The following proposition characterizes the solution to the extensive margin problem.

**Proposition 3.** Consider the setup above and suppose that

\[
\eta (\varepsilon - 1) < 1 \text{ and } \eta \gamma (\sigma - 1) < 1.
\]
Then, the firm’s profit maximization problem (28) has a unique solution for any value of \( \tilde{\varphi} \) and \( f \). The optimal mass of countries sourced is given by a function \( n(\tilde{\varphi}, f) \) and an efficiency cutoff \( \varphi(f) \) such that \( n(\tilde{\varphi}, f) = 0 \) for \( \varphi \leq \varphi(f) \) with \( \varphi(\cdot) \) increasing. Furthermore, \( n(\varphi, f) \) is increasing in efficiency \( \tilde{\varphi} \) and decreasing in the fixed costs of sourcing \( f \).

Proof. The firm’s maximization problem follow from (28), (29) and (30) as

\[
\pi = \max_n \left\{ B \times \tilde{\varphi}^{(\sigma-1)} \left( \frac{pD}{qD} \right)^{\gamma(1-\sigma)} \left( 1 + \frac{1-\beta}{\beta} \right)^\varepsilon \left( \frac{pD}{qD} \frac{1}{n^\eta} \right)^{\varepsilon-1} \right\}.
\]

Conditional on importing, the optimal mass of countries is characterized by the following first order condition:

\[
(1-\beta)^\varepsilon \gamma(\sigma-1) \times \left( \frac{\beta}{1-\beta} \right)^\varepsilon \left( \frac{qD}{pD} \right)^{\varepsilon-1} \frac{1}{n^\eta} = 1 - (nf + f I I (n > 0))
\]

(O49)

The second order condition is given by

\[
(1-\beta)^\varepsilon \gamma(\sigma-1) \times \left( \frac{\beta}{1-\beta} \right)^\varepsilon \left( \frac{qD}{pD} \right)^{\varepsilon-1} \frac{1}{n^\eta} = 1 - \eta \left( n \eta (\varepsilon-1) - 1 \right)
\]

(O50)

where

\[
l(n) \equiv \frac{z^{1-\varepsilon} n^\eta (\varepsilon-1)}{(1-\beta)^\varepsilon \left( \frac{qD}{pD} \right)^{\varepsilon-1} + z^{1-\varepsilon} n^\eta (\varepsilon-1)} \in [0, 1].
\]

It follows that (O.O8) is satisfied if and only if

(O51)

\[
\eta (\varepsilon-1) - 1 + (\gamma (\sigma-1) - \varepsilon + 1) \eta l(n) < 0.
\]

Because we allow for arbitrary values of \( \varphi \) and \( f \), we need to verify that (O51) holds for any value of \( n \). Sufficient conditions for this are given by

(O52)

\[
\eta (\varepsilon-1) < 1
\]

and

(O53)

\[
\eta \gamma (\sigma-1) < 1.
\]

If (O52) is not satisfied, there exists a range of values of \( n \) close enough to zero
such that (O51) is violated.\(^4\) (O52) is therefore necessary. If \(\gamma (\sigma - 1) - \varepsilon + 1 \leq 0\), then (O51) is satisfied for all \(n\). If \(\gamma (\sigma - 1) - \varepsilon + 1 > 0\), then (O51) holds for all \(n\) if and only if

\[
\eta (\varepsilon - 1) - 1 + (\gamma (\sigma - 1) - \varepsilon + 1) \eta l(1) < 0.
\]

As \(l(1) < 1\), a sufficient condition for (O54) is given by (O53). This proves that, under (O52)-(O53), the optimal mass of countries conditional on importing is uniquely characterized by (O49) for any values of \(\tilde{\varphi}\) and \(f\).\(^5\) The firm becomes an importer whenever \(\pi_I \geq \pi_D\), where \(\pi_I\) are optimal profits conditional on importing and \(\pi_D\) are profits as a non-importer. It can be shown that this condition is satisfied whenever

\[
\left[ \left( 1 + \left( \frac{1}{\beta} \right) ^{\varepsilon} \left( \frac{p_D}{q_D} z^{-1} n^\eta \right) ^{\varepsilon^{-1}} \right) ^{\frac{\gamma (\sigma - 1)}{\varepsilon}} - 1 \right] \frac{(q_D/p_D)^{\gamma (\sigma - 1)} \Gamma \varphi^{\sigma - 1}}{-fn - f_I} > 0,
\]

where \(n\) is the solution to (O49). It follows the firm’s profit maximization problem in (28) has a unique solution for any value of \(\varphi\) and \(f\).

Note that, under (O52)-(O53), the left hand side of (O49) is decreasing in \(n\). Therefore, the optimal mass of countries sourced is weakly increasing in \(\varphi\) and weakly decreasing in \(f\). Holding \(n\) fixed, an increase in \(\varphi\) tends to increase the left hand side of (O55). Additionally, \(\pi_I\) is increasing in \(\varphi\). It follows that \(\pi_I - \pi_D\) is increasing in \(\varphi\) for a given \(f\). This proves that \(n = 0\) if and only if \(\varphi = \tilde{\varphi}(f)\) where \(\tilde{\varphi}(\cdot)\) is implicitly defined as the value of \(\varphi\) that makes the left hand side of (O55) equal to zero. The fact that \(\tilde{\varphi}(f)\) is increasing in \(f\) follows from the fact that \(\pi_I - \pi_D\) is decreasing in \(f\) for a given \(\varphi\).\(^6\) This proves Proposition 3.

To solve for the aggregate allocations, we have to consider the general equilibrium of the economy. The formal derivation and the analytical characterization is contained in Section O.O10 in the Online Appendix.

\(^4\)This follows from the fact that \(l(n)\) is continuous and strictly increasing.

\(^5\)When the solution to (O49) exceeds unity, the solution is given by \(n = 1\). Clearly, \(n = 0\) cannot be a solution as the firm always prefers to be a non-importer and avoid payment of \(f_I\). Note that our calibrated and estimated parameters satisfy (O52)-(O53) - see Table 1.

\(^6\)To see why this is the case, note that the left hand side of (O55) is decreasing in \(f\) given \(\varphi\) and \(n\). Additionally, \(\pi_I\) is decreasing in \(f\).
9. Estimation of $\eta$

To solve for firms’ optimal domestic shares in the heterogeneous fixed cost model, we require an estimate for $\eta$. To do so, we need to take a stand on what the counterpart of the number varieties, $n$, is in the data. We focus on the number of countries the firm sources its products from, i.e. the number of foreign varieties.\(^7\) Using this assumption we can estimate $\eta$ from the cross-sectional relationship between firms’ domestic expenditure share and the number of sourcing countries, because the theory predicts a log-linear relationship between $n$ and $\frac{1-s_{D}}{s_{D}}$ (see (30)). Hence, we estimate $\eta$ from the following regression:

\[
\ln \left( \frac{1-s_{D_{ist}}}{s_{D_{ist}}} \right) = \delta_{s} + \delta_{t} + \delta_{NK} + \eta (\varepsilon - 1) \ln (n_{ist}) + u_{ist},
\]

where $n_{ist}$ denotes firm $i$’s average number of countries per product sourced, $\delta_{NK}$ contains a set of fixed effects for the number of products sourced and $\delta_{s}$ and $\delta_{t}$ are sector and year fixed effects. Hence, we identify $\eta$ from firms sourcing the same number of products from a different number of supplier countries. We measure products at the 8-digit level.

Table O8 contains the results of estimating (O56). Columns one and two show that it is important to control for the number of products sourced as import-intensive firms source both more varieties per-product and more products on international markets - without the product fixed effects, the estimated $\eta$ increases substantially.\(^8\) Columns three and four show that the estimate of $\eta$ is virtually unaffected by additional firm-level controls that can affect firms’ import behavior conditional on the number of varieties sourced. In column five, we focus on a subsample of firm-product pairs that source their respective products from at least two supplier countries. In this case, the estimated $\eta$ decreases as the single-variety importers have very high domestic shares in the data. For our quantitative analysis, we take column five as the benchmark.\(^9\) The implied value of $\eta$ is 0.382 and it is precisely estimated.

---

\(^7\)This notion of “varieties” is widely used in the literature - see e.g. Broda and Weinstein (2006) and Goldberg et al. (2010). Moreover, the choice of the number of products sourced may be determined to a large degree by technological considerations, while the demand for multiple supplier countries within a given product category may plausibly stem from love-for-variety effects, which are at the heart of the mechanism stressed by our theory. However, we note that the analysis that follows can be done under alternative interpretations of $n$.

\(^8\)Recall that $\eta$ is a combination of different structural parameters of the economy. While $\eta$ is sufficient to characterize the welfare gains from trade, one might be interested in decomposing the returns to international sourcing into the the elasticity of substitution across varieties $\kappa$, the dispersion in input quality $\theta$, and the elasticity of input prices with respect to quality $\nu$. To do so, we need two additional pieces of information: import prices (to identify $\nu$) and data on firms’ expenditure shares across trading partners (to identify $\theta$).

\(^9\)We are concerned that the single-variety observations may not help identify the extensive margin channel emphasized by our theory but rather pick-up other variation across firms. Additionally, a non-parametric regression shows that the log linear relation between $n$ and $\frac{1-s_{D}}{s_{D}}$ in (O56) fits the data better in the sample with at least two varieties than in the full sample (results available upon request).
<table>
<thead>
<tr>
<th></th>
<th>All Importers</th>
<th>&gt; 1 Variety</th>
<th>&gt; 2 Varieties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nb. of varieties (ln)</td>
<td>1.308***</td>
<td>0.707***</td>
<td>0.733***</td>
</tr>
<tr>
<td>Capital / Emp. (ln)</td>
<td>(0.009)</td>
<td>(0.010)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>Exporter dummy</td>
<td>-0.395***</td>
<td>-0.388***</td>
<td>-0.254***</td>
</tr>
<tr>
<td>International group</td>
<td>0.150***</td>
<td>0.174***</td>
<td>0.216***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.013)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>Implied $\eta$</td>
<td>0.950***</td>
<td>0.513***</td>
<td>0.532***</td>
</tr>
<tr>
<td></td>
<td>(0.260)</td>
<td>(0.142)</td>
<td>(0.147)</td>
</tr>
<tr>
<td>Control for nb. of products</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>120,344</td>
<td>120,344</td>
<td>120,344</td>
</tr>
</tbody>
</table>

**Note:** Robust standard errors in parentheses with ***, **, and * respectively denoting significance at the 1%, 5% and 10% levels. All regressions include year fixed effects and 3-digit industry fixed effects. The number of varieties is the average number of countries a firm sources its foreign products from. To back out the value for $\eta$, we use our benchmark value for $\varepsilon = 2.378$ from Section III.
We adopt a solution algorithm that allows us to bypass the computation of the general equilibrium variables within the calibration. Intuitively, we work with a normalized version of fixed costs, where these are scaled by an appropriate transformation of the general equilibrium variables. Because the equilibrium variables depend on firms’ import behavior only through the domestic shares, which are itself a calibration target, we can compute them after the calibration. That is, we can first ensure that the moments of the joint distribution of value added and domestic shares are matched\footnote{For this step, it is important that the dispersion and correlation moments are in logs. See below.}, and then back out the underlying general equilibrium variables required to compute welfare. We also show that the parameter $z$ is not required for the calibration.

We first start with three aggregate variables, which are determined in equilibrium. In the single-sector version of the model, characterized in Section O.O7 in the Online Appendix, we have that aggregate spending $S$ and the price level (which is also equal to the price of domestic varieties) is given by

\begin{align}
S &= \frac{\sigma}{(1-\gamma)(\sigma-1)} \left( L - \left( \int l_i \mathrm{d}i \right) \right) \\
P &= \left( \frac{\sigma}{\sigma-1} \left( \frac{1}{\gamma} \right) \left( \frac{1}{1-\gamma} \right) \left( \frac{1}{\tilde{q}_D} \right) \right) \left( \frac{1}{\sigma} \right),
\end{align}

where

\begin{align}
\Upsilon &= \left( \int_{\tilde{f}}^{\tilde{N}} \left( \frac{1}{\tilde{\varphi}_i} \left( s_D,i \right) ^{(\gamma-1)} \right) ^{1-\sigma} \mathrm{d}i \right)^{\frac{1}{1-\gamma}}.
\end{align}

We start by noting that the firm’s optimality conditions from the profit maximization problem, contained in Section O.O8, can be expressed in terms of $s_D$ instead of $n$. To see this, note that (30) and (27) imply

\begin{align}
n^{\eta(\varepsilon-1)} = \left( \frac{1-s_D}{s_D} \right) \left( \frac{\beta}{1-\beta} \right) ^{\varepsilon} z^{\varepsilon-1} \left( \frac{q_D}{p_D} \right) ^{\varepsilon-1}.
\end{align}

Substituting (O60) into the firm’s first order condition (O49), we obtain

\begin{align}
\frac{1-\gamma(\sigma-1)n}{s_D^{(\varepsilon-1)\eta}} \left( 1-s_D \right) ^{\frac{1}{\eta(\varepsilon-1)}} = \left( \frac{\beta}{1-\beta} \right) ^{\frac{\varepsilon}{\varepsilon-1}} \frac{1}{\tilde{\varphi}_D} \tilde{f} \quad \tilde{f} = f \times (zq_D)^{1/\eta} \frac{1}{\eta^\gamma (\sigma-1)} \frac{1}{\Theta} \times \frac{1}{P^{1/\eta} \Gamma},
\end{align}

where

\begin{align}
\tilde{f} \equiv f \times (zq_D)^{1/\eta} \frac{1}{\eta^\gamma (\sigma-1)} \frac{1}{\Theta} \times \frac{1}{P^{1/\eta} \Gamma},
\end{align}

\footnote{For this step, it is important that the dispersion and correlation moments are in logs. See below.}
where

\[ \Theta = \frac{1}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \left( \left( \frac{1}{\gamma} \right)^{1-\gamma} \left( \frac{1}{1-\gamma} \right)^{\gamma} \right) \left( \frac{1}{qD} \right)^{1-\sigma}, \]

\[ \Gamma = \frac{S}{P(1-\gamma)(1-\sigma)}. \]

Similarly, (O60) and the import status condition (O55) imply that the firm is an importer as long as

\[ \left[ \frac{\gamma^{(\sigma-1)}}{sD^{\frac{1}{1-\gamma}}} - 1 \right] \tilde{\varphi}^{(\sigma-1)} - \left( 1 - \frac{sD}{sD} \right)^{\frac{1}{\eta^{(\sigma-1)}}} \gamma (\sigma - 1) \eta \left( \frac{\beta}{1 - \beta} \right)^{\frac{1}{\eta^{(\sigma-1)}} - \gamma} \tilde{f} \]

\[ - \tilde{f}_I > 0, \]

where

\[ \tilde{f}_I \equiv \frac{1}{\Gamma \Theta} \times f_I. \]

(O61) and (O65) show that we can solve for firms' optimal domestic share and import status with knowledge of \( \tilde{\varphi}^{(\sigma-1)}, \tilde{f}\) and \( \tilde{f}_I\) only. Thus, we can work with the joint distribution of \( (\varphi, \tilde{f})\) to match the moments of the joint distribution of domestic shares and value added. We can then back out the exogenous component of fixed costs \( f_I\) and \( f\) from \( \tilde{f}_I\) and \( \tilde{f}\) using the equilibrium variables \( S\) and \( P\) and (O64).

To solve for \( S\), we require the aggregate resource loss of fixed costs (see (O57)). To do so, note that

\[ l_{\mathcal{G}_i} = l_i(sD_i) \]

\[ = f_i \times \left( \frac{sD_i}{1 - sD_i} \right)^{\frac{1}{\eta^{(1-\gamma)}}} \left( \frac{1}{P} \right)^{\frac{1}{\eta^{(1-\gamma)}}} \left( \frac{1}{qD} \right)^{\frac{1}{\eta^{(1-\gamma)}}} \left( \frac{\beta_i}{1 - \beta_i} \right)^{\frac{1}{\eta^{(1-\gamma)}}} + f_I \]

\[ = \Gamma \Theta \left\{ \frac{\eta \gamma (\sigma - 1) \times \tilde{f}_i \times \frac{sD_i}{1 - sD_i}}{\eta^{(1-\gamma)}} \left( \frac{1}{1 - \beta_i} \right)^{\frac{1}{\eta^{(1-\gamma)}}} + \tilde{f}_I \right\}. \]

Hence,

\[ \int_i^N l_{\mathcal{G}_i} \, di = \Gamma \Theta \left\{ \eta \gamma (\sigma - 1) \times \tilde{f}_i \left( \frac{sD_i}{1 - sD_i} \right)^{\frac{1}{\eta^{(1-\gamma)}}} \left( \frac{\beta_i}{1 - \beta_i} \right)^{\frac{1}{\eta^{(1-\gamma)}}} + \tilde{f}_I \left[ sD_i \right] \right\}. \]

The key is now to argue that \( \Gamma \) is known given the calibration. If so, we can calculate \( \int_i^N l_{\mathcal{G}_i} \, di \) from (O67) given the calibrated \( \tilde{f}\) and \( \tilde{f}_I\) and parameters, as

\[ \int_i^N l_{\mathcal{G}_i} \, di = \Gamma \times \Theta \times \Delta, \]
where
\[ \Delta \equiv \eta \gamma (\sigma - 1) \times \int_{i}^{N} \hat{f}_{i} \left( \frac{s_{Di}}{1 - s_{Di}} \right) \frac{1}{\eta (1 - \eta)} \left( \frac{\beta_{i}}{1 - \beta_{i}} \right) \frac{\varepsilon + \frac{1}{n}}{\varepsilon + \frac{1}{n}} di \]

\[ + \int_{i}^{N} \hat{f}_{i} \left[ s_{Di} \right] di. \]

(O68)

Recall that (O64) and (O57) imply that
\[ \Gamma = \frac{S}{P(1 - \gamma)(1 - \sigma)} = \frac{1}{P(1 - \gamma)(1 - \sigma)} \left( L - \left( \int_{i}^{N} l_{\gamma} di \right) \right) \]

\[ = \frac{1}{P(1 - \gamma)(1 - \sigma)} \frac{\sigma}{(1 - \gamma)(\sigma - 1)} L - \frac{1}{P(1 - \gamma)(1 - \sigma)} \frac{\sigma}{(1 - \gamma)(\sigma - 1)} \Gamma \Theta \Delta. \]

Solving for \( \Gamma \) yields
\[ \Gamma = \frac{1}{1 + \frac{1}{P(1 - \gamma)(1 - \sigma)} \frac{\sigma}{(1 - \gamma)(\sigma - 1)} \Theta \Delta L}. \]

(O69)

As \( L \) is a normalization (see below), (O69) shows that \( \Gamma \) is fully determined as \( P \) can be evaluated from the calibrated data on domestic shares (see (O58) and (O59)). Hence,
\[ \int_{i}^{N} l_{\gamma} di = \Gamma \Theta \Delta = \frac{1}{P(1 - \gamma)(1 - \sigma)} \frac{\sigma}{(1 - \gamma)(\sigma - 1)} \Theta \Delta L. \]

This implies that
\[ \frac{L - \int_{i}^{N} l_{\gamma} di}{L} = \frac{1}{1 + \frac{1}{P(1 - \gamma)(1 - \sigma)} \frac{\sigma}{(1 - \gamma)(\sigma - 1)} \Theta \Delta}, \]

(O70)

so that \( L \) is indeed a normalization. Finally we only have to show that (O70) does not depend on \( q_{D} \), even though \( \Theta \) does (see (O63)). However, it can easily be shown that
\[ \Theta (\sigma - 1) = \gamma^{\sigma - 1} \frac{1}{\sigma}. \]

Hence, the quality of domestic varieties \( q_{D} \) and the foreign price level \( z \) can be normalized for the calibration.

The five models we consider fit in this framework as follows:

1) The **aggregate model** assumes that \( \beta_{i} = \beta \) and \( f_{i} = f_{I} = 0 \). Hence, \( \int_{i}^{N} l_{\gamma} di = 0 \) and \( s_{Di} = s_{D} \) can be solved from (O60) using that \( n = 1 \) (as all firms are importers and import from every country). The level of \( \beta \) is chosen to match the aggregate domestic share. The dispersion in produc-
tivity $\sigma_\varphi$ is chosen to match the dispersion in value added.

2) The *homogeneous bias model* assumes that $\beta_i = \beta$ and $f_i = 0 < f_I$. Hence, conditional on importing, we have that $s_{Di} = s_D$, which can be solved from (O60) using that $n = 1$. The required level $\tilde{f}_I$ in (O65) is chosen to match the share of importers. Given a distribution of productivity $[\tilde{\varphi}_i]$, we can then calculate $\Delta$ from (O68), $P$ from (O58) and (O59) and hence $\Gamma$ from (O69). This is sufficient to calculate welfare using (O70) and $P^{Aut}/P$.

3) The *heterogeneous bias model* assumes that $\beta_i$ varies across firms and $f_i = 0 < f_I$. As for the case with fixed costs, it is useful to consider a scaled version of the home-bias $\tilde{\beta}_i = \frac{\beta_i}{1 - \beta}$. In particular, (O60) shows that $s_D$ only depends on $\beta^* = \left(\tilde{\beta}\right)^{e} \left(\frac{1}{p}\right)^{e-1}$ (again, we have $n = 1$ as there are no fixed costs per country). Hence, we draw $(\tilde{\varphi}, \beta^*)$ from a joint log-normal distribution. Using (O60), this generates a joint distribution of $(\tilde{\varphi}_i, s_{Di})$. We can then calibrate $\tilde{f}_I$ from (O65) to match the share of importers. Like for the case of the homogeneous bias model, we can then use (O68), $P$ and (O69) to compute all equilibrium objects.

4) For the *heterogeneous fixed cost model*, we draw $(\tilde{\varphi}_i, \tilde{f}_i)$ from a joint log-normal distribution. Using the (O61), this implies a joint distribution of $(\tilde{\varphi}_i, s_{Di})$. We can then calibrate $\tilde{f}_I$ from (O65) to match the share of importers. As above, we can then use (O68), $P$ and (O69) to compute all equilibrium objects.

5) The *homogeneous fixed cost model*, is a special case of the heterogeneous fixed cost model where $\tilde{f}_i = \tilde{f}$. Hence, the procedure is exactly the same given a marginal distribution for $\tilde{\varphi}_i$. 

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