Procurement design with corruption
Online Appendix

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July 6, 2016

Section 2.3

First, we show that concavity of $B(m)$ is sufficiently for the optimal choice of $m$ to be zero.

**Lemma 1** For any IC, IR direct mechanism, $(p, q, m)$, there exists an IC, IR mechanism with $m(\theta) = 0 \forall \theta \in [\underline{\theta}, \bar{\theta}]$, so that $E[q(\theta) - m(\theta) - p(\theta)]$ is higher for the latter.

**Proof.** Assume $m(\theta) > 0$ for some value $\theta$, and consider a change in the mechanism so that $q'(\theta) = q(\theta) - m(\theta)$, $m'(\theta) = 0$, and $p'(\theta) = p(\theta) - B(m(\theta))$. The profits of type $\theta$ do not change. Also, a type $\theta'$ imitating type $\theta$ could achieve

$$p(\theta) - \min_{z \in [0,q(\theta)]} \{ C(q(\theta) - z; \theta') + B(z) \},$$

with the original mechanism, whereas with the modified mechanism she can obtain

$$p'(\theta) - \min_{z \in [0,q'(\theta)]} \{ C(q'(\theta) - z; \theta') + B(z) \}$$

$$= p(\theta) - B(m(\theta)) - \min_{z \in [0,q(\theta) - m(\theta)]} \{ C(q(\theta) - m(\theta) - z; \theta') + B(z) \}$$

$$= p(\theta) - \min_{z \in [0,q(\theta) - m(\theta)]} C(q(\theta) - m(\theta) - z; \theta') + B(z) + B(m(\theta))$$

$$= p(\theta) - \min_{h \in [m(\theta),q(\theta)]} C(q(\theta) - h; \theta') + B(h - m(\theta)) + B(m(\theta)).$$

where we have used the change of variable $h = z + m(\theta)$. This expression is smaller since $B$ is concave and the choice set of $h$ is smaller than the choice set of $z$ in the
original mechanism. The profits of \( \theta' \) imitating any other type have not changed, and the profits of \( \theta \) imitating any other type are not larger.

Next, we prove the claim that the results in Proposition 3 extend to the concave case, provided assumptions A1, A2, and A3 are satisfied.

**Claim 2** Under concavity of \( B(m) \), A1, A2, and A3, if \( q^{NB}(\theta) \) violates (12) then there exist \( \theta^a \) and \( \theta^c \), with \( \bar{\theta} < \theta^a \leq \theta^c < \bar{\theta} \) such that at the optimal mechanism; (i) \( q(\theta) = 0 \) if \( \theta > \theta^c \); (ii) \( q(\theta) = q^{NB}(\theta) \) if \( \theta \in (\theta^a, \theta^c) \); and (iii) \( q(\theta) = q^{NB}(\theta^a) \) if \( \theta < \theta^a \).

**Proof.** Given an exogenous \( q(\bar{\theta}) \), the result is proved exactly as Proposition 3. Thus, we need only show that the sponsor’s surplus is maximized for \( q(\bar{\theta}) < q^{NB}(\bar{\theta}) \). The sponsor’s objective is still given by (22), and so its derivative at \( q^{NB}(\bar{\theta}) \) is also given by (23). Then, we only need show that \( \frac{d\theta^c}{dq(\bar{\theta})} < 0 \). Totally differentiating the equivalent now to (21),

\[
B(q(\bar{\theta})) - C(q(\bar{\theta}); \bar{\theta}) - \int_{\theta^a}^{\theta^c} C_\theta(q(\bar{\theta}); z)dz - \int_{\theta^a}^{\theta^c} C_\theta(q^{NB}(z); z)dz = 0,
\]

we have

\[
\frac{d\theta^c}{dq(\bar{\theta})} = \frac{B'(q(\bar{\theta})) - C_q(q(\bar{\theta}); \bar{\theta}) - \int_{\theta^a}^{\theta^c} C_{\theta q}(q(\bar{\theta}); z)dz}{C_\theta(q^{NB}(\theta^c); \theta^c)} < 0,
\]

where the inequality follows from A2 and the fact that \( C_{\theta q}(q; \theta) > 0 \). ■