A Revelation Principle

This section formulates the model as a game between the individuals and the government, and establishes that the no-reform constraint is satisfied in the government’s optimal equilibrium if $H(0) \geq \alpha$ and either of the following two conditions holds:

1. The reform threat is full equalization: $c^R_2(\theta) = RK$ for all $\theta$.

2. $H$ is the step function $H(x) = \mathbb{1}\{x \geq 0\}$ and the government is uncertain as to the reform threat and evaluates policies using a maxmin criterion over reform threats, where the set of possible reform threats contains only progressive reforms and contains full equalization.

Individuals’ and the government’s preferences are as in the text. The game will involve tax schedules $(T_y, T_k)$, which importantly are required to satisfy the resource constraint whatever production decisions individuals make. Formally, let $\mathcal{H}$ be the set of probability distributions on $\mathbb{R}_+$, corresponding to possible distributions of output or capital. A labor tax schedule $T_y$ is a map from $\mathbb{R}_+ \times \mathcal{H} \to \mathbb{R}$ such that $\int_{y \in \mathbb{R}_+} T_y(y, H) \ dH \geq 0$ for all $H \in \mathcal{H}$. A capital tax schedule $T_k$ is a map from $\mathbb{R}_+ \times \mathcal{H} \to \mathbb{R}$ such that $\int_{k \in \mathbb{R}_+} T_k(Rk, H) \ dH \geq 0$ for all $H \in \mathcal{H}$. A reform threat $T^R_2$ is now a mapping from status quo capital tax schedules $T_k$ to reform capital tax schedules $T^R_k$. The game is as follows:

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Period 1:

1. Taking as given an exogenous reform threat $T^R_2$, the government proposes a tax schedule $(T_y, T_k)$. The labor tax schedule $T_y$ is implemented (see below), and the period 2 status quo capital tax schedule is set to $T_k$.

2. Individuals produce, pay labor taxes, and consume in period 1, as follows:

   (a) Individuals simultaneously choose production $y$.

   (b) Given the resulting distribution of output $H^y$, an individual who produced $y$ pays labor tax $T_y(y, H^y)$.

   (c) An individual with after-tax income $y - T_y(y, H^y)$ chooses period 1 consumption $c_1 \in [0, y - T_y(y, H^y)]$. This leaves her with capital $k = y - T_y(y, H^y) - c_1 \geq 0$. Denote the resulting distribution of capital by $H^k$.

Period 2:

1. The reform threat $T^R_2$ combined with the status quo capital tax schedule $T_k$ generate a reform capital tax schedule $T^R_k$. Individuals decide whether to support the period 2 status quo capital tax schedule $T_k$ or the reform capital tax schedule $T^R_k$.

2. If at least $\alpha$ individuals support the status quo, then the capital tax schedule $T_k$ is implemented, meaning that an individual with capital $k$ consumes $Rk - T_k(Rk, H^k)$. If fewer than $\alpha$ individuals support the status quo, then the reform capital tax schedule $T^R_k$ is implemented, meaning that an individual with capital $k$ consumes $Rk - T^R_k(Rk, H^k)$.

A (symmetric, pure strategy, subgame perfect) equilibrium consists of a proposed tax schedule $(T_y, T_k)$, and production, consumption, and political support strategies $Y : T_y \times T_k \times \Theta \rightarrow y, C : T_y \times T_k \times \Theta \rightarrow c_1, S : T_k \times \mathbb{R}_+ \rightarrow \{0, 1\}$ (where $T_y$ and $T_k$ are the sets of possible tax schedules $T_y$ and $T_k$, and $S$ is a political support strategy), such that:

1. $(T_y, T_k)$ maximizes the government’s payoff given $(Y, C, S)$.

2. $(Y, C)$ maximizes the utility of each type $\theta$ given $(T_y, T_k)$ and given that other individuals follow $(Y, C, S)$.

3. $S$ is “sincere,” in that a type $\theta$ individual supports the period 2 status quo if and only if $c_2^{SQ}(\theta) + \varepsilon^{SQ} \geq c_2^R(\theta) + \varepsilon^R$. 

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4. \( C(T_y, T_k, \theta) \leq Y(T_y, T_k, \theta) - T_y \left( Y(T_y, T_k, \theta), H^Y(T_y, T_k, \theta) \right) \) for all \( T_y \in T_y, T_k \in T_k, \theta \in \Theta \) (i.e., individuals do not consume more than their after-tax incomes in period 1).

An allocation \((c_1 : \Theta \to \mathbb{R}_+, c_2 : \Theta \to \mathbb{R}_+, y : \Theta \to \mathbb{R}_+)\) is a mapping from types to period 1 consumption, period 2 consumption, and production. An allocation is feasible if it satisfies the intertemporal resource constraint \( \int (c_1(\theta) + c_2(\theta) / R) dF \leq \int y(\theta) dF \). An allocation is implementable if there exists an equilibrium \((T_y, T_k, Y, C, S)\) such that \( Y(T_y, T_k, \theta) = y(\theta) \forall \theta \in \Theta, C(T_y, T_k, \theta) = \alpha(\theta) \forall \theta \in \Theta, \) and \( D(T_y, T_k, Y, C, \theta) = c_2(\theta) \forall \theta \in \Theta, \) where \( D(T_y, T_k, Y, C, \theta) \) is the period 2 consumption of a type \( \theta \) individual in equilibrium \((T_y, T_k, Y, C, S)\).

In the text, attention is restricted to feasible, incentive-compatible direct mechanisms that satisfy the no-reform constraint \( \int H(c_2(\theta), R) dF \geq a \). With fully equalizing reforms, this approach is justified by the following result.\(^1\)

**Proposition 1** (Revelation Principle). With fully equalizing reforms, if Assumption 1 holds then every implementable allocation is feasible, incentive-compatible, and satisfies the no-reform constraint \((NR")\).

**Proof.** Showing that any implementable allocation is feasible is a simple accounting exercise. Any implementable allocation is incentive-compatible as a direct mechanism, by the usual revelation principle argument (whether or not it is implemented in an equilibrium in which a reform occurs): a unilateral deviation does not affect the implemented tax schedules or the resulting distributions \( G^y \) or \( G^k \), so if \( y(\theta) \) and \( c_1(\theta) \) are the optimal production and period 1 consumption choices of a type \( \theta \) individual given others’ behavior, then in particular she prefers \((c_1(\theta), c_2(\theta), y(\theta))\) to \((c_1(\theta'), c_2(\theta'), y(\theta'))\) for all \( \theta' \in \Theta \). Thus, it suffices to show that every implementable allocation satisfies the no-reform constraint \((NR")\) when viewed as a direct mechanism.

To see this, note that if an allocation is implemented in an equilibrium in which no period 2 reform occurs, then it satisfies the no-reform constraint when viewed as a direct mechanism, as the condition for no period 2 reform to occur in equilibrium is precisely the no-reform constraint for the corresponding direct mechanism. In addition, if an allocation \((c_1, c_2, y)\) is implemented in an equilibrium in which a period 2 reform does occur, then \( c \) is constant. In this case, Assumption 1 implies that \((c_1, c_2, y)\) satisfies the no-reform constraint when viewed as a direct mechanism. \(\square\)

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\(^1\)The converse also holds for monotone allocations, as shown in Section C.2.
With more general reforms, it is no longer true that every implementable allocation satisfies the no-reform constraint, as discussed in Section II.D of the paper. However, suppose now that there is a set of possible reform threats $T_k^R$, and that the government must propose a tax schedule $(T_y, T_k)$ without knowing which reform threat will arise.\footnote{In contrast, we assume that citizens learn the reform threat after the government proposes a tax schedule but before they make their period 1 production and consumption decisions.} We say that a tax schedule $(T_y, T_k)$ robustly attains value $V$ if, for every reform threat $T^R_2 \in T_k^R$, when the government proposes tax schedule $(T^*_y, T^*_k)$ and reform threat $T^R_2$ realizes, there exists a continuation equilibrium in which $\int v(\theta) \, dG \geq V$. Let $V^*$ be the supremum over values $V$ that can be robustly attained by any tax schedule. Finally, say that a tax schedule $(T^*_y, T^*_k)$ is maxmin optimal if it robustly attains value $V^*$.

Furthermore, assume that $T_k^R$ contains only progressive reforms and contains full equalization. We then obtain the following result:

**Proposition 2** (Maxmin Optimality). If $H$ is the step function $H(x) = \mathbb{I} \{x \geq 0\}$ then there exists a maxmin optimal tax schedule and a continuation equilibrium (independent of the realized reform threat $T_2^R$) such that the resulting allocation is feasible, incentive-compatible, and satisfies the no-reform constraint ($NR'$).

*Proof.* Let $(c^*_1, c^*_2, y^*)$ denote a solution to the government’s problem (which exists by Lemma 2), and let $V^{**}$ denote the corresponding value. By Proposition 1 (which applies as Assumption 1 is always satisfied when $H$ is the step function), when the reform threat is known to be full equalization, the value of $\int v(\theta) \, dG$ in any equilibrium is at most $V^{**}$. Hence, $V^* \leq V^{**}$. In addition, again by Proposition 1, when the reform threat is known to be full equalization, there exists an optimal tax schedule $(T^*_y, T^*_k)$ and a continuation equilibrium that leads to allocation $(c^*_1, c^*_2, y^*)$ and is feasible, incentive-compatible, and satisfies ($NR'$). As the condition for a reform to occur is the same under any progressive reform when $H$ is the step function, if the government announces tax schedule $(T^*_y, T^*_k)$ then this continuation strategy profile remains an equilibrium for any realized reform threat $T_2^R \in T_k^R$. In particular, this equilibrium yields value $V^{**}$ for every reform threat $T^R_2 \in T_k^R$, which by the definition of $V^*$ implies that $V^* \geq V^{**}$. Hence, $V^* = V^{**}$, and the tax schedule $(T^*_y, T^*_k)$ is maxmin optimal. \qed

**B Monotone Solution to the Government’s Problem**

The following inequality is due to Lorentz (1953).\footnote{Lorentz assumed that $\mu$ is Lebesgue measure, but the generalization to arbitrary non-atomic measures is straightforward. See Burchard and Hajaiej (2006) for further generalizations and discussion.}
Lemma 1. Let $\mu$ be a non-atomic measure on a set $\Theta \subseteq \mathbb{R}$, let $a$ and $b$ be measurable functions from $\Theta$ to $\mathbb{R}_+$, and let $w : \mathbb{R}_+^2 \rightarrow \mathbb{R}$ be continuous with increasing differences. Then

$$\int_{\Theta} w(a(\theta), b(\theta)) \, d\mu \leq \int_{\Theta} w(\bar{a}(\theta), \bar{b}(\theta)) \, d\mu,$$

where $\bar{a}$ and $\bar{b}$ are the non-decreasing rearrangements of $a$ and $b$, defined by

$$\bar{a}(\theta) = \inf \{ a_0 \in \mathbb{R}_+ : \mu(\{ \theta' : a(\theta') \leq a_0 \}) > \mu((\infty, \theta]) \},$$

$$\bar{b}(\theta) = \inf \{ b_0 \in \mathbb{R}_+ : \mu(\{ \theta' : b(\theta') \leq b_0 \}) > \mu((\infty, \theta]) \}.$$

We use Lemma 1 to establish the existence of a monotone solution to the government’s problem.

Lemma 2. There exists a solution to the government’s problem in which $c_2(\theta)$ is non-decreasing.

Proof. Recall that the government’s problem is $\max_{c_1, c_2, y} \int v(\theta) \, dG$ subject to (RC), (IC), and (NR'). That some solution $(c_1^*, c_2^*, y^*)$ to this problem exists follows because the objective is continuous and the constraint set is closed and can be bounded using the Inada conditions on $u$.\(^4\) Given such a solution, let $U^*(\theta) = u(c_1^*(\theta)) + \beta u(c_2^*(\theta))$, let $\bar{c}_2$ be the non-decreasing rearrangement of $c_2^*$, and consider the allocation given by

$$(\tilde{c}_1(\theta), \tilde{c}_2(\theta), y^*(\theta)) = \left( u^{-1}(U^*(\theta) - \beta u(c_2^*(\theta))), \bar{c}_2(\theta), y^*(\theta) \right)$$

for all $\theta$.

This alternative allocation leaves consumption utility $U(\theta)$ and production $y(\theta)$ unchanged for every type, and therefore yields the same value as $(c_1^*, c_2^*, y^*)$ and satisfies (IC). As $\bar{c}_2$ and $c_2^*$ are equimeasurable, it also satisfies (NR’). Finally, note that

$$\int \left( u^{-1}(U(\theta) - \beta u(c_2(\theta))) + \frac{1}{R} c_2(\theta) \right) \, d\mu$$

has decreasing differences in $\theta$ and $c_2$ for any non-decreasing $U(\theta)$, as $u$ is concave. As

\(^4\)This follows by standard arguments if the Pareto weights are bounded away from zero. With zero Pareto weights, one might worry that the government would want to give unboundedly negative utility to some types. However, the Inada conditions imply an upper bound on $y(\theta)$ for the government’s most-favored type $\theta$, and (IC) then implies that the utility of any type $\theta'$ can be lower than type $\theta$’s utility by at most $h(y(\theta), \theta') - h(y(\theta), \theta)$, a bounded number.
\( U^\ast \) is non-decreasing by (IC), Lemma 1 implies that

\[
\int \left( c_1^\ast (\theta) + \frac{1}{R} c_2^\ast (\theta) \right) dF = \int \left( u^{-1} (U^\ast (\theta) - \beta u (c_2^\ast (\theta))) + \frac{1}{R} c_2^\ast (\theta) \right) dF \\
\geq \int \left( u^{-1} (\bar{U}^\ast (\theta) - \beta u (\bar{c}_2^\ast (\theta))) + \frac{1}{R} \bar{c}_2^\ast (\theta) \right) dF \\
= \int \left( u^{-1} (U^\ast (\theta) - \beta u (\bar{c}_2^\ast (\theta))) + \frac{1}{R} \bar{c}_2^\ast (\theta) \right) dF \\
= \int \left( \tilde{c}_1 (\theta) + \frac{1}{R} \tilde{c}_2 (\theta) \right) dF,
\]

where \( \bar{U}^\ast \) is the non-decreasing rearrangement of \( U^\ast \), which equals \( U^\ast \) almost everywhere as \( U^\ast \) is non-decreasing. Thus, the alternative allocation \((\tilde{c}_1(\theta), \tilde{c}_2(\theta), y^\ast(\theta))\) also satisfies (RC), and it therefore constitutes a monotone solution to the government’s problem.

C  Calibration and Tax Implementation

This appendix describes the calibration underlying the numerical results in Section IV of the paper and relates the capital and labor wedges to marginal tax rates in an explicit tax implementation.

C.1 Calibration

We consider iso-elastic preferences of the form \( u(c) = c^{1-\sigma} / (1 - \sigma) \) and \( h(l) = \gamma l^{1+1/\varepsilon} / (1 + 1/\varepsilon) \) where \( l = y/\theta \), so the Frisch elasticity of labor supply is constant and given by \( \varepsilon \). We set \( \varepsilon = 1 \), consistent with evidence in Kimball and Shapiro (2010) and Erosa, Fuster and Kambourov (2011). We interpret a model period as \( T = 30 \) years and accordingly set \( \beta = 0.95^{30} \) and \( R = 1/\beta \) (so the optimum under full commitment involves consumption smoothing with \( c_1(\theta) = c_2(\theta) \)). For the skill distribution \( F \), we follow Mankiw, Weinzierl and Yagan (2009), who fit a lognormal distribution to the empirical wage distribution from the 2007 Current Population Survey and append a Pareto distribution for the upper tail of wages to obtain asymptotic marginal tax rates as in Saez (2001). We extend their numerical procedure for a static Mirrlees model to our dynamic setting in order to compute both \( \tau_l(\theta) \) and \( \tau_k(\theta) \) and follow them in setting \( \sigma = 1.5 \) and \( \gamma = 2.55 \). We assume that the distribution of taste shocks \( H \) is normal with mean 0 and standard deviation 1 (corresponding to just over 10% of mean utility).
C.2 Tax Implementation

Our analysis in the main text has implicitly considered direct mechanisms, where the government allocates \( c_1(\theta), c_2(\theta), \) and \( y(\theta) \) conditional on individual reports about \( \theta \), taking into account technological, incentive compatibility, and political sustainability constraints. It is straightforward to show that these allocations can alternatively—and more realistically—be implemented through a tax system where each individual is confronted with the same budget set and picks her preferred allocation within this set. In particular, with a non-linear labor income tax \( T_y \) and a non-linear capital income tax \( T_k \), individuals are faced with the budget constraint \( c_1 + k \leq y - T_y(y) \) in period 1 and \( c_2 \leq Rk - T_k(Rk) \) in period 2 and choose \( c_1, c_2, y, k \) to maximize \( u(c_1) + \beta u(c_2) - h(y, \theta) \) subject to these two constraints.

By Proposition 3 of Farhi et al. (2012), any incentive compatible allocation \( (c_1(\theta), c_2(\theta), y(\theta)) \) that is non-decreasing in \( \theta \) can be implemented using such a tax system. Since we show that \( c_2(\theta) \) is non-decreasing in an optimal allocation and \( y(\theta) \) is non-decreasing by incentive compatibility, their result can be applied to our framework.\(^5\) Moreover, the first-order conditions from the above utility-maximization problem imply

\[
u'(c_1(\theta)) = \beta R(1 - T'_k(Rk(\theta)))u'(c_2(\theta))\]

for all \( \theta \) whenever \( T_k \) is differentiable, so the wedge \( \tau_k(\theta) \) defined in equation (1) of the paper and characterized throughout this paper coincides with the actual marginal capital income tax rate \( T'_k(Rk(\theta)) \) faced by individuals of type \( \theta \) in this implementation.\(^6\)

D Extensions

This appendix briefly presents two extensions of the model that serve as robustness checks on our results. Section D.1 considers a setting where the full commitment benchmark involves non-zero capital taxes. Section D.2 extends our two-period model to overlapping generations.

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\(^5\)The statement of Proposition 3 in Farhi et al. (2012) also requires \( c_1(\theta) \) to be non-decreasing, which holds at the optimum in their model but may or may not hold in ours. However, inspecting their proof reveals that this condition is not actually needed for the result.

\(^6\)If \( T_k(Rk(\theta)) \) is not differentiable because there is pooling at consumption level \( c_2(\theta) \) (so \( T_k \) has a convex kink), \( \tau_k(\theta) \) is still bounded between the (well-defined) left- and right-derivatives of \( T_k \).
D.1 Heterogeneity in Savings Propensity

We have derived our results in a dynamic Mirrlees model where, in the absence of political constraints, optimal capital taxes are zero. It is natural to ask whether and how they extend to settings where the Atkinson-Stiglitz theorem does not apply to the full commitment benchmark. For example, this is the case when individuals differ in their propensity to save rather than their ability, as emphasized recently by Farhi and Werning (2013) and Piketty and Saez (2013). We briefly demonstrate that, in this case, our results extend in the sense that the pattern of capital taxes found in Section III.A of the paper can be interpreted as the optimal addition to the full commitment capital tax benchmark.

To this purpose, we consider a modification of our basic two-period model without labor supply, where individuals simply consume in both periods and all start out with the same initial wealth $K_1$. Private heterogeneity enters exclusively in the form of savings propensity, so that the wealthy individuals in period 2 will be those who are more patient. Formally, preferences are $u(c_1) + \beta u(c_2)$ and $\beta$ is distributed according to $F$ with support contained in $(0, 1)$. An allocation $(c_1(\beta), c_2(\beta))$ is resource feasible if

$$\int c_1(\beta) dF + K_2 \leq RK_1 \quad \text{and} \quad \int c_2(\theta) dF \leq RK_2$$

for some $K_2 \geq 0$ and is incentive compatible if

$$u(c_1(\beta)) + \beta u(c_2(\beta)) \leq u(c_1(\beta')) + \beta u(c_2(\beta')) \quad \forall \beta, \beta'.$$  

Considering the case with $F = G$ for simplicity, the government solves in period 1

$$\max_{c_1, c_2, K_2} \int [u(c_1(\beta)) + \beta u(c_2(\beta))] dF$$

s.t. (1), (2), and

$$\int H(u(c_2(\beta)) - u(RK_2)) dF \geq \alpha.$$  

It is obvious that implicit capital taxes $\tau_k(\beta)$ (defined as in equation (1) of the paper) are no longer necessarily zero even without the political constraint (3), because these wedges are the only way to achieve redistribution across individuals with different savings propensities $\beta$ in period 1. However, the following useful decomposition of overall marginal capital taxes in any solution without bunching can be established using standard steps:

$$\frac{\tau_k(\beta)}{1 - \tau_k(\beta)} = \chi(\beta) - \frac{R\eta}{\lambda} [H'(u(c_2(\beta)) - u(RK_2))u'(c_2(\theta)) - H_K],$$

(4)
where $H_K$ is defined as in equation (3) of the paper and

$$\chi(\beta) = \frac{R u'(c_2(\beta))}{f(\beta)} \int_{1}^{\beta} \left( \frac{1}{u'(c_1(\beta))} - \frac{1}{\lambda} \right) dF$$

is the formula for the optimal $\tau_k/(1 - \tau_k)$ under full commitment, with

$$\lambda = \left( \int_{0}^{1} \frac{1}{u'(c_1(\beta))} dF \right)^{-1}.$$

As can be seen from (4), exactly the same formula for the optimal marginal capital tax as in Lemma 1 of the paper appears, with the only difference that it gets added to the (no longer necessarily zero) benchmark marginal capital tax $\chi(\beta)$ that is present even in the absence of political constraints.\(^7\) In this sense, our results extend in a transparent way to situations where the Atkinson-Stiglitz conditions are not met under full commitment.

### D.2 Overlapping Generations

There is also a straightforward extension of our two-period model to an infinite-horizon overlapping generations (OLG) setting. In particular, consider the model from Section I of the paper, except that a new generation is born in every period $t = 1, 2, \ldots$. As before, individuals live for two periods, produce only when young, and consume in both periods. Each period begins with a capital stock $R K_t$ and a status quo consumption schedule for the old $\tilde{c}_O^t$. The timing in each period $t$ is as follows:

1. Old individuals decide whether to support the status quo.

2. The government chooses a vector $\left( y_t, c_Y^t, \tilde{c}_O^{t+1} \right)$ corresponding to production and consumption for the period $t$ young and status quo consumption for the period $t + 1$ old, subject to the resource constraints

$$\int c_Y^t(\theta_t)dF + K_{t+1} = \int y_t(\theta_t)dF \quad \text{and} \quad \int \tilde{c}_O^{t+1}(\theta_t)dF = R K_{t+1}. \quad (5)$$

If fewer than $\alpha$ old individuals support the status quo, a reform consumption schedule $\tilde{c}_O^t$ for the period $t$ old is implemented, which must satisfy the resource constraint

$$\int \tilde{c}_O^t(\theta_{t-1})dF \leq R K_t. \quad (6)$$

\(^7\)Of course, this interpretation of an additive adjustment to the capital tax under full commitment holds only in terms of the formula (4), since both $c_1$ and $c_2$ change when we introduce political constraints.
Otherwise, consumption for the period $t$ old is given by the status quo $\tilde{c}_t^O$.

The interpretation is that, in each period $t$, the government sets policies for the currently young generation, namely a labor income tax for the young in $t$ and a capital tax for when they will be old in $t+1$, which will become the status quo for the next period. If there is enough support among the currently old, it can also reform their status quo capital tax (which was set in the preceding period) by redistributing their wealth. Note that the resource constraints (5) and (6) rule out intergenerational transfers; we briefly comment on this below.\footnote{Given this, the young generation is indifferent between supporting the status quo or the reform in each period, and we can assume that they always support the status quo. Hence, assuming that only the old generation’s political support matters amounts to a renormalization of $\alpha$ and is thus without loss of generality.}

A Markov equilibrium of this model is one where individuals born in period $t$ condition their behavior when young (i.e., production and consumption) only on $(y_t, c_t^Y)$ and $K_t$ and, when old, condition their support for the status quo only on $\tilde{c}_{t+1}^O$ and $K_{t+1}$. All of our results for the two-period model of the paper immediately extend to the Markov equilibria of this OLG model.

The model is more complicated when intergenerational transfers and non-Markov equilibria are allowed. In non-Markov equilibria, the government could be punished for implementing a reform, or for setting particular allocations for the young, which provides an additional source of commitment power. Such history-dependence could also arise with intergenerational transfers, as in this case the period $t$-young would also have to condition their behavior on the status quo $\tilde{c}_t^O$, which was set by the government in period $t-1$. In particular, it is not clear that a suitable definition of Markov equilibrium exists in this case.

**Online Appendix References**


