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Putting Ricardo to Work

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1 Mathematical Appendix

1.1 Derivations for the Probabilistic Model

Let's begin with Eaton and Kortum (2002), who express the Ricardian model in terms of labor efficiency $z_i(j)$ for workers in country i to produce good j. There we assume that these efficiencies are drawn from the Fréchet distribution:

$$\Pr\left[z_i(j) \leq z\right] = e^{-T_i z^{-\theta}}.$$

In the current paper we work with labor requirements $a_i(j) = 1/z_i(j)$, whose distribution is thus

$$\begin{aligned} \Pr[a_i(j) &\leq a] = \Pr[z_i(j) \geq 1/a] \\ &= 1 - \Pr[z_i(j) \leq 1/a] \\ &= 1 - e^{-(A_i a)^{\theta}}, \end{aligned}$$

where $A_i = T_i^{1/\theta}$.[1]

[1] To clarify the role of the two parameters, define:

$$x_i(j) = \left[A_i a_i(j)\right]^{\theta},$$

which normalizes the worker requirement $a_i(j)$ for each good j in country i by the overall productivity in country i. In that case $x_i(j)$ has a unit exponential distribution (i.e., an exponential distribution with parameter 1) so that:

$$\Pr[x_i(j) \leq x] = 1 - e^{-x}.$$

We can then write the labor requirement (in logs) as:

$$\ln a_i(j) = \frac{1}{\theta} \ln x_i(j) - \ln A_i.$$

Thus, for the log of the labor requirement $\ln a_i(j)$, $\ln A_i$ is a mean shifter and $1/\theta$ a variance shifter. Note the analogy to the Normal distribution in which a family of distributions

We can use the distribution of labor requirements to derive the distribution of the cost $c_{ni}(j) = w_i d_{ni} a_i(j)$ of good j if supplied by i to n:

$$\Pr[c_{ni}(j) \leq c] = 1 - e^{-(A_{ni}c)^{\theta}},$$

where $A_{ni} = A_i/(w_i d_{ni})$. Taking the minimum cost of delivery to n, the distribution of prices is given by:

$$\begin{aligned} F(p) &= \Pr[p_n(j) \leq p] \\ &= 1 - \prod_i \Pr[c_{ni}(j) > p] \\ &= 1 - e^{-\left(\bar{A}_n p\right)^{\theta}}, \end{aligned}$$

where:

$$\bar{A}_n = \left[\sum_{i=1}^{I} (A_{ni})^{\theta}\right]^{1/\theta}. \tag{1a}$$

Integrating over the distribution of prices gives the price index:

$$p_n = \exp\left(\int_0^{\infty} \ln(p)\, dF(p)\right) = \frac{\gamma}{\bar{A}_n}, \tag{2}$$

where $\gamma = \exp(-\varepsilon/\theta)$ and $\varepsilon = 0.5772...$ is Euler's constant.

The probability π_{ni} that a particular country i is the lowest cost source of a good in country n is:

$$\pi_{ni} = \left(\frac{A_{ni}}{\bar{A}_n}\right)^{\theta}. \tag{3}$$

With a unit continuum of goods and Cobb Douglas preferences, this probability becomes the trade share, the fraction of country n's expenditure devoted to goods produced in country i.

Evaluating (3) at $n = i$, applying (2), and rearranging, we get the simple relationship between the real wage, productivity, and the home share π_{ii}, described in the text:

$$\frac{w_i}{p_i} = \gamma^{-1} A_i \pi_{ii}^{-1/\theta}. \tag{4}$$

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emerge by applying an additive shifter μ and multiplicative shifter σ to a standard Normal random variable (with mean zero and variance 1).

The model is closed with conditions for labor market equilibrium. Denoting the labor endowment in country i by L_i and the trade deficit in country n by D_n, these conditions are

$$w_i L_i = \sum_{n=1}^{I} \pi_{ni}(w_n L_n + D_n). \tag{5}$$

1.2 A Convenient Special Case

In general, solving these equations forces us to put numbers into a computer. But the special case in which there are no trade barriers, that is, with $d_{ni} = 1$ for all i and n, delivers a simple and insightful solution (even though it takes us very far from reality).

In this case we can write $A_{ni} = A_i/w_i$ so that $\pi_{ni} = \pi_i$, that is, country i has the same share in every destination n including itself. As we see below, π_i turns out to be not only country i's share in each individual market n but also its share in world income Y.

Looking at (5) we can factor out π_i from the sum on the right-hand side. That sum is the same for any other source. Doing the same exercise for some other source i' and taking the ratio, the sum cancels, leaving us with:

$$\frac{Y_{i'}}{Y_i} = \frac{w_{i'} L_{i'}}{w_i L_i} = \frac{\pi_{i'}}{\pi_i} = \left(\frac{A_{i'}/w_{i'}}{A_i/w_i}\right)^{\theta}. \tag{6}$$

the relative GDP of countries i' and i. Solving for the wage ratio:

$$\frac{w_i'}{w_i} = \left(\frac{A_{i'}^{\theta}/L_{i'}}{A_i^{\theta}/L_i}\right)^{1/(1+\theta)}.$$

To get the share of country i in world income Y, sum (6) across all sources i' giving us world GDP Y in the numerator. Inverting gives us:

$$\pi_i = \frac{Y_i}{Y} = \frac{(A_i L_i)^{\theta/(1+\theta)}}{\sum_{n=1}^{I} (A_n L_n)^{\theta/(1+\theta)}}.$$

Substituting this expression for the home share into (4), the real wage is thus:

$$\frac{w_i}{p} = \gamma^{-1} A_i^{\theta/(1+\theta)} \left[\sum_{k=1}^{I} \left(\frac{A_k L_k}{L_i}\right)^{\theta/(1+\theta)}\right]^{1/\theta}. \tag{7}$$

1.3 Derivations for the Quantitative Model

The model used in the quantitative applications is augmented in several ways. First, we include intermediate inputs in production. The price index for these intermediates is p_i and their share in production is $1-\beta$ so that

$$A_{ni} = \frac{A_i}{w_i^{\beta} p_i^{1-\beta} d_{ni}}.$$

Second, we include non-manufactured services. Their share in final spending is $1-\alpha$ so that (5) becomes

$$\alpha \left(w_i L_i + D_i \right) - D_i^M = \sum_{n=1}^{I} \pi_{ni} \left[\alpha (w_n L_n + D_n) - (1-\beta) D_n^M \right],$$

where D_i^M is country i's trade deficit in manufactures. While we do not model trade in services, we acknowledge trade imbalances outside of manufacturing, so that D_i^M may differ from D_i.

We can express the equations of the model in terms of changes in the endogenous variables resulting from a counterfactual experiment. Substituting (1a) into (2), exponentiating, and then substituting in (3) we get:

$$\hat{p}_n^{-\theta} = \sum_{k=1}^{N} \pi_{nk} \left(\frac{\hat{A}_k}{\hat{w}_k^{\beta} \hat{p}_k^{(1-\beta)} \hat{d}_{nk}} \right)^{\theta}. \tag{8}$$

By a similar set of steps, and also using $Y_i = w_i L_i$ we can write the labor market equilibrium conditions as

$$\begin{aligned} & \alpha \left(\hat{w}_i Y_i + D_i^{'} \right) - D_i^{M'} \\ = & \sum_{n=1}^{N} \pi_{ni} \left(\frac{\hat{A}_i \hat{p}_n}{\hat{w}_i^{\beta} \hat{p}_i^{(1-\beta)} \hat{d}_{ni}} \right)^{\theta} \left[\alpha \left(\hat{w}_n Y_n + D_n^{'} \right) - (1-\beta) D_n^{M'} \right]. \end{aligned} \tag{9}$$

These two sets of equations, (8) and (9), are what we solve for when conducting a counterfactual experiment. In these equations, the π_{nk} are calibrated to data on trade shares and the Y_n to data on GDP.

We calculate the change in a country's price level (including the prices of services) as

$$\hat{P}_i = \hat{p}_i^{\alpha} \hat{w}_i^{1-\alpha}.$$

For counterfactuals in which trade deficits remain, we keep them at a constant value in terms of this overall price level. The real wage in our counterfactual results is defined as

$$\frac{\hat{w}_i}{\hat{P}_i} = \left(\frac{\hat{w}_i}{\hat{p}_i}\right)^{\alpha}.$$

In the tables and figures we express the change in any variable x in percentage terms: $100 \times (\hat{x} - 1)$.