Competition with Exclusive Contracts and Market-Share Discounts

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Online Appendix

Proof of Proposition 4. Given its rival's strategy, firm i behaves like a monopolist facing a buyer with a suitably defined indirect utility function and reservation utility. The reservation utility is obtained when $q_i = 0$. In this case, the buyer does not pay the shopping cost and so obtains

$$v(0,\theta) = \max_x \left[u(0,x,\theta) - \max\left\{0, P^*(x) - P^*\left[q^*(\tilde{\theta}_z)\right]\right\}\right].$$

The indirect utility function is the maximum utility buyer θ can obtain by purchasing $q_i > 0$ and then trading optimally with firm $-i$. We must distinguish between two cases. If the buyer prefers not to buy from firm $-i$, his indirect utility function is simply $v(q,\theta) = u(q,0,\theta)$. Notice that this can never exceed $v(0,\theta)$ if $q \leq q^*(\tilde{\theta}_z)$. If instead the buyer purchases a positive quantity of good $-i$, his indirect utility function is

$$v(q,\theta) = \max_{x>0} \left[u(q,x,\theta) - \max\left\{0, P^*(x) - P^*\left[q^*(\tilde{\theta}_z)\right]\right\} - z\right].$$

For future reference, we notice that

$$v(q^*(\tilde{\theta}_z),\tilde{\theta}_z) = u\left[q^*(\tilde{\theta}_z), q^*(\tilde{\theta}_z), \tilde{\theta}_z\right] - z$$

since $q^*(\tilde{\theta}_z)$ is offered at zero price. By definition, $u(q^*(\tilde{\theta}_z), 0, \tilde{\theta}_z) = u\left[q^*(\tilde{\theta}_z), q^*(\tilde{\theta}_z), \tilde{\theta}_z\right] - z$ which implies

$$u(q^*(\tilde{\theta}_z), 0, \tilde{\theta}_z) = v(q^*(\tilde{\theta}_z), \tilde{\theta}_z).$$

In other words, at $q_i = q^*(\tilde{\theta}_z)$ and $\theta = \tilde{\theta}_z$, the buyer is exactly indifferent between purchasing $q^*(\tilde{\theta}_z)$ of good $-i$ and not purchasing good $-i$ at all. Clearly, then, any buyer $\theta < \tilde{\theta}_z$ would strictly prefer not to buy product $-i$. In other words, $v\left[q^*(\tilde{\theta}_z),\theta\right] = u(q^*(\tilde{\theta}_z), 0, \theta)$ for $\theta < \tilde{\theta}_z$.Next consider the case $q_i = q^E(\theta) = \arg\max_q u(q,0,\theta)$. Recall that by definition $q^E(\hat{\theta}_z) = q^*(\tilde{\theta}_z)$. This means that no buyer $\theta < \hat{\theta}_z$ would purchase a positive quantity of product $-i$ when $q_i = q^E(\theta)$. That is, $v\left[q^E(\theta),\theta\right] = u(q^E(\theta), 0, \theta)$ for $\theta < \hat{\theta}_z$.

Obviously, firm i's profits vanish when $q_i = 0$. In maximizing its profits, firm i can therefore proceed as if the buyer had the utility function $v(q,\theta)$ and a type-dependent reservation utility $v(0,\theta)$. This maximization problem can be stated as follows:

$$\begin{aligned} \max_{P(q)} \pi_i &= \int_0^1 P[q(\theta)] d\theta \\ \text{s. t. } q(\theta) &= \arg\max_q \left[v(q,\theta) - P(q)\right] \\ \max_q \left[v(q,\theta) - P(q)\right] &\geq v(0,\theta). \end{aligned}$$

We can restate this optimal control problem using $q(\theta)$ as our control variable and $U(\theta) = \max_q \left[v(q,\theta) - P(q)\right]$ as the corresponding state variable. The problem then becomes to maximize $\int_0^1 \left[v(q(\theta),\theta) - U(\theta)\right] d\theta$ subject to $U(\theta) \geq v(0,\theta)$.

As in the proofs of Proposition 2 and 3, we simply guess the solution and use sufficiency arguments to show that we have guessed correctly. Our guess now is

$$\tilde{q}(\theta) = \begin{cases} q^E(\theta) = \arg\max_q u(q,0,\theta) & \text{for} 0 \leq \theta \leq \hat{\theta}_z \\ q^*(\tilde{\theta}_z) \ \text{ for } \hat{\theta}_z \leq \theta \leq \tilde{\theta}_z \\ q^*(\theta) \quad \text{for } \tilde{\theta}_z \leq \theta \leq 1, \end{cases}$$

where $\hat{\theta}_z$ is implicitly defined by the condition $q^E(\hat{\theta}_z) = q^*(\tilde{\theta}_z)$, with the associated utility $\tilde{U}(\theta) = v\left[\tilde{q}(\theta),\theta\right] - \max\left\{P^*\left[\tilde{q}(\theta)\right] - P^*\left[q^*(\tilde{\theta}_z)\right], 0\right\}$. Theorem 1 in Seierstad and Sydsaeter (1987 p. 317) implies that a sufficient condition for this to be a maximum is that there exists a continuous and piecewise differentiable function $\xi(\theta)$ such that $\xi'(\theta) \geq 0$ and $\xi'(\theta)\left[U(\theta) - v(0,\theta)\right] \equiv 0$, and such that $\tilde{q}(\theta)$ and $\tilde{U}(\theta)$ maximize the Lagrangian

$$\mathcal{L} = \int_0^1 \left\{\left[v(q(\theta),\theta) - U(\theta)\right] + \xi'(\theta)\left[U(\theta) - v(0,\theta)\right]\right\} d\theta.$$

Integrating by parts and normalizing $\xi(\theta_{\max})$ to zero, we get

$$\mathcal{L} = \int_0^1 \left[v(q(\theta),\theta) - (1-\theta)v_\theta(q(\theta),\theta) - \xi(\theta)v_\theta(q(\theta),\theta) - \xi'(\theta)v(0,\theta)\right] d\theta.$$

Now consider the following function $\xi(\theta)$:

$$\tilde{\xi}(\theta) = \begin{cases} -(1-\theta) \quad \text{for } 0 \leq \theta \leq \hat{\theta}_z \\ \frac{v_q(q^*(\tilde{\theta}_z),\theta)}{v_{q\theta}(q^*(\tilde{\theta}_z),\theta)} - (1-\theta) \quad \text{for } \hat{\theta}_z \leq \theta \leq \tilde{\theta}_z \\ 0 \quad \text{for } \tilde{\theta}_z \leq \theta \leq 1. \end{cases}$$

Since $v\left[q^*(\tilde{\theta}_z),\theta\right] = u(q^*(\tilde{\theta}_z),0,\theta)$ when $\hat{\theta}_z \leq \theta \leq \tilde{\theta}$, we have $v_q(q^*(\tilde{\theta}_z),\theta) = \theta - (1-\gamma)q^*(\tilde{\theta}_z)$ and $v_{q\theta}(q^*(\tilde{\theta}_z),\theta) = 1$. Clearly, this implies that $\tilde{\xi}'(\theta) \geq 0$

everywhere. By setting $\xi(\theta)=\tilde{\xi}(\theta)$, we can rewrite $\mathcal{L}$ as $\mathcal{L}=\mathcal{A}+\mathcal{B}+\mathcal{C}$, where:

$$\begin{aligned}
\mathcal{A} &= \int_0^{\hat{\theta}_z}\left[v(q(\theta),\theta)-(1-\theta)v_\theta(q(\theta),\theta)-\tilde{\xi}(\theta)v_\theta(q(\theta),\theta)-\tilde{\xi}'(\theta)v(0,\theta)\right]d\theta \\
\mathcal{B} &= \int_{\hat{\theta}_z}^{\tilde{\theta}_z}\left[v(q(\theta),\theta)-(1-\theta)v_\theta(q(\theta),\theta)-\tilde{\xi}(\theta)v_\theta(q(\theta),\theta)-\tilde{\xi}'(\theta)v(0,\theta)\right]d\theta \\
\mathcal{C} &= \int_{\tilde{\theta}_z}^{1}\left[v(q(\theta),\theta)-(1-\theta)v_\theta(q(\theta),\theta)\right]d\theta.
\end{aligned}$$

To prove that our guess is correct, we must show that (i) $q^E(\theta)$ maximizes $\mathcal{A}$; (ii) $q^*(\tilde{\theta}_z)$ maximizes $\mathcal{B}$, (iii) $q^*(\theta)$ maximizes $\mathcal{C}$; (iv) $\tilde{\xi}(\theta)$ is everywhere continuous.

(i) When $\xi(\theta)=-(1-\theta)$, $\mathcal{A}$ becomes

$$\int_0^{\hat{\theta}_z}\left[v(q(\theta),\theta)-\tilde{\xi}'(\theta)v(0,\theta)\right]d\theta.$$

Since $v\left[q^E(\theta),\theta\right]=u(q^E(\theta),0,\theta)$, it is immediate to verify that the derivative of the term inside square brackets with respect to q vanishes at $q=q^E(\theta)$, so $q^E(\theta)$ pointwise maximizes $\mathcal{A}$.

(ii) When $\xi(\theta)=\frac{v_q(q^*(\tilde{\theta}_z),\theta)}{v_{q\theta}(q^*(\tilde{\theta}_z),\theta)}-(1-\theta)$, $\mathcal{B}$ becomes

$$\int_{\hat{\theta}_z}^{\tilde{\theta}_z}\left[v(q(\theta),\theta)-\frac{v_q\left[q^*(\tilde{\theta}_z),\theta\right]}{v_{\theta q}\left[q^*(\tilde{\theta}_z),\theta\right]}v_\theta(q(\theta),\theta)-\tilde{\xi}'(\theta)v(0,\theta)\right]d\theta.$$

It is immediate to verify that the derivative of the term inside square brackets with respect to q vanishes at $q=q^*(\tilde{\theta}_z)$, so $q^*(\tilde{\theta}_z)$ pointwise maximizes $\mathcal{B}$.

(iii) By definition, the non-linear pricing equilibrium quantities $q^*(\theta)$ and the associated equilibrium schedule $P^*(q)$ maximize a firm's profit given the pricing strategy of its rival in the non-linear pricing game without shopping costs. That is, they solve the problem

$$\begin{aligned}
\max_{P(q)}\pi_i &= \int_0^1 P[q(\theta)]d\theta \\
\text{s. t. } q(\theta) &= \arg\max_q\left[v^*(q,\theta)-P(q)\right] \\
\max_q\left[v^*(q,\theta)-P(q)\right] &\geq v^*(0,\theta).
\end{aligned}$$

where

$$v^*(q,\theta) = \max_x \{u(q,x,\theta) - P^*(x)\}$$

is the indirect utility function in the non-linear pricing game without shopping costs. Once again, we can restate this optimal control problem using $U(\theta)$ as our state variable and $q(\theta)$ as the corresponding control variable. Using these variables, the firm's objective becomes to maximize

$$\int_0^1 \left[v^*(q(\theta),\theta) - U(\theta)\right] f(\theta) d\theta$$

subject to $U(\theta) \geq v^*(0,\theta)$. As in Martimort and Stole (2009), this latter constraint is never binding and so can be neglected. Integrating by parts the objective becomes

$$\int_0^1 \left[v^*(q(\theta),\theta) - (1-\theta)v^*_\theta(q(\theta),\theta)\right] d\theta.$$

By construction, $q^*(\theta)$ maximizes this function. In fact, since $\frac{dq^*(\theta)}{d\theta} > 0$, $q^*(\theta)$ must pointwise maximizes any function $\int_{\tilde{\theta}}^1 \left[v^*(q(\theta),\theta) - (1-\theta)v^*_\theta(q(\theta),\theta)\right] d\theta$. This implies that $q^*(\theta)$ pointwise maximizes $\mathcal{C}$ with respect to $q(\theta)$.

(iii) Since by construction $q^E(\hat{\theta}_z) = q^*(\tilde{\theta}_z)$, we have $v_q(q^*(\tilde{\theta}_z),\theta) = 0$, which implies that $\tilde{\xi}(\theta)$ is continuous at $\theta = \hat{\theta}_z$. Again by construction, $\frac{v_q(q^*(\tilde{\theta}_z),\tilde{\theta}_z)}{v_{q\theta}(q^*(\tilde{\theta}_z),\tilde{\theta}_z)} = (1-\tilde{\theta}_z)$, which implies that $\tilde{\xi}(\theta)$ is continuous at $\theta = \tilde{\theta}_z$. It follows that $\xi(\theta)$ is everywhere continuous. This completes the proof of the Proposition. ■

Proof of Proposition 5

The proof is similar to that of Proposition 2, except that the preference for variety is now effectively $\ell(q,\theta) - z$. This implies that for sufficiently low types there is no preference for variety to be extracted, and hence both exclusive and non-exclusive prices must vanish. ■