TRADING AWAY WIDE BRANDS FOR CHEAP BRANDS

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Online Appendix

Appendix A. Theoretical & Empirical Results

A.1. **CES and Logit Preferences: Invariance of Innovation to Trade Liberalization.** I introduce process innovation in the standard multiproduct Krugman-Melitz setting with nested CES preferences. In this setup, innovation is invariant to trade liberalization. A similar result holds with several variants of nested Logit preferences.

A.1.1. *Consumers.* Consider the same economic structure as in Section I. The only difference is that the utility function has a nested CES form of Allanson and Montagna (2005).

$$U \equiv \left(\int q_j^{(\sigma-1)/\sigma} dj\right)^{\sigma/(\sigma-1)} \qquad q_j \equiv \left(\int q_{ij}^{(\varepsilon-1)/\varepsilon} di\right)^{\varepsilon/(\varepsilon-1)} \quad \varepsilon > \sigma > 1$$

Let $Q \equiv U$. Then the inverse demand for product i produced by firm j is

$$p_{ij} = IQ^{-\frac{\sigma-1}{\sigma}} q_j^{-\frac{\varepsilon-\sigma}{\varepsilon\sigma}} q_{ij}^{-1/\varepsilon}$$

The parameter σ captures the degree of across-firm substitutability while ε captures the degree of within-firm substitutability. Let $\gamma \equiv \varepsilon - \sigma$. Then γ captures the degree of cannibalization with $\gamma = 0$ implying no cannibalization. When $\gamma > 0$, there is within-brand cannibalization.

A.1.2. *Firms.* I start with homogeneous firms. The process innovation FOC is similar to Section I, $c'(\omega)(q^d + \tau q^x) = r_\omega$ where τ is an iceberg transport cost. Optimal price of a product supplied to the domestic and export markets are $p^d = \sigma c/(\sigma-1)$ and $p^x = \sigma\tau c/(\sigma-1)$ reflecting the usual CES property of constant markups.[1] Optimal product range h is given by

(A.1)
$$[p^d - c]q^d + [p^x - \tau c]q^x - r_\omega \omega - r_h + \underbrace{h\{q^d(\partial p^d/\partial q_j^d)(\partial q_j^d/\partial h) + q^x(\partial p^x/\partial q_j^x)(\partial q_j^x/\partial h)\}}_{\text{Cannib. Effect}} = 0$$

With CES preferences, the rate of cannibalization from total firm quantity q_j is constant and given by $(\partial p/\partial q_j)q_j/p_{ji} = -\gamma/\varepsilon\sigma$. Product innovation h directly affects profits through cannibalization (since $\partial h/\partial\gamma \neq 0$) while process innovation does not.

[1] I assume a firm can anticipate the effect of its decisions on its own products. Allanson and Montagna (2005) and Agur (2007) implicitly assume that a firm prices each of its products separately, implying $p = \varepsilon c/(\varepsilon - 1)$. I note that my result regarding insensitivity of innovation choices is valid even when this pricing strategy is considered.

A.1.3. *Product and Process Innovation with Trade Liberalization.* CES preferences imply constant markups and a constant cannibalization rate γ. This results in the following quantity per product $q^d + \tau q^x$:

$$[c(\omega)(q^d + \tau q^x)]/(\varepsilon - 1) - r_\omega \omega = r_h \tag{A.2}$$

First, note that firm scale $q^d + \tau q^x$ and hence process innovation does not depend on intra-brand cannibalization γ. Second, Equation (A.2) fixes scale and process innovation implying they are invariant to trade liberalization.

In a Krugman economy (with symmetric firms), free entry implies $h = f(\varepsilon - \gamma - 1)/\gamma c(q^d + 1_x \tau q^x)$. Unlike process innovation, product innovation cannibalizes directly and indirectly as $dh/d\gamma < 0$. As with process innovation, product innovation is also unaffected by trade liberalization. For completeness, I note that in a Melitz economy with fixed exporting costs $f_x > 0$ and heterogeneous firms, exporters increase their product range after trade liberalization but process innovation continues to be unresponsive as shown earlier. These results are summarized in a Proposition below.

Proposition. *In a Krugman-Melitz economy, trade liberalization has no impact on process innovation. In the absence of fixed exporting costs, trade liberalization has no impact on product innovation either.*

This result is similar to Atkeson and Burstein (2010) and Desmet and Parente (2010) for process innovation in single-product firms. They do not consider product innovation but find process innovation is invariant to trade liberalization for single product firms in the absence of fixed exporting costs. Finally, the reader may verify that innovation is invariant to trade liberalization when the standard multiproduct nested Logit demand of Anderson et al. (1992) is considered instead (see pp. 250). The reason is similar: markups and the rate of cannibalization are exogenously fixed by taste for diversity in products and brands. This is not surprising given the close relationship between the logit and CES framework (see Verboven 1996 for an equivalence relationship).

A.2. **Multiproduct Linear Demand Model.** This sub-section provides a detailed proof of optimal firm decisions in the multiproduct linear demand model. I assume a symmetric equilibrium across firms. Firm j chooses $(h_j, \{q_{ij}, \omega_{ij}\})$ to solve the following objective function:

$$\Pi_j \equiv \int_0^{h_j} \pi_{ij} di - f \equiv \int_0^{h_j} \left([p_{ij} - c(\omega_{ij})]q_{ij} - r_\omega \omega_{ij} - r_h\right) di - f$$

Let $\alpha > c + 2(\gamma f/L)^{1/2}, 2\eta^{1/2}$ to ensure consumption of both homogeneous and differentiated goods in equilibrium, as in Melitz and Ottaviano (2008). For well-defined profits and pro-competitive effects, $\gamma + \kappa Q_i > 0$ and $\eta + 2\kappa hq > 0$.[2] I assume $\delta/L > c'(\omega)^2[a - c(\omega)]/2c''(\omega)[r_\omega \omega + r_h]$ for all $\omega > 0$ to ensure a strictly concave firm problem. Sufficient conditions in terms of primitives are provided in the proof below. I start with finding the optimal process and quantity for each product (ω_{ij}, q_{ij}), given aggregate firm quantity q_j and product range h_j. Next, I solve for the optimal q_j given product range h_j. Finally, I solve for the optimal product range. These firm decisions are optimal for a given value of a which is verified to be consistent with firm decisions.

[2]A sufficient condition is $\kappa \geq -\eta\gamma/4(\alpha - c)L$.

A.2.1. *Product Quantity and Production Process.* The firm chooses $(\omega_{ij}, q_{ij}) \in \Omega \times \mathscr{Q} \equiv \bigcup_{i\in[0,h]} \Omega_{[0,h]} \times \mathscr{Q}_{[0,h]}$ where

$$\Omega_{[0,h]} \equiv \{\omega \in \mathscr{C}^1, \geq 0 \text{ on } [0,h] \text{ and } 0 \text{ otherwise}\}$$

$$\mathscr{Q}_{[0,h]} \equiv \{q \in \mathscr{C}^1, > 0 \text{ on } [0,h] \text{ and } 0 \text{ otherwise}\}$$

so $\Omega_{[0,h]}$ and $\mathscr{Q}_{[0,h]}$ denote all smooth, strictly positive process and quantity allocations on $[0,h_j]$. Given h_j and q_j, the optimal (ω_{ij}, q_{ij}) choices are found by maximizing $\Pi_j + \lambda[q_j - \int_0^{h_j} q_{ij} di]$.

For each fixed (h_j, q_j), the problem of finding $(\omega^*_{ij}, q^*_{ij}) \in \Omega \times \mathscr{Q}$ is

$$\max_{(\omega_{ji}, q_{ji}) \in \Omega\times\mathscr{Q}} \Pi(\omega_{ij}, q_{ij}) \text{ subject to } H(\omega_{ij}, q_{ij}) \equiv \int_0^{h_j} q_{ij} di = q_j$$

The FOCs for $\Pi(\omega_{ij}, q_{ij}) - \lambda H(q_{ij})$ with respect to ω_{ij} and q_{ij} are

$$p_{ij} - c(\omega^*_{ij}) - \frac{\delta}{L} q^*_{ij} = \lambda$$
$$-c'(\omega^*_{ij}) q^*_{ij} - r_\omega = 0$$

The FOCs yield a global max with $(\omega^*_{ij}, q^*_{ij})$ since $\Pi((\omega_{ij}, q_{ij})) - \lambda H(q_{ij})$ is strictly concave. This follows from the linearity of constraint $H(q)$ and strict concavity of $\Pi(\omega_{ij}, q_{ij})$. Strict concavity of Π is shown below. Suppressing the ij subscripts for brevity, the Hessian is

$$H = \begin{pmatrix} -2\delta/L & -c'(\omega) \\ -c'(\omega) & -c''(\omega)q \end{pmatrix}$$

First, $\pi_{qq} = -2\delta/L$ and $\pi_{\omega\omega} = -c''(\omega)q$ are negative for all $\omega \geq 0$ and $q > 0$. Second $|H| > 0$ if $2(\delta/L)c''(\omega)q - c'(\omega)^2 > 0$ for all feasible values of q and ω. A firm ceases production of a product with $\pi < 0$ implying $\pi = [a - (\gamma + \kappa Q_i)q_j/L - \delta q/L - c(\omega)]q - r_\omega \omega - r_h \geq 0$. Therefore, $q \geq (r_\omega \omega + r_h)/[a - (\gamma + \kappa Q_i)q_j/L - \delta q/L - c(\omega)] \geq (r_\omega \omega + r_h)/[a - c(\omega)]$ and we obtain a lower bound for q. The Hessian condition can be written as $\delta/L > c'(\omega)^2/2c''(\omega)q$. Substituting for the lower bound for q, the Hessian inequality is $\delta/L > [c'(\omega)^2/2c''(\omega)][a - c(\omega)]/[r_\omega \omega + r_h]$. As long as this condition holds for all $\omega > 0$, we have a strictly concave problem and the global max is given by the process and quantity choices satisfying the FOCs.

Consider the specific functional form $c(\omega) = c - c\omega^{1/2}$ for $\omega \in [0,1]$. In an interior equilibrium, $a = c(\omega^*) + 2(\delta + \gamma h^* + \kappa h^* Q^*_i)q^*/L$ implying $a = c + 2q(\delta/L - c^2/4r_\omega) + 2(\gamma + \kappa Q_i)hq/L$. Substituting for optimal ω and hq, $a = c + 2r_h^{1/2}(\delta/L - c^2/4r_\omega)^{1/2} + 2(\gamma + \kappa Q_i)^{1/2}(f/L)^{1/2}$. The Hessian condition is

$$\delta/L > c\omega^{1/2}\left[r_h^{1/2}(\delta/L - c^2/4r_\omega)^{1/2} + (\gamma f/L)^{1/2} + c\omega^{1/2}/2\right]/[r_\omega \omega + r_h]$$

for all $\omega > 0$ and $\kappa \leq 0$. The term outside square brackets on the RHS $c\omega^{1/2}/(r_\omega \omega + r_h)$ is zero at $\omega = 0$, $c/(r_\omega + r_h)$ at $\omega = 1$ and $c/2r_\omega^{1/2} r_h^{1/2}$ at the interior critical point of $\omega = r_h/r_\omega$. As the arithmetic mean is always greater than the geometric mean, $(r_\omega + r_h)/2 \geq r_\omega^{1/2} r_h^{1/2}$ implying the first term is at most $c/2r_\omega^{1/2} r_h^{1/2}$. The last term in square brackets on the RHS is at most c and a sufficient condition for an interior solution is $\delta/L > c\left[r_h^{1/2}(\delta/L - c^2/4r_\omega)^{1/2} + (\gamma f/L)^{1/2} + c/2\right]/2r_\omega^{1/2} r_h^{1/2}$. I assume this condition holds so that the FOCs guarantee a global argmax.

The condition $p-c(\omega)-(\delta/L)q=\lambda$ is equivalent to $q^*=L(a-c(\omega^*)-(\gamma+\kappa Q_i)q_j/L-\lambda)/2\delta$ where $-c'(\omega^*)q^*=r_\omega$. From above, this q^* serves as a solution to $\max\Pi$ provided that $H(q^*)=q_j$. This will be satisfied by appropriate choice of λ since for fixed λ we have

$$H(q^*) \;=\; \frac{L}{2\delta}\int_0^h [a-\frac{\gamma+\kappa Q_i}{L}q_j-\lambda-c(\omega^*)]di=\frac{L}{2\delta}h[a-\frac{\gamma+\kappa Q_i}{L}q_j-\lambda-c(\omega^*)]$$

Choosing $\lambda^*\equiv a-(2\delta+\gamma h+\kappa hQ_i)q_j/hL-c(\omega^*)$, a globally optimal solution satisfying the resource constraint is $q^*=q_j/h$ and $\omega^*=c'^{-1}(-r_\omega/q^*)$ for each fixed (h,q_j).

A.2.2. *Firm Quantity.* Substituting for optimal q^* and ω^*,

$$\Pi\equiv\int_0^{h_j}\pi_{ij}di-f=h_j\pi_{ij}-f=h\left\{[a-\frac{\delta+\gamma h+\kappa hQ_i}{hL}q_j-c(\omega^*)]q_j/h-r_\omega\omega^*-r_h\right\}$$

Given h, each firm chooses its aggregate quantity q_j. The FOC for q_j is $\Pi_{q_j}=\int_0^h\pi_{q_j}di=0$ which is given by $\pi_{q_j}=-(\delta+\gamma h+\kappa hQ_i)q_j/h^2L+[p-c(\omega)]/h=0$. For any $h\geq 0$, the objective function is strictly concave as the SOC is $\pi_{q_jq_j}=-2(\delta+\gamma h+\kappa hQ_i)/h^2L<0$. Therefore, the FOC yields the global max and $(\delta+\gamma h+\kappa hQ_i)q^*/L=[a-c(\omega^*)]/2$.

A.2.3. *Products.* Substituting for optimal choices in $\pi=(\delta+\gamma h+\kappa hQ_i)q^2/L-r_\omega\omega-r_h$, the firm problem is

$$\max_h\Pi=h\pi-f=h\frac{L[a-c(\omega^*)]^2}{4(\delta+\gamma h+\kappa hQ_i)}-f$$

The FOC is $\Pi_h=\pi+h\pi_h=0$ and $\pi_h=-(\gamma+\kappa Q_i)L[a-c(\omega^*)]^2/4(\delta+\gamma h+\kappa hQ_i)^2$ (which follows from $\pi_\omega(\omega^*)=0$ by the envelope theorem). Substituting for optimal q_j, $\pi_h==-(\gamma+\kappa Q_i)(q_j/h)^2/L$. Consequently, the FOC is $\Pi_h=\pi-h(\gamma+\kappa Q_i)(q_j/h)^2/L=0$.

The SOC is $\Pi_{hh}=\pi_h-(\gamma+\kappa Q_i)(q/L)\left[q+2h(dq/dh)\right]=-2(\gamma+\kappa Q_i)(q^2/L)\left(1+(h/q)(dq/dh)\right)<0$. From the ω FOC, $d\omega/dh=-[c'(\omega)/c''(\omega)q](dq/dh)$. From the q FOC, $(\delta+\gamma h+\kappa hQ_i)q/L=[a-c(\omega)]/2$ implying $(\delta+\gamma h+\kappa hQ_i)(dq/dh)/L=-c'(\omega)(d\omega/dh)/2-(\gamma+\kappa Q_i)q/L$. Substituting for $d\omega/dh=(d\omega/dq)(dq/dh)$,

$$1+\frac{h}{q}\frac{dq}{dh}=1-\frac{(\gamma h+\kappa hQ_i)/L}{(\delta+\gamma h+\kappa hQ_i)/L-c'(\omega)^2/2c''(\omega)q}$$

I already assumed $\delta/L>c'(\omega)^2/2c''(\omega)q$ for all feasible values of ω and q. Therefore, $1+(h/q)(dq/dh)>0$ and $\Pi_{hh}<0$. Note that product range is a continuous variable so choosing $h=0$ gives zero profits to the firm.

A.3. **Impact on Product Innovation.** From the product FOC, $\Pi=\left[\gamma+\kappa Q_i+\theta^2\left(\gamma+\kappa Q_i^*\right)\right](hy)^2/(1+\theta)^2$ implying $2\Pi d\ln h/dt^*=\Pi_{t^*}-\kappa\left(hq^d\right)^2\left(1-\theta^2A^x/A^d\right)(dQ_i/dt^*)$. Substituting for Π_{t^*}, the change in product range can be written as

$$-\frac{2\Pi}{h^2q^d}\frac{dh}{dt^*}=\theta+\left[\eta\left(1-\theta A^x/A^d\right)+2\kappa hq^d\left(1-\theta^2A^x/A^d\right)\right](dQ_i/dt^*)$$

As $\eta+2\kappa hq^d(1+\theta)>0$, the term in square brackets is positive. For a non-exporter, $\theta=0$ and $\eta+2\kappa hq^d>0$. Therefore, $dh/dt^*>0$ implying non-exporters reduce product innovation with a reduction in foreign tariffs. For exporters, note that the RHS is decreasing in c because $\theta'(c)<0$ and the term in square brackets is increasing in c. Let $B(c)$ denote the term in square brackets.

Then $B'(c) = -(\eta + 2\kappa hq^x)\,\theta'(c)A^x/A^d + 2\kappa\left(1-\theta^2 A^x/A^d\right) dhq^d/dc - 2\theta'(c)\kappa hq^x A^x/A^d$. From the q^d FOC, $2(\gamma+\kappa Q_i)dhq^d/dc = (-1+\omega'(c))\left[1+\delta\theta(1-\theta)/(1+\theta^2)(\delta+\gamma h+\kappa hQ_i)\right] < 0$. Differentiating with respect to c, $\theta'(c) = (-1+\omega'(c))\,(1-\theta)/2q^d(\delta+\gamma h+\kappa hQ_i) < 0$. Substituting for dhq^d/dc and $\theta'(c)$ in the last two terms of $B(c)$ gives

$$\frac{\kappa\,(-1+\omega'(c))}{\gamma+\kappa Q_i}\left(1-\theta^2 A^x/A^d\right)\left[1+\frac{\delta}{\delta+\gamma h+\kappa hQ_i}\frac{\theta(1-\theta)}{1+\theta^2} - \frac{\gamma+\kappa Q_i}{\delta+\gamma h+\kappa hQ_i}\frac{\theta(1-\theta)A^x/A^d}{1-\theta^2 A^x/A^d}\right]$$

The last term is increasing in A^x/A^d and reaches a maximum at $\theta/(1+\theta) < 1$. The term in square brackets above is positive and the last two terms of $B'(c)$ are positive for $\kappa < 0$ and $-1+\omega'(c) < 0$. We already know that the first term of $B'(c)$ is positive as $\theta'(c) < 0$. Finally, $-\left(2\Pi/h^2q^d\right)(dh/dt^*)$ is decreasing in c. For the marginal exporters, we know $-dh/dt^* < 0$. For firms with lower cost draws, the RHS increases implying dh/dt^* keeps falling. Therefore, exporters engage in greater product innovation than non-exporters. At $c = 0$,

$$-\frac{2\Pi}{h^2q^d}\frac{dh(0)}{dt^*} = \theta_0 - \frac{A^x}{A^d}\frac{1-\theta_0 A^x/A^d}{1-(A^x/A^d)^2}\left[\frac{\eta+\kappa(hq^d)_0\cdot 2\frac{1-\theta_0^2 A^x/A^d}{1-\theta_0 A^x/A^d}}{\int hq^x(\eta+\kappa hq^x)/\int hq^x}\right]$$

The fraction in the numerator is increasing in A^x/A^d and reaches its minimum at $A^x/A^d = 0$. At this point the fraction becomes 1. As $\eta+\kappa hq^x \le \eta+\kappa(hq^d)_0$, multiplying by hq^x on both sides and integrating shows the last term in square brackets is less than 1. As long as $\theta_0 \ge A^x/A^d$, the RHS is $\theta_0 - (\le\theta_0)(<1)(<1)$ and the RHS is positive. When $\kappa = 0$, $A^x/A^d = \int hq^x dG/\int hq^d dG$ which is the average export to domestic sales ratio in the economy. To conclude, the lowest cost firms have $dh/dt^* < 0$ and increase their product innovation following a reduction in foreign tariffs.

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