Corrective Taxation versus Liability

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Online Appendix

This appendix contains the proof of the part of Proposition 4 that is not shown in the text, namely, under the optimal joint tax and liability regime, the optimal tax $t^{**} < E(y)$ provided that injurer benefits $b(x)$ display decreasing absolute risk aversion.

To establish this claim, observe that under the joint tax and liability regime, social welfare as a function of the tax $t$ is

$$W_{TL}(t) = \int_0^m [b(x^*(t + py)) - x^*(t + py)y]f(y)dy$$

(1)

since it was shown already that $\lambda = 1$. Hence

$$W_{TL}(t) = \int_0^m x^*(t + py)[b'(x^*(t + py)) - y]f(y)dy,$$

(2)

where $x^*(t + py)$ is the derivative of $x^*(t + py)$ with respect to $t$. It will be shown that

$$W_{TL}(E(y)) = \int_0^m x^*(E(y) + py)[b'(x^*(E(y) + py)) - y]f(y)dy < 0.$$  

(3)

As I will note below, an essentially identical argument to what I am about to give will prove also that $W_{TL}(t) < 0$ for any $t > E(y)$. Hence, it will follow that the optimal tax $t^{**}$ must be less than $E(y)$.

Observe first that since the optimal tax under a tax only regime is $E(y)$ (from Proposition 1), $b(x^*(t)) - x^*(t)E(y)$ is maximized at $t = E(y)$. Therefore, $b'(x^*(E(y)))x^*(E(y)) - x^*(E(y))E(y) = 0$, which implies that $b'(x^*(E(y))) - E(y) = 0$. This is equivalent to

$$\int_0^m [b'(x^*(E(y)) - y]f(y)dy = 0.$$  

(4)

It will now be shown that (4) implies

$$\int_0^m [b'(x^*(E(y) + py) - y]f(y)dy > 0.$$  

(5)

To this end, rewrite (4) as

$$\int_{E(y)}^0 [b'(x^*(E(y)) - y]f(y)dy + \int_{E(y)}^m [b'(x^*(E(y)) - y]f(y)dy = 0.$$  

(6)

The first term in (6) is positive, since the integrand is positive for each $y < E(y)$. This claim about (6) is readily seen from Figure 1. In particular, in region A, an upward movement in the line $x^*(E(y))$ brings $x$ closer to the optimum $x^*(y)$ at each $y$, and given the concavity of welfare $b(x) - xy$ in $x$, this change in $x$ increases welfare.  

The second term in (6) is negative, since the integrand is negative for each $y > E(y)$. The explanation is analogous to what was just stated; in regions B and C, an upward movement in the line $x^*(E(y))$ makes $x$ more distant from $x^*(y)$ and thus lowers welfare at each $y$.

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1 That is, $b'(x) - y = 0$ at $x^*(y)$, and thus $b'(x) - y > 0$ for $x < x^*(y)$ since $b''(x) < 0$. Hence, $b'(x^*(E(y)) - y > 0$, since, for each $y$ in A, $x^*(E(y)) < x^*(y)$. I will omit further explanations like this one that are easy to verify from concavity of $b(x) - xy$ in $x$. 

1
Figure 1

Level of activity

$E(y)$

$E(y)/(1-p)$

$y$

Expected harm per unit of activity

Figure 1
Next, observe that
\[
\int_0^0 [b'(x^*(E(y) + py)) - y] f(y) dy > \int_0^0 [b'(x^*(E(y)) - y] f(y) dy. \tag{7}
\]
This also can be seen from Figure 1. In region A, an increase in \(x\) from \(x^*(E(y) + py)\) will increase welfare more than an increase in \(x\) from \(x^*(E(y))\) since the former is more distant from \(x^*(y)\) and welfare is concave in \(x\). Similarly, we have that
\[
\int_0^m [b'(x^*(E(y) + py)) - y] f(y) dy > \int_0^m [b'(x^*(E(y)) - y] f(y) dy. \tag{8}
\]
To explain, in region B, an increase in \(x\) from \(x^*(E(y) + py)\) will raise welfare since \(x\) will become closer to \(x^*(y)\), whereas an increase in \(x\) from \(x^*(E(y))\) will lower welfare since \(x\) will become farther from \(x^*(y)\). In region C, an increase in \(x\) from \(x^*(E(y) + py)\) will reduce welfare by less than an increase in \(x\) from \(x^*(E(y))\) since the former is closer to \(x^*(y)\). Hence, over both regions B and C, \(b'(x^*(E(y) + py)) - y > b'(x^*(E(y)) - y\), from which (8) follows. Finally, (7) and (8) imply (5).

I now show that (5) implies (3) given the assumption that \(b\) displays decreasing absolute risk aversion. Note first that the integrand of (3) equals the integrand of (5) multiplied by \(x^*(E(y) + py)\).

I first claim that \(x^*(E(y) + py) < 0\) and that it increases with \(y\). To verify this, observe first that \(x^*(E(y) + py) = 1/b''(x^*(E(y) + py)) < 0\), for differentiation of \(b''(x(t)) = t + py\) with respect to \(t\) gives \(x'(t) = 1/b''(x(t))\). Second, note that \(x^*(E(y) + py)\) will increase with \(y\) if \(b''(x) > 0\). In particular, differentiation of \(x'(t) = 1/b''(x(t))\) gives \(x''(t) = -b''''(x(t))/[b''(x(t))]^2\), so that the sign of \(x'(t)\) equals the sign of \(b''''(x(t))\). The assumption of decreasing absolute risk aversion implies that \(b''''(x(t)) > 0\), for this assumption means that \(-b''(x)/b'(x)\) decreases with \(x\).

I now show that (3) holds. Recall that I demonstrated above that \([b'(x^*(E(y) + py)) - y] f(y) dy > 0\). It will follow that for any function \(w(y)\) such that \(w(y) > 0\) and \(w'(y) < 0\), we must have
\[
\int_0^0 w(y)[b'(x^*(E(y) + py)) - y] f(y) dy > 0. \tag{9}
\]
To show (9), let \(w^*\) equal \(w(E(y)/(1 - p))\), namely, the value of \(w\) at the point between regions B and C. Then we have
\[
\int_0^0 w(y)[b'(x^*(E(y) + py)) - y] f(y) dy > \int_0^0 w^*[b'(x^*(E(y) + py)) - y] f(y) dy, \tag{10}
\]
since \(w(y)[b'(x^*(E(y) + py)) - y] f(y) > w^*[b'(x^*(E(y) + py)) - y] f(y)\) for \(y < E(y)/(1 - p)\) (because for such \(y\), \(w(y) > w^*\) and \([b'(x^*(E(y) + py)) - y] f(y) > 0\) as well as for \(y > E(y)/(1 - p)\) (because for such \(y\), \(w(y) < w^*\) and \([b'(x^*(E(y) + py)) - y] f(y) < 0\). But
\[
\int_0^0 w^*[b'(x^*(E(y) + py)) - y] f(y) dy = w^* \int_0^0 [b'(x^*(E(y) + py)) - y] f(y) dy > 0, \tag{11}
\]
since \(w^* > 0\) and (5) holds. Hence, (9) is established. Now since \(x^*(E(y) + py) < 0\) and increases with \(y\), we know that \(-x^*(E(y) + py) > 0\) and decreases with \(y\). Thus, \(-x^*(E(y) + py)\) may play the role of \(w(y)\), so that (9) implies
\[
\int_0^0 -x^*(E(y) + py)[b'(x^*(E(y) + py) - y] f(y) dy > 0, \tag{12}
\]
which is equivalent to (3).

Finally, the argument that has been given would apply essentially unchanged for any \(t > E(y)\) and would show that \(W_{TL}(t) < 0\). The only difference would be that the
graph of $x^*(t + py)$ would lie below that of $x^*(E(y) + py)$ in Figure 1; but this would not affect the logic of any of the steps of the proof.