IV. Online Mathematical Appendix

Proof of the condition for \( s^* < s^{**} \). From (6)-(7) we first easily check that, if \( s^* = +\infty \), then \( s^{**} = +\infty \). Next, when both \( s^* \) and \( s^{**} \) are finite, \( s^* < s^{**} \) if and only if

\[
\begin{align*}
(y_B - 2v_L)[v_H + \lambda v_L + (1 - \lambda)\bar{u} - y_B] > 0 \\
(y_B - v_H - v_L)[2(\lambda v_L + (1 - \lambda)v_H) - y_B] \iff \Delta = v_H - v_L, \\
\end{align*}
\]

Denoting \( \Delta \equiv v_H - v_L \), this becomes

\[
\begin{align*}
(y_B - 2v_L)[2v_L + \Delta + (1 - \lambda)\Delta/2 - y_B] > 0 \\
(y_B - 2v_L - \Delta)[2v_L + (1 - \lambda)\Delta - y_B] \iff \Delta = v_H - v_L, \\
(y_B - 2v_L)[\Delta + (1 - \lambda)\Delta/2 - 2(1 - \lambda)\Delta] > 0 \\
-\Delta[2v_L + 2(1 - \lambda)\Delta] \iff \Delta = v_H - v_L, \\
(y_B - 2v_L)[-3/2 - \lambda/2 + 2\lambda] + 2(1 - \lambda)\Delta > 0
\end{align*}
\]

or, finally, \( 2v_H + v_L > (3/2)y_B \).

Proof of Proposition 1. The result for \( s < s^* \) was shown in the text. The others follow from Lemmas 1 and 2 below.

**Lemma 1:** For \( s > s^* \), HL pairs must split.

(a) One cannot have both HL and LL agreeing since this requires \( s \leq \min \{s^*, s^{**}\} \).

(b) One also cannot have HL agreeing and LL splitting. Otherwise, let \( \theta^*_H, \theta^*_L = 1 - \theta^*_H \) be the shares agreed to in an HL pair and \( \theta^*, \theta^* \), with \( \theta^* + \theta^* > 1 \) the incompatible shares demanded in an LL pair. If neither of \( \theta^* \) nor \( \theta^* \) equals \( \theta^*_H \), by deviating to \( \theta^*_H \) the L in an HL pair can achieve a gain of\( s(1 - \lambda)(v_H - v_L) > 0 \). Therefore, it must be that \( \theta^*_H \in \{\theta^*, \theta^*\} \), say \( \theta^* = \theta^*_H \). But the other partner can then deviate from \( \theta^* \) to \( 1 - \theta^* = \theta^*_H \), i.e. concede; he will remain identified as an L, but now achieve \( (1 + s)\theta^*_H y_B \geq (1 + \lambda s)v_L + \lambda s\bar{u} > (1 + s)v_L \), where the first inequality must hold in order for the L partner in an HL pair to agree. The deviation is thus profitable, so once again LL pairs cannot be sustained.

It follows from the Lemma that, for \( s^* < s \leq s^{**} \), at most the LL matches can be sustained; and indeed, we showed in the text that in this region the shares \( (1/2, 1/2) \) allow these pairs to reach agreement.

**Lemma 2:** For \( s > s^{**} \), LL pairs must split.

(a) Once again HL and LL cannot both agree, as this requires \( s \leq \min \{s^*, s^{**}\} \).

(b) We also cannot have LL agreeing and HL splitting. Otherwise, let \( \theta^*_H, \theta^*_L \) with \( \theta^*_H + \theta^*_L > 1 \) be the incompatible shares demanded by \( H \) and \( L \) respectively in an unbalanced pair and \( \theta^*, 1 - \theta^* \) the shares agreed to in an LL pair, with \( \theta^* \geq 1/2 \). Consider now a deviation by the partner who was getting \( 1 - \theta^* \), to some \( \theta'' > \theta^* \) and \( \theta'' \neq \{\theta^*_H, \theta^*_L\} \), and distinguish the following cases:

(i) If \( \theta'' = \theta^*_H \) the non-deviating partner, who is still asking for the equilibrium share \( \theta^* \), remains unambiguously identified as L (by the first of our refinements), and the deviating partner as an H (by the second refinement, or by D1), thus achieving \( (1 + \lambda s)v_L + s(1 - \lambda)v_H > (1 + s)y_B/2 \), since \( s > s^{**} \). A fortiori, this is better than his equilibrium utility \( (1 + s)(1 - \theta'')y_B \).

(ii) If \( \theta'' = \theta^*_H \), this implies \( \theta''_H \geq 1/2 \). The LL partner receiving \( 1 - \theta''_H \) in equilibrium (say, Player 1) can profitably deviate to \( \theta'''' \equiv \theta''_H > 0 \), otherwise LL pairs are unsustainable, with \( \theta'''' \neq \theta''_L \). Indeed, by our first refinement Player 2 is then presumed to have played according to equilibrium (which stipulates \( \theta''_H \) for both H types and one side in LL pairs), while the fact that Player 1 broke the match identifies him, by D1, as an H type. Since \( s > s^{**} \) this is again a profitable deviation.

It follows from the two Lemmas that, for \( s > s^{**} \), no matches can be sustained, even through asymmetric equilibria.