A Appendix

Steady-State Distribution of Agent Types:

The steady-state distribution of agent types is given by the following system of equations. The economic intuition for these equations is discussed in connection with Equation (5) in the paper.

\[
\begin{align*}
0 &= \dot{\mu}_{lo}(t) = -2\lambda \mu_{hn}(t)\mu_{lo}(t) - \lambda_u \mu_{lo}(t) + \lambda_d \mu_{ho}(t) \\
0 &= \dot{\mu}_{hn}(t) = -2\lambda \mu_{hn}(t)\mu_{lo}(t) - \lambda_d \mu_{hn}(t) + \lambda_u \mu_{ln}(t) \\
0 &= \dot{\mu}_{ho}(t) = 2\lambda \mu_{hn}(t)\mu_{lo}(t) - \lambda_d \mu_{ho}(t) + \lambda_u \mu_{lo}(t) \\
0 &= \dot{\mu}_{ln}(t) = 2\lambda \mu_{hn}(t)\mu_{lo}(t) - \lambda_u \mu_{ln}(t) + \lambda_d \mu_{hn}(t).
\end{align*}
\]

(A.1)

Two of the equations in (A.1) are redundant, so that, together with (3)–(4), (A.1) forms a well-posed system. The system can be reduced to the quadratic equation

\[
0 = 2\lambda \mu_{hn}^2 + \left(2\lambda \left(\frac{\Theta}{\tilde{\theta}} - \frac{\lambda_u}{\lambda_d + \lambda_u}\right) + \lambda_u + \lambda_d\right) \mu_{hn} - \lambda_u \left(1 - \frac{\Theta}{\tilde{\theta}}\right).
\]

(A.2)

We use below the following result, which follows from calculations in Duffie, Gârleanu, and Pedersen (forthcoming).

**Lemma 3** If \(\tilde{\theta} \geq \Theta\), the system of equations (3), (4), and (A.1) has a unique solution in \([0,1]^4\). The steady-state fraction of sellers \(\mu_{lo}\) increases with \(\lambda_d\) and decreases with \(\lambda_u\), while the fraction of buyers \(\mu_{hn}\) decreases with \(\lambda_d\) and increases with \(\lambda_u\). Both \(\mu_{lo}\) and \(\mu_{hn}\) decrease with the meeting intensity \(\lambda\). Furthermore, \(\mu_{lo}\) increases, while \(\mu_{hn}\) decreases with \(\tilde{\theta}\).
Proof of Proposition 1:

The value function coefficients are given by

\[ 0 = rv_{lo} - \lambda_u (v_{ho} - v_{lo}) - 2\lambda \mu_{hn} (p - v_{lo} + v_{ln}) + \delta \]
\[ 0 = rv_{ln} - \lambda_u (v_{hn} - v_{ln}) \]
\[ (A.3) \]
\[ 0 = rv_{ho} - \lambda_d (v_{lo} - v_{ho}) \]
\[ 0 = rv_{hn} - \lambda_d (v_{ln} - v_{hn}) - 2\lambda \mu_{ho} (v_{ho} - v_{hn} - p) \]
\[ p = (v_{lo} - v_{ln})(1 - q) + (v_{ho} - v_{hn})q. \]

The first equation means that an agent of type \( lo \) has a zero change in steady-state utility. The change in his utility is due to opportunity cost \(-rv_{lo}\), expected change in intrinsic-type \( \lambda_u (v_{ho} - v_{lo}) \), trade \( 2\lambda \mu_{hn} (p - v_{lo} + v_{ln}) \), and holding cost \(-\delta\). The next three equations are similar. Direct solution of this system yields

\[ (A.4) \]
\[ p = \frac{\delta r (1 - q) + \lambda_d + 2\lambda \mu_{lo} (1 - q)}{r + \lambda_d + 2\lambda \mu_{lo} (1 - q) + \lambda_u + 2\lambda \mu_{hn} q}. \]

Given the dependence of \( P(X_t) \) on \( X_t \), it is immediate that

\[ \text{var}_t(P(X_{t+\tau}) - P(X_t)) = \frac{\sigma_X^2}{r^2 \tau} \]
for constant $\tau$. If $\tau$ is randomly distributed with constant arrival intensity $\lambda \mu_{hn}$,

$$\begin{align*}
\text{var}_t(P(X_{t+\tau}) - P(X_t)) &= \frac{1}{r^2} \text{var}_t(X_{t+\tau} - X_t) \\
&= \frac{1}{r^2} [E_t(\text{var}_t(X_{t+\tau} - X_t)) + \text{var}_t(E_t(X_{t+\tau} - X_t))] \\
&= \frac{1}{r^2} \sigma_X^2 E_t(\tau) = \frac{\sigma_X^2}{r^2 \lambda \mu_{hn}},
\end{align*}$$

and it is clear that, when the VaR constraint (2) binds, the equilibrium holding $\theta$ is given by (9) or (10), depending on the nature of risk management.

□

**Proof of Proposition 2:** The equilibrium with the two kinds of risk management is given by $f_i(\bar{\theta}) = \frac{\bar{\theta} \bar{\sigma}}{\sigma_X}$, where $f_0(\bar{\theta}) = \bar{\theta} \sqrt{r}$ and $f_1(\bar{\theta}) = \frac{\bar{\theta}}{\sqrt{2 \lambda \mu_{hn}(\theta)}}$. Clearly, $f_0 = f_1$ when $\tau = \frac{1}{2 \lambda \mu_{hn}}$, so that the two equilibria are identical.

The sensitivity $\bar{\theta}'$ of the equilibrium position $\bar{\theta}$ to the ratio $\frac{\bar{\sigma}}{\sigma_X}$ is given by $f_i' \bar{\theta}' = r$. With simple risk management, it is clear that $f_0' > 0$, so that the equilibrium position $\bar{\theta}$ decreases if the volatility $\sigma_X$ increases or the risk limit $\bar{\sigma}$ decreases. A decreasing $\bar{\theta}$ leads, in turn, to an increasing expected search time for sellers $(2 \lambda \mu_{hn})^{-1}$ and a decreasing price, because $\partial \mu_{hn}/\partial \bar{\theta} > 0$ and $\partial \mu_{hn}/\partial \bar{\theta} < 0$, as stated by Lemma 3.

With liquidity-adjusted risk management, $f_1' > 0$ by the definition of a stable equilibrium, and, since $\partial \mu_{hn}/\partial \bar{\theta} > 0$, $f_1' < f_0'$. Hence, with liquidity-adjusted risk management, the effects of $\sigma_X$ on the equilibrium quantities are larger in absolute value and of the same sign as with simple risk management. A stable equilibrium exists because $f_1 < \infty$ on $(\Theta, \infty)$, while $\lim_{x \to \Theta} f_1(x) = \lim_{x \to \infty} f_1(x) = \infty$, given that $\mu_{hn}(\Theta) = 0$ and $\lim_{x \to \infty} \mu_{hn}(x) > 0$. 

3
Consider now the dependence on the meeting intensity $\lambda$. It holds that

$$
0 = \frac{df_i}{d\lambda} = \frac{\partial f_i}{\partial \bar{\theta}} \frac{d\bar{\theta}}{d\lambda} + \frac{\partial f_i}{\partial \mu_{hn}} \frac{d\mu_{hn}}{d\lambda} + \frac{\partial f_i}{\partial \mu_{hn}} \left( \frac{d\mu_{hn}}{d\lambda} + \frac{\partial \mu_{hn}}{\partial \bar{\theta}} \frac{d\bar{\theta}}{d\lambda} \right),
$$

which can be solved for $\frac{d\bar{\theta}}{d\lambda}$.

With simple risk management, it follows that $\frac{d\bar{\theta}}{d\lambda} = 0$, as $\frac{\partial f_0}{\partial \lambda} = \frac{\partial f_0}{\partial \mu_{hn}} = 0$. With liquidity-adjusted risk management, $\frac{d\bar{\theta}}{d\lambda} > 0$, since $\frac{\partial f_1}{\partial \lambda} + \frac{\partial f_1}{\partial \mu_{hn}} \frac{d\mu_{hn}}{d\lambda} > 0$ for a stable equilibrium.

We also use the fact that $\frac{\partial f_1}{\partial \lambda} + \frac{\partial f_1}{\partial \mu_{hn}} \frac{d\mu_{hn}}{d\lambda} < 0$, which holds because $\frac{d(\lambda \mu_{hn})}{d\lambda} > 0$, as can be shown based on (A.2). Since $\frac{\partial f_1}{\partial \lambda} < 0$, the result on selling times also obtains.

The price effects follows from

\begin{equation}
(A.5) \quad \frac{dP}{d\lambda} = \frac{\partial P}{\partial \lambda} + \frac{\partial P}{\partial \bar{\theta}} \frac{d\bar{\theta}}{d\lambda}.
\end{equation}

The first term gives the impact with simple risk management, while the second captures the additional impact introduced by adjusting risk management to liquidity. Since $\frac{\partial P}{\partial \bar{\theta}} > 0$ from Proposition 1 and (A.2) (a complete proof can be given along the lines of Duffie, Gârleanu, and Pedersen (forthcoming), the second term has the same sign as $\frac{d\bar{\theta}}{d\lambda}$. This sign was shown above to be positive, as is the sign of $\frac{\partial P}{\partial \lambda}$. The total effect is therefore larger with risk-adjusted risk management.

Similar reasoning establishes the results concerning the dependence on $\lambda_d$ and $\lambda_u$. 

\[ \square \]

4