Proof of Corollary 1 to Proposition 4.

First, some definitions. Total date 2 output of produced goods is

\[ X_2[\mu^B] = \frac{1}{1-t}(\theta^G C + (1-\theta^G)(\alpha^B C + (1-\alpha^B)\mu^B c)) \]

and on date 4 it is

\[ X_4[\mu^B] = \frac{C}{1-t}(1-\theta^G)(1-\alpha^B)(1-\mu^B) \].

Total date 2 liquidity is the sum of cash goods and produced goods, \( L[\mu^B] = q_1 + X_2[\mu^B] \). Let \( M_0 \) and \( B_2 \) be the quantity of money and bonds after the open market operations. We assume that the ratio of bond to money when refinancing maturing debt at date 2, \( \frac{B_2}{M_2 + B_4} \), is a constant, independent of changes in date 0 monetary policy. The value of all nominal claims (money plus bonds) at date 2 is

\[ N_2[r_{24}, \mu^B] = tX_2[\mu^B] + \frac{1}{r_{24}} \text{Max}\{q_3 + \frac{B_4}{M_2 + B_4} tX_4[\mu^B], tX_4[\mu^B] \} \]

and the date 2 present value of a type B bank’s loans is \( \Gamma[r_{24}, \mu^B] = \alpha^B \gamma C + \mu^B (1-\alpha^B)c + (1-\mu^B)(1-\alpha^B)\frac{\gamma C}{r_{24}} \). The value of all of a type B bank’s assets at date 2 is

\[ V_2[\mu^B, M_0, B_2] = \frac{M_0}{P_{02}} + \frac{B_2}{P_{12} + \Gamma[r_{24}, \mu^B]} \]

For a type B bank to be solvent with no restructuring and for the goods market to clear:

\[ \frac{\delta_0}{P_{02}} = \delta_0 \text{Max}\{\frac{q_1}{M_0}, N_2[r_{24} = 1, \mu^B = 0]\} \leq \text{Min}\{X_2[\mu^B = 0] + q_1, V_2[\mu^B, r_{24} = 1, \mu^B = 0, M_0, B_2]\} \]

For the goods market to clear with restructuring of \( \mu^B \in (0,1] \) the condition is:
\[
\frac{\delta_0}{P_{02}} = \delta_0 \max \left\{ \frac{q_1 N_2[r_{24}, \mu^B]}{M_0}, \frac{\frac{\gamma C}{c} \cdot \mu^B}{M_0 + B_2} \right\} = X_2[\mu^B] + q_1 \leq V_2^B[r_{24}, \mu^B, M_0^', B_2^'] \}. \]

Depending on which min and max is binding, there are several cases. Rather than detail all the cases, we sketch one. Let \( q_1 \geq N_2[r_{24}, \mu^B] \), then \( \frac{q_1}{M_0} > \frac{N_2[r_{24}, \mu^B]}{M_0 + B_2} \) for all \( M_0 \) and \( B_2 \), and the nominal interest rate always exceeds one. Let \( X_2[\mu^B] + q_1 < V_2^B[r_{24}, \mu^B, M_0^', B_2^'] \). The money supply after open market operations to keep the banking system solvent has to be \( M_0^' \geq \frac{\delta_0 q_1}{\min \left\{ X_2[\mu^B] + q_1, V_2^B[r_{24}, \mu^B, M_0^', B_2^'] \right\}} = \frac{\delta_0 q_1}{X_2[\mu^B] + q_1} \). Let us determine if this can be achieved with an open market operation. Given an open market operation at the market clearing bond price from Proposition 1, of \( \frac{N_2[r_{24}, \mu^B] M_0^'}{q_1 (M_0^' + B_2^')} \), the quantities of money and bonds outstanding adjust by \( (M_0^' - M_0) q_1 (M_0^' + B_2^') = (B_2 - B_2^') N_2[r_{24}, \mu^B] M_0^' \). The feasibility of open market operations then turns on whether there are enough bonds to buy back given the needed monetary expansion. Solving for \( B_2^' \), substituting \( M_0^' = \frac{\delta_0 q_1}{X_2[\mu^B] + q_1} \), and then imposing the non-negativity condition, we require \( B_2 \geq \frac{q_1 (\delta_0 q_1 - M_0^' (X_2[\mu^B] + q_1))}{N_2[\mu^B, r_{24}] (q_1 + X_2[\mu^B])} \). If does not hold, open market operations will repurchase bonds down to zero and additional helicopter drops of money will be necessary to ensure the banking system is solvent.