APPENDIX: OMITTED PROOFS

PROOF OF LEMMA 1:

Suppose to the contrary that $y(k+1) \leq y(k)$. Since $y(k+1)$ is optimal when a poll has outcome $k+1$, it follows from (1) that:

$$\int_{0}^{1} U_1 (y(k+1), \theta) \frac{g(\theta | \langle n, k+1 \rangle)}{g(\theta | \langle n, k \rangle)} g(\theta | \langle n, k \rangle) d\theta = 0.$$  \hspace{1cm} (A1)

Furthermore, there exists a unique state, $\theta = \theta_{k+1}$, where $U_1 (y(k+1), \theta) = 0$. Using this fact, we can rewrite the left-hand side of (A1) as

$$\int_{0}^{\theta_{k+1}} U_1 (y(k+1), \theta) \frac{g(\theta | \langle n, k+1 \rangle)}{g(\theta | \langle n, k \rangle)} g(\theta | \langle n, k \rangle) d\theta$$

$$\quad + \int_{\theta_{k+1}}^{1} U_1 (y(k+1), \theta) \frac{g(\theta | \langle n, k+1 \rangle)}{g(\theta | \langle n, k \rangle)} g(\theta | \langle n, k \rangle) d\theta$$

$$\quad > \int_{0}^{\theta_{k+1}} U_1 (y(k+1), \theta) \frac{g(\theta_{k+1} | \langle n, k+1 \rangle)}{g(\theta_{k+1} | \langle n, k \rangle)} g(\theta | \langle n, k \rangle) d\theta$$

$$\quad + \int_{\theta_{k+1}}^{1} U_1 (y(k+1), \theta) \frac{g(\theta_{k+1} | \langle n, k+1 \rangle)}{g(\theta_{k+1} | \langle n, k \rangle)} g(\theta | \langle n, k \rangle) d\theta$$

$$= \frac{g(\theta_{k+1} | \langle n, k+1 \rangle)}{g(\theta_{k+1} | \langle n, k \rangle)} \int_{0}^{1} U_1 (y(k+1), \theta) g(\theta | \langle n, k \rangle) d\theta,$$

where the strict inequality follows from two facts: First, when $\theta < \theta_{k+1}$, $U_1 (y(k+1), \theta) < 0$ and when $\theta > \theta_{k+1}$, $U_1 (y(k+1), \theta) > 0$. Second, since the posterior beliefs of the policy maker are Beta distributed, the family of posterior densities $\{g(\cdot | \langle n, k \rangle)\}$ satisfies the strict monotone likelihood ratio property. Accordingly, when $\theta < \theta_{k+1}$, $g(\theta_{k+1} | \langle n, k+1 \rangle) / g(\theta_{k+1} | \langle n, k \rangle) > g(\theta | \langle n, k+1 \rangle) / g(\theta | \langle n, k \rangle)$ and when $\theta > \theta_{k+1}$, then $g(\theta_{k+1} | \langle n, k+1 \rangle) / g(\theta_{k+1} | \langle n, k \rangle) < g(\theta | \langle n, k+1 \rangle) / g(\theta | \langle n, k \rangle)$.

Finally, we claim

$$\frac{g(\theta_{k+1} | \langle n, k+1 \rangle)}{g(\theta_{k+1} | \langle n, k \rangle)} \int_{0}^{1} U_1 (y(k+1), \theta) g(\theta | \langle n, k \rangle) d\theta \geq 0.$$  

The likelihood ratio $g(\theta_{k+1} | \langle n, k+1 \rangle) / g(\theta_{k+1} | \langle n, k \rangle)$ is positive. Further, the strict concavity of the policy maker’s payoff function implies that for all $y \leq y(k)$,

$$\int_{0}^{1} U_1 (y, \theta) g(\theta | \langle n, k \rangle) d\theta \geq 0.$$
Since, by assumption, \( y (k + 1) \leq y (k) \), then \( \int_{0}^{1} U_1 (y (k + 1), \theta) g (\theta|\langle n, k \rangle) \geq 0 \). Therefore,

\[
\int_{0}^{1} U_1 (y (k + 1), \theta) \frac{g(\theta|\langle n, k+1 \rangle)}{g(\theta|\langle n, k \rangle)} g (\theta|\langle n, k \rangle) d\theta > 0,
\]

which contradicts (A1).

**PROOF OF LEMMA 2:**

To establish the result, we will show that for any truthful sequence, \( k_n \),

\[
\lim_{n \to \infty} \frac{g(\theta|\langle n, k_n \rangle)}{g(\theta|\langle n, k_n+1 \rangle)} = 1.
\]

Therefore

\[
\lim_{n \to \infty} \frac{g(\theta|\langle n, k_n \rangle)}{g(\theta|\langle n, k_n+1 \rangle)} = \left(1 - \frac{\theta}{\theta_0}\right) \lim_{n \to \infty} \left( \frac{f(t(n+1)+a-1(1-t)^n-\beta-1 dt)}{f(t(n+1)+a-1(1-t)^n-\beta-1 dt)} \right)
\]

\[
= \left(1 - \frac{\theta}{\theta_0}\right) \lim_{n \to \infty} \left( \frac{\Gamma(a+k_n+1)\Gamma(n+\beta-k_n-1)}{\Gamma(a+k_n)\Gamma(n+\beta-k_n)} \right)
\]

\[
= \left(1 - \frac{\theta}{\theta_0}\right) \lim_{n \to \infty} \left( \frac{\alpha + k_n}{\beta + n - k_n + 1} \right)
\]

\[
= \left(1 - \frac{\theta}{\theta_0}\right) \lim_{n \to \infty} \left( \frac{n + 1 - \frac{k_n}{n+1}}{\beta + n - k_n + 1} \right)
\]

\[
= 1,
\]

where we use the fact that \( \lim_{n \to \infty} k_n/n = \theta \). Since \( \lim_{n \to \infty} g (\theta|\langle n, k_n \rangle)/g (\theta|\langle n, k_n+1 \rangle) = 1 \) for all \( \theta \), the optimal policy following a poll having outcome \( k_n \) converges to that of the optimal policy following a poll having outcome \( k_n + 1 \). Thus, for any \( \varepsilon > 0 \), there exists a sufficiently large \( n \) such that \( y (k + 1) - y (k) < \varepsilon \).

**PROOF OF PROPOSITION 2:**

Consider a right-biased constituent with ideology \( b_i = b \). Clearly, when this constituent receives a signal \( s_i = 1 \), he can do no better than to report truthfully. For truth-telling to be incentive compatible, a constituent \( i \) receiving a signal \( s_i = 0 \) must prefer to report truthfully than to dissemble, that is,

\[
\sum_{k=0}^{n-1} \Pr (\langle n - 1, k \rangle | s_i = 0) \int_{0}^{1} (U (y (k), \theta, b) - U (y (k + 1), \theta, b)) g (\theta|\langle n, k \rangle) d\theta
\]

\[
= \sum_{k=0}^{n-1} \Pr (\langle n - 1, k \rangle | s_i = 0) ((y (k + 1) - y (k) - 2b) (y (k + 1) - y (k)))
\]
\[ \sum_{k=0}^{n-1} \Pr (\langle n-1, k \rangle | s_i = 0) \left( \frac{1}{n+\alpha+\beta} \right) \left( \frac{1}{n+\alpha+\beta} - 2b \right) \geq 0. \]

Therefore, for incentive compatibility, we must have

\[ (A2) \quad n \leq \frac{1}{2b} - \alpha - \beta. \]

An identical inequality occurs for left-biased constituents to tell the truth when \( s_i = 1 \). If the right-hand side of (A2) is negative, no poll (or equivalently a \( \bar{n} = 0 \) size poll) is consistent with a truth-telling equilibrium. If the right-hand side of (A2) is positive, then the largest poll where truth-telling is an equilibrium is \( \bar{n} = \left\lfloor \frac{1}{2b} - \alpha - \beta \right\rfloor \).

PROOF OF PROPOSITION 3:

Given the construction, we need to establish that no polled constituent can profitably deviate. We drop subscripts when doing so should not cause confusion. There are four cases to consider. First, consider the incentive constraint of a centrist with ideology \( b > 0 \). For truth-telling to be optimal requires that, having received the signal \( s = 0 \), the payoff to that constituent is greater by reporting \( m = 0 \) than from reporting \( m = 1 \); that is,

\[ \sum_{k=0}^{n-1} \Pr [\langle n-1, k \rangle | s = 0] (y(k+1) - y(k)) \times (y(k+1) + y(k) - 2E [\theta | \langle n-1, k \rangle, s = 0] - 2b) \geq 0. \]

Recall that optimality on the part of the policy maker requires that \( y(k) = E [\theta | \langle n, k \rangle] \) for all \( k \).

We make the following claim:

CLAIM 1: In a centrist-extremist equilibrium under stratified polling, for any constituent who is a centrist with signal \( s = 0 \), \( E [\theta | \langle n-1, k \rangle, s = 0] = E [\theta | \langle n, k \rangle] \).

PROOF OF CLAIM 1:

To establish the claim, notice that
\[ E[\theta \mid (n-1, k), s = 0] \]
\[ = \int_0^1 \theta^{(c-1)\rho - 1} (1-\theta)^{\rho-1}(1-\theta)^{-\beta-1} d\theta \]
\[ = \frac{k-\rho+\alpha}{c+\alpha+\beta} \]
\[ = E[\theta \mid (n, k)]. \]

Using this claim, the incentive constraint of a centrist with signal \( s = 0 \) reduces to

\[ \sum_{k=0}^{n-1} \text{Pr}[(n-1, k) \mid s = 0] (y(k+1) - y(k)) (y(k+1) - y(k) - 2b) \geq 0. \]

Clearly, this constraint is most binding for the right-most centrist. Substituting for \( y(k+1) - y(k) = 1/(n-2(n-r)+\alpha+\beta) \) and for \( b \) using the index \( r_j \) yields

\[ \sum_{k=0}^{n_j-1} \text{Pr}[(n_j-1, k) \mid s = 0] \left( \frac{1}{n_j-2(n_j-r_j)+\alpha+\beta} - 2 \left( \frac{2(r_j-1)-(n_j-1)}{(n_j-1)} \right) \right). \]

The sign of this expression depends solely on the sign of

\[ \frac{1}{n_j-2(n_j-r_j)+\alpha+\beta} - 2 \left( \frac{2(r_j-1)-(n_j-1)}{(n_j-1)} \right). \]

After substituting for \( n_j \) and \( r_j \), this expression reduces to zero. Hence, no centrist with signal \( s = 0 \) would profitably deviate.

Second, consider a centrist with signal \( s = 1 \) and ideology \( b < 0 \). The incentive constraint is

\[ \sum_{k=0}^{n-1} \text{Pr}[(n-1, k) \mid s = 1] (y(k+1) - y(k)) \]
\[ \times (2E[\theta \mid (n-1, k), s = 1] - y(k+1) - y(k) + 2b) \geq 0. \]

CLAIM 2: In a centrist-extremist equilibrium under stratified polling, for any constituent who is a centrist with signal \( s = 1 \), \( E[\theta \mid (n-1, k), s = 1] = E[\theta \mid (n, k+1)]. \)

PROOF OF CLAIM 2:
To establish the claim, notice that

\[
E \left[ \theta \mid \langle n - 1, k \rangle , s = 1 \right] = \frac{\int_0^1 \theta^{k-\rho} (1-\theta)^{c-1-(k-\rho)\theta} \times \theta^{\alpha-1}(1-\theta)^{\beta-1} \, d\theta}{\int_0^1 (c-1-(k-\rho)\theta)^{\alpha-1}(1-\theta)^{\beta-1} \, d\theta}
\]

\[
= \frac{k+1-\rho+\alpha}{\alpha+\beta+1}
\]

\[
= E \left[ \theta \mid \langle n, k + 1 \rangle \right].
\]

Hence, the incentive constraint of a centrist with signal \( s = 1 \) reduces to

\[
\sum_{k=0}^{n-1} \Pr \left[ \langle n - 1, k \rangle \mid s = 1 \right] \left( y(k+1) - y(k) \right) \left( y(k+1) - y(k) + 2b \right) \geq 0,
\]

and, by symmetry, this constraint is satisfied with equality.

Third, consider a right-wing extremist with signal \( s = 0 \). His incentive constraint is

\[
\sum_{k=0}^{n-1} \Pr \left[ \langle n - 1, k \rangle \mid s = 0 \right] \left( y(k+1) - y(k) \right)
\times \left( y(k+1) + y(k) - 2E \left[ \theta \mid \langle n - 1, k \rangle , s = 0 \right] - 2b \right)
\]

\[
< 0.
\]

CLAIM 3: In a centrist-extremist equilibrium under stratified polling, for any constituent who is a right-wing extremist with signal \( s = 0 \), \( E \left[ \theta \mid \langle n - 1, k \rangle , s = 0 \right] > E \left[ \theta \mid \langle n, k \rangle \right] \).

PROOF OF CLAIM 3:

To establish the claim, notice that

\[
E \left[ \theta \mid \langle n - 1, k \rangle , s = 0 \right] = \frac{\int_0^1 \theta^{k-\rho} (1-\theta)^{c-1-(k-\rho)\theta} \times \theta^{\alpha-1}(1-\theta)^{\beta-1} \, d\theta}{\int_0^1 (c-1-(k-\rho)\theta)^{\alpha-1}(1-\theta)^{\beta-1} \, d\theta}
\]

\[
= \frac{k-\rho+\alpha+1}{\alpha+\beta+1}.
\]

Define the function \( \psi(z) = (k - \rho + \alpha + z) / (c + \alpha + \beta + z) \), and notice that it is increasing in \( z \). Then

\[
E \left[ \theta \mid \langle n, k \rangle \right] = \psi(0)
\]

\[
< \psi(1) = E \left[ \theta \mid \langle n - 1, k \rangle , s = 0 \right].
\]

Hence, it follows that
\[ \sum_{k=0}^{n-1} \Pr[\langle n-1, k \rangle | s = 0] (y(k+1) - y(k)) \times (y(k+1) + y(k) - 2E[\theta | \langle n-1, k \rangle, s = 0] - 2b) \]

\[ < \sum_{k=0}^{n-1} \Pr[\langle n-1, k \rangle | s = 0] (y(k+1) - y(k))(y(k+1) - y(k) - 2b) \]

\[ < 0, \]

where the last inequality follows from the fact that right-wing extremists have higher values of \( b \) than do centrists. Hence, there is no profitable deviation.

Fourth, consider a left-wing extremist with signal \( s = 1 \). His incentive constraint is

\[ \sum_{k=0}^{n-1} \Pr[\langle n-1, k \rangle | s = 1] (y(k+1) - y(k)) \times (2E[\theta | \langle n-1, k \rangle, s = 1] - y(k+1) - y(k) + 2b) \]

\[ < 0. \]

CLAIM 4: In a centrist-extremist equilibrium under stratified polling, for any constituent who is a left-wing extremist with signal \( s = 1 \), \( E[\theta | \langle n-1, k \rangle, s = 1] < E[\theta | \langle n, k+1 \rangle] \).

PROOF OF CLAIM 4:

To establish the claim, observe that

\[ E[\theta | \langle n-1, k \rangle, s = 1] = \frac{\int_0^1 \theta (c^{k-\rho}(1-\theta)^{c-(k-\rho)}\theta^\alpha-1(1-\theta)^\beta-1d\theta}{\int_0^1 t^{k-\rho}(1-t)^{c-(k-\rho)}t^\alpha-1(1-t)^\beta-1dt} \]

\[ = \frac{k-\rho+\alpha+1}{c+\alpha+\beta+1}. \]

Next, notice

\[ E[\theta | \langle n-1, k \rangle, s = 1] = \frac{k-\rho+\alpha+1}{c+\alpha+\beta+1} \]

\[ < \frac{k-\rho+\alpha+1}{c+\alpha+\beta+1} \]

\[ = E[\theta | \langle n, k+1 \rangle]. \]

Hence, it follows that

\[ \sum_{k=0}^{n-1} \Pr[\langle n-1, k \rangle | s = 1] (y(k+1) - y(k)) \times (2E[\theta | \langle n-1, k \rangle, s = 1] - y(k+1) - y(k) + 2b) \]
\[
< \sum_{k=0}^{n-1} \Pr \left[ (n-1, k) \mid s = 1 \right] (y(k+1) - y(k)) (y(k+1) - y(k) + 2b) \\
< 0,
\]

where the last inequality follows from the fact that left-wing extremists have lower values of \( b \) than do centrists. Hence, there is no profitable deviation.

**PROOF OF PROPOSITION 6:**

We first establish that \( \phi \) is strictly decreasing when \( k < n/2 \) and is strictly increasing when \( k > n/2 \); the case where \( k = n/2 \) is impossible since \( n \) is odd. Fix two values, \( k \) and \( k' \), and evaluate \( \phi(\cdot) \) at these points; notice that

\[
\phi(k') - \phi(k) = 2 \left( (n-r)^2 + 3r^2 \right) (k-k') (n-k-k').
\]

When \( k < k' < n/2 \), then \( \phi(k') - \phi(k) < 0 \). When \( n/2 < k < k' \), then \( \phi(k') - \phi(k) > 0 \). Next, when \( k = r \) or \( n-r \), then \( \phi(k) = r (2r-n)^3 > 0 \). Hence, \( n-r < k_0 < k_1 < r \). Thus, we have shown that for \( k < k_0 \) and \( k > k_1 \), \( \phi(k) > 0 \) while for \( k \in [k_0, k_1] \), \( \phi(k) \leq 0 \).

**PROOF OF PROPOSITION 7:**

The proof relies on the following three lemmas.

**LEMMA 3:** \( E[\theta \mid (n, k+1)] < E[\theta \mid (n-1, k), s = 1] \).

**PROOF OF LEMMA 3:**

Fix the strategy of all individuals at \((q_c, q_r)\). Suppose the policy maker simultaneously conducts two polls: one consisting of surveying a single constituent, and one consisting of surveying \( n-1 \) constituents. Suppose the policy maker observes an outcome of 1 in the first poll and an outcome of \( k \) in the second poll. Clearly, this is equivalent to conducting a single poll consisting of \( n \) constituents and observing an outcome of \( k+1 \). Hence, \( E[\theta \mid (n, k+1)] = E[\theta \mid (1, 1), (n-1, k)] \).

Next, suppose that the policy maker learned when conducting the poll with the single constituent that the constituent was telling the truth. Then the change to the policy
maker’s posterior beliefs would be greater than if she were uncertain about the constituent’s truthfulness. Hence, \( E[\theta|\langle 1, 1 \rangle, \langle n - 1, k \rangle] < E[\theta|s = 1, \langle n - 1, k \rangle] \).

**Lemma 4:** \( E[\theta|\langle n, k \rangle] > E[\theta|\langle n - 1, k \rangle, s = 0] \).

**Proof of Lemma 4:**

Fix the strategy of all individuals at \((q_c, q_r)\). Suppose the policy maker simultaneously conducts two polls: one consisting of a single constituent, and one consisting of \(n - 1\) constituents. Suppose the policy maker observes an outcome of 0 in the first poll and an outcome of \(k\) in the second poll. Clearly, this is equivalent to conducting a single poll consisting of \(n\) constituents and observing an outcome of \(k\). Hence, \( E[\theta|\langle n, k \rangle] = E[\theta|\langle 1, 0 \rangle, \langle n - 1, k \rangle] \).

Next, suppose that the policy maker learned when conducting the poll with a single constituent that the constituent was telling the truth. Then the change to the policy maker’s posterior beliefs would be greater than if she were uncertain about the constituent’s truthfulness. Hence, \( E[\theta|\langle 1, 0 \rangle, \langle n - 1, k \rangle] > E[\theta|s = 0, \langle n - 1, k \rangle] \).

**Lemma 5:** In any centrist-extremist equilibrium, a constituent with index \(b_i\) is a right-wing extremist only if \(b_i > 0\). Likewise, a constituent with index \(b_i\) is a left-wing extremist only if \(b_i < 0\).

**Proof of Lemma 5:**

Suppose not. Consider the case of a constituent with index \(b_i > 0\) who is supposed to be a left-wing extremist. Then, when \(s_i = 1\), it must be the case that

\[
\sum_{k=0}^{n-1} \Pr[\langle n - 1, k \rangle|s_i = 1] \left( E[\theta|\langle n, k + 1 \rangle] - E[\theta|\langle n, k \rangle] \right) \\
\times \left( -E[\theta|\langle n, k + 1 \rangle] - E[\theta|\langle n, k \rangle] + 2E[\theta|\langle n - 1, k \rangle, s = 1] + 2b_i \right) < 0.
\]

However, since

\[
E[\theta|\langle n - 1, k \rangle, s = 1] \geq E[\theta|\langle n, k + 1 \rangle] \geq E[\theta|\langle n, k \rangle],
\]

it then follows that, for all \(k\),
\[ -E[\theta|\langle n, k+1 \rangle] - E[\theta|\langle n, k \rangle] + 2E[\theta|\langle n-1, k \rangle, s=1] + 2b_i \geq 0, \]

and hence

\[ \sum_{k=0}^{n-1} Pr[\langle n-1, k \rangle|s_i=1] (E[\theta|\langle n, k+1 \rangle] - E[\theta|\langle n, k \rangle]) \times (-E[\theta|\langle n, k+1 \rangle] - E[\theta|\langle n, k \rangle] + 2E[\theta|\langle n-1, k \rangle, s=1] + 2b_i) \geq 0, \]

which is a contradiction. The proof for the reverse case is identical.

From lemma 5, we may conclude that as \( q_c \to 0 \), \( q_r \to 1/2 \). As \( q_c \to 1 \), \( q_r \to 0 \).

We are now in a position to prove the main result. First, fix \( b_r \in [0,1] \). Consider the incentive compatibility constraint of a constituent with ideology \( b_l \) given in (21). Suppose \( b_l = -1 \). Substituting this value for \( b_l \) yields

\[ (-y(k+1) - y(k) + 2E[\theta|\langle n-1, k \rangle, s=1] + 2b_l) \]
\[ = (-y(k+1) - y(k) + 2E[\theta|\langle n-1, k \rangle, s=1] - 2) < 0, \]

because \( y(k+1), y(k), E[\theta|\langle n-1, k \rangle, s=1] > 0 \). Because \( (y(k+1) - y(k)) > 0 \), it follows that the left-hand side of (21) is negative. Now suppose \( b_l = 0 \). Then substituting this value for \( b_l \) yields

\[ (-y(k+1) - y(k) + 2E[\theta|\langle n-1, k \rangle, s=1] + 2b_l) \]
\[ = -E[\theta|\langle n, k+1 \rangle] - E[\theta|\langle n, k \rangle] + 2E[\theta|\langle n-1, k \rangle, s=1] \]
\[ > 0, \]

where the inequality follows from the fact that \( E[\theta|\langle n, k \rangle] < E[\theta|\langle n, k+1 \rangle] < E[\theta|\langle n-1, k \rangle, s=1] \) established in Lemma 3. Thus, the left-hand side of (21) is positive. Finally, since the left-hand side of (21) is continuous in \( b_l \) it follows from the intermediate value theorem that there exists a value for \( b_l \) such that

\[ \sum_{k=0}^{n-1} (Pr[\langle n-1, k \rangle|s=1] (E[\theta|\langle n, k+1 \rangle] - E[\theta|\langle n, k \rangle]) \times (-E[\theta|\langle n, k+1 \rangle] - E[\theta|\langle n, k \rangle] + 2E[\theta|\langle n-1, k \rangle, s=1] + 2b_l)) \]
Second, fix \( b_r \in [-1, 0] \). Consider the incentive compatibility constraint of a constituent with ideology \( b_r \) given in (20). When \( b_r = 0 \), then

\[
(y (k + 1) + y (k) - 2E [\theta | \langle n - 1, k \rangle, s = 0] - 2b_r) \\
= y (k + 1) + y (k) - 2E [\theta | \langle n - 1, k \rangle, s = 0] \\
> 0,
\]

because \( E [\theta | \langle n, k + 1 \rangle] > E [\theta | \langle n, k \rangle] > E [\theta | \langle n - 1, k \rangle, s = 0] \), which was established in Lemma 4. It then follows that the left-hand side of (20) is positive. When \( b_r = 1 \), then

\[
(y (k + 1) + y (k) - 2E [\theta | \langle n - 1, k \rangle, s = 0] - 2b_r) \\
= y (k + 1) + y (k) - 2E [\theta | \langle n - 1, k \rangle, s = 0] - 2 \\
< 0,
\]

and the left-hand side of (20) is negative. Finally, since the incentive compatibility constraint is continuous in \( b_r \) it follows from the intermediate value theorem that there exists a value for \( b_r \) such that

\[
\sum_{k=0}^{n-1} (\text{Pr}[\langle n - 1, k \rangle | s = 0] (E [\theta | \langle n, k + 1 \rangle] - E [\theta | \langle n, k \rangle])) \\
\times (E [\theta | \langle n, k + 1 \rangle] + E [\theta | \langle n, k \rangle] - 2E [\theta | \langle n - 1, k \rangle, s = 0] - 2F^{-1} (1 - q_r))) \\
= 0.
\]

Combining the first and second arguments together with Kakutani’s fixed point theorem implies the existence of a centrist-extremist equilibrium.

**PROOF OF PROPOSITION 8:**

Consider a convergent subsequence of \( \{q_{c,n}\} \); note that because \( \{q_{c,n}\} \) is bounded, such a subsequence exists. Call this subsequence \( \{q_{c,n_i}\} \) with subsequence limit point \( L \). We claim that for such subsequences, the subsequence limit \( L = 0 \). To see this, suppose to the contrary that the limit of a convergent subsequence was \( \lim_{n \to \infty} \{q_{c,n_i}\} = L > 0 \). In that
case, either $\lim_{n \to \infty} \{q_{r,n}\} < 1/2$, or $\lim_{n \to \infty} (1 - \{q_{r,n}\} - \{q_{c,n}\}) < 1/2$. Suppose, without loss of generality that $\lim_{n \to \infty} \{q_{r,n}\} < 1/2$. It then follows that the associated bound on the ideology of the right-most centrist satisfies $\lim_{n \to \infty} \{b_{r,n}\} = \bar{b}_r > 0$.

Now observe that the incentive constraint of the right-most centrist, given in (20), may be written as

$$(A3) \quad b_r = \left[ \sum_{k=0}^{n-1} \Pr \left[ \langle n-1, k \rangle | \delta = 0 \right] \left( E[\theta|\langle n, k+1 \rangle] - E[\theta|\langle n, k \rangle] \right) \times \left( E[\theta|\langle n, k+1 \rangle] + E[\theta|\langle n, k \rangle] - 2E[\theta|\langle n-1, k \rangle, \delta = 0] \right) \right] / \left[ 2 \sum_{k=0}^{n-1} \Pr \left[ \langle n-1, k \rangle | \delta = 0 \right] \left( E[\theta|\langle n, k+1 \rangle] - E[\theta|\langle n, k \rangle] \right) \right].$$

Since $\lim_{n \to \infty} \{q_{c,n}\} = L > 0$, it follows from the strong law of large numbers that

$$\Pr (\lim_{n \to \infty} (E[\theta|\langle n, k \rangle] - \theta) = 0) = \Pr (\lim_{n \to \infty} (E[\theta|\langle n-1, k \rangle, \delta = 0] - \theta) = 0) = 1.$$  

Therefore, $\Pr (\lim_{n \to \infty} (E[\theta|\langle n, k \rangle] - E[\theta|\langle n-1, k \rangle, \delta = 0]) = 0) = 1$. The relevant incentive constraint thus simplifies in the limit, and it follows almost surely that

$$\lim_{n \to \infty} \frac{\sum_{k=0}^{n-1} \Pr[\langle n-1,k \rangle|\delta=0](E[\theta|\langle n,k+1 \rangle]-E[\theta|\langle n,k \rangle])^2}{2 \sum_{k=0}^{n-1} \Pr[\langle n-1,k \rangle|\delta=0](E[\theta|\langle n,k+1 \rangle]-E[\theta|\langle n,k \rangle])} = 0,$$

which is a contradiction the fact that $\bar{b}_r > 0$.

Thus, every convergent subsequence of $\{q_{c,n}\}$ converges to zero. It then follows from Walter Rudin (1976, 51) that if every subsequence of $\{q_{c,n}\}$ converges to $q$, then $\{q_{c,n}\}$ converges to $q$. This result implies $\{q_{c,n}\}$ converges to zero.

**PROOF OF PROPOSITION 9:**

If information aggregates, then $\lim_{n \to \infty} \Pr [\|Y_n - \theta\| < \varepsilon] = 1$, where $Y_n$ is the equilibrium policy rule (which depends on the realization $k$) for a poll of size $n$. Suppose to the contrary that information aggregation does not occur, then either the expected number of centrists is finite in the limit or the sampling variance overwhelms the information contained in the reports of an unbounded number of centrists. We will show that either case contradicts the
result that the fraction of centrists goes to zero in the limit. To see this, recall that the incentive compatibility condition for the right-most centrist in (20) may be written as

\[(A4) \quad F^{-1}(1 - q_r) = \left[ \sum_{k=0}^{n-1} \Pr[\langle n-1, k \rangle | s = 0] \left( E[\theta|\langle n, k+1 \rangle] - E[\theta|\langle n, k \rangle] \right) \right. \]
\[\times \left. \left( E[\theta|\langle n, k+1 \rangle] + E[\theta|\langle n, k \rangle] - 2E[\theta|\langle n-1, k \rangle, s = 0] \right) \right] \]
\[/ \left[ 2 \sum_{k=0}^{n-1} \Pr[\langle n-1, k \rangle | s = 0] \left( E[\theta|\langle n, k+1 \rangle] - E[\theta|\langle n, k \rangle] \right) \right].\]

Since information does not aggregate, a given constituent’s signal affects his posterior beliefs even conditional on the given outcome of the poll. This implies

\[\lim_{n \to \infty} (E[\theta|\langle n, k \rangle] - E[\theta|\langle n-1, k \rangle, s = 0]) > 0,\]

and hence

\[\lim_{n \to \infty} (E[\theta|\langle n, k+1 \rangle] + E[\theta|\langle n, k \rangle] - 2E[\theta|\langle n-1, k \rangle, s = 0]) > 0\]

for all \(k\). Let

\[\delta = \min_k \left( \lim_{n \to \infty} (E[\theta|\langle n, k+1 \rangle] + E[\theta|\langle n, k \rangle] - 2E[\theta|\langle n-1, k \rangle, s = 0]) \right).\]

It follows that if we let \(RHS\) denote the right-hand side of (A4), then \(\lim_{n \to \infty} RHS \geq \delta/2 > 0\). Proposition 8 established that \(\lim_{n \to \infty} \{q_c,n\} = 0\), which implies \(\lim_{n \to \infty} \{q_r,n\} = 1/2\) from the symmetry of \(F\). Hence, the left-hand side of (A4) equals zero, which is a contradiction.

**PROOF OF PROPOSITION 11:**

Let \(k_{piv}^n\) denote the \(K\) rule used for a poll of size \(n\). It follows from the optimality of the policy maker’s choice that under truth-telling

\[k_{piv}^n = \left[ \frac{(y_1 + y_0)}{2} \right] (n + \alpha + \beta) - \alpha].\]

Without loss of generality, fix a constituent’s ideology \(b > 0\), and consider the case where this constituent receives a signal \(s = 0\). We will show that for \(n\) sufficiently large, truth-telling will not be incentive compatible. The constituent’s incentive constraint is
\[
\Pr \left[ \langle n-1, k_{\text{piv}}^n \rangle | s = 0 \right] (y_1 - y_0) (y_1 + y_0 - 2E \left[ \theta \mid \langle n-1, k_{\text{piv}}^n \rangle, s = 0 \right] - 2b).
\]

This expression takes the sign of
\[
y_1 + y_0 - 2E \left[ \theta \mid \langle n-1, k_{\text{piv}}^n \rangle, s = 0 \right] - 2b.
\]

Using the fact that, under truth-telling \( E \left[ \theta \mid \langle n-1, k_{\text{piv}}^n \rangle, s = 0 \right] = E \left[ \theta \mid \langle n, k_{\text{piv}}^n \rangle \right] \) and (2), we obtain
\[
y_1 + y_0 - 2 \left( \frac{k_{\text{piv}}^n + \alpha}{n + \alpha + \beta} \right) - 2b.
\]

On substituting \( k_{\text{piv}}^n \), we obtain
\[
y_1 + y_0 - 2 \left( \frac{\left( \frac{n+y_1+y_0}{2} \right)(n+\alpha+\beta)-\alpha}{n+\alpha+\beta} + \alpha \right) - 2b
\]
\[
< y_1 + y_0 - 2 \left( \frac{\left( \frac{n+y_1+y_0}{2} \right)(n+\alpha+\beta)-\alpha-1+\alpha}{n+\alpha+\beta} \right) - 2b
\]
\[
= 2 \left( \frac{1}{n+\alpha+\beta} \right) - 2b,
\]

where the strict inequality follows from the definition of an integer floor function. Notice that for a given \( b > 0 \), there exists \( \bar{n} \) such that for all \( n > \bar{n} \) the above expression is strictly negative. This, however, implies that the constituent can profitably deviate from truth-telling.

**PROOF OF PROPOSITION 12:**

Following Feddersen and Pesendorfer (1997), define
\[
v(\theta, b) = -(y_1 - (\theta + b))^2 + (y_0 - (\theta + b))^2
\]
\[
= (y_1 - y_0) (2b + 2\theta - y_0 - y_1),
\]

and notice that \( v(\theta, b) \) is increasing linearly in \( \theta \) and \( b \).

Next, observe that, for all \( \theta \), \( v(\theta, -1) < 0 \) and \( v(\theta, 1) > 0 \). This implies that there is a set of types \([-1, -1 + \varepsilon] \) for whom it is a dominant strategy to report \( m = 0 \) and similarly a set of types \([1 - \varepsilon, 1] \) for whom it is a dominant strategy to always report \( m = 1 \).
Next, the incentive compatibility condition requires that there exists a constituent type such that $E[v(\theta, b) | \langle n - 1, K \rangle, s = 0, b] = 0$, where $b$ denotes a putative equilibrium strategy, $(b_l, b_r)$, played by all the other constituents. Since $v$ is increasing in $b$, then, together with the conditions on $v(\theta, -1)$ and $v(\theta, 1)$ implies that there exists a single value $b'_r$ such that the above expression is equal to zero. Similarly, there exists a unique value $b'_l$ such that $E[v(\theta, b) | \langle n - 1, K \rangle, s = 1, b] = 0$. Finally, since the posterior distribution $G(\theta | \langle n - 1, K \rangle, s, b)$ is stochastically ordered in $s$, it follows that $b'_l < b'_r$.

Hence, for any putative equilibrium strategy $b$, a unique pair of cutpoints result. Next, define the function that yields the unique set of cutpoints associated with the putative equilibrium strategy $b$ as $\psi$, and notice that $\psi : b \rightarrow b$. Furthermore, it is apparent that $\psi$ is continuous in the vector $b$ (from the Theorem of the Maximum). Hence, by Kakutani’s fixed point theorem, this mapping has a fixed point which comprises a non-degenerate centrist-extremist equilibrium.

**PROOF OF PROPOSITION 13:**

Suppose not. Then there exists a centrist-extremist equilibrium where $c > \bar{n}$ constituents are centrists and the remainder are extremists. Suppose that constituent $i$ with ideology $b_i = b$ and signal $s_i = 0$ is a centrist. Then, under the putative equilibrium, such a constituent must weakly prefer to reveal truthfully than to report $m_i = 1$. This choice requires that

$$\sum_{k=0}^{c-1} \Pr(\langle c - 1, k \rangle | s_i = 0) \int_0^1 (U(y(k), \theta, b)) g(\theta | \langle n, k \rangle) d\theta$$

$$\geq \sum_{k=0}^{c-1} \Pr(\langle c - 1, k \rangle | s_i = 0) \int_0^1 (U(y(k + 1), \theta, b)) g(\theta | \langle n, k \rangle) d\theta.$$  

In the quadratic loss specification, this inequality reduces to the condition that

$$\sum_{k=0}^{c-1} \Pr(\langle c - 1, k \rangle | s_i = 0) \left( \frac{1}{c + \alpha + \beta} - 2b \right) \left( \frac{1}{c + \alpha + \beta} \right) \geq 0,$$

which further reduces to $1/(c + \alpha + \beta) \geq 2b$. However, since $c > \bar{n}$, it follows that

$$\frac{1}{c + \alpha + \beta} \leq \frac{1}{(\bar{n} + 1) + \alpha + \beta},$$
and, by the definition of $\bar{n}$,

$$\frac{1}{((\bar{n}+1)+\alpha+\beta)} < 2b.$$ 

Therefore, $1/(c + \alpha + \beta) < 2b$, which is a contradiction.