Methodological Appendix

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“The Interaction of Public and Private Insurance: Medicaid and the Long-Term Care Insurance Market”

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Our model considers a 65-year old individual who chooses an optimal consumption path to maximize expected discounted lifetime utility. The per-period utility function is defined on a monthly basis, with a maximum lifespan of 105, resulting in 480 periods denoted by t. In each month, the individual may be in one of five possible states of care, denoted by s: (1) at home receiving no care, (2) at home receiving paid home health care, (3) in residence at an assisted living facility, (4) in residence in a nursing home, or (5) dead. The cumulative probability of being in each state of care s at time t is denoted $Q_{t,s}$. Utility is a function of ordinary consumption $C_{s,t}$ as well as the consumption value (if any) derived from long-term care expenditures $F_{s,t}$. The individual discounts future utility at the monthly time preference rate $\rho$.

The general model also permits the consumption value of long-term care expenditures to vary depending on whether they are paid by Medicaid or by private insurance. We capture this difference in consumption value through the parameter $\alpha_s$. In particular, if $\alpha_s=1$, the assumption is that the consumption value of care is the same whether paid for by Medicaid or from private insurance. In contrast, $\alpha_s>1$ would be consistent with a model in which private insurance allows one to purchase higher quality care, which thus provides higher consumption value. Although the baseline model assumes $\alpha_s=1$, we discuss results for $\alpha_s>1$ in section 6.2.

The consumer’s utility function is therefore:

\[
U\left( C_{s,t} + I_{s,t}^M \cdot F_{s,t} + \left(1 - I_{s,t}^M\right) \cdot \alpha_s \cdot F_{s,t} \right)
\]
where $I^M_{s,t}$ is an indicator variable for whether or not the person is receiving Medicaid while in state $s$ in period $t$. We assume that the utility function exhibits constant relative risk aversion, such that:

$$U(C_{s,t} + I^M_{s,t} \cdot F_{s,t} + (1 - I^M_{s,t}) \cdot \alpha_s \cdot F_{s,t}) = \frac{\left(\frac{C_{s,t} + I^M_{s,t} \cdot F_{s,t} + (1 - I^M_{s,t}) \cdot \alpha_s \cdot F_{s,t}}{1 - \gamma}\right)^{1/\gamma} - 1}{1 - \gamma}$$

The consumer’s constrained dynamic optimization problem is therefore:

$$\max \sum_{s,t=1}^{480} \frac{Q_{s,t}}{(1 + \rho)^t} \cdot U(C_{s,t} + I^M_{s,t} \cdot F_{s,t} + (1 - I^M_{s,t}) \cdot \alpha_s \cdot F_{s,t})$$

subject to

(Ai) $W_0$ is given

(Aii) $W_t \geq 0 \ \forall \ t$

(Aiii) $W_{t+1} = [W_t + A_t + \min(B_{s,t}, X_{s,t}) - C_{s,t} - X_{s,t} - P_{s,t}] (1 + r)$ if $I^M_{s,t} = 0$

(Aiv) $W_{t+1} = [W_t - \max(W_t - W, 0) + (C - C_t)] (1 + r)$ if $I^M_{s,t} = 1$

where $W_0$ is pre-determined financial wealth at 65, $A_t$ denotes annuity income, $B_{s,t}$ denotes the daily benefit cap on the private insurance payments, $X_{s,t}$ denotes long-term care expenditures, $P_{s,t}$ denotes the premium on the private insurance policy, and $r$ is the monthly real rate of interest.

To be eligible for Medicaid (i.e. $I^M_{s,t} = 1$), the individual must:

(i) Be receiving care, i.e., $s \in \{2,3,4\}$

(ii) Meet the asset test, i.e., $W_t < W$

(iii) Meet the income test: $A_t + \min(B_{s,t}, X_{s,t}) + r^t W_{t-1} - X_{s,t} < C_t$

Where $W$ is the asset eligibility threshold and $C_t$ is the income eligibility threshold for care state $s$. Note that Medicaid eligibility at any given point in time is thus endogenous to consumption choices.

The solution to the constrained dynamic optimization problem (A1) involves the choice of a
consumption plan at time 0, with the consumer’s knowledge that he will be able to choose a new plan at time 1, and so on, until the final period. To solve this stochastic dynamic decision problem, we employ stochastic dynamic programming methods, as discussed in Blanchard & Fischer (1989) which reduce the multi-period problem to a sequence of simpler two-period decision problems. We begin by introducing a value function $V_{s,t}(W_t; A)$ for state $s$ and time $t$ that represents the present discounted value of expected utility evaluated along the optimal consumption path. This value depends on financial wealth ($W_t$), annuity income ($A_t$), and state of care ($s$) in which the individual finds himself, all at the start of period $t$.

The value function satisfies the recursive Bellman equation:

$$\begin{align*}
\max_{C_{s,t}} V_{s,t}(W_t; A) = & \max_{C_{s,t}} \left( C_{s,t} + I_{s,t}^M \cdot F_{s,t} + \left( 1 - I_{s,t}^M \right) \cdot \alpha_s \cdot F_{s,t} \right) + \sum_{\sigma=1}^{5} q_{s,t+1}^{s,\sigma} \left( 1 + \rho \right) V_{\sigma,t+1}(W_{t+1}; A) \\
\end{align*}$$

where $q_{s,t+1}^{s,\sigma}$ the conditional probability that an individual who is in care state $s$ at time $t$ is in care state $\sigma$ at time $t+1$.

We solve this problem using standard dynamic programming techniques (e.g. Stokey and Lucas, 1989). We begin by solving for the last period’s problem at age 105, which produces a matrix of optimal consumption decisions, one for each combination of discrete value of wealth and state of care. We discretize wealth quite finely, down to $10$ increments at low levels of wealth, and gradually rising at higher levels of wealth, but never exceeding 0.2% of starting wealth. (Thus for example, for the median household, for whom initial financial wealth is approximately $89,000$, the maximum distance between two points on the financial wealth grid is $130$.) In the final period of life, age 105, all remaining wealth is consumed, which maps into a value function matrix that is $N_w \times N_s$, where $N_w$ is the number of discrete wealth points evaluated on the grid (for a median wealth household, $N_w$ is over $1,400$) and $N_s = 4$ (assuming no bequest motives, only 4 of the 5 states of the world have value).
For each element in the state spaces, we continue to solve the model backwards, collecting separate decision rules and value functions for every month-by-care-state combination back to age 65. Given our discretization methods and the number of periods and states in the problem, a single set of parameters involves solving our model for approximately 3.5 million discrete points. This is implemented using a program written for Gauss.

References
