Appendix - Derivation of the Likelihood Function

First note that if driver $i$ stops after trip $t$, he did not stop after the first $t - 1$ trips. The probability of observing driver $i$ stop after the $t^{th}$ trip conditional on a particular value of $T_{ij}$ is

$$Q_{ijt}|T_{ij} = (1 - (P_{ijt}|T_{ij})) \prod_{k=1}^{t-1} P_{ijk}|T_{ij}$$

(A.1) $$= (1 - \Phi[X_{ijt}\beta + \delta I[T_{ij} > Y_{ij}]] \prod_{k=1}^{t-1} \Phi[X_{ijk}\beta + \delta I[T_{ij} > Y_{ijk}]]$$

In order to derive the unconditional shift probability, I use the distribution of $T_{ij}$ to “integrate out” the random component in the reference level ($\mu_{ij}$ in equation (8)) as follows. For a driver who stops after the $t^{th}$ trip, there are $t+1$ possible intervals for the reference level of income to fall relative to accumulated income after each trip during the shift. The reference level of income may be

- less than $Y_{ij1}$,

- in one of the $t$-1 intervals $Y_{ij(k-1)} < T_{ij} \leq Y_{ijk}$, or

- above $Y_{ijt}$.

Suppose driver $i$ on shift $j$ stops after trip $t_{ij}$. Using the information on accumulated income after each trip on a shift, the unconditional shift probability associated with driver $i$ on shift $j$ is

$$Q_{ij} = (Q_{ijt_{ij}}|T_{ij} \leq Y_{ij1}) \cdot Pr(T_{ij} \leq Y_{ij1})$$

$$+ \sum_{h=2}^{t_{ij}} [(Q_{ijt_{ij}}|Y_{ij(h-1)} < T_{ij} < Y_{ijh}) \cdot Pr(Y_{ij(h-1)} \leq T_{ij} < Y_{ijh})]$$
The conditional shift probabilities in this expression follow from equation A.1:

- The probability of observing a driver stop after trip \( t_{ij} \) conditional on the reference income level being less than income after the first trip is

\[
Q_{ijt_{ij}}| (T_{ij} \leq Y_{ij1}) = (1 - \Phi[X_{ijt_{ij}} \beta]) \cdot \prod_{k=1}^{t_{ij}-1} \Phi[X_{ijk} \beta].
\]

- The probability of observing a driver stop after trip \( t_{ij} \) conditional on the reference income level being in the one of the \( t_{ij} - 1 \) possible intervals \( Y_{ij(h-1)} \) to \( Y_{ijh} \) is

\[
Q_{ijt_{ij}}| (Y_{ij(h-1)} < T_{ij} \leq Y_{ijh}) = (1 - \Phi[X_{ijt_{ij}} \beta]) \cdot \prod_{k=1}^{h-1} \Phi[X_{ijk} \beta + \delta] \cdot \prod_{k=h}^{t_{ij}-1} \Phi[X_{ijk} \beta].
\]

- The probability of observing a driver stop after trip \( t \) conditional on the reference income level being greater than income after trip \( t \) is

\[
Q_{ijt_{ij}}| (T_{ij} > Y_{ijt}) = (1 - \Phi[X_{ijt_{ij}} \beta + \delta]) \cdot \prod_{k=1}^{t_{ij}-1} \Phi[X_{ijk} \beta + \delta].
\]

It remains to write the probabilities of the reference income falling in each of the \( t + 1 \) intervals. These probabilities follow from the definition of \( T_{ij} \) in equation (8):

- The probability that the reference income level is no greater than income after the first trip is

\[
Pr(T_{ij} \leq Y_{ij1}) = \Phi[(Y_{ij1} - \theta_i)/\sigma].
\]

- The probability that the reference income level lies in one of the \( t - 1 \) possible intervals \( Y_{ij(k-1)} \) to \( Y_{ijk} \) is

\[
Pr(Y_{ij(k-1)} < T_{ij} \leq Y_{ijk}) = Pr(T_{ij} \leq Y_{ijk}) - Pr(T_{ij} \leq Y_{ij(k-1)})
\]

\[
= \Phi[(Y_{ijk} - \theta_i)/\sigma] - \Phi[(Y_{ij(k-1)} - \theta_i)/\sigma].
\]
The probability that the reference income level is greater than the income after trip $t$ is

$$Pr(T_{ij} > Y_{ijt}) = 1 - \Phi[(Y_{ijt} - \theta_i)/\sigma].$$

The probabilities defined in equations A.3-A.8 specify the components of the unconditional probability $Q_{ij}$, defined in equation A.2, for driver $i$ observed to end shift $j$ after trip $t_{ij}$. Assuming each shift for a driver is an independent observation, the likelihood function appropriate to this model is defined as

$$L = \prod_{i=1}^{n} \prod_{j=1}^{m_i} Q_{ij},$$

where $n$ denotes the number of drivers in the sample and $m_i$ is the number of shifts for driver $i$ in the sample.