Model appendix
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1 Decision problems of firms and households and equilibrium conditions

1.1 Final goods producers

The final good $Y_t$ is a composite made of a continuum of intermediate goods $Y_t(i)$ as in Kimball (1995). The final good producers buy intermediate goods on the market, package $Y_t$, and resell it to consumers, investors and the government in a perfectly competitive market.

The final good producers maximize profits. Their problem is:

$$\max_{Y_t, Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

subject to:

$$\int_0^1 G \left( \frac{Y_t(i)}{Y_t}, \lambda_{p,t} \right) di = 1 \quad (\mu_{f,t})$$

where $P_t$ and $P_t(i)$ are the price of the final and intermediate goods respectively, and $G$ is a strictly concave and increasing function characterised by $G(1) = 1$. $\epsilon_t^p$ is an exogenous process that reflects shocks to the aggregator function that result in changes in the elasticity of demand and therefore in the markup. We will constrain $\epsilon_t^p \in (0, \infty)$. $\epsilon_t^p$ follows the exogenous ARMA process:

$$\ln \epsilon_t^p = (1 - \rho_p) \ln \epsilon_{t-1}^p + \rho_p \ln \epsilon_{t-1}^p - \theta_p \eta^p_{t-1} + \eta^p_t, \quad \eta_{p,t} \sim N(0, \sigma_p)$$

To simplify notation, in what follows we leave out this argument.

The FOCs are:

$$\left( \partial Y_t \right)$$

$$P_t = \frac{\mu_{f,t}}{Y_t} \int_0^1 G' \left( \frac{Y_t(i)}{Y_t} \right) \frac{Y_t(i)}{Y_t} di$$

$$\left( \partial Y_t(i) \right)$$

$$P_t(i) = \mu_{f,t} G' \left( \frac{Y_t(i)}{Y_t} \right) \frac{1}{Y_t}$$

resulting in

$$Y_t(i) = Y_t G' \left[ \frac{P_t(i)}{P_t} \int_0^1 G' \left( \frac{Y_t(i)}{Y_t} \right) \frac{Y_t(i)}{Y_t} di \right]$$

As in Kimball (1995), the assumptions on $G()$ imply that the demand for input $Y_t(i)$ is decreasing in its relative price, while the elasticity of demand is a positive function of the relative price (or a negative function of the relative output).
1.2 Intermediate goods producers

Intermediate good producer \( i \) uses the following technology:

\[
Y_t(i) = \varepsilon_t^a K_t^s(i)^\alpha \left[ \gamma^l L_t(i) \right]^{1-\alpha} - \gamma^l \Phi
\]

where \( K_t^s(i) \) is capital services used in production, \( L_t(i) \) is aggregate labour input and \( \Phi \) is a fixed cost. \( \gamma^l \) represents the labour-augmenting deterministic growth rate in the economy and \( \varepsilon_t^a \) is total factor productivity and follows the process:

\[
\ln \varepsilon_t^a = (1 - \rho_2) \ln \varepsilon_{t-1}^a + \rho_2 \ln \varepsilon_{t-1}^a + \eta_t^a, \eta_t^a \sim N(0, \sigma_a)
\]

The firm’s profit is given by:

\[
P_t(i)Y_t(i) - W_t L_t(i) - R_k^t K_t(i).
\]

where \( W_t \) is the aggregate nominal wage rate and \( R_k^t \) is the rental rate on capital.

Cost minimization yields the conditions:

\[
(\partial L_t(i)) : \Theta_t(i) \gamma^{(1-\alpha)l} (1 - \alpha) \varepsilon_t^a K_t^s(i)^\alpha L_t(i)^{-\alpha} = W_t
\]

\[
(\partial K_t^s(i)) : \Theta_t(i) \gamma^{(1-\alpha)l} \alpha \varepsilon_t^a K_t^s(i)^{\alpha-1} L_t(i)^{1-\alpha} = R_k^t
\]

where \( \Theta_t(i) \) is the Lagrange multiplier associated with the production function and equals marginal cost \( MC_t \).

Combining these FOCs and noting that the capital-labour ratio is equal across firms implies:

\[
K_t^s = \frac{\alpha}{1 - \alpha} \frac{W_t}{R_k^t} L_t
\]

The marginal cost \( MC_t \) is the same for all firms and equal to:

\[
MC_t = \alpha^{-(1-\alpha)} (1 - \alpha)^{-1} W_t^{1-\alpha} R_k^{\alpha(1-\alpha)} \gamma^{-(1-\alpha)} \left( \varepsilon_t^a \right)^{-1}
\]

Under Calvo pricing with partial indexation, the optimal price set by the firm that is allowed to re-optimize results from the following optimisation problem:

\[
\max_{\tilde{P}_t(i)} \mathbb{E}_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \mathbb{E}_{t+s} P_t \left[ \tilde{P}_t(i) \left( \Pi_{t=1}^{\infty} \pi_t^{1-\xi_p^s} \right) - MC_{t+s} \right] Y_{t+s}(i)
\]

s.t. \( Y_{t+s}(i) = Y_t X_{t+s}^{\gamma^{s-1}} \left( \frac{P_t(i) X_{t+s}}{P_{t+s}} \right) \pi_{t+s} \)

where \( \tilde{P}_t(i) \) is the newly set price, \( \xi_p \) is the Calvo probability of being allowed to optimise
one’s price, $\pi_t$ is inflation defined as $\pi_t = P_t/P_{t-1}$, $[\frac{\beta^s \pi_{t+s} P_t}{\pi^{1-\eta} P_t}]$ is the nominal discount factor for firms (which equals the discount factor for the households that are the final owners of the firms), $\tau_t = \int_0^1 G'(\frac{Y_t(i)}{Y_t}) \frac{Y_t(i)}{Y_t} di$ and

$$X_{t,s} = \begin{cases} 1 & \text{for } s = 0 \\ (\Pi_{s=1}^s \pi^{1-\eta}_{t+s-1}) & \text{for } s = 1, \ldots, \infty \end{cases}$$

The first-order condition is given by:

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\beta^s \pi_{t+s} P_t}{\pi^{1-\eta} P_t} Y_{t+s}(i) \left[ X_{t,s} \tilde{P}_t(i) + \left( \tilde{P}_t(i) X_{t,s} - MC_{t+s} \right) \frac{1}{G^{r-1}(z_{t+s}) G''(x_{t+s})} - \right] = 0 \tag{7}$$

where $x_t = G^{r-1}(z_t)$ and $z_t = \frac{P_t(i)}{P_t} - \tau_t$.

The aggregate price index is in this case given by:

$$P_t = (1 - \xi_p) P_t(i) G^{r-1} \left[ \frac{P_t(i) \tau_t}{P_t} \right] + \xi_p \pi^{1-\eta} P_{t-1} G^{r-1} \left[ \frac{\pi^{1-\eta} P_{t-1} \tau_t}{P_t} \right] \tag{8}$$

### 1.3 Households

Household $j$ chooses consumption $C_t(j)$, hours worked $L_t(j)$, bonds $B_t(j)$, investment $I_t(j)$ and capital utilisation $Z_t(j)$, so as to maximise the following objective function:

$$E_t \sum_{s=0}^{\infty} \beta^s \left[ \frac{1}{1 - \sigma_c} (C_{t+s}(j) - \lambda C_{t+s-1})^{1 - \sigma_c} \right] \exp \left( \frac{\sigma_c - 1}{1 + \sigma_c} L_{t+s}(j)^{1 + \sigma_c} \right)$$

subject to the budget constraint:

$$C_{t+s}(j) + I_{t+s}(j) + \frac{B_{t+s}(j)}{\varepsilon_t R_{t+s} P_{t+s}} - T_{t+s} \leq \frac{B_{t+s-1}(j)}{P_{t+s}} + \frac{W^h_{t+s}(j) L_{t+s}(j)}{P_{t+s}} + \frac{R^k_{t+s} Z_{t+s}(j) K_{t+s-1}(j)}{P_{t+s}} - a(Z_{t+s}(j)) K_{t+s-1}(j) + \frac{Div_{t+s}}{P_{t+s}} \tag{9}$$

and the capital accumulation equation:

$$K_t(j) = (1 - \delta) L_{t-1}(j) + \varepsilon_t \left[ 1 - S \left( \frac{I_t(j)}{I_{t-1}(j)} \right) \right] I_t(j)$$

There is external habit formation captured by the parameter $\lambda$. The one-period bond is expressed on a discount basis. $\varepsilon_t^b$ is an exogenous premium in the return to bonds, which might reflect inefficiencies in the financial sector leading to some premium on the deposit rate versus the risk free rate set by the central bank, or a risk premium that households require to hold the one period bond. $\varepsilon_t^b$ follows the stochastic process:

$$\ln \varepsilon_t^b = \rho_b \ln \varepsilon_{t-1}^b + \eta_t^b, \eta_t^b \sim N(0, \sigma_b) \tag{10}$$
\( \delta \) is the depreciation rate, \( S(\cdot) \) is the adjustment cost function, with \( S(\gamma) = 0, \quad S'(\gamma) = 0, \quad S''(\cdot) > 0 \), and \( \varepsilon_i^t \) is a stochastic shock to the price of investment relative to consumption goods and follows an exogenous process:

\[
\ln \varepsilon_i^t = \rho_i \ln \varepsilon_{i-1}^t + \eta_i^t, \quad \eta_i^t \sim N(0, \sigma_i)
\]

\( T_{t+s} \) are lump sum taxes or subsidies and \( \text{Div}_t \) are the dividends distributed by the labour unions.

Finally, households choose the utilisation rate of capital. The amount of effective capital that households can rent to the firms is:

\[
K^*_t(j) = Z_t(j)K_{t-1}(j)
\]  

The income from renting capital services is \( R^k_t Z_t(j)K_{t-1}(j) \), while the cost of changing capital utilisation is \( P_t a(Z_t(j))K_{t-1}(j) \).

In equilibrium households will make the same choices for consumption, hours worked, bonds, investment and capital utilisation. The first-order conditions for consumption, hours worked, bond holdings, investment, capital and capital utilisation can be written as (dropping the \( j \) index):

\[
(\partial C_t) \quad \Xi_t = \exp \left( \frac{\sigma_c-1}{1+\sigma_c} L_t(j)^{1+\sigma_c} \right) (C_t - \chi C_{t-1})^{-\sigma_c}
\]

\[
(\partial L_t) \quad \Xi_t = \beta \varepsilon_t R_t E_t \left( \frac{L_{t+1}}{L_{t+1}} \right)
\]

\[
(\partial I_t) \quad \Xi_t = \Xi_t^k \varepsilon_t^i \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right)
\]

\[
+ \beta E_t \left[ \Xi_{t+1}^k \varepsilon_{i+1}^i S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right) \right]
\]

\[
(\partial K_t) \quad \Xi_t^k = \beta E_t \left[ \Xi_{t+1}^k \left( \frac{R^k_{t+1} Z_{t+1} - a(Z_{t+1})}{P_{t+1}} \right) + \Xi_{t+1}^k (1 - \delta) \right]
\]

\[
(\partial u_t) \quad \frac{R^k_t}{P_t} = a'(Z_t)
\]

where \( \Xi_t \) and \( \Xi_t^k \) are the Lagrange multipliers associated with the budget and capital accumulation constraint respectively. Tobin’s is \( Q_t = \Xi_t^k / \Xi_t \) and equals one in the absence of adjustment costs.
1.4 Intermediate labour union sector

Households supply their homogenous labour to an intermediate labour union which differentiates the labour services and sets wages subject to a Calvo schemeiates about the wage with the intermediate labour packers.

Before going into the household’s and the union’s decision on labour supply and wage setting, more details on the labor market are needed. Labor used by the intermediate goods producers $L_t$ is a composite:

$$L_t = \left[ \int_0^1 L_t(l) \frac{1}{1 + \lambda_{w,t}} dl \right]^{1 + \lambda_{w,t}}. \quad (16)$$

There are labor packers who buy the labor from the unions, package $L_t$, and resell it to the intermediate goods producers. Labor packers maximize profits in a perfectly competitive environment. From the FOCs of the labor packers one obtains:

$$L_t(l) = \left( \frac{W_t(l)}{W_t} \right)^{1 + \lambda_{w,t}} L_t \quad (17)$$

Combining this condition with the zero profit condition one obtains an expression for the wage cost for the intermediate goods producers:

$$W_t = \left[ \int_0^1 W_t(l) \frac{1}{1 + \lambda_{w,t}} dl \right]^{\lambda_{w,t}} \quad (18)$$

We assume that $\lambda_{w,t}$ follows the exogenous ARMA process:

$$ln\lambda_{w,t} = (1 - \rho_w)ln\lambda_w + \rho_w ln\lambda_{w,t-1} - \theta_w \epsilon_{w,t-1} + \epsilon_{w,t}; \epsilon_{w,t} \sim . \quad (19)$$

Labor packers buy the labour from the unions. The unions are an intermediate between the households and the labor packers. The unions allocate and differentiate the labour services from the households and have market power: they can choose the wage subject to the labour demand equation 17. The household’s budget constraint now also contains the dividends of the union distributed to the households:

$$C_t + s(j) + I_t + s(j) + \frac{B_{t+s}(j)}{b_t^2 R_{t+s} P_{t+s}} + A_{t+s}(j) - T_{t+s} \leq \frac{B_{t+s-1}(j)}{P_{t+s}} + \frac{W_{h_{t+s}}(j) L_{t+s}(j)}{P_{t+s}} + \frac{D_{n_{t+s}}(j) K_{t+s-1}(j)}{P_{t+s}} - a(u_{t+s}(j)) K_{t+s-1}(j)$$

The household labour supply decision is the same for all households and is given by the following FOC:

$$\frac{W_h}{P_t} = -\left[ \frac{1}{1 - \sigma_h} (C_t - h C_{t-1})^{1-\sigma_h} \right] \exp \left( \frac{\sigma_h-1}{1+\sigma_h} L_t^{1+\sigma_h} \right) (\sigma_h - 1) L_t^{\sigma_h} \quad (20)$$
The real wage desired by the households here reflects the marginal rate of substitution between leisure and consumption. The marginal disutility of labour is equal across households and equal to:

\[ U_{l,t} = \]

Labour unions take this marginal rate of substitution as the cost of the labour services in their negotiations with the labour packers. The markup above the marginal disutility is distributed to the households. However, the union is also subject to nominal rigidities á la Calvo. Specifically, unions can readjust wages with probability \( 1 - \zeta_w \) in each period. For those that cannot adjust wages, \( W_t(l) \) will increase at the deterministic growth rate \( \gamma \) and weighted average of the steady state inflation \( \pi_s \) and of last period's inflation (\( \pi_{t-1} \)). For those that can adjust, the problem is to choose a wage \( \tilde{W}_t(l) \) that maximizes the wage income in all states of nature where the union is stuck with that wage in the future:

\[
\max_{\tilde{W}_t(l)} E_t \sum_{s=0}^{\infty} \zeta_w^s \frac{\beta^s \pi_{t+s} P_t}{\pi_{t+s} P_{t+s}} \left[ W_{t+s}(l) - W_{t+s}^h \right] L_{t+s}(l)
\]

and \( L_{t+s}(l) = \left( \frac{W_{t+s}(l)}{W_{t+s}^h} \right)^{1+\lambda_{w,t+s}} L_{t+s} \) with \( W_{t+s}(l) = \tilde{W}_t(l) \left( \Pi_{t=1}^{s} \gamma \pi_{t+s-1}^{l-1} \pi_s^{1-l} \right) \) for \( s = 1, \ldots, \infty \).

The first order condition becomes:

\[
(\partial W_t) = E_t \sum_{s=0}^{\infty} \zeta_w^s \frac{\beta^s \pi_{t+s} P_t}{\pi_{t+s} P_{t+s}} \left[ (W_{t+s}(l) - W_{t+s}^h) \left( X_{t,s} \tilde{W}_t(l) \right)^{1+\lambda_{w,t+s}} L_{t+s} - X_{t,s} L_{t+s}(l) \right]
\]

where

\[ X_{t,s} = \begin{cases} 1 & \text{for } s = 0 \\ \left( \Pi_{t=1}^{s} \gamma \pi_{t+s-1}^{l-1} \pi_s^{1-l} \right) & \text{for } s = 1, \ldots, \infty \end{cases} \]

Simplifying by substituting for the individual labour and multiplying with the optimal wage

\[
E_t \sum_{s=0}^{\infty} \zeta_w^s \frac{\beta^s \pi_{t+s} P_t}{\pi_{t+s} P_{t+s}} \left[ (X_{t,s} \tilde{W}_t(l) - W_{t+s}^h)(-\frac{1 + \lambda_{w,t+s}}{\lambda_{w,t+s}})L_{t+s}(l) - X_{t,s} \tilde{W}_t(l) L_{t+s}(l) \right] = 0
\]

or

\[
E_t \sum_{s=0}^{\infty} \zeta_w^s \frac{\beta^s \pi_{t+s} P_t}{\pi_{t+s} P_{t+s}} L_{t+s}(l) \frac{1}{\lambda_{w,t+s}} \left[ (1 + \lambda_{w,t+s})W_{t+s}^h - X_{t,s} \tilde{W}_t(l) \right] = 0 \quad (20)
\]

The aggregate wage expression is

\[
W_t = [(1 - \zeta_w)\tilde{W}_t^{\chi_{w,t}} + \zeta_w(\gamma \pi_{t-1}^{l-1} \pi_s^{1-l} W_{t-1})^{\frac{1}{\lambda_{w,t}}} \tilde{W}_t^{\chi_{w,t}}]. \quad (21)
\]
1.5 Government Policies

The central bank follows a nominal interest rate rule by adjusting its instrument in response to deviations of inflation and output from their respective target levels:

$$\frac{R_t}{R^*} = \left( \frac{R_{t-1}}{R^*} \right)^{\rho_R} \left[ \left( \frac{\pi_t}{\pi^*} \right)^{\psi_1} \left( \frac{Y_t}{Y^*_t} \right)^{\psi_2} \right]^{1-\rho_R} \left( \frac{Y_t/Y_{t-1}}{Y^*_t/Y^*_{t-1}} \right)^{\psi_3} r_t$$  \hspace{1cm} (22)

where $R^*$ is the steady state nominal rate (gross rate) and $Y^*_t$ is the natural output. The parameter $\rho_R$ determines the degree of interest rate smoothing. The monetary policy shock $r_t$ is determined as

$$\ln r_t = \rho_r \ln r_{t-1} + \epsilon_{r,t}$$  \hspace{1cm} (23)

The central bank supplies the money demanded by the household to support the desired nominal interest rate.

The government budget constraint is of the form

$$P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_t}$$  \hspace{1cm} (24)

where $T_t$ are nominal lump-sum taxes (or subsidies) that also appear in household’s budget constraint. Government spending expressed relative to the steady state output path $g_t = G_t/(Y^*)$ follows the process:

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \rho_{ga} \ln Z_t - \rho_{ga} \ln Z_{t-1} + \epsilon_{g,t}; \epsilon_{g,t} \sim ...$$  \hspace{1cm} (25)

where we allow for a reaction of government spending to respond on the productivity process. Or a slightly different specification:

$$\ln g_t - \rho_{ga} \ln Z_t = (1 - \rho_g) \ln g + \rho_g (\ln g_{t-1} - \rho_{ga} \ln Z_{t-1}) + \epsilon_{g,t}; \epsilon_{g,t} \sim ...$$

1.6 The natural output level

The natural output level is defined as the output in the flexible price and wage economy. The question is which shocks need to be taken into account. More in particular the question is whether the markup shocks in prices and wages must be taken into account in the calculation of the natural output level. If the markup shock are not taken into account this will lead to a trade-off problem between output-gap stabilisation and inflation stabilisation. Persistent markup shocks might result in persistent conflicts between the two objectives and therefore in persistent deviations of inflation from the inflation target.
1.7 Resource constraints

To obtain the market clearing condition for the final goods market first integrate the HH budget constraint across households, and combine it with the government budget constraint:

\[ P_t C_t + P_t I_t + P_t G_t \leq \Pi_t + \int W_t(j) L_t(j) dj + R^h_t \int K_t(j) dj - P_t a(u_t) \int \bar{K}_{t-1}(j) dj. \]

or in case of the labour unions:

\[ P_t C_t + P_t I_t + P_t G_t \leq \Pi_t + \int W^h_t(j) L_t(j) dj + Div_t + R^k_t \int K_t(j) dj - P_t a(u_t) \int \bar{K}_{t-1}(j) dj. \]

Next, realize that

\[ \Pi_t = \int \Pi_t(i) di = \int P_t(i) Y_t(i) di - W_t L_t - R^h_t K_t, \]

where \( L_t = \int L(i) di \) is total labor supplied by the labor packers (and demanded by the firms), and \( K_t = \int K_t(i) di = \int K_t(j) dj \). Now replace the definition of \( \Pi_t \) into the HH budget constraint, realize that by the labor and goods' packers' zero profit condition \( W_t L_t = \int W_t(j) L_t(j) dj \), or \( = \int W^h_t(j) L_t(j) dj + Div_t \) and \( P_t Y_t = \int P(i) Y(i) di \) and obtain:

\[ P_t C_t + P_t I_t + P_t G_t + P_t a(u_t) \bar{K}_{t-1} = P_t Y_t, \]

or

\[ C_t + I_t + G_t + a(u_t) \bar{K}_{t-1} = Y_t \tag{26} \]

where \( Y_t \) is defined by eq(1).

In the data we do observe \( Y_t \) and \( L_t \) instead of \( Y_t \) and \( L_t \) where

\[ Y_t = \int Y_t(i) di \]

Starting from eq (1), the relationship between output and the aggregate inputs, labor and capital, is:

\[ Y_t = \int \left( \frac{P_t(i)}{P_t} \right)^{-1+\lambda_{p,t}} Y_t di \]

\[ = Y_t \left( \frac{1}{P_t} \right)^{-1+\lambda_{p,t}} \int P_t(i) \left( \frac{1}{P_t} \right)^{-1+\lambda_{p,t}} di \]

\[ = Y_t \left( \frac{1}{P_t} \right)^{-1+\lambda_{p,t}} \int P_t(i) \left( \frac{1}{P_t} \right)^{-1+\lambda_{p,t}} di \]

\[ = Y_t \left( \frac{1}{P_t} \right)^{-1+\lambda_{p,t}} \frac{1}{P_t} \left( \frac{1}{P_t} \right)^{-1+\lambda_{p,t}} \]

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where \( P_t = \left[ \int P_t(i) \frac{1 + \lambda_{p,t}}{\lambda_{p,t}} \, di \right]^{-\frac{\lambda_{p,t}}{1 + \lambda_{p,t}}} \)

and

\[
\bar{L}_t = \int L_t(j) \, dj \\
= \int \left( \frac{W_t(j)}{W_t} \right)^{-\frac{1 + \lambda_{w,t}}{\lambda_{w,t}}} L_t \, dj \\
= L_t(W_t)^{\frac{1 + \lambda_{w,t}}{\lambda_{w,t}}} W_t^{-\frac{1 + \lambda_{w,t}}{\lambda_{w,t}}}
\]

where \( W_t = \left[ \int W_t(j) \frac{1 + \lambda_{w,t}}{\lambda_{w,t}} \, dj \right]^{-\frac{\lambda_{w,t}}{1 + \lambda_{w,t}}} \)

In the first order approximation the barred concepts will equal the unbarred.
1.8 Exogenous Processes

There are seven exogenous processes in the model:

- **Technology process:**
  \[ \ln Z_t = (1 - \rho_z) \ln Z + \rho_z \ln Z_{t-1} + \epsilon_{z,t} \]

- **Investment relative price process:**
  \[ \ln \mu_t = (1 - \rho_\mu) \ln \mu + \rho_\mu \ln \mu_{t-1} + \epsilon_{\mu,t} \]

- **Intertemporal preference shifter (financial risk premium process):**
  \[ \ln b_t = (1 - \rho_b) \ln b + \rho_b \ln b_{t-1} + \epsilon_{b,t} \]

- **Government spending process:**
  \[ \ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \rho_{ga} \ln Z_t - \rho_{ga} \ln Z_{t-1} + \epsilon \]
  \[ \ln g_t - \rho_{ga} \ln Z_t = (1 - \rho_g) \ln g + \rho_g (\ln g_{t-1} - \rho_{ga} \ln Z_{t-1}) + \epsilon_{g,t} \]

- **Monetary Policy Shock:**
  \[ \ln r_t = \rho_r \ln r_{t-1} + \epsilon_{r,t} \]

- **Price Mark-up shock:**
  \[ \ln \lambda_{p,t} = (1 - \rho_p) \ln \lambda_p + \rho_p \ln \lambda_{p,t-1} - \theta_p \epsilon_{p,t-1} + \epsilon_{p,t} \]

- **Wage Mark-up shock:**
  \[ \ln \lambda_{w,t} = (1 - \rho_w) \ln \lambda_w + \rho_w \ln \lambda_{w,t-1} - \theta_w \epsilon_{w,t-1} + \epsilon_{w,t} \]

and where the innovations \( \epsilon \) are distributed as i.i.d. Normal innovations:
\[ \epsilon_{i,t} \sim N(0, \sigma_i) \]

2 Detrending and steady state

2.1 Intermediate goods producers

The model can be detrended with the deterministic trend \( \gamma \) and nominal variables can be replaced by their real counterparts. Lower case variables represent detrended real variables which can be considered as stationary processes that have a well defined steady state: for instance
\[ k_t = K_t/\gamma^t, w_t = W_t/(P_t \gamma^t), r_t^k = R_t^k/P_t, \xi_t = \Xi_t \gamma^{\sigma_c t} \]

The aggregate production function 2 becomes
\[ y_t(i) = Z_t k_t(i)^{\alpha}(L_t(i))^{1-\alpha} - \Phi \] (27)

Equation 6 becomes:

\[ k_t = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t^L} L_t \] (28)

and at st.st.:

\[ k_s = \frac{\alpha}{1-\alpha} \frac{w_s}{r_s^L} L_s \]

The marginal cost expression becomes:

\[ mc_t = \frac{MC_t}{P_t} = \frac{w_t^{1-\alpha} r_t^k}{\alpha(1-\alpha)(1-\alpha)} Z_t \] (29)

**Option 1:** Expression becomes

\[ \frac{1}{\lambda_{p,t}} \left( \tilde{P}_t - (1 + \lambda_{p,t})MC_t \right) Y_t(i) + \]

\[ E_t \sum_{s=1}^{\infty} \zeta_p^{s^p \gamma (1-\sigma_s)} \xi_{t+s} \frac{1}{\lambda_{p,t+s}} \left( \tilde{P}_t(i)(\Pi_{t=1}^{s^p} i_{t+1}^{s^p} \pi_1^{1-\gamma s} - (1 + \lambda_{p,t+s})MC_{t+s}) \right) Y_{t+s}(i) \]

becomes:

\[ \frac{1}{\lambda_{p,t}} \left( \tilde{p}_t - (1 + \lambda_{p,t})mc_t \right) y_t(i) + \]

\[ E_t \sum_{s=1}^{\infty} \zeta_p^{s^p \gamma (1-\sigma_s)} \xi_{t+s} \frac{1}{\lambda_{p,t+s}} \left( \tilde{p}_t(i)(\Pi_{t=1}^{s^p} i_{t+1}^{s^p} \pi_1^{1-\gamma s} - (1 + \lambda_{p,t+s})mc_{t+s}) \right) y_{t+s}(i) \]

\[ = 0 \]

or

\[ \frac{1}{\lambda_{p,t}} \left( \tilde{p}_t - (1 + \lambda_{p,t})mc_t \right) y_t(i) + \]

\[ E_t \sum_{s=1}^{\infty} \zeta_p^{s^p \gamma (1-\sigma_s)} \xi_{t+s} \frac{1}{\lambda_{p,t+s}} \left( \tilde{p}_t(i)(\Pi_{t=1}^{s^p} i_{t+1}^{s^p} \pi_1^{1-\gamma s} - (1 + \lambda_{p,t+s})mc_{t+s}) \right) y_{t+s}(i) \]

\[ = 0 \] (30)

where \( \tilde{p}_t = \tilde{P}_t / P_t \), \( \beta' = \beta^s \gamma - \sigma_s \) and \( \xi_{t+s} = \xi_{t+s} \gamma \sigma_s(t+s) \) is the real discount factor.

Note that in case \( \sigma_c = 1 \) the expression simplifies to the standard problem with \( \beta \) as the discount factor. This implies that in steady state:

\[ \tilde{p}_s = (1 + \lambda_p)\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}w_s^{1-\alpha} r_s^k a z_s^{-1} \]

Expression becomes:

\[ 1 = \left[ (1 - \zeta_p)\tilde{p}_t^{1-\alpha} + \zeta_p(\Sigma_{i=1}^{s^p} \pi_1^{1-\gamma s_i} \pi_1^{1-\gamma s_i}) \right] \lambda_{p,t} \] (31)
which means that:

\[ \hat{p}_s = 1 \]

**Option 2: Equation (7)**

\[
E_t \sum_{s=0}^{\infty} \xi_p^s \beta \Xi_t Y_t(i) \left[ \left( 1 + \frac{1}{G'(z_{t+s})} \frac{G''(x_{t+s})}{G''(x_{t+s})} \right) \tilde{p}_t(i) x_{t,s} - \left( 1 + \frac{1}{G'(z_{t+s})} \frac{G''(x_{t+s})}{G''(x_{t+s})} \right) MC_{t+s} \right] = 0
\]

\[
E_t \sum_{s=0}^{\infty} \xi_p^s \gamma^s \xi_t Y_t(i) \left[ \left( 1 + \frac{1}{G'(z_{t+s})} \frac{G''(x_{t+s})}{G''(x_{t+s})} \right) \tilde{p}_t(i) x_{t,s} - \left( 1 + \frac{1}{G'(z_{t+s})} \frac{G''(x_{t+s})}{G''(x_{t+s})} \right) mc_{t+s} \right] = 0
\]

where

\[ X_{t+s}^p = 0 \quad \text{for} \quad s = 0 \quad \text{or else} \quad (\Pi^p_{t=0} \pi_{t+i}) \]

In steady state this gives (using \( G'(1) = x = 1 \))

\[
1 + [1 - mc] \frac{1}{G'(1)} = 0
\]

\[
1 + \frac{G''(1)}{G'(1)} = mc
\]

The implied markup is equal to

\[
m_u(1) = \frac{1}{1 + \frac{G''(1)}{G'(1)}} = \frac{G'(1)}{G'(1) + 1} = \frac{\varepsilon(1)}{\varepsilon(1) - 1} = (1 + \lambda_p)
\]

which corresponds with the elasticity of demand \( \varepsilon(1) = -\frac{G'(1)}{G''(1)} \)

The aggregate price index in the case of the Calvo model becomes (8):

\[
1 = (1 - \zeta_p) p_t(i) G'' \left[ p_t(i) \tau \right] + \zeta_p \pi_{t-1} \pi_{t-1}^{-1} \pi_{t-1} \tau \left[ \pi_{t-1} \pi_{t-1}^{-1} \pi_{t-1} \tau \right]
\]

In steady-state\(^1\)

\[
1 = (1 - \zeta_p) p_s(i) G'' \left[ p_s(i) \tau \right] + \zeta_p G'' \left[ \pi_s \right]
\]

\[
1 = p_s(i) G'' \left[ \pi_s \right]
\]

\[
1 = p_s(i)
\]

\(^1\)Recall that \( z_t = p_t(i) \tau \), hence \( \pi_s = p_s(i) \tau \) and \( G''(z_*) = x_* = 1 \), \( \tau = G'(1) \leftrightarrow G''(1) = 1 \).
Recall that aggregate profits are equal to:

$$\Pi_t = P_t Y_t - W_t L_t - R_t^k K_t.$$ 

In terms of detrended variables we then have, using 2:

$$\frac{\Pi_t}{P_t \gamma^t} = y_t - w_t L_t - r_t^k k_t$$

$$= Z_t k_t^\alpha L_t^{-\alpha} - \Phi - w_t L_t - \frac{\alpha}{1 - \alpha} w_t L_t$$

$$= \left[ Z_t k_t^\alpha L_t^{-\alpha} - \frac{1}{1 - \alpha} w_t \right] L_t - \Phi$$

$$= \left[ \alpha^\alpha (1 - \alpha)^{-\alpha} Z_t w_t^\alpha (r_t^k)^{-\alpha} - \frac{w_t}{1 - \alpha} \right] L_t - \Phi$$

$$= \left[ \alpha^\alpha (1 - \alpha)^{1-\alpha} Z_t w_t^\alpha (r_t^k)^{-\alpha} - 1 \right] \frac{w_t L_t}{1 - \alpha} - \Phi$$

$$= \left( \frac{1}{m c_t} - 1 \right) \frac{w_t L_t}{1 - \alpha} - \Phi$$

At steady state we can use 1 = \( p_s(i) = (1 + \lambda_p) m c_s \) to get st. st. profits:

$$\Pi_* = (1 + \lambda_p - 1) \frac{w_s L_*}{1 - \alpha} - \Phi$$

$$= \lambda_p \frac{w_s L_*}{1 - \alpha} - \Phi$$

### 2.2 Households

Expression ?? and ?? become:

$$\xi_t = b_t^1 \exp \left( \frac{\sigma_c - 1}{\nu} L_t^{1+\nu} \right) \left( c_t - (h/\gamma) c_{t-1} \right)^{-\sigma_c}$$

(34)

$$\xi_t = b_t^2 R_t E_t \left[ \xi_{t+1} \pi_{t+1}^{-1} \right]$$

(35)

respectively with where \( \xi_t = \Xi_t \gamma c_t \) and \( \beta = (\beta/\gamma^c) \). At steady state:

$$\xi_* = b_t^1 \exp \left( \frac{\sigma_c - 1}{\nu} L_*^{1+\nu} \right) c_*^{-\sigma_c} (1 - (h/\gamma))^{-\sigma_c}$$

$$R_* = \beta^{-1} \pi_*$$

Equation 12 and ?? become:

$$k_t = w_t k_{t-1}/\gamma,$$

(36)

$$\bar{k}_t = [(1 - \delta)/\gamma] \bar{k}_{t-1} + \mu_t \left( 1 - S(\gamma \bar{\iota} k_{t-1}) \right) i_t$$

(37)
which deliver the steady state relationships:

\[ k_s = \frac{u_s \bar{k}_s}{\gamma} = \bar{k}_s/\gamma \]

\[ i_s = \frac{(1/\mu_s)(1 - (1 - \delta)/\gamma)}{\bar{k}_s} \]

under the assumption that \( S(\gamma) = 0, u_s = 1, \) and \( \mu_s = 1. \)

The FOC with respect to investment, capital, and capital utilization are:

\[ Q_t^k \mu_t \left[ 1 - S \left( \frac{i_t \gamma}{i_{t-1}} \right) - S' \left( \frac{i_t \gamma}{i_{t-1}} \right) \right] \]

\[ + \left( \frac{\beta}{\gamma^{\sigma_c}} \right) \frac{\xi_t}{\xi_{t-1}} \left[ \frac{Q_{t+1}^k \mu_{t+1} S' \left( \frac{i_{t+1} \gamma}{i_t} \right) \left( \frac{i_{t+1} \gamma}{i_t} \right)^2}{\xi_{t+1}^{\sigma_c}} \right] \}

\[ \frac{\xi_{t+1}^{\sigma_c}}{\xi_t^{\sigma_c}} \left[ \frac{Q_{t+1}^k \mu_{t+1} S' \left( \frac{i_{t+1} \gamma}{i_t} \right) \left( \frac{i_{t+1} \gamma}{i_t} \right)^2}{\xi_{t+1}^{\sigma_c}} \right] \}

\[ \frac{\xi_t}{\xi_{t-1}} \left[ \frac{Q_{t+1}^k \mu_{t+1} S' \left( \frac{i_{t+1} \gamma}{i_t} \right) \left( \frac{i_{t+1} \gamma}{i_t} \right)^2}{\xi_{t+1}^{\sigma_c}} \right] \}

\[ Q_t = \left( \frac{\beta}{\gamma^{\sigma_c}} \right) \frac{\xi_t}{\xi_{t-1}} \left[ \frac{r_{t+1}^k u_{t+1} - a(u_{t+1}) + Q_{t+1}(1 - \delta)}{\xi_t^{\sigma_c}} \right] \]

\[ r_t^k = a'(u_t) \]

with \( \Xi_t^k/\Xi_t = Q_t \) so that

\[ (\partial_i) 1 = Q_t^k \mu_t \left[ 1 - S \left( \frac{i_t \gamma}{i_{t-1}} \right) - S' \left( \frac{i_t \gamma}{i_{t-1}} \right) \right] \]

\[ + \left( \frac{\beta}{\gamma^{\sigma_c}} \right) \frac{\xi_t}{\xi_{t-1}} \left[ \frac{Q_{t+1}^k \mu_{t+1} S' \left( \frac{i_{t+1} \gamma}{i_t} \right) \left( \frac{i_{t+1} \gamma}{i_t} \right)^2}{\xi_{t+1}^{\sigma_c}} \right] \}

\[ \frac{\xi_t}{\xi_{t-1}} \left[ \frac{Q_{t+1}^k \mu_{t+1} S' \left( \frac{i_{t+1} \gamma}{i_t} \right) \left( \frac{i_{t+1} \gamma}{i_t} \right)^2}{\xi_{t+1}^{\sigma_c}} \right] \]

\[ \frac{\xi_t}{\xi_{t-1}} \left[ \frac{Q_{t+1}^k \mu_{t+1} S' \left( \frac{i_{t+1} \gamma}{i_t} \right) \left( \frac{i_{t+1} \gamma}{i_t} \right)^2}{\xi_{t+1}^{\sigma_c}} \right] \]

so that in steady state where \( Q_s = 1, u_s = 1, \) and \( a(u_s) = 0: \)

\[ 1 = \bar{\beta}(r_s^k + (1 - \delta)) \]

In the variant for the utilisation cost, the accumulated utilisation costs becomes

\[ ak_t = \frac{a(u_{t+s}(j))}{\gamma^{1-s}} \Xi_{t+s}^{l+1}(j) + \left( \frac{1 - \delta}{\gamma} \right) ak_{t-1} \]

**Option 1:** Expressed in terms of detrended variables, equations ?? and 21 become:

\[ E_t \sum_{s=0}^{\infty} \lambda_{w,t+s} \Xi_{t+s} L_{t+s}(j) \frac{1}{\lambda_{w,t+s}} \left[ \frac{U_{t+s}^\prime(j)}{\Xi_{t+s}} (1 + \lambda_{w,t+s}) + \frac{X_{t,s}}{P_{t,s}} \bar{W}_t(j) \right] = 0 \]

where

\[ \frac{U_{t,s}^\prime}{\Xi_t} = b_l^1 \frac{\left( \frac{1}{1-\sigma_c} (C_t - hC_{t-1})^{1-\sigma_c} + \ell_t \right) \exp \left( \frac{\sigma_c-1}{1-\gamma} L_{t}^{1+\nu} \right) (\sigma_c-1) L_{t}^{\nu}}{b_l^1 \exp \left( \frac{\sigma_c-1}{1-\gamma} L_t^{1+\nu} \right) (C_t - hC_{t-1})^{-\sigma_c}} \]

\[ = \left[ \frac{1}{1-\sigma_c} (C_t - hC_{t-1})^{1-\sigma_c} + \ell_t \right] (\sigma_c-1) L_{t}^{\nu} \]

\[ = - [(C_t - hC_{t-1}) L_{t}^{\nu}] \]

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for $\ell_t = 0$.

$$
E_t \sum_{s=0}^{\infty} \zeta_w^s \beta^s \gamma^{t+s} (\xi_{t+s}) L_{t+s}(j) \frac{1}{\lambda_{w,t+s}} \left[ - [(c_t - (h/\gamma)c_{t-1}) L_t^{l_t} (1 + \lambda_{w,t+s}) + \frac{\left( \prod_{t=0}^{6} \pi_{t+1}^{1-l_w} \pi_s^{1-t_w} \right)^{\lambda_{w,t}}}{(\Pi_{t=0}^{6} \pi_{t+1}^{1-l_w})} \tilde{w}_t(j) \right] = 0
$$

$$
E_t \sum_{s=0}^{\infty} \zeta_w^s (\beta^s / \gamma^{s,s}) \gamma^{s} (\sigma^{s,t+s} \xi_{t+s}) L_{t+s}(j) \frac{1}{\lambda_{w,t+s}} \left[ - [(c_t - (h/\gamma)c_{t-1}) L_t^{l_t} (1 + \lambda_{w,t+s}) + \frac{\left( \prod_{t=0}^{6} \pi_{t+1}^{1-l_w} \pi_s^{1-t_w} \right)^{\lambda_{w,t}}}{(\Pi_{t=0}^{6} \pi_{t+1}^{1-l_w})} \tilde{w}_t(j) \right] = 0
$$

or option 2: from (20)

$$
E_t \sum_{s=0}^{\infty} \zeta_w^s (\beta^s / \gamma^{s,s}) \gamma^{s} (\xi_{t+s}) L_{t+s}(l) \frac{1}{\lambda_{w,t+s}} \left[ (1 + \lambda_{w,t+s}) W_t^h - X_{t,s} \tilde{W}_t(l) \right] = 0
$$

which becomes

$$
E_t \sum_{s=0}^{\infty} \zeta_w^s \beta^s \gamma^{s(1-c_0)} \xi_{t}^{s} L_{t+s}(l) \frac{1}{\lambda_{w,t+s}} \left[ (1 + \lambda_{w,t+s}) w_t^{h} - \frac{\left( \prod_{t=0}^{6} \pi_{t+1}^{1-l_w} \pi_s^{1-t_w} \right)^{\lambda_{w,t}}}{(\Pi_{t=0}^{6} \pi_{t+1}^{1-l_w})} \tilde{w}_t(l) \right] = 0
$$

(41)

$$
E_t \sum_{s=0}^{\infty} \zeta_w^s \beta^s \gamma^{s} \xi_{t+s} L_{t+s}(l) \frac{1}{\lambda_{w,t+s}} \left[ (1 + \lambda_{w,t+s}) w_t^{h} - \frac{\left( \prod_{t=0}^{6} \pi_{t+1}^{1-l_w} \pi_s^{1-t_w} \right)^{\lambda_{w,t}}}{(\Pi_{t=0}^{6} \pi_{t+1}^{1-l_w})} \tilde{w}_t(l) \right] = 0
$$

(42)

With

$$
w_t^{h} = - \frac{U_{l,t}}{\gamma^{l_t}} = \left[ (c_t - (h/\gamma)c_{t-1}) \right] L_t^{l_t}
$$

in steady state

$$(1 + \lambda_{w,s}) w_t^{h} = \tilde{w}_s
$$

with $w_t^{h} = \frac{U_{l,s}}{\gamma^{l_t}} = \left[ (c_s - (h/\gamma)c_s) \right] L_s^{l_s} = (1 - h/\gamma)c_s L_s^{l_s}$

and where

$$
W_t = \left[ (1 - \zeta_w) \tilde{W}_t^{l_t} + \zeta_w (\gamma^{l_t} \pi_{t-1}^{1-l_w} \pi_{l-t}^{1-t_w}) \right]^{\lambda_{w,t}}
$$

becomes

$$
w_t = \left[ (1 - \zeta_w) \tilde{w}_t^{l_t} + \zeta_w (\gamma^{l_t} \pi_{t-1}^{1-l_w} \pi_{l-t}^{1-t_w}) \right]^{\lambda_{w,t}}
$$

(43)

which imply at steady state:

$$
w_s = \tilde{w}_s = (1 + \lambda_w) w_s^{h}
$$
2.3 Resource constraints

The resource constraint(s) become:

\[ C_t + I_t + G_t + a(u_t) \bar{K}_{t-1} = Y_t \]
\[ c_t + i_t + y_t g_t + a(u_t) \bar{K}_{t-1}/\gamma = y_t. \]  

(44)

and

\[ \dot{y}_t = Z_t k_t^\alpha L_t^{1-\alpha} - \Phi. \]

\[ Y_t = \left( \frac{\dot{P}_t}{P_t} \right)^{\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} \dot{Y}_t \]

becomes

\[ y_t = (\dot{p}_t)^{\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} y_t \]

where

\[ \dot{p}_t = \left( \frac{\dot{P}_t}{P_t} \right)^{\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} \]

\[ = [(1 - \zeta_p)(\frac{\dot{P}_t}{P_t})^{\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} + \zeta_p(\frac{\dot{r}_t}{r_t})^{\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} - \lambda_{p,t}^{1+\lambda_{p,t}}] \]

\[ = [(1 - \zeta_p)\dot{P}_t^{\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} + \zeta_p(\frac{\dot{r}_t}{r_t})^{\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} - \lambda_{p,t}^{1+\lambda_{p,t}}] \]

While

\[ L_t = \left( \frac{\dot{W}_t}{W_t} \right)^{\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} (L_t) \]

becomes

\[ L_t = (\dot{w}_t)^{\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} (L_t) \]

where

At steady state we have:

\[ c_s + i_s + g_s y_s = y_s. \]

and

\[ y_s = Z_s k_s^\alpha L_s^{1-\alpha} - \Phi. \]

and

\[ \dot{y}_s = y_s, \hat{L}_s = L_s. \]

2.4 Government Policies

The Taylor rule becomes:

\[ \frac{R_t}{R_s} = \left( \frac{R_{t-1}}{R_s} \right)^{\rho_R} \left[ \left( \frac{\pi_t}{\pi_s} \right)^{\psi_1} \left( \frac{y_t}{y_s} \right)^{\psi_2} \right]^{1-\rho_R} \left( \frac{y_t/y_{t-1}}{y_s/y_{s-1}} \right)^{\psi_3} r_t \]  

(45)

\[ ^2 \text{Using } g_t = \frac{G_t}{Y_t} \Rightarrow \frac{G_t}{Y_t} = Y_t g_t. \]
3 Steady state

Combining the steady state expressions:

From (28)

\[ k_s = \frac{\alpha}{1 - \alpha} \frac{w_s}{r_s^k} L_s. \]

From (30)

\[ \hat{p}_s = 1 = (1 + \lambda_p)\alpha^{-\alpha}(1 - \alpha)^{-(1 - \alpha)}w_s^{1 - \alpha}(r_s^k)^\alpha Z_s^{-1} \]

From (35)

\[ R_s = \frac{1}{\pi_s} \]

From (36) and (37):

\[ \bar{k}_s = \gamma k_s, \]
\[ i_s = (1 - (1 - \delta)/\gamma) \bar{k}_s. \]

From (42):

\[ w_s = \tilde{w}_s = (1 + \lambda_w)w_s^h \]

with

\[ w_s^h = \frac{U_{t^*}}{\Xi_s} = [(c_s - (h/\gamma)c_s)] L_s^{\nu_t} = (1 - h/\gamma)c_s L_s^{\nu_t} \]

From (34):

\[ \Xi_s = b_1^h \exp \left( \frac{\sigma_c - 1}{1 + \nu_t} L_s^{1 + \nu_t} \right) c_s^{-\sigma_c} (1 - (h/\gamma))^{-\sigma_c} \]

From (39) and (40)

\[ 1 = (\beta)[(r_s^k) + (1 - \delta)] \]
\[ r_s^k = \beta^{-1} - (1 - \delta) \]
\[ = \beta^{-1}\gamma^\sigma - (1 - \delta) \]

and

\[ r_s^k = a'(u_s) \]

From (27):

\[ y_s = Z_s k_s L_s^{1 - \alpha} - \Phi. \]

From (44):

\[ \frac{c_s}{y_s} + \frac{i_s}{y_s} + g_s = 1 \]
The ratio
\[
\frac{w^h L_s}{c_s} = \frac{1}{1 + \lambda_w} \frac{w_s L_s}{c_s} = \frac{1}{1 + \lambda_w} \frac{1 - \alpha r^k k_s}{\alpha c_s} = \frac{1}{1 + \lambda_w} \frac{1 - \alpha r^k k_s y_s}{\alpha y_s c_s}
\]

with
\[
k_s = \frac{y_s + \Phi}{y_s} \left( \frac{L_s}{k_s} \right)^{\alpha - 1}
\]
and
\[
c_s = \frac{y_s}{y_s} = \left( 1 - g - \frac{i_s}{y_s} \right) = \left( 1 - g - \frac{i_s k_s}{k_s y_s} \right)
\]

4 Log-linearized model

Eq. (29) becomes:
\[
\hat{m} c_t = (1 - \alpha) \hat{w}_t + \alpha \hat{r}^k_t - \hat{Z}_t \quad (46)
\]

Eq. (31) becomes:
\[
\hat{p}_t = \frac{\zeta_p}{1 - \zeta_p} \left( \hat{p}_t - \tau_p \hat{p}_{t-1} \right) \quad (47)
\]

- **Option1:**

Eq. (30) becomes:
\[
d(\tilde{p}_t) - (1 + \lambda_p) d(m c_t) - m c d(\lambda_{p,t}) +
\]
\[
E_t \sum_{s=1}^{\infty} \zeta_p \beta^s \gamma^s \left( d(\tilde{p}_t) + d\left( \frac{1}{\Pi_{l=1}^{s-1} \pi_{p+t}^{1-t_p} \pi_{s-t+l}^{1-t_p}} \right) - (1 + \lambda_p) d(m c_{t+s}) - m c d(\lambda_{p,t+s}) \right) = 0
\]

minus the same expression for time \( t+1 \) multiplied with \( \zeta_p \beta \gamma \)
\[
\frac{1}{(1 - \zeta_p \beta \gamma)} d(\tilde{p}_t) - (1 + \lambda_p) d(m c_t) - m c d(\lambda_{p,t}) - \frac{\zeta_p \beta \gamma}{(1 - \zeta_p \beta \gamma)} (d(\tilde{p}_{t+1}))
\]
\[
+ \frac{\zeta_p \beta \gamma}{(1 - \zeta_p \beta \gamma)} \left( \tau_p \frac{1-t_p}{\pi_s} d(\pi_t) - \left( \frac{1-t_p}{\pi_s} \pi_{s-t+l}^{1-t_p} \right) d(\pi_{t+1}) \right) = 0
\]

in terms of deviations from steady state this becomes:
\[
\frac{1}{(1 - \zeta_p \beta \gamma)} \tilde{p}_t - (1 + \lambda_p) m c (\tilde{m} c_t) - m c \lambda_p \tilde{p}_{t+1} - \frac{\zeta_p \beta \gamma}{(1 - \zeta_p \beta \gamma)} \tilde{p}_{t+1}
\]
\[
+ \frac{\zeta_p \beta \gamma}{(1 - \zeta_p \beta \gamma)} (\tau_p \tilde{p}_t - \tilde{p}_{t+1}) = 0
\]
using 47 to substitute the optimal price expression

\[
\frac{1}{(1 - \zeta_p \bar{\beta} \gamma) (1 - \zeta_p)} \left( \hat{\pi}_t - t_p \hat{\pi}_{t-1} \right) - (\tilde{mc}_t) - mc \lambda_p \hat{\lambda}_{p,t} - \frac{\zeta_p \bar{\beta} \gamma}{(1 - \zeta_p \bar{\beta} \gamma) (1 - \zeta_p)} \left( \hat{\pi}_{t+1} - t_p \hat{\pi}_t \right) + \frac{\zeta_p \bar{\beta} \gamma}{(1 - \zeta_p \bar{\beta} \gamma) (1 - \zeta_p)} (t_p \hat{\pi}_t - \hat{\pi}_{t+1}) = 0
\]

regrouping

\[
(\hat{\pi}_t - t_p \hat{\pi}_{t-1}) - \zeta_p \bar{\beta} \gamma (\hat{\pi}_{t+1} - t_p \hat{\pi}_t) + (\bar{\beta} \gamma - \bar{\beta} \gamma \zeta_p) (t_p \hat{\pi}_t - \hat{\pi}_{t+1}) = 0
\]

Finally we obtain:

\[
\hat{\pi}_t = \frac{(1 - \zeta_p \bar{\beta} \gamma)(1 - \zeta_p)}{(1 + t_p \bar{\beta} \gamma) \zeta_p} \left[ \tilde{mc}_t + \frac{\lambda_p}{1 + \lambda_p \hat{\lambda}_{p,t}} \right] + \frac{t_p}{1 + t_p \bar{\beta} \gamma} \hat{\pi}_{t-1} + \frac{\bar{\beta} \gamma}{1 + t_p \bar{\beta} \gamma} E_t[\hat{\pi}_{t+1}] \quad (48)
\]

if \( \sigma_c = 1 \), the expression \( \bar{\beta} \gamma = \beta \) and the traditional expression is obtained again (see also DSSW).

**Option 2:**

For the flexible price case the derivation is relative simple. Remember the presence of the markup shock in the aggregator function results in:\(^3\)

\[
d(\frac{G^{-(1)}(z_t)G''(x_t)}{\tau}) - d(\lambda_{p,t}) + d(p_t(i)) - d(mc_t) = 0
\]

\[
\frac{G''(x_t)}{\tau} d(\frac{G^{-(1)}(z_t)}{\tau}) + \frac{G'^{-1}(z_t)}{\tau} d(G''(x_t)) - d(\lambda_{p,t}) + d(p_t(i)) - d(mc_t) = 0
\]

\[
\frac{G''(x_t)}{\tau} \frac{1}{G''(x_t)} d(z_t) + \frac{G'^{-1}(z_t)}{\tau} G''(x_t) d(x_t) - d(\lambda_{p,t}) + d(p_t(i)) - d(mc_t) = 0
\]

\[
\frac{G''(x)}{\tau} \frac{1}{G''(x)} d(p_t) + \frac{G'^{-1}(z)}{G''(x)} G''(x) d(p_t) - d(\lambda_{p,t}) + d(p_t(i)) - d(mc_t) = 0
\]

\[
d(p_t) + \frac{1}{G''(x)} G''(x) d(p_t) - d(\lambda_{p,t}) + d(p_t(i)) - d(mc_t) = 0
\]

\[
(2 + \frac{1}{G''(x)} G''(x)) \hat{p}_t(i) - mc \hat{\lambda}_{p,t} - mc \tilde{mc}_t = 0
\]

\(^3\)The shock has been scaled with \(- \frac{G''(z)G'^{-1}(z)}{\tau} = - \frac{G''(z)}{G''(z)}\).
where the deviations in the markup are expressed in percentage of the marginal cost

\[
\hat{p}_t(i) = \left[1 + \frac{G''(1)}{G'(1)}\right] \left(\bar{m}_c + \lambda_{p,t}\right)
\]

Linearization the aggregate price expression (33) (in stst: \(G'^{-1}[z] = x = 1\) and \(\tau = G'(1)\) and \(\pi = 0\))

\[
0 = (1 - \zeta_p)\left(\hat{p}_t(i) + \frac{G'(1)}{G'(1)} \hat{p}_t(i)\right) + \zeta_p \left(t_p \hat{u}_{t-1} - \hat{u}_t + t_p G'(1) \hat{u}_{t-1} - \frac{G'(1)}{G'(1)} \hat{u}_t\right)
\]

\[
0 = (1 - \zeta_p) \hat{p}_t(i) + \zeta_p (t_p \hat{u}_{t-1} - \hat{u}_t)
\]

\[
\hat{p}_t(i) = \frac{\zeta_p}{1 - \zeta_p} (\hat{u}_t - t_p \hat{u}_{t-1})
\]

Linearize the FOC for the Calvo wage with indexation (32):

\[
E_t \sum_{s=0}^{\infty} \zeta_p^s \gamma^s y_{t+s} \left(\frac{1}{G'^{-1}(z_{t+s}) G''(x_{t+s})}\right) \left[1 + G'^{-1}(z_{t+s}) \frac{G''(x_{t+s})}{G'(x_{t+s})} \frac{\hat{p}_t(i) X_{t,s}}{X_{t+s}} - m_{c_{t+s}}\right] = 0
\]

(49)

\[
E_t \sum_{s=0}^{\infty} \zeta_p^s \gamma^s y_{t+s} \left(\frac{1}{G'^{-1}(z_{t+s}) G''(x_{t+s})}\right) \left[\frac{\hat{p}_t(i) X_{t,s}}{X_{t+s}} + \frac{G'^{-1}(z_{t+s}) G''(x_{t+s})}{\tau} \frac{\lambda_{p,t+s}}{X_{t,s}} - m_{c_{t+s}}\right] = 0
\]

(50)

\[
E_t \sum_{s=0}^{\infty} \zeta_p^s \gamma^s \left[d(\hat{p}_t(i)) + d\left(\frac{X_{t,s}}{X_{t+s}}\right) - d(m_{c_{t+s}}) - d(\lambda_{p,t+s}) + \frac{G''(x)}{\tau} d(z_{t+s})\right] = 0
\]

(51)

\[
E_t \sum_{s=0}^{\infty} \zeta_p^s \gamma^s \left[d(\hat{p}_t(i)) + d\left(\frac{X_{t,s}}{X_{t+s}}\right) - d(m_{c_{t+s}}) - d(\lambda_{p,t+s}) + \frac{G''(x)}{\tau} d(p_{t+s})\right] = 0
\]

(52)
\[ E_t \sum_{s=0}^{\infty} \zeta_p^s \gamma_s \left[ d(\tilde{p}_t(i)) + d\left( \frac{X_{t,s}}{X_{t+s}} \right) - d(\lambda_{p,t+s}) - d(mc_{t+s}) + \frac{G'''}{G''} d(p_{t+s}) + d(p_{t+s}) \right] = 0 \]

\[ E_t \sum_{s=0}^{\infty} \zeta_p^s \gamma_s \left[ (2 + \frac{\varphi'''}{G''}) d(\tilde{p}_t(i)) + (2 + \frac{\varphi'''}{G''}) d\left( \frac{X_{t,s}}{X_{t+s}} \right) - (mc_{t+s}) - d(\lambda_{p,t+s}) \right] = 0 \]

where

\[
d\left( \frac{X_{t,s}}{X_{t+s}} \right) = d\left( \frac{\Pi_{t=1}^{t} \pi_{t+1} \pi_{t+1}^{1-i_p}}{\Pi_{t=1}^{t} \pi_{t+1}} \right) = t_p \frac{\pi_{t}^{1-i_p}}{\pi_{t}} d(\pi_{t}) + t_p \frac{\pi_{t}^{1-i_p}}{\pi_{t}} d(\pi_{t+1}) + \ldots \]

\[- \frac{(\pi_{t}^{1-i_p})}{(\pi_{t})^2} d(\pi_{t+1}) - \frac{(\pi_{t}^{1-i_p})}{(\pi_{t})^2} d(\pi_{t+2}) - \ldots \]

minus the same equation for \( t+1 \) multiplied with \( \zeta_{p,t} \gamma \)

\[ 0 = (2 + \frac{\varphi'''}{G''}) \frac{1}{1 - \zeta_{p,t}} d(\tilde{p}_t(i)) - (2 + \frac{\varphi'''}{G''}) \frac{\zeta_{p,t} \gamma}{1 - \zeta_{p,t}} d(\tilde{p}_{t+1}(i)) - d(mc_t) - d(\lambda_{p,t}) \]

\[ + \frac{\zeta_{p,t} \gamma}{1 - \zeta_{p,t}} \left( 2 + \frac{\varphi'''}{G''} \right) \left( t_p \frac{\pi_{t}^{1-i_p}}{\pi_{t}} d(\pi_{t}) - \frac{(\pi_{t}^{1-i_p})}{(\pi_{t})^2} d(\pi_{t+1}) \right) \]

\[ 0 = (2 + \frac{\varphi'''}{G''}) \frac{1}{1 - \zeta_{p,t}} \tilde{p}_t(i) - (2 + \frac{\varphi'''}{G''}) \frac{\zeta_{p,t} \gamma}{1 - \zeta_{p,t}} \tilde{p}_{t+1}(i) - (1 + \frac{\varphi'''}{G''}) (mc_t + \lambda_{p,t}) \]

\[ + \frac{\zeta_{p,t} \gamma}{1 - \zeta_{p,t}} \left( 2 + \frac{\varphi'''}{G''} \right) \left( t_p \frac{\pi_{t}^{1-i_p}}{\pi_{t}} \tilde{p}_t - \frac{(\pi_{t}^{1-i_p})}{(\pi_{t})^2} \tilde{p}_{t+1} \right) \]

using the expression for \( \tilde{p}_t(i) \)

\[ 0 = \frac{1}{1 - \zeta_{p,t}} \left( \frac{\zeta_{p,t} \gamma}{1 - \zeta_{p}} \left( \tilde{p}_t - t_p \tilde{p}_{t-1} \right) - \frac{\zeta_{p,t} \gamma}{1 - \zeta_{p,t}} \left( \tilde{p}_{t+1} - t_p \tilde{p}_t \right) \right) \]

\[- \frac{(1 + \frac{\varphi'''}{G''})}{(2 + \frac{\varphi'''}{G''})} (mc_t + \lambda_{p,t}) + \frac{\zeta_{p,t} \gamma}{1 - \zeta_{p,t}} \left( t_p \frac{\pi_{t}^{1-i_p}}{\pi_{t}} \tilde{p}_t - \frac{(\pi_{t}^{1-i_p})}{(\pi_{t})^2} \tilde{p}_{t+1} \right) \]

\[ 0 = (\tilde{p}_t - t_p \tilde{p}_{t-1}) - \frac{\zeta_{p,t} \gamma}{1 - \zeta_{p}} (\tilde{p}_{t+1} - t_p \tilde{p}_t) \]

\[ + \frac{\zeta_{p,t} \gamma (1 - \zeta_{p})}{\zeta_{p}} (t_p \tilde{p}_t - \tilde{p}_{t+1}) \]

\[ + \frac{(1 + \frac{\varphi'''}{G''})}{(2 + \frac{\varphi'''}{G''})} (\tilde{p}_t) (1 - \zeta_{p,t}) \]

\[ - \frac{1}{\zeta_{p}} (mc_t + \lambda_{p,t}) \]
\[
0 = \hat{\pi}_t - t_p\hat{\pi}_{t-1} - \zeta_p\beta\gamma\hat{\pi}_{t+1} + \zeta_p\beta\gamma t_p\hat{\pi}_t \\
+ \beta\gamma (t_p\hat{\pi}_t - \hat{\pi}_{t+1}) - \beta\gamma \zeta_p (t_p\hat{\pi}_t - \hat{\pi}_{t+1}) \\
- \left( \frac{1 + \zeta_p^{''}}{G''} \right) \left( 1 - \zeta_p \beta \gamma \right) (1 - \zeta_p) \left( \hat{m}c_t + \lambda_{p,t} \right)
\]

\[
(1 + \beta\gamma t_p) \hat{\pi}_t = t_p\hat{\pi}_{t-1} + \beta\gamma \hat{\pi}_{t+1} + A \left( 1 - \zeta_p \beta \gamma \right) (1 - \zeta_p) \left( \hat{m}c_t + \lambda_{p,t} \right)
\]

(53)

with \( A = \frac{1 + \zeta_p^{''}}{G''} \)

If we consider the special case in which the aggregator function \( G \) reduces to the Dixit-Stiglitz aggregator, we can see that option 1 and option 2 results in the same NKPC: in that case \( G(x) = (x)^{1/(1+\lambda)} \) and \( A = 1 \). The following relation between \( A \) and the curvature holds:

\[
\epsilon = \frac{d(\epsilon)}{d(x)} = \frac{d(\zeta_p^{''})}{d(\beta\gamma)} = 1 + \epsilon + \epsilon G''
\]

Eq. 28 becomes:

\[
\hat{k}_t = \hat{u}_t - \hat{r}_t^k + \hat{L}_t.
\]

(54)

Eq. 34 becomes:

\[
(1 - h/\gamma)\hat{\xi}_t = (1 - h/\gamma)\hat{b}_t - \sigma_c \hat{c}_t + (h/\gamma)\hat{c}_{t-1} + (1 - h/\gamma) (\sigma_c - 1)(1 + \gamma^u) \hat{L}_t
\]

using \( w^h_t = \frac{\nu^h_t}{\epsilon} = [(c_t - (h/\gamma)c_{t-1})] L^u_t \):

\[
(1 - h/\gamma)\hat{\xi}_t = (1 - h/\gamma)\hat{b}_t - \sigma_c \hat{c}_t + (h/\gamma)\hat{c}_{t-1} + (\sigma_c - 1)(w^h_t L/c_s) \hat{L}_t
\]

(55)

\[
\left[ \frac{w^h_t L}{c_s} = \frac{1}{1 + \lambda_{\gamma}} \frac{1 - \alpha}{\alpha} \frac{1 - k_{s} \lambda_{\gamma} t}{\lambda_{\gamma} k_{s} t} \right]
\]

Eq. 35 becomes:

\[
\hat{\xi}_t = \hat{b}_t^2 + \hat{R}_t + E_t[\hat{\xi}_{t+1}] - E_t[\hat{\pi}_{t+1}].
\]

(56)

Eq. 36 becomes:

\[
\hat{k}_t = \hat{u}_t + \hat{k}_{t-1}.
\]

Eq. 37 becomes:

\[
\hat{k}_t = (1 - \frac{i_s}{K_s}) \hat{k}_{t-1} + \frac{i_s}{K_s} \mu_t + \frac{i_s}{K_s} \nu_t.
\]

(57)
Eq. 38 becomes:
\[ Q^k_t \mu_t \left[ 1 - S \left( \frac{i_t}{i_{t-1}} \right) - S' \left( \frac{i_t}{i_{t-1}} \right) \frac{i_t}{i_{t-1}} \right] + \beta E_t \left\{ \xi_{t+1} Q^k_{t+1} \mu_{t+1} S' \left( \frac{i_{t+1}}{i_t} \right) \left( \frac{i_{t+1}}{i_t} \right) \right\} = 1 \]

becomes:
\[ \frac{\hat{Q}_t^k}{\hat{Q}_{t+1}^k} + \hat{\mu}_t - \gamma^2 S'\hat{H} + \gamma^2 S'\hat{H}_{t-1} + (\beta) \gamma^2 S'H_{t+1} - (\beta) \gamma^2 S'H_t = 0 \]
\[ \hat{i}_t = \frac{1}{(1 + \beta)} \hat{i}_{t-1} + (\beta) \hat{i}_{t+1} + \frac{1}{\gamma^2 S'} \hat{Q}_t^k + \frac{1}{\gamma^2 S'} \hat{H}_t \]  
(58)

Eq. 39 becomes:
\[ \hat{Q}_t^k = -\xi_t + E_t[\xi_{t+1}] + \frac{r^k}{r^k + (1 - \delta)} E_t[r^k_{t+1}] + \frac{(1 - \delta)}{r^k + (1 - \delta)} E_t[Q^k_{t+1}] \]  
(59)

where the steady state expression \( \beta(r^k + (1 - \tau)) = 1 \) is used.

Eq. 40 becomes:
\[ r^k_{t+1} = a^u \hat{u}_t. \]  
(60)

or in case of accumulated costs (assuming \( s = 0 \)) expressed relative to output:
\[ ak_t = \frac{a'(u_*) \tilde{K}_t \hat{u}_t + 1 - \delta}{\gamma} ak_{t-1} \]
\[ = r^k_{t+1} \hat{u}_t + \frac{1 - \delta}{\gamma} ak_{t-1} \]

Option 2 for wage setting:

Eq. 42 becomes:
\[ (1 + \lambda_w) \hat{w}^h + \lambda_w \hat{w}^l \hat{w}_t \]  
\[ - \frac{\hat{w}}{1 - \zeta_w \beta \gamma} \hat{w}_t + \frac{\zeta_w \beta \gamma \hat{w}}{1 - \zeta_w \beta \gamma} \hat{w}_{t+1} \]
\[ = 0 \]

Using Eq. 43:
\[ \frac{1}{1 - \zeta_w} \hat{w}_t \]  
\[ - \frac{\zeta_w}{1 - \zeta_w} \hat{w}_{t-1} + \frac{\zeta_w}{1 - \zeta_w} \hat{\pi}_t - \frac{\zeta_w \hat{\pi}}{1 - \zeta_w} \hat{\pi}_{t-1} = \hat{w}_t. \]

and
\[ \hat{w}_t^h = \frac{1}{1 - \frac{h}{\gamma}} \hat{\pi}_t - \frac{h}{1 - h \gamma} \hat{\pi}_{t-1} + \nu \hat{\pi}_t \]

results in
\[(1 + \lambda_w)w^h \hat{\omega}_t^h + \lambda_w w^h \hat{\lambda}_{w,t} - \hat{\omega} \hat{w}_t + \hat{\lambda} \hat{w}_t\]
\[- \frac{1}{(1 - \zeta_w \overline{\beta} \gamma)} \left( \hat{\omega}_t - \zeta_w \hat{\omega}_{t-1} + \zeta_w \hat{\pi}_t - \zeta_w \hat{\pi}_{t-1} \right)\]
\[+ \frac{1}{(1 - \zeta_w \overline{\beta} \gamma)} \left( \hat{w}_{t+1} - \zeta_w \hat{w}_t + \zeta_w \hat{\pi}_{t+1} - \zeta_w \hat{\pi}_t \right)\]
\[- \frac{\zeta_w \overline{\beta} \gamma \hat{w}}{(1 - \zeta_w \overline{\beta} \gamma)} \hat{w} - \frac{\zeta_w \overline{\beta} \gamma \hat{w} \hat{w}_{t+1}}{(1 - \zeta_w \overline{\beta} \gamma)} \hat{w} + \frac{\zeta_w \overline{\beta} \gamma \hat{w} (1 - \zeta_w) \hat{\pi}_{t+1}}{(1 - \zeta_w \overline{\beta} \gamma)} \hat{\pi}_{t+1}\]
\[= 0\]

\[(1 - \zeta_w \overline{\beta} \gamma)(1 - \zeta_w) \left[ \hat{\omega}_t^h + \frac{\lambda_w}{(1 + \lambda_w) \hat{\lambda}_{w,t} - \hat{\omega}_t} \right]\]
\[+ (1 - \zeta_w \overline{\beta} \gamma - \zeta_w + \zeta_w \overline{\beta} \gamma \zeta_w) \hat{\omega}_t\]
\[- \left[ \hat{\omega}_t - \zeta_w \hat{\omega}_{t-1} + \zeta_w \hat{\pi}_t - \zeta_w \hat{\pi}_{t-1} \right] \]
\[+ \zeta_w \overline{\beta} \gamma [\hat{\omega}_{t+1} - \zeta_w \hat{\omega}_t + \zeta_w \hat{\pi}_{t+1} - \zeta_w \hat{\pi}_t] \]
\[- \zeta_w \overline{\beta} \gamma \epsilon \hat{w}(1 - \zeta_w) \hat{\pi}_t + \zeta_w \overline{\beta} \gamma (1 - \zeta_w) \hat{\pi}_{t+1}\]
\[= 0\]

\[= \frac{(1 + \overline{\beta} \gamma) \hat{\omega}_t - \hat{\omega}_{t-1} - \overline{\beta} \gamma \hat{\omega}_{t+1}}{(1 - \zeta_w \overline{\beta} \gamma)(1 - \zeta_w) \hat{\omega}_t^h + \frac{\lambda_w}{(1 + \lambda_w) \hat{\lambda}_{w,t} - \hat{\omega}_t}}\]
\[-(1 + \overline{\beta} \gamma \epsilon \hat{w}) \hat{\pi}_t + \epsilon \hat{\pi}_{t-1} + \overline{\beta} \gamma \hat{\pi}_{t+1}\]  (61)

Eq. (44) becomes:
\[\hat{y}_t = \hat{y}_t + \frac{c_s}{y_s} \hat{a}_t + \frac{i_s}{y_s} \hat{a}_t + \frac{r_s k_s}{y_s} \hat{a}_t.\]  (62)

Eq. (27) becomes (remember \(\hat{y}_t = \tilde{y}_t\)):
\[\tilde{y}_t = \frac{\alpha}{y_s} y_s + \frac{\Phi}{y_s} \hat{R}_t + (1 - \alpha) \frac{y_s + \Phi}{y_s} \hat{L}_t + \frac{y_s + \Phi}{y_s} \tilde{Z}_t\]  (63)

Eq. (45) becomes:
\[\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R)(\psi_1 \hat{\pi}_t + \psi_2 (\hat{y}_t - \hat{y}^{\text{flex}}_t))\]
\[+ \psi_3 (\hat{y}_t - \hat{y}_{t-1} - (\hat{y}^{\text{flex}}_{t-1} - \hat{y}^{\text{flex}}_t) + r_t\]  (64)
5 Estimated equations and rescaling residuals

5.1 For the sticky economy

Eq. (46):
\[ \hat{m}_t = (1 - \alpha) \hat{w}_t + \alpha \hat{r}_t^k - \hat{Z}_t \]

Eq (53)

\[
(1 + \bar{\beta}\gamma u_p) \widehat{\pi}_t = \nu_p \widehat{\pi}_{t-1} + \bar{\beta}\gamma E_t[\widehat{\pi}_{t+1}] + A \frac{(1 - \zeta_p \bar{\beta}\gamma)(1 - \zeta_p)}{\zeta_p} (\hat{m}_t) + \lambda_{p,t}
\]

Eq (61)

\[
(1 + \bar{\beta}\gamma) \hat{w}_t - \hat{w}_{t-1} - \bar{\beta}\gamma E_t[\hat{w}_{t+1}] = \frac{(1 - \zeta_w \bar{\beta}\gamma)(1 - \zeta_w)}{\zeta_w} \left[ \frac{1}{1 - h/\gamma} \hat{c}_t - \frac{h/\gamma}{1 - h/\gamma} \hat{c}_{t-1} + \nu_t \hat{L}_t - \hat{w}_t \right] - (1 + \bar{\beta}\gamma \nu_w) \hat{t}_t + \nu_w \hat{t}_{t-1} + \bar{\beta}\gamma E_t[\hat{t}_{t+1}] + \lambda_{w,t}
\]

Eq (56)

\[
\hat{c}_t = \frac{1}{(1 + (h/\gamma))} E_t [\hat{c}_{t+1}] + \frac{(h/\gamma)}{(1 + (h/\gamma))} \hat{c}_{t-1} - \frac{(1 - h/\gamma)}{\sigma_c (1 + (h/\gamma))} (\hat{c}_t^0 + \hat{R}_t - E_t [\hat{c}_{t+1}]) - \frac{(\sigma_c - 1)(w_k^c L/c_a)}{\sigma_c (1 + (h/\gamma))} (E_t [\hat{L}_{t+1}] - \hat{L}_t).
\]

Eq (54)

\[
\hat{k}_t = \hat{w}_t - \hat{r}_t^k + \hat{L}_t.
\]

Eq (60)

\[
r_s^k r_t^k = a^n \hat{u}_t.
\]

Eq (57)

\[
\hat{k}_t = (1 - \frac{i_s}{k_s}) \hat{k}_{t-1} + \frac{i_s}{k_s} \hat{\mu}_t + \frac{i_s \gamma}{k_s}. \hat{\gamma}_t.
\]

Eq (58)

\[
\hat{i}_t = \frac{1}{(1 + \bar{\beta}\gamma)} \hat{i}_{t-1} + (\bar{\beta}\gamma) E_t [\hat{i}_{t+1}] + \frac{1}{\gamma S^u} \hat{Q}_t^k + \frac{1}{\gamma S^u} \hat{\mu}_t.
\]
Eq (59)
\[ \hat{Q}_t^k = -b_t^2 - (\hat{R}_t - E_t[\hat{\pi}_{t+1}]) + \frac{r^k}{r^k + (1 - \delta)} E_t[r_{t+1}^k] + \frac{(1 - \delta)}{r^k + (1 - \delta)} E_t[Q_{t+1}^k]. \]

Eq. (64):
\[ \hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R)(\psi_1 \hat{\pi}_t + \psi_2(\hat{y}_t - \hat{y}_t^{\text{flex}})) + \psi_3(\bar{y}_t - \bar{y}_{t-1} - (\hat{y}_t^{\text{flex}} - \hat{y}_{t-1}^{\text{flex}})) + r_t \]

Eq. (63):
\[ \hat{y}_t = \alpha y_s + \Phi \hat{\pi}_t + (1 - \alpha) \frac{y_s + \Phi \hat{L}_t}{y_s} + \frac{y_s + \Phi \hat{Z}_t}{y_s} \]

Eq. (62):
\[ \hat{y}_t = \hat{y}_t + \frac{c_s}{y_s} \hat{c}_t + \frac{i_s}{y_s} \hat{i}_t + \frac{r^k}{y_s} \hat{\pi}_t. \]

5.2 For the flexible economy