**Online Appendix to**

**Partnerships versus Corporations: Moral Hazard, Sorting and Ownership Structure**

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Appendix A formally defines an equilibrium in our model, Appendix B presents omitted proofs, and Appendix C establishes compatibility of the restrictions maintained throughout the paper, shows that these restrictions hold for wide range of parameter values, and discusses a natural modification of the model under which some of the restrictions can be dropped. Throughout, the equations from the main text are referenced as they are labeled there, i.e. (1), (2), etc., while the equations that are introduced in the Appendix are labeled as (A1), (A2), etc.

# 1 Appendix A: Definition of equilibrium

Let the set of all agents (four workers and the entrepreneur) be represented by $\mathbf{K}$. Any subset $K$ of $\mathbf{K}$ is called a coalition. A payoff allocation $\{u_i\}_{i\in K}$ for a coalition $K$ is feasible if these payoffs can be obtained through production and surplus allocation, respecting limited liability when applicable. Let $V(K)$ stand for the set of feasible payoff vectors for a coalition $K$. A payoff allocation $\{u_i\}_{i\in\mathbf{K}}$ for the grand coalition is said to be blocked by a coalition $K$ if there exists a feasible allocation $\{u_i'\}_{i\in K}$ such that $u_i < u_i'$ for each $i \in K$. A payoff allocation $\{u_i\}_{i\in\mathbf{K}}$ is stable if and only if it cannot be blocked by any coalition $K$.

An equilibrium consists of a stable payoff allocation, a matching (specifying which coalitions are formed) and contracts that deliver the equilibrium payoffs.

The description of $V(K)$ for any $K$ follows from the discussion in the text. We make the following observations:

- Since production is undertaken in pairs, the only relevant coalitions are those consisting of (i) two workers; (ii) all four workers; (iii) two workers and the entrepreneur; (iv) all four workers and the entrepreneur; (v) the entrepreneur alone.

- When $K$ consists of the entrepreneur alone $V(K) = \{\Phi\}$.

- When $K$ consists of workers only (coalitions of types (i) or (ii)) ,$V(K)$ can be described by the maximum surplus that the workers can obtain via choice of effort and sorting pattern (for type-(ii) coalition) since utility is fully transferable among workers. The relevant maximal surplus is characterized in (5).

- When $K$ consists of the entrepreneur together with some or all of the workers (coalitions of type (iii) or (iv)), the feasible payoff vector consists of profits for the entrepreneur and payoffs for the agents that are obtained as a result of a contract that specifies: (i) effort recommendations; (ii) payments to the workers contingent on output; (iii) a sorting pattern, while satisfying incentive compatibility and limited liability in the obvious sense.[1]

# 2 Appendix B: Omitted proofs

## 2.1 Equilibrium conditions

In this section, we formalize the discussion in Section IV.A of the main text characterizing the equilibrium conditions. We start by formally stating our assumption that guarantees that the only possible types of equilibria are corporation equilibria and partnership equilibria. In other words, hybrid equilibria, in which both forms of ownership structure co-exist, are not possible.

**Assumption: Entrepreneur obtains positive profits from a team of identical types**

$$\text{For all } m \in \{L, H\}, \quad y_{mm} + 2p_m(\Delta - 2b_m) - 2\underline{w} > 0. \tag{A1}$$

This assumption guarantees that the entrepreneur's payoff from entering is maximized if he hires all workers: On the one hand, if he hires only, say, the $L$ types, then he can make additional profits by hiring the $H$ types, as well. On the other hand, if it is profitable for him to enter by hiring one worker of each type, it must be that he is making positive profits from this choice (since $\Phi > 0$). In this case, he can double his profits by hiring the remaining two workers.

Before proving the main result of this section, we reproduce the following definitions and conditions from the main text.

- Feasibility of outside options / equilibrium payoffs:

$$u_H + u_L \leq \max\{S_{LL}/2 + S_{HH}/2, S_{HL}\}. \tag{A2}$$

- No-blocking condition:

$$u_H \geq S_{HH}/2, \quad u_L \geq S_{LL}/2 \quad \text{and} \quad u_H + u_L \geq S_{HL}. \tag{A3}$$

- Entrepreneur's maximum profits:

$$\Pi^E(u_H, u_L) = \max\{\Pi_{HH}(u_H, u_H) + \Pi_{LL}(u_L, u_L), 2\Pi_{HL}(u_H, u_L)\},$$

where

$$\Pi_{mn}(u_m, u_n) = Ey_{mn} - \max\{u_m + c\ ,\ \underline{w} + (p_n + p_m)b_m\} - \max\{u_n + c\ ,\ \underline{w} + (p_n + p_m)b_n\}. \tag{A4}$$

[1]Note that due to limited liability, utility is not fully transferable from the workers to the entrepreneur. Therefore, the Pareto frontier of the set of feasible payoffs for coalitions of this type cannot be described solely by its level.

The next Lemma establishes some useful properties that the entrepreneur's profit function $\Pi^E$ exhibits under our maintained assumptions.

**Lemma 1** *Fix technology parameters* $(y_{mn}, \Delta, p_m, c, b_H, b_L)$. *Take* $\underline{w}$ *that satisfies Assumption 2 (Limited liability is always binding or is always slack). Then,* $\Pi^E(u_H, u_L)$ *is constant over* $(u_H, u_L)$ *satisfying (11) and (12). Moreover, when* (A1) *holds,*

$$\begin{aligned}\Pi^E(u_H, u_L) &> \Pi_{HH}(u_H, u_H) \\ &> \Pi_{LL}(u_L, u_L) \\ &> \Pi_{HL}(u_H, u_L).\end{aligned} \tag{A5}$$

**Proof:** Note that (11) together with (12) imply that $u_H + u_L = \max\{S_{LL}/2 + S_{HH}/2, S_{HL}\} \equiv U^*$ is a constant. First, if limited liability is always slack, then

$$\Pi^E(u_H, u_L) = Ey_{HH} + Ey_{LL} - 2U^*,$$

which is a constant. Next, if limited liability is always binding, then $\Pi^E(u_H, u_L)$ is independent of $u_H, u_L$, and, therefore, constant since the workers' compensations exceed their outside options.

To verify (A5), note that, by definition,

$$\Pi^E(u_H, u_L) \geq \Pi_{HH}(u_H, u_H) + \Pi_{LL}(u_L, u_L).$$

When Assumption 2 holds, $\Pi_{mm}(u_m, u_m) > 0$, either because of Assumption 1 (when limited liability is always slack) or because of (A1) (when limited liability always binds). Therefore, by (A1), $\Pi^E(u_H, u_L) > \Pi_{HH}(u_H, u_H)$ and $\Pi^E(u_H, u_L) > \Pi_{LL}(u_L, u_L)$. Moreover, this implies that $\Pi^E(u_H, u_L) > 0$. Then, if $\Pi_{HL}(u_H, u_L) \leq 0$, the third inequality is trivially satisfied. However, if $\Pi_{HL}(u_H, u_L) > 0$, then the third inequality in (A5) follows because, by definition, $\Pi^E(u_H, u_L) \geq \Pi_{HL}(u_H, u_L)$. ■

Now we are ready to state the main result.

**Proposition 1 (Equilibrium characterization)** *Fix technology parameters* $(y_{mn}, \Delta, p_m, c, b_H, b_L)$. *Take* $\underline{w}$ *that satisfies* (A1) *and Assumption 2 (Limited liability is always binding or is always slack). Letting* $\Pi^* \equiv \Pi^E(u_H, u_L)$ *for all* $(u_H, u_L)$ *satisfying (11) and (12),*

*(i) There exists a corporation equilibrium if and only if* $\Pi^* \geq \Phi$.

*(ii) There exists a partnership equilibrium if and only if* $\Pi^* \leq \Phi$.

*(iii) All equilibria are either partnership equilibria or corporation equilibria; i.e., no hybrid equilibria exist.*

**Proof:** For any $u_H, u_L$, let $v_H, v_L$ stand for the payoff of a worker under the optimal contract attaining profit $\Pi^E(u_H, u_L)$ for the entrepreneur. Notice that $u_m = v_m$ if limited liability is slack and $v_m \geq u_m$, if it is binding. Also note that, $\Pi^E(u_L, u_H) = \Pi^E(v_L, v_H)$.

(i) Corporation equilibrium:

- *Sufficiency:* Assume that $\Pi^* \geq \Phi$. Take some $u_H$, $u_L$ satisfying (11) and (12). The allocation that delivers the entrepreneur $\Pi^*$ and type $m$ worker $v_m$ is feasible and can be achieved by the entrepreneur hiring all four workers. This allocation cannot be blocked by the entrepreneur alone since $\Pi^* \geq \Phi$. It cannot be blocked by any two workers or all four workers by (12). It cannot be blocked by the entrepreneur and two workers by Lemma 1. Therefore, there exists a corporation equilibrium.
- *Necessity:* Assume that there exists a corporation equilibrium. Let $\pi$ represent the payoff of the entrepreneur in this equilibrium, and $v_L, v_H$ the payoffs of the workers of type $L$ and $H$, respectively. First, it is necessary that $\pi \geq \Phi$ because, otherwise, the entrepreneur can unilaterally block this allocation. Also, $v_L, v_H$ must satisfy (12) because, otherwise, a subset of the workers can block this allocation by forming a partnership. Then, $\pi \leq \Pi^*$. Then, since $\pi \geq \Phi$, we have $\Pi^* \geq \Phi$.

(ii) Partnership equilibrium:

- *Sufficiency:* Assume that $\Pi^* \leq \Phi$. Consider an allocation with $u_H$, $u_L$ satisfying (11) and (12), and the entrepreneur's payoff equal to $\Phi$. This is achievable by all workers organizing in partnerships and, hence, is feasible. By Lemma 1, and the fact that $\Pi^* \leq \Phi$, any coalition that includes the entrepreneur cannot block this allocation. By (12), no coalition of two or four workers can block this allocation. Therefore, there exists a partnership equilibrium.
- *Necessity:* Assume that there exists a partnership equilibrium. Let $u_H, u_L$ be the payoffs of workers of type $H$ and $L$, respectively, in this equilibrium. The entrepreneur's payoff is $\pi = \Phi$. Feasibility guarantees that $(u_H, u_L)$ satisfy (11). Moreover, $(u_H, u_L)$ must satisfy (12) because, otherwise, a subset of the workers can block this allocation. Therefore, for the grand coalition, a payoff allocation that gives $\Pi^E(u_H, u_L) - \varepsilon = \Pi^* - \varepsilon$ to the entrepreneur and $v_H + \varepsilon' > u_H, v_L + \varepsilon'' > u_L$ to the workers is feasible. Then, for this coalition not to block the partnership equilibrium payoff allocation, it must be that $\Pi^* - \varepsilon \leq \Phi$ for any $\varepsilon > 0$; i.e., $\Pi^* \leq \Phi$.

(iii) Suppose there exists a hybrid equilibrium in which the entrepreneur hires two workers of types $m$ and $n$. The entrepreneur's payoff, $\pi$, in this equilibrium must satisfy $\pi \geq \Phi$.

- First, assume that $m = n$. Then, the payoff of a type $k \neq m = n$ worker in this candidate equilibrium is $u_k = S_{kk}/2$. Then, by Assumption 2 and (A1), $\Pi_{kk}(u_k, u_k) > 0$, regardless of whether $\underline{w}$ is such that limited liability is always slack or is always binding. Let $v_m$ and $\pi$ represent the equilibrium payoffs of type $m$ workers and the entrepreneur, respectively. Then, for the grand coalition, a payoff allocation that gives the $m$-types a payoff of $v_m + \varepsilon_1$, the $k$-types a payoff of $\max\{u_k + \varepsilon_2, \underline{w} + 2p_k b_k - c\} > u_k$, and the entrepreneur a payoff of $\pi + \Pi_{kk}(u_k, u_k) - \varepsilon_3 > \pi$—with appropriately chosen and sufficiently small $\varepsilon_1, \varepsilon_2, \varepsilon_3 > 0$—is feasible: It can be achieved when all workers are hired by the entrepreneur and are sorted positively. Therefore, the candidate allocation is blocked.

- Now, assume $m \neq n$. Let $u_H$ and $u_L$ be the payoffs of the two workers of types $H$ and $L$, respectively, who are not employed by the entrepreneur. Let $u'_H, u'_L$ be the payoffs of the workers employed by the entrepreneur. Note that $u'_m \geq u_m$, for $m\{L, H\}$ because, otherwise, a coalition consisting of type-$m$ worker employed by the entrepreneur and a type-$n \neq m$ worker not employed by the entrepreneur can block this allocation. This implies that $\pi \leq \Pi_{HL}(u_H, u_L)$, which, in turn, implies that $\Phi \leq \Pi_{HL}(u_H, u_L)$. Since $\Phi > 0$, this, in particular, implies that $\Pi_{HL}(u_H, u_L) > 0$.

  If the entrepreneur hires all workers, and sorts them negatively, the following payoff allocation is feasible for appropriately chosen and small enough $\varepsilon, \varepsilon' > 0$: The worker of type $m$, who is already hired by the entrepreneur in the original allocation, receives $u'_m + \varepsilon$; the worker of type $m$, who was not hired by the entrepreneur, receives $v_m + \varepsilon' > u_m$; and the entrepreneur receives $\pi + \Pi_{HL}(u_H, u_L) - 2\varepsilon - 2\varepsilon' > \pi$. Therefore, the grand coalition can block the original allocation. ■

## 2.2 Proofs of Proposition 3 and Proposition 4

Before formally proving the propositions, a few remarks are useful. Fix a vector of technology parameters $(y_{mn}, \Delta, p_m, c, \bar{b}, \underline{b})$.

- Let $S^{subst}_{mn}$ and $S^{compl}_{mn}$ represent the surplus that a partnership of a type $m$ and a type $n$ worker can generate under substitutes and complements specifications, respectively. Then, simple algebra leads to the following conclusions:

$$S^{subst}_{HL} = S^{compl}_{HL} \equiv S_{HL}; \tag{A6}$$

$$S^{subst}_{HH} + S^{subst}_{LL} - 2S_{HL} = \Theta + |\Lambda|; \tag{A7}$$

$$S^{compl}_{HH} + S^{compl}_{LL} - 2S_{HL} = \Theta - |\Lambda|. \tag{A8}$$

- Take $\underline{w}$ such that the limited liability is binding under either specification for all $(u_H, u_L)$ satisfying (11) and (12). Let $\Pi^{subset}_{mn}$ and $\Pi^{compl}_{mn}$ represent the maximum profit that the entrepreneur can extract from a team of workers of type $m$ and $n$, under the substitutes and complements specifications, respectively. Then, the following can be obtained by simple algebra:

$$\Pi_{HL}^{subst} = \Pi_{HL}^{compl} \equiv \Pi_{HL} \tag{A9}$$

$$\Pi_{HH}^{subst} + \Pi_{LL}^{subst} - 2\Pi_{HL} = \Theta - |\Lambda|; \tag{A10}$$

$$\Pi_{HH}^{compl} + \Pi_{LL}^{compl} - 2\Pi_{HL} = \Theta + |\Lambda|. \tag{A11}$$

**Proof of Proposition 3:** Fix a vector of technology parameters $(y_{mn}, \Delta, p_m, c, \bar{b}, \underline{b})$. Assume that $\underline{w}$ is such that Assumption 2 holds under both the substitutes and the complements specifications.[2] Let $\Pi_{subst}^*$ and $\Pi_{compl}^*$ be the entrepreneur's profit when the workers' outside options are given by $u_H, u_L$ satisfying (11) and (12) under the two specifications.[3] In what follows, we argue that $\Pi_{subst}^* < \Pi_{compl}^*$, which completes the proof, since, by the discussion in the previous section, $\Phi_j^* = \Pi_j^*$, $j = subst, comp$. Consider two cases:

- Limited liability is always slack under either specification. Then, under either specification, and for any pair of outside options $u_H, u_L$ satisfying (11) and (12), the entrepreneur's profits are maximized under positive sorting. Therefore,

  $$\Pi_j^* = Ey_{HH} + Ey_{LL} - 4c - \max\{2S_{HL}, S_{HH}^j + S_{LL}^j\}, \quad j = subst,\ comp.$$

  The claim follows by (A6) - (A8).

- Limited liability is always binding under either specification. Then,

  $$\Pi_j^* = \max\{\Pi_{HH}^j + \Pi_{LL}^j, 2\Pi_{HL}\}, \quad j = subst,\ comp.$$

  The claim follows by (A9) - (A11). ■

**Proof of Proposition 4:**

- First, consider an appropriate variation in $y_{HL}$. Let $\Phi_j^*(y_{HL}), j = subst, comp$, represent the cutoff level of the outside option under specification $j$ and when the technology parameter is given by $y_{HL}$. Take $y'_{HL} > y''_{HL}$. Let $\Theta' = y_{HH} + y_{LL} - 2y'_{HL}$ and $\Theta'' = y_{HH} + y_{LL} - 2y''_{HL}$. Note that $\Theta' < \Theta''$.

  Consider two cases:

  1. Limited liability is always slack under either specification.

[2] It is easy to see that for a given technology, if all limited liability conditions are binding in the substitutes specification, it cannot be that they are all slack in the complements specification and vice versa. Therefore, it is sufficient to consider only those environments where limited liability is always binding under both specifications and those where limited liability is always slack under both specifications.

[3] Under Assumption 2, these profits are constant by Lemma 1.

Note that $S^j_{mm}$, $(m = H, L,\ j = comp, subst,)$ is independent of $y_{HL}$. Let $S_{HL}(y_{HL})$ represent the appropriate surplus when the technology parameter is given by $y_{HL}$. Note that this is independent of technology specification $j = comp, subst$, by (A6).

- First, consider the substitutes specification. By (A7),

$$\max\{2S_{HL}(y_{HL}), S^{subst}_{HH} + S^{subst}_{LL}\} = S^{subst}_{HH} + S^{subst}_{LL},$$

  for any $y_{HL}$, and, therefore,

$$\Phi^*_{subst}(y'_{HL}) = \Phi^*_{subst}(y''_{HL}) = Ey_{HH} + Ey_{LL} - 4c - S^{subst}_{HH} - S^{subst}_{LL}.$$

- Now, consider the complements specification.
  (1) If $|\Lambda| \leq \Theta' < \Theta''$, then

$$\max\{2S_{HL}(y_{HL}), S^{compl}_{HH} + S^{compl}_{LL}\} = S^{compl}_{HH} + S^{compl}_{LL},$$

  for $y_{HL} = y'_{HL}, y''_{HL}$, and, therefore,

$$\Phi^*_{compl}(y'_{HL}) = \Phi^*_{compl}(y''_{HL}) = Ey_{HH} + Ey_{LL} - 4c - S^{compl}_{HH} - S^{compl}_{LL}.$$

  (2) If $|\Lambda| > \Theta'$,

$$2S_{HL}(y'_{HL}) > S^{compl}_{HH} + S^{compl}_{LL},$$

  and

$$2S_{HL}(y'_{HL}) > 2S_{HL}(y''_{HL}).$$

  Therefore,

$$\Phi^*_{compl}(y'_{HL}) = Ey_{HH} + Ey_{LL} - 4c - 2S_{HL}(y'_{HL}) \ldots$$
$$\ldots < Ey_{HH} + Ey_{LL} - 4c - \max\{S^{compl}_{HH} + S^{compl}_{LL}, 2S_{HL}(y''_{HL})\} = \Phi^*_{compl}(y''_{HL}).$$

2. Limited liability is always binding under either specification.

   Let $\Pi^j_{mn}(y_{HL})$ be the maximum surplus that the entrepreneur can extract from a team of workers of types $m$ and $n$ when their compensation is determined by limited liability, under specification $j = comp, subst$, and when the technology parameter is given by $y_{HL}$. By (A9), $\Pi^j_{HL}(y_{HL})$ is independent of $j$, so we drop this superscript. Also, it is easy to see that $\Pi^j_{mm}(y_{HL})$ is independent of $y_{HL}$ for $j = subst, comp$, and $m = L, H$. Therefore, we simply write $\Pi^j_{mm}$.

   - First, consider the substitutes specification.

(1) If $|\Lambda| \leq \Theta' < \Theta''$, then

$$\max\{2\Pi_{HL}(y_{HL}), \Pi_{HH}^{subst} + \Pi_{LL}^{subst}\} = \Pi_{HH}^{subst} + \Pi_{LL}^{subst},$$

for $y_{HL} = y'_{HL}, y''_{HL}$, and, therefore,

$$\Phi^*_{subst}(y'_{HL}) = \Phi^*_{subst}(y''_{HL}) = \Pi_{HH}^{subst} + \Pi_{LL}^{subst}.$$

(2)If $|\Lambda| > \Theta'$, then

$$2\Pi_{HL}(y'_{HL}) > \Pi_{HH}^{subst} + \Pi_{LL}^{subst},$$

and

$$2\Pi_{HL}(y'_{HL}) > 2\Pi_{HL}(y''_{HL}).$$

Therefore,

$$\Phi^*_{subst}(y'_{HL}) = 2\Pi_{HL}(y'_{HL}) > \max\{\Pi_{HH}^{subst} + \Pi_{LL}^{subst}, 2\Pi_{HL}(y''_{HL})\} = \Phi^*_{subst}(y''_{HL}).$$

– Now, consider the complements specification. By (A11),

$$\max\{2\Pi_{HL}(y_{HL}), \Pi_{HH}^{compl} + \Pi_{LL}^{compl}\} = \Pi_{HH}^{compl} + \Pi_{LL}^{compl},$$

for any $y_{HL}$, and, therefore,

$$\Phi^*_{compl}(y'_{HL}) = \Phi^*_{compl}(y''_{HL}) = \Pi_{HH}^{compl} + \Pi_{LL}^{compl}.$$

- Now, consider an appropriate variation in $\Delta b$. Let $\Phi^*_j(\Delta b), j = subst, comp$, represent the cutoff level of the outside option under specification $j$ and when the technology parameter is given by $\Delta b$. Take $\Delta b' > \Delta b''$. Let $|\Lambda'| = 2(p_H - p_L)\Delta b'$ and $|\Lambda''| = 2(p_H - p_L)\Delta b''$. Then, $|\Lambda'| > |\Lambda''|$. Consider two cases:

1. Limited liability is always slack under either specification.

   First, note that $S_{HL}$ is independent of $\Delta b$ and depends only on $\underline{b} + \bar{b}$, which is constant. Let $S_{mm}(\Delta b)^j$ represent the appropriate surplus when the technology parameter is given by $\Delta b$ and $j = subst, comp$.

   Note that by (A7) and (A8),

$$S_{HH}^{compl}(\Delta b') + S_{LL}^{compl}(\Delta b') < S_{HH}^{compl}(\Delta b'') + S_{LL}^{compl}(\Delta b''),$$

   while

$$S_{HH}^{subs}(\Delta b') + S_{LL}^{subs}(\Delta b') > S_{HH}^{subs}(\Delta b'') + S_{LL}^{subs}(\Delta b'').$$

– First, consider the substitutes specification. By (A7),

$$\max\{2S_{HL}, S_{HH}^{subst}(\Delta b) + S_{LL}^{subst}(\Delta b)\} = S_{HH}^{subst}(\Delta b) + S_{LL}^{subst}(\Delta b),$$

for any $\Delta b$, and, therefore,

$$\begin{aligned}\Phi^*_{subst}(\Delta b') =& Ey_{HH} + Ey_{LL} - 4c - S_{HH}^{subst}(\Delta b') - S_{LL}^{subst}(\Delta b') < ...\\ ... <& Ey_{HH} + Ey_{LL} - 4c - S_{HH}^{subst}(\Delta b'') - S_{LL}^{subst}(\Delta b'') = \Phi^*_{subst}(\Delta b'').\end{aligned}$$

– Now, consider the complements specification.
(1) If $\Theta \leq |\Lambda''|$,

$$\max\{2S_{HL}, S_{HH}^{compl}(\Delta b) + S_{LL}^{subst}(\Delta b)\} = 2S_{HL},$$

for $\Delta b = \Delta b', \Delta b''$, and, therefore,

$$\Phi^*_{compl}(\Delta b') = \Phi^*_{compl}(\Delta b'') = Ey_{HH} + Ey_{LL} - 4c - 2S_{HL}.$$

(2) If $\Theta > |\Lambda''|$,

$$S_{HH}^{compl}(\Delta b'') + S_{LL}^{compl}(\Delta b'') > 2S_{HL},$$

and

$$S_{HH}^{compl}(\Delta b'') + S_{LL}^{compl}(\Delta b'') > S_{HH}^{compl}(\Delta b') + S_{LL}^{compl}(\Delta b').$$

Therefore,

$$\begin{aligned}\Phi^*_{compl}(\Delta b'') =& Ey_{HH} + Ey_{LL} - 4c - S_{HH}^{compl}(\Delta b'') + S_{LL}^{compl}(\Delta b'')...\\ ... <& Ey_{HH} + Ey_{LL} - 4c - \max\{S_{HH}^{compl}(\Delta b') + S_{LL}^{compl}(\Delta b'), 2S_{HL}\} = \Phi^*_{compl}(\Delta b').\end{aligned}$$

2. Limited liability is always binding under either specification.
Let $\Pi^j_{mn}(\Delta b)$ be the maximum surplus that the entrepreneur can extract from a team of workers of types $m$ and $n$ when their compensation is determined by limited liability, under specification $j = comp, subst$.
Note that $\Pi^j_{HL}$ is independent of $j$ and independent of $\Delta b$. Also, by (A10) and (A11),

$$\Pi_{HH}^{compl}(\Delta b') + \Pi_{LL}^{compl}(\Delta b') > \Pi_{HH}^{compl}(\Delta b'') + \Pi_{LL}^{compl}(\Delta b''),$$

while

$$\Pi_{HH}^{subs}(\Delta b') + \Pi_{LL}^{subs}(\Delta b') < \Pi_{HH}^{subs}(\Delta b'') + \Pi_{LL}^{subs}(\Delta b'').$$

– First, consider the substitutes specification.

(1) If $\Theta \leq |\Lambda''|$,

$$\max\{2\Pi_{HL}, \Pi_{HH}^{subst}(\Delta b) + \Pi_{LL}^{subst}(\Delta b)\} = 2\Pi_{HL},$$

for $\Delta b = \Delta b', \Delta b''$, and, therefore,

$$\Phi_{subst}^{*}(\Delta b') = \Phi_{subst}^{*}(\Delta b'') = 2\Pi_{HL}.$$

(2) If $\Theta > |\Lambda''|$,

$$\Pi_{HH}^{subst}(\Delta b'') + \Pi_{LL}^{subst}(\Delta b'') > 2\Pi_{HL},$$

and

$$\Pi_{HH}^{subst}(\Delta b'') + \Pi_{LL}^{subst}(\Delta b'') > \Pi_{HH}^{subst}(\Delta b') + \Pi_{LL}^{subst}(\Delta b').$$

Therefore,

$$\Phi_{subst}^{*}(\Delta b'') = \Pi_{HH}^{subst}(\Delta b'') + \Pi_{LL}^{subst}(\Delta b'')...$$
$$... > \max\{\Pi_{HH}^{subst}(\Delta b') + \Pi_{LL}^{subst}(\Delta b'), 2\Pi_{HL}\} = \Phi_{subst}^{*}(\Delta b').$$

– Now, consider the complements specification. By (A11),

$$\max\{2\Pi_{HL}, \Pi_{HH}^{compl}(\Delta b) + \Pi_{LL}^{compl}(\Delta b)\} = \Pi_{HH}^{compl}(\Delta b) + \Pi_{LL}^{compl}(\Delta b),$$

for any $\Delta b$. Therefore,

$$\Phi_{compl}^{*}(\Delta b'') = \Pi_{HH}^{compl}(\Delta b'') + \Pi_{LL}^{compl}(\Delta b'') < \Pi_{HH}^{compl}(\Delta b') + \Pi_{LL}^{compl}(\Delta b') = \Phi_{compl}^{*}(\Delta b').$$

This completes the proof. ■

# 3 Appendix C: Parameter restrictions

In this section, we discuss the parameter restrictions that we invoke throughout the text, show that they are compatible when workers' types are sufficiently close, and address the implied restrictions on type-heterogeneity. These restrictions refer to either only the technology parameters $(y_{mn}, \Delta, p_m, c, \bar{b}, \underline{b})$ or both the technology parameters and the limited liability parameter $\underline{w}$. It is useful to start by classifying them into three groups:

1. Conditions that involve only technology parameters:
   (a) effort is socially optimal;
   (b) moral hazard is costly in partnerships;

(c) effort by both partners is optimal in partnerships in the presence of moral hazard.

2. Conditions that involve $\underline{w}$, *except* the limited liability conditions:

   (a) Effort by both partners is optimal in any team in a corporation in the presence of moral hazard;

   (b) Profits from any two identical workers are positive, i.e. (A1) holds.

3. The limited liability conditions (Assumption 2 in the text), which restrict the analysis to one of two ranges of $\underline{w}$:

   (a) Limited liability is not binding for any type, under any sorting pattern, for either specification of the technology;

   **OR**

   (b) Limited liability is binding for all types, under either sorting pattern, for either specification of the technology.

In the ensuing subsections, we discuss the extent of heterogeneity that is allowed by these restrictions. Since the types are multi-dimensional ($p_m, q_m$ and, therefore, $b_m$, as well as $y_{mn}$ all vary with the type of the workers), there is no obvious measure of heterogeneity. Our analysis below reveals that none of these conditions imply any restrictions on the variation of $y_{mn}$ by type, while the variations in $p_m$, $q_m$ and $b_m$ are restricted. To have a sense of how restricted they are, we undertake a series of numerical exercises and, fixing $p_H$ and $p_L$, report the largest values of $\bar{b}/\underline{b}$ that is allowed by the restrictions. In addition, we report the largest possible value of

$$\delta = |\Lambda|/l^P(H,L) = 2\frac{(p_H - p_L)(\bar{b} - \underline{b})}{l^P(H,L)}. \tag{A12}$$

The latter is the possible loss in terms of moral hazard cost due to mis-sorting, as a percentage of moral hazard cost for a team of heterogenous workers. In addition to being a "combined measure" of type-heterogeneity (it is small if $p_H - p_L$ or $\bar{b} - \underline{b}$ is small), it can be viewed as a measure of how significant the impact of sorting is and, therefore, how important the channel we are highlighting in this paper is. It is worth emphasizing that an analogous measure for the cost of mis-sorting in terms of expected output loss is

$$\frac{\Theta}{y_{HL}} = \frac{y_{HH} + y_{LL} - 2y_{HL}}{y_{HL}},$$

which is not restricted by any of the assumptions we make.

We proceed in two steps: First, in Subsection 3.1, we discuss the restrictions imposed only by the technology parameters (i.e. 1(a)-1(c)), and show that substantial heterogeneity is allowed when considering only these. Second, in the same subsection, we argue that, for any technology that satisfies 1(a)-1(c), the set of $\underline{w}$ satisfying $2(a), 2(b)$ and 3(a) is non-empty. That is, no further restriction on type heterogeneity is imposed by these conditions. In this case – i.e., when 3(a), as opposed to 3(b), is required – sorting effects the surplus only in partnerships, but it is still sufficient to establish the results presented in Propositons 3 and 4. Yet, it may be of interest to know when the set of $\underline{w}$ that satisfy 2(a) and 2(b), also satisfies 3(b) (i.e., binding limited liability), since this is the case in which the sorting has an impact on the profits of the corporation as well, and, therefore, all possible channels driving the results in Propositions 3 and 4 are active. Accordingly, in Subsection 3.2, we take up this case: first, we present numerical exercises that demonstrate the allowed variation in types when this restriction is imposed. We find that type-heterogeneity needs to be mild; in other words, not a lot of variation in types is allowed. Finally, in the same subsection we discuss a plausible variation of the model under which the channels, via which endogenous ownership structure is affected, remain unchanged while we are able to relax some of the restrictions imposed on the parameters, and, therefore, allow for substantial heterogeneity.

### 3.1 Limits on heterogeneity

Below, we formally state conditions 1(a)-1(c) for a given vector of technology parameters $(y_{mn}, \Delta, p_m, c, \bar{b}, \underline{b})$.

1(a) Effort is socially optimal:

$$\Delta \geq \bar{b} \tag{A13}$$

1(b) Moral hazard is costly in partnerships:

$$\Delta \leq 2\underline{b} \tag{A14}$$

1(c) Effort by both partners is optimal in partnerships in the presence of moral hazard: For all $m, n \in \{L, H\}$,

$$y_{mn} + \Delta - 2c - (1 - p_m - p_n)(b_m + b_n) \geq y_{mn} + (p_m + q_n)\Delta - c \tag{A15}$$

$$\geq y_{mn} + (q_m + q_n)\Delta, \tag{A16}$$

with $b_m, b_n \in \{\bar{b}, \underline{b}\}$.

Here, (A15) implies (A16) since $(p_m - q_m)\Delta > c$. Further, (A15) can be rearranged as

$$\Delta \geq b_m + b_n - \frac{b_m}{b_n} \cdot \frac{c}{1 - p_m - p_n + c/b_n}, \text{ for all } b_m, b_n \in \{\bar{b}, \underline{b}\} \text{ and } m, n \in \{L, H\}. \tag{A17}$$

Note that, when there is no variation in types (i.e., when $p_L = p_H$ and $\underline{b} = \bar{b}$), (A13), (A14) and (A17) are obviously compatible. When types are different, it can be verified that the two most restrictive lower bounds on $\Delta$ imposed by (A17) are the ones with (i) $m = n = L$ and $b_m = b_n = \bar{b}$ and (ii) $m = H, n = L$ and $b_m = \underline{b}, b_n = \bar{b}$. This leads to the following result:

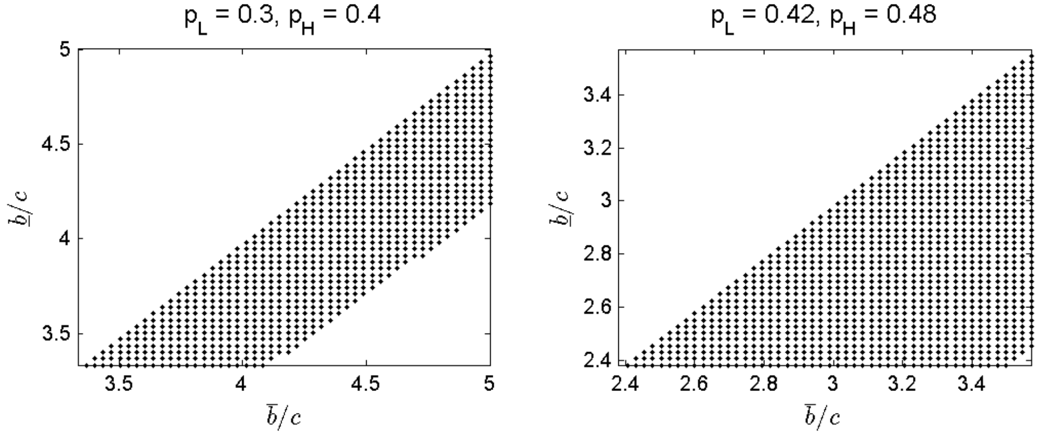


Figure 1: Ranges of $(\bar{b}, \underline{b})$ for which (A18) can be satisfied for some $\Delta$, for a given $(p_H, p_L)$.

**Lemma 2** *Fix a vector of technology parameters* $(y_{mn}, \Delta, p_m, c, \bar{b}, \underline{b})$. *Conditions 1(a), 1(b) and 1(c) are satisfied if, and only if,*

$$\max\left\{\bar{b}, \quad 2\bar{b} - \frac{c}{1 - 2p_L + c/\bar{b}}, \quad \bar{b} + \underline{b} - \underline{b}/\bar{b}\frac{c}{1 - p_L - p_H + c/\bar{b}}\right\} \leq \Delta \leq 2\underline{b}. \tag{A18}$$

*Moreover, if* $p_H = p_L$ *and* $\bar{b} = \underline{b}$*, the range of* $\Delta$ *satisfying* (A18) *is non-empty.*

Figure 1 illustrates combinations of $(\bar{b}, \underline{b})$ for which (A18) is satisfied, for two different pairs of $(p_H, p_L)$.[4] As can be seen, (A18) allows for substantial amount of heterogeneity, both in terms of probabilities of success $p_H$ and $p_L$, and in terms of incentive-inducing wedges $\bar{b}$ and $\underline{b}$. In addition, we note that,

- for the specification $p_H = .4, p_L = .3$, within the range of $(\bar{b}, \underline{b})$ illustrated on the graph, the ratio $\bar{b}/\underline{b}$ can be as large as 1.23, while $\delta$ defined in (A12) can be as large as 0.67;[5]

- for he specification $p_H = .48, p_L = .42$, the ratio $\bar{b}/\underline{b}$ can be as large as 1.46, while $\delta$ defined in (A12) can be as large as 1.2.

Next, we consider the conditions involving $\underline{w}$. We start by formally stating these conditions:

2 Conditions that involve $\underline{w}$, *except* the limited liability conditions:

[4] From (A18), one can see that the larger $p_H$, $p_L$ and $\bar{b}$ are, the bigger are the differences between $\bar{b}$ and $\underline{b}$ allowed by (A18). Also, note that $\bar{b}$ can be made arbitrarily large by making $q_m$ close to $p_m$.

[5] Note that the percentage change in the moral hazard cost in partnerships due to switching from negative to positive sorting is measured by $\delta/2 \cdot 100\%$.

(a) Effort by both partners is optimal in any team in a corporation in the presence of moral hazard: For all $m, n \in \{L, H\}$,

$$y_{mn} + (\Delta - b_m - b_n)(p_m + p_n) - 2\underline{w} \geq y_{mn} + \Delta(p_m + q_n) - c - u_m - u_n, \quad \text{(A19)}$$
$$\geq y_{mn} + \Delta(q_m + q_n) - u_m - u_n. \quad \text{(A20)}$$

for all $u_m, u_n$ satisfying (11) and (12) and $b_m, b_n \in \{\underline{b}, \bar{b}\}$.

(b) Profits from any two identical workers are positive, i.e. (A1) holds:

$$y_{mm} + 2p_m(\Delta - 2b_m) - 2\underline{w} \geq 0, \text{ for any } m \in \{L, H\} \text{ and } b_m \in \{\underline{b}, \bar{b}\}.$$

3 The limited liability conditions:

(a) Limited liability is not binding for any type, under any sorting pattern, for either specification of the technology:
For all $m, n \in \{L, H\}$ and for all $u_m, u_n$ satisfying (11) and (12),

$$\underline{w} + (p_m + p_n)b_m - c \leq u_m,$$

with $b_m \in \{\underline{b}, \bar{b}\}$.
**OR**

(b) Limited liability is binding for all types, under either sorting pattern, for each specification of the technology:
For all $m, n \in \{L, H\}$ and for all $u_m, u_n$ satisfying (11) and (12),

$$\underline{w} + (p_m + p_n)b_m - c \geq u_m,$$

with $b_m \in \{\underline{b}, \bar{b}\}$.

Fixing a vector of technology parameters satisfying 1(a)-1(c), to decide whether a given value of $\underline{w}$ satisfies all the restrictions, it is sufficient to verify 2(a) and 2(b), along with 3(a). Inspecting these conditions immediately reveals that each of them imposes an upper bound on $\underline{w}$, while there are no lower bounds. Letting $\underline{w}^*$ represent the smallest of these upper bounds, then any $\underline{w} \in (-\infty, \underline{w}^*]$ satisfies all the restrictions of the model. Clearly, such an interval is never empty, and, therefore, for any vector of technology parameters satisfying 1(a)-1(c), the set of economic environments to which our results apply is non-empty (and, moreover, is "large").

### 3.2 Discussion: binding limited liability

In our paper, the channel that endogenously determines the ownership structure is the impact of sorting on the owners' payoffs that can be obtained for either type of ownership structure. The key is the presence of the tradeoff between the goals of minimizing the moral hazard cost and maximizing the expected output in one or the other ownership structure. Even though, for our results to go through, it is sufficient that this trade-off exists only for partnerships, it is still of interest to know when the set of environments in which this trade-off also exists for the corporations is non-empty. This trade-off may exists within a corporation only under the binding limited liability constraints. Next, we discuss the conditions on the technology parameters under which the set of $\underline{w}$ satisfying 1(a)-1(c), 2(a), 2(b) along with 3(c) is non-empty.

As discussed in the previous subsection, for a given vector of technology parameters satisfying 1(a)-1(c), the conditions 2(a) and 2(b) impose upper bounds on $\underline{w}$, whereas 3(b) (the binding limited liability condition) imposes a lower bound.

Firstly, comparing the binding limited liability restriction across all possible cases (i.e., all team combinations for both complements and substitutes specifications) reveals that the two largest lower bounds on $\underline{w}$ are imposed when $b_H = \underline{b}$ and $b_L = \bar{b}$ and either (i) $m = H$, $n = L$ and $u_H = S_{HL} - S_{LL}/2$ or (ii) $m = H$, $n = L$ and $u_H = S_{HH}/2$. Thus, limited liability always binds if

$$\begin{aligned} 2\underline{w} &\geq y_{HH} + \Delta - 2\underline{b} + 2(p_H - p_L)\underline{b} && \text{(A21)} \\ &\geq 2y_{HL} - y_{LL} + \Delta - 2\underline{b} + 2(p_H - p_L)\bar{b}. && \text{(A22)} \end{aligned}$$

Secondly, it can be verified that the smallest upper bound imposed on $2\underline{w}$ by condition 2(a) is computed for $m = n = L$, $u_m = u_n = S_{LL}/2$ and $b_m = b_n = \bar{b}$, in which case it becomes

$$2\underline{w} \leq y_{LL} + \Delta - c - 2\bar{b} + c \cdot \Delta/\bar{b}. \tag{A23}$$

Finally, it is easy to see that the two smallest upper bounds imposed by 2(b) are realized when $m = L, H$ and $b_m = \bar{b}$, i.e. these bounds are given by

$$2\underline{w} \leq \min\left\{y_{LL} - 2p_L(2\bar{b} - \Delta), y_{HH} - 2p_H(2\bar{b} - \Delta)\right\}. \tag{A24}$$

The following Lemma summarizes the above analysis:

**Lemma 3** *Fix a vector of technology parameters $(y_{mn}, \Delta, p_m, c, \bar{b}, \underline{b})$. Then conditions 2(a), 2(b) and*

*3(b) are satisfied if*

$$y_{HH}+\Delta-2\underline{b}+2(p_H-p_L)\underline{b}\leq 2\underline{w} \tag{A25}$$

$$2y_{HL}-y_{LL}+\Delta-2\underline{b}+2(p_H-p_L)\overline{b}\leq 2\underline{w} \tag{A26}$$

$$2\underline{w}\leq y_{LL}+\Delta-c-2\overline{b}+c\cdot\Delta/\overline{b} \tag{A27}$$

$$y_{LL}+\Delta-\underline{b}(p_H-p_L)-2\underline{b}+2c\cdot\underline{b}/\overline{b} \tag{A28}$$

$$2\underline{w}\leq y_{LL}-2p_L(2\overline{b}-\Delta) \tag{A29}$$

$$2\underline{w}\leq y_{HH}-2p_H(2\overline{b}-\Delta). \tag{A30}$$

*Moreover, if* (A18) *holds and all workers are identical, the range of* $\underline{w}$ *satisfying* (A25)-(A30) *is non-empty.*

The last statement of Lemma 3 is trivially verified by substituting $y_{mn}=y$, $p_m=p$ and $\underline{b}=\overline{b}=b$ for all $m,n\in\{L,H\}$ in (A25)-(A30) and verifying that the two lower bounds are strictly below each of the four upper bounds. By continuity, (A25)-(A30) can be satisfied for some $\underline{w}$ if the types are sufficiently close to each other. In what follows, we provide a few calculations illustrating how much heterogeneity between the types can be allowed while satisfying conditions (A25)-(A30) along with (A18).

Note that (A18) and (A25)-(A30) can be re-written in terms of $y_{mn}/c$, $b_m/c$ and $\Delta/c$. Differences in output $y_{mn}$ are restricted only by the conditions in Lemma 3, and, if (A25)-(A30) strictly hold for $y_{mn}=y$, arbitrarily large differences between $y_{mn}$ can be achieved by choosing $c$ sufficiently large. Thus, we only need to verify that (A25)-(A30) can be satisfied for some $\underline{w}$ under $y_{mn}=y$ for all $m,n\in\{L,H\}$. In this case, the compatibility of the bounds in (A25)-(A30) is independent of $y$. Moreover, (A25) is implied by (A26), and (A29) is implied by (A30). Thus, compatibility of the upper and lower bounds on $2\underline{w}$ boils down to verifying that there exist $(p_L,p_H,\overline{b},\underline{b},\Delta)$ such that

$$2(p_H-p_L)\overline{b}\leq c(\Delta/\overline{b}-1)-2(\overline{b}-\underline{b}) \tag{A31}$$

$$\leq 2c\cdot\overline{b}/\underline{b}-\underline{b}(p_H-p_L) \tag{A32}$$

$$\leq (2\underline{b}-\Delta)(1-2p_H)-4p_H(\overline{b}-\underline{b}). \tag{A33}$$

Here, the first and the last inequalities impose, respectively, extra (in addition to (A18)) upper and lower bounds on $\Delta$:

$$\overline{b}\cdot\left[1+2(p_H-p_L)\overline{b}/c+2(\overline{b}-\underline{b})/c\right]\leq\Delta\leq 2\underline{b}-\frac{2(p_H-p_L)\overline{b}+4p_H(\overline{b}-\underline{b})}{1-2p_H}, \tag{A34}$$

and (A32) is an additional restriction on $(\overline{b},\underline{b})$ for any $(p_H,p_L)$.

Figure 2 illustrates the ranges of $(\overline{b},\underline{b})$ for which, for a given pair $(p_H,p_L)$, conditions stated in Lemmas 2 and 3 (dark shaded areas), as well as in Lemma 3 alone (all shaded areas), can be satisfied for some $(y_{mn},\Delta,\underline{w})$. To satisfy (A25)-(A30), in addition to (A18), only relatively small differences between $p_H$

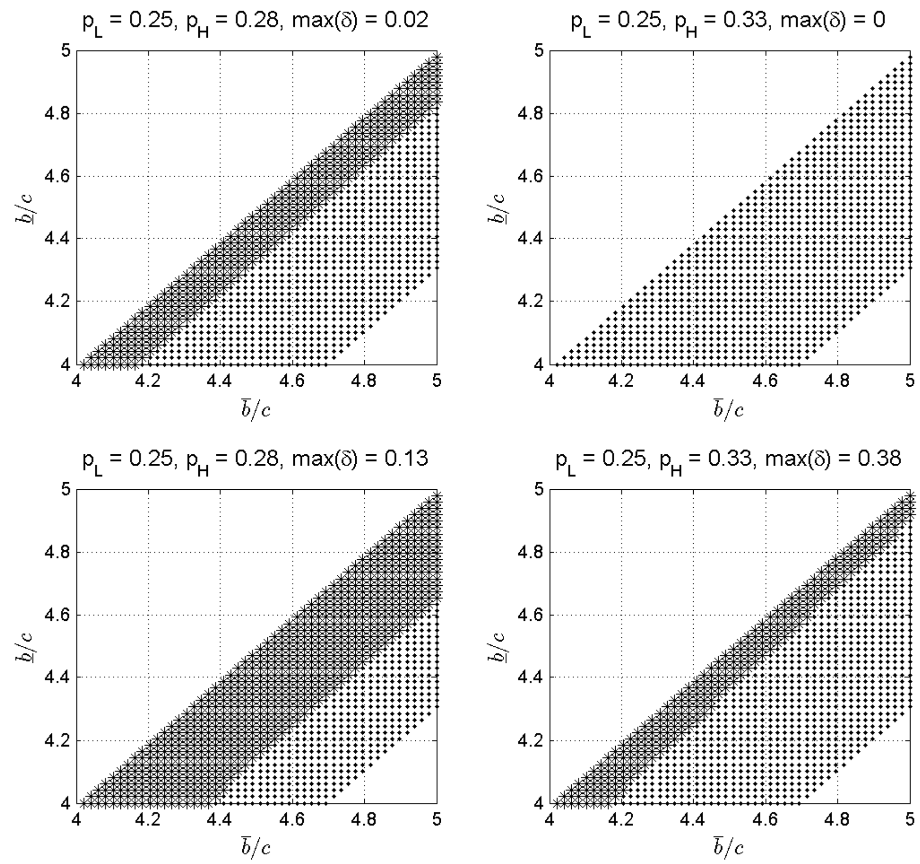


Figure 2: Ranges of $(\bar{b}, \underline{b})$ for which conditions stated in Lemmas 2 and 3 can be satisfied for some $(y_{mn}, \Delta, \underline{w})$, for a given $(p_H, p_L)$. The top two panels correspond to the specification of the model in the main text. The bottom two panels correspond to the modification discussed in Section 3.2.1.

and $p_L$are allowed: the dark shaded area in the top left plot, where $p_L = 0.25$ and $p_H = 0.28$, is non-empty, but it disappears in the top right plot where $p_H$ rises to 0.33. In the former case, $\bar{b}/\underline{b}$ can be as large as 1.05, and $\delta = |\Lambda|/l^P(H, L)$ can be as large as 0.02, suggesting that conditions stated in Lemmas 2 and 3 can be satisfied only if the degree of heterogeneity is rather mild.

### 3.2.1 A modification of the model

There exists a natural modification of our modelling environment in which binding limited liability, along with all other conditions, can be maintained for a considerably wider set of parameters. To a large extent, the severity of the imposed restrictions is driven by conditions (A29) and (A30) imposed by the requirement that the firm generates positive profits from each team of identical workers. These conditions could be relaxed by assuming that the firm has some technological advantage, e.g., due to a cost advantage, and operates the technology with $\hat{y}_{mn} > y_{mn}$ for all $m, n \in \{H, L\}$.[6] Then, the aforementioned requirement can be satisfied by choosing $\hat{y}_{mn}$ sufficiently high, rather than imposing an upper bound on $\underline{w}$. This modification leaves all other conditions unchanged (since all other appearances of $y_{mn}$ are due to the presence of workers' outside options $u_m$ or $u_n$ in the corresponding conditions) and, therefore, (A29) and (A30) can be dropped. The two bottom panels on Figure 2 illustrate how the ranges of $(\bar{b}, \underline{b})$ are affected by such a modification. As can be seen, the extent of 'allowed' heterogeneity under $p_L = 0.25$ and $p_H = 0.28$ is considerably greater (the maximum $\delta$ in the dark shaded area rises to 0.13, while $\bar{b}/\underline{b}$ can be as high as 1.09); and much bigger differences between $p_L$ and $p_H$ can now be attained (the right bottom plot demonstrates that for $p_L = 0.25$ and $p_H = 0.33$, all the restrictions can be satisfied for $\bar{b}/\underline{b}$ as high as 1.04, with maximum $\delta$ equal to 0.38). Moreover, as noted in footnote 14 in the main text, under this modification, Assumption 1 (costly moral hazard in partnerships) can be dropped altogether, i.e. the upper bound on $\Delta$ in (A18) can be removed. In this case, to guarantee that limited liability always binds, the differences between the types are only restricted by (A32), which is easily satisfied.

[6]For instance, one can assume that a fixed per-firm cost must be incurred, implying that corporations, hiring more than one team of workers, face lower per-team costs.