Two Perspectives on Preferences and Structural Transformation

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Online Appendix

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Online Appendix A: Aggregation of Demand Functions

Consider N households indexed by $n = 1, ..., N$. Each household solves:

$$\max_{c_a^n, c_m^n, c_s^n} \left[\omega_a^{\frac{1}{\sigma}} (c_a^n + \bar{c}_a)^{\frac{\sigma-1}{\sigma}} + \omega_m^{\frac{1}{\sigma}} (c_m^n + \bar{c}_m)^{\frac{\sigma-1}{\sigma}} + \omega_s^{\frac{1}{\sigma}} (c_s^n + \bar{c}_s)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$
$$\text{s.t.} \quad p_a c_a^n + p_m c_m^n + p_s c_s^n \leq C_n.$$

Let the parameters and the income distribution be such that for all $n \in \{1, ..., N\}$ household expenditure exceed a minimum level:

$$C_n > \sum_{i=a,m,s} p_i \max\{-\bar{c}_i, 0\}. \tag{1}$$

Then the solution to each household's problem is interior and the first–order conditions are

$$\left(\frac{\omega_a}{\omega_s}\right)^{\frac{1}{\sigma}} \left(\frac{c_a^n + \bar{c}_a}{c_s^n + \bar{c}_s}\right)^{-\frac{1}{\sigma}} = \frac{p_a}{p_s},$$
$$\left(\frac{\omega_m}{\omega_s}\right)^{\frac{1}{\sigma}} \left(\frac{c_m + \bar{c}_m}{c_s^n + \bar{c}_s}\right)^{-\frac{1}{\sigma}} = \frac{p_m}{p_s},$$

which can be rewritten as

$$\frac{p_a}{p_s} \frac{c_a^n + \bar{c}_a}{c_s^n + \bar{c}_s} = \frac{\omega_a}{\omega_s} \left(\frac{p_a}{p_s}\right)^{1-\sigma},$$
$$\frac{p_m}{p_s} \frac{c_m^n + \bar{c}_m}{c_s^n + \bar{c}_s} = \frac{\omega_m}{\omega_s} \left(\frac{p_m}{p_s}\right)^{1-\sigma}.$$

This gives the demand functions

$$
\begin{aligned}
p_a(c_a^n + \bar{c}_a) &= \frac{p_a(c_a^n + \bar{c}_a) + p_m(c_m^n + \bar{c}_m) + p_s(c_s^n + \bar{c}_s)}{1 + \dfrac{\omega_m}{\omega_a}\left(\dfrac{p_m}{p_a}\right)^{1-\sigma} + \dfrac{\omega_s}{\omega_a}\left(\dfrac{p_s}{p_a}\right)^{1-\sigma}}, \\
p_m(c_m^n + \bar{c}_m) &= \frac{p_a(c_a^n + \bar{c}_a) + p_m(c_m^n + \bar{c}_m) + p_s(c_s^n + \bar{c}_s)}{1 + \dfrac{\omega_a}{\omega_m}\left(\dfrac{p_a}{p_m}\right)^{1-\sigma} + \dfrac{\omega_s}{\omega_m}\left(\dfrac{p_s}{p_m}\right)^{1-\sigma}}, \\
p_s(c_s^n + \bar{c}_s) &= \frac{p_a(c_a^n + \bar{c}_a) + p_m(c_m^n + \bar{c}_m) + p_s(c_s^n + \bar{c}_s)}{1 + \dfrac{\omega_a}{\omega_s}\left(\dfrac{p_a}{p_s}\right)^{1-\sigma} + \dfrac{\omega_m}{\omega_s}\left(\dfrac{p_i}{p_s}\right)^{1-\sigma}}.
\end{aligned}
$$

Adding up over all households, we obtain:

$$
\begin{aligned}
p_a(c_a + N\bar{c}_a) &= \frac{p_a(c_a + N\bar{c}_a) + p_m(c_m + N\bar{c}_m) + p_s(c_s + N\bar{c}_s)}{1 + \dfrac{\omega_m}{\omega_a}\left(\dfrac{p_m}{p_a}\right)^{1-\sigma} + \dfrac{\omega_s}{\omega_a}\left(\dfrac{p_s}{p_a}\right)^{1-\sigma}}, \\
p_m(c_m + N\bar{c}_m) &= \frac{p_a(c_a + N\bar{c}_a) + p_m(c_m + N\bar{c}_m) + p_s(c_s + N\bar{c}_s)}{1 + \dfrac{\omega_a}{\omega_m}\left(\dfrac{p_a}{p_m}\right)^{1-\sigma} + \dfrac{\omega_s}{\omega_m}\left(\dfrac{p_s}{p_m}\right)^{1-\sigma}}, \\
p_s(c_s + N\bar{c}_s) &= \frac{p_a(c_a + N\bar{c}_a) + p_m(c_m + N\bar{c}_m) + p_s(c_s + N\bar{c}_s)}{1 + \dfrac{\omega_a}{\omega_s}\left(\dfrac{p_a}{p_s}\right)^{1-\sigma} + \dfrac{\omega_m}{\omega_s}\left(\dfrac{p_i}{p_s}\right)^{1-\sigma}},
\end{aligned}
$$

where

$$
c_i \equiv \sum_{n=1}^{N} c_i^n.
$$

Let $C \equiv \sum_{n=1}^{N} C^n$. If the stand–in household solves

$$
\begin{aligned}
&\max_{c_a, c_m, c_s} \left[\omega_a(c_a + N\bar{c}_a)^{\frac{\sigma-1}{\sigma}} + \omega_m(c_m + N\bar{c}_m)^{\frac{\sigma-1}{\sigma}} + \omega_s(c_s + N\bar{c}_s)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
&\qquad \text{s.t.} \quad p_a c_a + p_m c_m + p_s c_s \le C,
\end{aligned}
$$

then its choices satisfy

$$
c_i = \sum_{n=1}^{N} c_i^n.
$$

In other words, there is aggregation.

Online Appendix B: Calculating Consumption Value Added

B.1: Constructing Final Expenditure in Producer's Prices

B.1.1: Disaggregation to seven sectors

To obtain final consumption expenditure in producer's prices from the available data on final consumption expenditure in purchaser's prices, we need to remove the distribution costs from the different goods categories and move them to services. For two reasons, this requires further disaggregation. First, we calculate the distribution costs for retail, wholesale and transportation services from the expenditure on the sector Trade and Transport. We therefore, need to separate Trade and Transport from the rest of services. Second, the expenditure on mining involve distribution costs whereas those on construction do not, so we need to separate the two from other manufacturing. We therefore consider the following seven sectors: Agriculture, Mining, Construction, Durable Manufacturing, Nondurable Manufacturing, Trade and Transport, and Services excluding Trade and Transport, which we index by $i \in \{Ag, Mi, Co, MaD, MaN, TT, Se\}$, which aggregate to our model sectors in the obvious way: $a = \{Ag\}$, $m = \{Mi, Co, MaD, MaN\}$, $s = \{TT, Se\}$. Note that while we use the BEA classification for Agriculture, Mining, Construction, and Manufacturing , the sector Trade and Transport combines "Wholesale Trade", "Retail Trade" and "Transportation and Warehousing".

We should mention a potential problem that arises from the reclassification of industries over time. In particular, while the BEA now publishes GDP by industry data based on the NAICS for the whole period 1947–2010, it still publishes the underlying input–output tables for the subperiod 1947–1997 based on the different SIC's. Fortunately, many of the reclassifications from the SIC's to the NAICS happened at finer levels of disaggregation than we study here, and so they do not affect the aggregates of the six sectors we have just introduced.

B.1.2: Removing distribution costs from personal consumption expenditure

We now explain how to remove distribution costs from personal consumption expenditure.

The expenditure side of GDP values personal consumption expenditure at purchaser's prices and it disaggregates them into the expenditure on durable goods, nondurable goods, trade and transportation, and services excluding trade and transportation:

$$PC^{Pu} = PC^{Pu}_{DG} + PC^{Pu}_{NDG} + PC^{Pu}_{TT} + PC^{Pu}_{Se}.$$

Nondurable goods consist of "food and beverages purchased for off-premises consumption" and "nondurable goods" excluding "food and beverages purchased for off-premises consumption", trade and transportation consists of "public transportation", and services consist of "services" excluding "public transportation".

We start by removing distribution costs from personal consumption expenditure on durable goods and non-durable goods. We assume that distribution margin is the same across different goods. To go from purchaser's to producer's prices, we calculate the distribution margins DM_{PC_G} by using the fact that in the IO Tables personal consumption expenditure on trade and transportation consists of all transportation expenditure whereas PC^{Pu}_{TT} consists only of "public transportation" that households explicitly purchase. Hence, the difference between the two equals the distribution costs of goods that household purchase indirectly when purchasing goods, and so:

$$\begin{aligned}
DM_{PC_{Gs}} &= \frac{(PC_{TT}^{IO} - PC_{TT}^{Pu})}{(PC_{TT}^{IO} - PC_{TT}^{Pu}) + (PC_{Ag}^{IO} + PC_{Mi}^{IO} + PC_{Co}^{IO} + PC_{MaD}^{IO} + PC_{MaN}^{IO})},\\
PC_{DG}^{Pr} &= (1 - DM_{PC_{Gs}})PC_{DG}^{Pu},\\
PC_{NDG}^{Pr} &= (1 - DM_{PC_{Gs}})PC_{NDG}^{Pu}.
\end{aligned}$$

We continue by removing distribution costs from personal consumption expenditure on services. This is straightforward because the IO Tables suggest that personal consumption expenditure on services involve negligible distribution costs. Therefore:

$$PC_{Se}^{Pr} = PC_{Se}^{Pu}.$$

Given that we have calculated PC_{DG}^{Pr} and PC_{NDG}^{Pr}, we now disaggregate it into the components PC_{Ag}^{Pr}, PC_{Mi}^{Pr}, PC_{Co}^{Pr}, PC_{MaD}^{Pr} and PC_{MaN}^{Pr}. The IO Tables report that PC_{Mi}^{IO} are very small and that PC_{Co}^{IO} are zero in all years. We therefore set $PC_{Mi}^{Pr} = PC_{Co}^{Pr} = 0$. We set $PC_{DG}^{Pr} = PC_{MaD}^{Pr}$. This leaves us with the task of splitting PC_{NDG}^{Pr} between PC_{Ag}^{Pr} and PC_{MaN}^{Pr}. First, we calculate expenditures on food at producer prices, PC_{Food}^{Pr}. Expenditure on food is "food and beverages purchased for off-premises consumption". We remove distribution costs by applying the distribution margin of goods that we calculated above, $PC_{Food}^{Pr} = (1 - DM_{PC_{Gs}})PC_{Food}^{Pu}$. Next, since PC_{Food}^{Pr} contains both unprocessed and processed food, we need to take processed food out to obtain the expenditure on agricultural commodities. We use that PC_{Ag}^{IO} are the expenditure on agricultural goods without processed food. Defining $\Phi_1 \equiv PC_{Ag}^{IO}/PC_{Food}^{Pr}$, we have

$$PC_{Ag}^{Pr} = \Phi_1 PC_{Food}^{Pr}.$$

In sum, the components of personal consumption expenditure in producer's prices are obtained as follows:

$$\begin{aligned}
PC_{Ag}^{Pr} &= \Phi_1 PC_{Food}^{Pr}\\
PC_{Mi}^{Pr} &= 0,\\
PC_{Co}^{Pr} &= 0,\\
PC_{MaD}^{Pr} &= (1 - DM_{PC_{Gs}})PC_{DG}^{Pu},\\
PC_{MaN}^{Pr} &= (1 - DM_{PC_{Gs}})PC_{NDG}^{Pu} - PC_{Ag}^{Pr},\\
PC_{TT}^{Pr} &= PC_{TT}^{Pu} + DM_{PC_{Gs}}(PC_{MaD}^{Pu} + PC_{MaN}^{Pu}),\\
PC_{Se}^{Pr} &= PC_{Se}^{Pu}.
\end{aligned}$$

B.1.3: Removing distribution costs from government consumption expenditure

We now explain how to remove distribution costs from final expenditure on government consumption.

In the IO Tables, the general government appears as a production industry and as a commodity. In the expenditure side of GDP, government consumption expenditure at purchaser's prices are defined as the gross output of the general government industry minus own account investment and sales to other sectors.

The treatment of the gross output of the general government industry changed in 1998. Before 1998, it was defined as its value added GC_{VA}^{Pu} (compensation of general government employees plus consumption of general government fixed capital). All intermediate inputs were consequently treated as final government expenditure on these goods. Since 1998, the gross output of the general government industry has included intermediate goods, that is, it is defined as the sum of value added GC_{VA}^{Pu}, purchased intermediate durable and nondurable goods, GC_{DG}^{Pu}, GC_{NDG}^{Pu}, and purchased intermediate services, GC_{Se}^{Pu}.

We start with the period 1947–1997. During this period, the IO Tables show that GC_{Ag}^{IO} and GC_{Mi}^{IO} are small, so we set $GC_{Ag}^{Pr} = GC_{Mi}^{Pr} = 0$. The distribution margins of government consumption expenditure in the 1997 IO Tables on average equal 18% of the distribution margins of personal consumption expenditure, so we set $DM_{GC_{Gs}} = 0.18 \cdot DM_{PC_{Gs}}$. Next we calculate GC_{Co}^{Pr}. The raw IO Tables distinguish between government expenditure on "maintenance and repair construction" and on "new construction". First, we calculate

$$\Phi_2 \equiv \frac{\text{government expenditure on maintenance and repair construction}}{\text{depreciation on government structures}},$$

where the depreciation on government structures is taken from Table 7.3: "Current-Cost Depreciation of Government Fixed Assets" of the BEA Fixed Assets Tables. We then calculate GC_{Co}^{Pr} by multiplying Φ_2 with depreciation on government structures.

In sum, for the period 1947–1997, we calculate the variables of interest as:

$$GC_{Ag}^{Pr} = 0, \tag{2a}$$
$$GC_{Mi}^{Pr} = 0, \tag{2b}$$
$$GC_{Co}^{Pr} = \Phi_2 \cdot \text{Depreciation on government structures}, \tag{2c}$$
$$GC_{MaD}^{Pr} = (1 - DM_{GC_{Gs}})GC_{DG}^{Pu}, \tag{2d}$$
$$GC_{MaN}^{Pr} = (1 - DM_{GC_{Gs}})GC_{NDG}^{Pu} - GC_{Co}^{Pr}, \tag{2e}$$
$$GC_{TT}^{Pr} = DM_{GC_{Gs}}(GC_{DG}^{Pu} + GC_{NDG}^{Pu}), \tag{2f}$$
$$GC_{Se}^{Pr} = GC_{VA}^{Pu} + GC_{Se}^{Pu} - \text{Sales to other sectors} - \text{Own account investment}. \tag{2g}$$

The last equation expresses that government expenditure on services are equal to the value added representing the service flow from government capital and employees plus the services purchased as intermediate input net of what is invested on own account sold to other sectors, which typically are general government services. Own account investment is best viewed as services because it typically involves government capital and labor which creates investment goods such developing softwares. The equation also reflects that typically services do not have distribution costs, so services evaluated at producer's and purchaser's prices are the same.

For the period 1998–2010, government consumption expenditure in the IO Tables almost exclusively consist of expenditure on general government services. Since services have no distribution costs, we set:

$$GC_{Ag}^{Pr} = GC_{Mi}^{Pr} = GC_{Co}^{Pr} = GC_{Ma}^{Pr} = GC_{TT}^{Pr} = 0,$$
$$GC^{Pr} = GC^{Pu}.$$

B.2: Linking Consumption Expenditures to Value Added

The total requirement matrix (henceforth TR Matrix) links the income and the expenditure side of GDP. We now explain how to use the TR Matrix to obtain the value added in producer's prices that are generated by the final expenditure on consumption in producer's prices, which we have just constructed in the previous subsection. We use the language and the notation of the BEA to the extent possible. For further explanation see Bureau of Economic Analysis (2006). The way in which the TR Matrix is calculated changed in 1972. So for years prior to 1972, the IO Tables assumed that each industry produces one commodity and that each commodity is produced in exactly one industry. For years after 1972 the IO Tables have taken account of the fact that industries can produce more than one commodity and that the same commodity can be produced in different industries.

We start by explaining the TR Matrix prior to 1972. We denote the number of industries by n, which before 1972 equals the number of commodities. Domestically produced commodities are purchased either by domestic industries (intermediate expenditure) or by final users (final uses or final expenditure). Final uses include both domestic final uses and exports, where exports can be either intermediate or final foreign uses. Domestic industries produce gross output and the difference between gross output and intermediate expenditure is industry value added.

Let $\mathbf{A}$ denote the $(n \times n)$ transaction matrix.[1] Rows are associated with commodities and columns with industries: entry ij shows the dollar amount of commodity i that industries j uses per dollar of output it produces. Note that these commodities may have been produced domestically or imported. Let $\mathbf{q}$ denote the $(n \times 1)$ output vector of domestically produced commodities. Element i records the sum of the dollar amounts of commodity i that are delivered to other domestic industries as intermediate inputs and to final uses. Let $\mathbf{g}$ denote the $(n \times 1)$ industry output vector. Element j records the dollar amount of output of industry j. Let $\mathbf{e}$ denote the $(n \times 1)$ vector of expenditures on final uses. Element i records the dollar amount of final uses of the domestically produced commodity i, so component e_i reports domestic private and public consumption, domestic investment, and net exports of commodity i.

Two identities link these vectors and with the TR Matrix:

$$\mathbf{q} = \mathbf{A}\mathbf{g} + \mathbf{e}, \tag{3}$$

$$\mathbf{q} = \mathbf{g}. \tag{4}$$

The first identity states that the dollar amount of domestically produced output of each commodity equals the sum of intermediate uses plus the final uses of that commodity. The second identity states that total value of output of industry i equals to the total value of commodity i, which is trivially true here because each industry is assumed to produce one distinct commodity. We can solve these two equations for g:

$$\mathbf{g} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{e}, \tag{5}$$

where $\mathbf{I}$ is the $(n \times n)$ identity matrix (1 in the diagonal and zero elsewhere). $\mathbf{R} \equiv (\mathbf{I} - \mathbf{A})^{-1}$ is called the total requirements matrix. Rows are associated with industries and columns with commodities. Entry ji shows the dollar value of industry j's production that is required, both directly and indirectly, to deliver one dollar of the domestically produced commodity i to final uses including net exports.

We continue by explaining the TR Matrix after 1972, so now the IO Tables take account of the fact that an industry may produce more then one commodity and that a commodity may be

[1] Matrices and vectors are in bold symbol throughout the paper.

produced in different industries. In general, the number of industries n will then differ from the number of commodities. We call the number of commodities m. This implies that we no longer have one transaction matrix, but a use and a make matrix. $\mathbf{B}$ denotes the $(m \times n)$ use matrix. Entry ij shows the dollar amount of commodity i that industries j uses per dollar of output it produces. Again note that these commodities may have been produced domestically or imported. $\mathbf{W}$ denotes the $(n \times m)$ make matrix. Rows are associated with industries and columns with commodities: entry ji shows which share of one dollar of the domestically produced commodity i industry j makes. Two identities link these matrices and vectors:

$$\mathbf{q} = \mathbf{Bg} + \mathbf{e}, \tag{6a}$$

$$\mathbf{g} = \mathbf{Wq}. \tag{6b}$$

The first identity says that the dollar amount of each domestically produced commodity equals the sum of the dollar amount of that commodity that the different domestic industries use as intermediate goods plus the dollar amount of final uses of that commodity. Note again that final uses are for domestic private and public consumption, domestic investment, and net exports. The second identity says the dollar output of each industry equals the sum of that industry's contribution to the outputs of the different domestically produced commodities.

To eliminate $\mathbf{q}$ from these identities, we substitute (6b) into (6a) to obtain $\mathbf{q} = \mathbf{BWq} + \mathbf{e}$. We then solve this for $\mathbf{q}$ and substitute the result back into (6b). This gives:

$$\mathbf{g} = \mathbf{W}(\mathbf{I} - \mathbf{BW})^{-1}\mathbf{e}. \tag{7}$$

$\mathbf{R} \equiv \mathbf{W}(\mathbf{I} - \mathbf{BW})^{-1}$ is called the industry–by–commodity total requirements matrix. Rows are associated with industries and columns with commodities. Entry ji shows the dollar value of industry j's production that is required, both directly and indirectly, to deliver one dollar of the domestically produced commodity i to final uses including net exports.

Let $\mathbf{v}$ denote the $(1 \times n)$ vector of industry value added per unit of industry output, which is easily calculated from the IO Tables by dividing industry value added by industry output. To obtain the value added, $\mathbf{va}$, that is generated by the domestically produced final expenditure vector, $\mathbf{e}$, we multiply $\mathbf{R}$ (as defined either in (5) or in (7)) with $\mathbf{e}$:

$$\mathbf{va} = <\mathbf{v}> \mathbf{Re},$$

where $<\mathbf{v}>$ denotes the diagonal matrix with vector $\mathbf{v}$ in its diagonal. It is important to realize that this formula works for any domestically produced final expenditure vector, and so in principle we could use it for the domestically produced final consumption vector. However, we don't know this vector because we do not know the share of imports that is consumed. Instead, we only know the final consumption vector, $\mathbf{e}_c$. Component i of this vector reports the final consumption of commodity i, which may either be produced domestically or be imported. Assuming that imported commodities are produced with the same input requirements as in the U.S., we can use the total requirement matrix together with the vector $\mathbf{e}_c$. This gives us the consumption value added vector we are looking for:

$$\mathbf{c} = <\mathbf{v}> \mathbf{Re}_c. \tag{8}$$

In words, the vector on the left–hand side reports the value added in the different industries that is generated by the final consumption expenditure vector $\mathbf{e}_c$. Aggregating the components of this vector into our three broad sectors agriculture, manufacturing, and services gives us the consumption value added used in the text.

Finally, the sum of the components of $\mathbf{e}_c$ equals to the sum of the components of $\mathbf{c}$ by construction. However, this only holds if one has the correct $\mathbf{e}_c$ vector that corresponds to the matrix $\mathbf{R}$. Constructing $\mathbf{e}_c$ from final expenditures from NIPA is only an approximation of the correct $\mathbf{e}_c$. Therefore the sum of the components of $\mathbf{c}$ will not be exactly equal to the sum of the components of $\mathbf{c}$. There will be small discrepancies between the two. Therefore for consistency reason we proportionally scale the components of $\mathbf{c}$ such that the sum of its components exactly equal to the sum of the components of $\mathbf{e}_c$.

Online Appendix C: Approximate Aggregation of Chained Quantity Indices

Chain indices relate the value of an index number to its value in the previous period. In contrast, fixed–base indices relate the value of an index number to its value in a fixed base period. While chain indices are preferable to fixed–base indices when prices change considerably over time, using them may lead to problems because real quantities are not additive in general, that is, the real quantity of an aggregate does not equal the sum of the real quantities of its components. In practice, this becomes relevant when one is interested in the real quantity of an aggregate, but the statistical agencies only report the real quantities of the components of this aggregate. This appendix explains how to construct the real quantity of the aggregate according to the so called cyclical expansion procedure.

Let Y_{it} be the nominal value, y_{it} the real value, Q_{it} the chain–weighted quantity index, and P_{it} the chain–weighted price index for variable $i \in \{1, \ldots, n\}$ in period t. Let $t = b$ be the base year for which we normalize $Q_{ib} = P_{ib} = 1$. The nominal and real values of variable i in period t are then given by:

$$
\begin{aligned}
Y_{it} &= P_{it}\frac{Q_{it}}{Q_{ib}}Y_{ib} = P_{it}Q_{it}Y_{ib},\\
y_{it} &= \frac{Y_{it}}{P_{it}} = Q_{it}Y_{ib}.
\end{aligned}
$$

Let $Y_t = \sum_{i=1}^{n} Y_{it}$ and suppose that the statistical agency reports y_{it}, Q_{it} and P_{it} for all components i but not y_t, Q_t and P_t. Since in general $y_t \neq \sum_i y_{it}$, we need to find a way of calculating y_t.

We start by approximating Q_t using the "chain–summation" method:[2]

$$
\frac{Q_t}{Q_{t-1}} = \sqrt{\frac{\sum_i P_{it-1}y_{it}}{\sum_i P_{it-1}y_{it-1}}\frac{\sum_i P_{it}y_{it}}{\sum_i P_{it}y_{it-1}}}.
$$

Using this expression iteratively, we obtain Q_t as:

$$
Q_t = \frac{Q_t}{Q_{t-1}}\frac{Q_{t-1}}{Q_{t-2}}\cdots\frac{Q_{b+1}}{Q_b}Q_b = \frac{Q_t}{Q_{t-1}}\frac{Q_{t-1}}{Q_{t-2}}\cdots\frac{Q_{b+1}}{Q_b},
$$

[2]This is only an approximation because sums like $\sum_i P_{it-1}y_{it}$ are not directly observable and the statistical agency typically uses more disaggregate categories than $i \in \{1, \ldots, n\}$ to calculate them.

where the last step used the normalization $Q_b = 1$. The real value and the price in period t then follow as:

$$y_t = Q_t Y_b,$$
$$P_t = \frac{Y_t}{Q_t Y_b}.$$

References

Bureau of Economic Analysis. 2006. “Concepts and Methods of the U.S. Input–Output Accounts.” http://bea.gov/bea/papers/IOmanual_092906.pdf.