1 Online Appendix for "News, Noise, and Fluctuations"

1.1 More on non invertibility

This is the result discussed at the end of Section 2.2. The result is derived for the theoretical impulse responses, ignoring sampling error.

Proposition 1 *Suppose the econometrician observes (c_t, a_t, s_t) or (c_t, a_t). The impulse response of c_t to any identified shock from a structural VAR are either permanent and flat or zero.*

Proof. Let w_t be an identified shock, corresponding to a linear combination of current and past observables. Denote by $\mathcal{I}_t$ the consumer's information and by $\mathcal{I}_t^e$ the econometrician's information. Applying the law of iterated expectations we get

$$E\left[c_{t+k}|w_t, \mathcal{I}_{t-1}^e\right] = E[\lim_{j\to\infty} E\left[a_{t+k+j}|\mathcal{I}_{t+k}\right]|w_t, \mathcal{I}_{t-1}^e] = \lim_{j\to\infty} E\left[a_{t+j}|w_t, \mathcal{I}_{t-1}^e\right],$$

for all $k \geq 0$ and, similarly,

$$E\left[c_{t+k}|\mathcal{I}_{t-1}^e\right] = \lim_{j\to\infty} E\left[a_{t+j}|\mathcal{I}_{t-1}^e\right].$$

It follows that the response of consumption to w_t is constant and equal to

$$E\left[c_{t+k}|w_t, \mathcal{I}_{t-1}^e\right] - E\left[c_t|\mathcal{I}_{t-1}^e\right] = \lim_{j\to\infty} E\left[a_{t+j}|w_t, \mathcal{I}_{t-1}^e\right] - \lim_{j\to\infty} E\left[a_{t+j}|\mathcal{I}_{t-1}^e\right],$$

for all $k \geq 0$. ■

1.2 DSGE Model

1.2.1 Setup

Households. The household preferences are given by (13) in the text. The household budget constraint is

$$P_t C_t + P_t I_t + T_t + B_t + P_t \mathcal{C}(U_t)\bar{K}_{t-1} = R_{t-1}B_{t-1} + \Upsilon_t + \int_0^1 W_{jt}N_{jt}dj + R_t^k K_t \qquad (26)$$

where P_t is the price level, T_t is a lump sum tax, B_t are holdings of one period bonds, R_t is the one period nominal interest rate, Υ_t are aggregate profits, W_{jt} is the wage of specialized labor of type j, R_t^k is the capital rental rate. The investment-specific technology parameter $d_t = \log D_t$ follows the stochastic process

$$d_t = \rho_d d_{t-1} + \varepsilon_{dt}.$$

ε_{dt} and all the variables denoted ε from now on are i.i.d. shocks.

Households choose consumption, bond holdings, capital utilization, and investment each period so as to maximize their expected utility subject to (26) and a standard no-Ponzi condition. Nominal bonds are in zero net supply, so market clearing in the bonds market requires $B_t = 0$.

Production and labor services. Consumption and investment are in terms of a final good which is produced by competitive final good producers using the CES production function

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{1}{1+\mu_{pt}}} dj \right)^{1+\mu_{pt}},$$

which employs a continuum of intermediate inputs. Y_{jt} is the quantity of input j employed and μ_{pt} captures a time-varying elasticity of substitution across goods, where $\log(1+\mu_{pt}) = \log(1+\mu_p) + m_{pt}$ and m_{pt} follows the process

$$m_{pt} = \rho_p m_{pt-1} + \varepsilon_{pt} - \psi_p \varepsilon_{pt-1}.$$

The production function for intermediate good j is

$$Y_{jt} = \left(K_{jt}\right)^{\alpha} \left(A_t L_{jt}\right)^{1-\alpha}, \tag{27}$$

where K_{jt} and L_{jt} are, respectively, capital and labor services employed. As in our baseline model, the technology parameter a_t follows the process (1)-(3) and the representative consumer does not observe x_t and z_t separately, but observes a_t and the signal s_t given by (6). However, here we treat explicitly the constant term in TFP growth by letting

$$A_t = \Gamma^t e^{a_t}.$$

Intermediate good prices are sticky with price adjustment as in Calvo (1983). Each period intermediate good firm j can freely set the nominal price P_{jt} with probability $1-\theta_p$ and with probability θ_p is forced to keep it equal to P_{jt-1}. These events are purely idiosyncratic, so θ_p is also the fraction of firms adjusting prices each period.

Labor services are supplied to intermediate good producers by competitive labor agencies that combines specialized labor of types in $[0,1]$ using the technology

$$N_t = \left[\int_0^1 N_{jt}^{\frac{1}{1+\mu_{wt}}} dj \right]^{1+\mu_{wt}},$$

where $\log(1+\mu_{wt}) = \log(1+\mu_w) + m_{wt}$ and m_{wt} follows the process

$$m_{wt} = \rho_w m_{wt-1} + \varepsilon_{wt} - \psi_w \varepsilon_{wt-1}.$$

Specialized labor wages are also sticky and set by the household. For each type of labor j, the household can freely set the price W_{jt} with probability $1-\theta_w$ and has to keep it equal to W_{jt-1} with probability θ_w.

Market clearing in the final good market requires

$$C_t + I_t + \mathcal{C}\left(U_t\right)\bar{K}_{t-1} + G_t = Y_t, \tag{28}$$

market clearing in the market for labor services requires $\int L_{jt}dj = N_t$.

Government spending and monetary policy. Government spending is set as a fraction of output and the ratio of government spending to output is $G_t/Y_t = \psi + g_t$, where g_t follows the stochastic process

$$g_t = \rho_g g_{t-1} + \varepsilon_{gt}.$$

Monetary policy follows the interest rate rule

$$r_t = \rho_r r_{t-1} + \left(1-\rho_r\right)\left(\gamma_\pi \pi_t + \gamma_y \hat{y}_t\right) + q_t \tag{29}$$

where $r_t = \log R_t - \log R$ and $\pi_t = \log P_t - \log P_{t-1} - \pi$, π is the inflation target, $\hat{y}_t$ is defined below and q_t follows the process

$$q_t = \rho_q q_{t-1} + \varepsilon_{qt}.$$

1.2.2 Optimality conditions

Households. Define the marginal utility of consumption

$$\Lambda_t = \frac{1}{C_t - hC_{t-1}} - \beta h E_t\left[\frac{1}{C_{t+1} - hC_t}\right]. \tag{30}$$

The consumer's Euler equation is then

$$\Lambda_t = \beta R_t E_t\left[\Lambda_{t+1}\frac{P_t}{P_{t+1}}\right]. \tag{31}$$

The optimality conditions for $\bar{K}_{t-1}$ and I_t are

$$\Phi_t = \beta E_t\left[\Lambda_{t+1}\left(R^k_{t+1}U_{t+1} - P_t\mathcal{C}\left(U_{t+1}\right)\right)\right] + \left(1-\delta\right)\beta E_t\Phi_{t+1}, \tag{32}$$

$$P_t\Lambda_t = \Phi_t D_t\left[1 - \mathcal{G}_t - \frac{I_t}{I_{t-1}}\mathcal{G}'_t\right] + \beta E_t\left[\Phi_{t+1}D_{t+1}\left(\frac{I_{t+1}}{I_t}\right)^2\mathcal{G}'_{t+1}\right], \tag{33}$$

where Φ_t is the Lagrange multiplier on the capital accumulation constraint. The optimality condition for capacity utilization U_t is

$$R^k_t = \mathcal{C}'\left(U_t\right). \tag{34}$$

Firms. The optimality condition for the final good producer yields the demand for intermediate good j,

$$Y_{jt} = Y_t \left(P_{jt}/P_t\right)^{-\left(1+\mu_{pt}\right)/\mu_{pt}},$$

while the zero profit condition gives the final good price $P_t = (\int P_{jt}^{1/\mu_{pt}} dj)^{\mu_{pt}}$.

Cost minimization by the intermediate good producers and constant returns to scale imply that the nominal cost of producing Y_{jt} is $M_t Y_{jt}$, where M_t is the marginal cost

$$M_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} \left(R_t^k\right)^{\alpha} \left(W_t/A_t\right)^{1-\alpha}. \tag{35}$$

Cost minimization also implies that all intermediate good firms choose the same capital-labor ratio

$$\frac{K_t}{N_t} = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k}. \tag{36}$$

Calvo pricing implies that the firms changing their price in period t choose the price P_t^* that maximizes

$$E_t\left[\sum_{\tau=0}^{\infty} \theta_p^{\tau}\beta^{\tau}\frac{\Lambda_{t+\tau}}{P_{t+\tau}}\left(P_t^* - M_{t+\tau}\right) Y_{jt+\tau}\right]$$

subject to $Y_{jt+\tau} = Y_{t+\tau}\left(P_t^*/P_{t+\tau}\right)^{-\left(1+\mu_{pt+\tau}\right)/\mu_{pt+\tau}}$. Their optimality condition is thus

$$E_t\left[\sum_{\tau=0}^{\infty} \theta_p^{\tau}\beta^{\tau}\frac{\Lambda_{t+\tau}}{P_{t+\tau}}\left(\frac{1}{\mu_{pt+\tau}} - \frac{1+\mu_{pt+\tau}}{\mu_{pt+\tau}}\frac{M_{t+\tau}}{P_t^*}\right) Y_{it+s}\right] = 0. \tag{37}$$

The problem of the labor agency is similar to that of the final good producer, so the demand for specialized labor of type j is

$$N_{jt} = N_t\left(W_{jt}/W_t\right)^{-(1+\mu_{wt})/\mu_{wt}},$$

and the price of labor services is $W_t = (\int W_{jt}^{1/\mu_{wt}} dj)^{\mu_{wt}}$. Calvo pricing for wage setters implies that workers who can adjust their wage, maximize

$$E_t\left[\sum_{\tau} \theta_w^{\tau}\beta^{\tau}\left[\frac{\Lambda_{t+\tau}}{P_{t+\tau}} W_t^* N_{jt+\tau} - \frac{1}{1+\zeta} N_{jt+\tau}^{1+\zeta}\right]\right],$$

and their optimality condition is

$$E_t\left[\sum_{\tau=0}^{\infty} \theta_w^{\tau}\beta^{\tau}\left(\frac{\Lambda_{t+\tau}}{P_{t+\tau}}\frac{1}{\mu_{wt+\tau}} - \frac{1+\mu_{wt+\tau}}{\mu_{wt+\tau}}\frac{N_{jt+\tau}^{\eta}}{W_t^*}\right) N_{jt+\tau}\right] = 0. \tag{38}$$

1.2.3 Log-linear approximation

First, we need to normalize some variables to ensure their stationarity. In particular, we define

$$\hat{c}_t = \log(C_t/A_t) - \log(C/A)$$

where C/A denotes the value of C_t/A_t in the deterministic version of the model in which A_t grows at the constant growth rate Γ. Analogous definitions apply to the quantities $\hat{y}_t, \hat{k}_t, \hat{\bar{k}}_t, \hat{i}_t$. The quantities N_t and U_t are already stationary, so we have

$$n_t = \log N_t - \log N,$$

and a similar expression for u_t. For nominal variables, we also need to take care of non-stationarity in the price level, so we define

$$\begin{aligned} \hat{w}_t &= \log\left(W_t/(A_t P_t)\right) - \log\left(W/\left(AP\right)\right), \qquad r_t^k = \log(R_t^k/P_t) - \log(R^k/P), \\ m_t &= \log(M_t/P_t) - \log(M/P), \qquad r_t = \log R_t - \log R, \qquad \pi_t = \log(P_t/P_{t-1}) - \pi. \end{aligned}$$

Finally, for the Lagrange multipliers we define

$$\hat{\lambda}_t = \log(\Lambda_t A_t) - \log\left(\Lambda A\right), \qquad \hat{\phi}_t = \log(\Phi_t A_t/P_t) - \log(\Phi A/P).$$

Notice that the hat is only used for variables normalized by A_t.

Optimality conditions. Conditions (30)-(36) can be log-linearized to yield the following seven conditions

$$\begin{aligned} \hat{\lambda}_t &= \frac{h\beta\Gamma}{(\Gamma - h\beta)(\Gamma - h)} E_t\hat{c}_{t+1} - \frac{\Gamma^2 + h^2\beta}{(\Gamma - h\beta)(\Gamma - h)}\hat{c}_t + \frac{h\Gamma}{(\Gamma - h\beta)(\Gamma - h)}\hat{c}_{t-1} + \\ &\quad + \frac{h\beta\Gamma}{(\Gamma - h\beta)(\Gamma - h)} E_t[\Delta a_{t+1}] - \frac{h\Gamma}{(\Gamma - h\beta)(\Gamma - h)}\Delta a_t, \qquad (39) \\ \hat{\lambda}_t &= r_t + E_t\left[\hat{\lambda}_{t+1} - \Delta a_{t+1} - \pi_{t+1}\right], \qquad (40) \\ \hat{\phi}_t &= (1-\delta)\beta\Gamma^{-1} E_t\left[\hat{\phi}_{t+1} - \Delta a_{t+1}\right] \\ &\quad + \left(1 - (1-\delta)\beta\Gamma^{-1}\right) E_t[\hat{\lambda}_{t+1} - \Delta a_{t+1} + r_{t+1}^k], \qquad (41) \\ \hat{\lambda}_t &= \hat{\phi}_t + d_t - \chi\Gamma^2\left(\hat{i}_t - \hat{i}_{t-1} + \Delta a_t\right) + \beta\chi\Gamma^2 E_t\left[\hat{i}_{t+1} - \hat{i}_t + \Delta a_{t+1}\right], \qquad (42) \\ r_t^k &= \xi u_t, \qquad (43) \\ m_t &= \alpha r_t^k + (1-\alpha)\,\hat{w}_t, \qquad (44) \\ r_t^k &= \hat{w}_t - \hat{k}_t + n_t. \qquad (45) \end{aligned}$$

Resource constraints and inflation. Log-linearizing conditions (15) and (14) yields

$$\hat{k}_t = u_t + \hat{\bar{k}}_{t-1} - \Delta a_t, \tag{46}$$

$$\hat{\bar{k}}_t = (1-\delta)\,\Gamma^{-1}\left(\hat{\bar{k}}_{t-1} - \Delta a_t\right) + \left(1 - (1-\delta)\,\Gamma^{-1}\right)\left(d_t + \hat{i}_t\right). \tag{47}$$

Approximating and aggregating (27) over intermediate good producers and using the final good production function yields

$$\hat{y}_t = \alpha \hat{k}_t + (1-\alpha)\, n_t. \tag{48}$$

Market clearing in the final good market (28) yields

$$(1-\psi)\,\hat{y}_t = \frac{C}{Y}\hat{c}_t + \frac{I}{Y}\hat{i}_t + \frac{R^k K}{PY} u_t + g_t. \tag{49}$$

$C/Y, I/Y$ and $R^k K/(PY)$ are all equilibrium ratios in the deterministic version of the model in which A_t grows at the constant rate Γ.

Aggregating individual optimality conditions for price setters yields

$$\pi_t = \beta E_t \pi_{t+1} + \kappa m_t + \kappa m_{pt}, \tag{50}$$

where $\kappa = (1-\theta\beta)(1-\theta)/\theta$. Aggregating individual optimality conditions for wage setters yields

$$\begin{aligned} \hat{w}_t = {} & \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta} E_t \hat{w}_{t+1} - \frac{1}{1+\beta}\left(\pi_t + \Delta a_t\right) + \frac{\beta}{1+\beta} E_t\left(\pi_{t+1} + \Delta a_{t+1}\right) \\ & -\kappa_w\left(\hat{w}_t - \zeta n_t + \hat{\lambda}_t\right) + \kappa_w m_{wt}, \end{aligned} \tag{51}$$

where

$$\kappa_w = \frac{(1-\theta_w\beta)(1-\theta_w)}{\theta_w(1+\beta)\left(1+\zeta\left(1+\frac{1}{\mu_w}\right)\right)}.$$

Summing up, we have 23 variables: the 9 exogenous variables

$$d_t, a_t, x_t, z_t, s_t, m_{pt}, m_{wt}, g_t, q_t,$$

and the 14 endogenous variables

$$\hat{c}_t, \hat{i}_t, \hat{y}_t, n_t, \hat{\bar{k}}_t, \hat{k}_t, u_t, \hat{\lambda}_t, \hat{\phi}_t, r_t^k, \hat{w}_t, \pi_t, m_t, r_t.$$

The model dynamics are given by the exogenous processes for the exogenous variables above and by the 14 equations (29) and (39)-(51).

The observables that are matched to the data are $\Delta y_t, \Delta c_t, \Delta i_t, \Delta w_t, \Delta n_t, r_t, \pi_t$, where the first four variables are obtained by adding $\Delta \hat{a}_t$ to the first differences of the corre-

sponding variables, e.g.,

$$\Delta y_t = \hat{y}_t - \hat{y}_{t-1} + \Delta \hat{a}_t.$$

1.2.4 New Keynesian model

The standard new Keynesian model is a special case of the DSGE model above when: (i) the capital stock and capacity utilization are fixed (and hence the investment-specific technology shock is absent), (ii) the habit parameter in the preferences h is set to zero, (iii) labor is homogeneous and wages are flexible. Furthermore, we consider a simple new Keynesian model with no fiscal policy ($G_t = 0$) and a simplified monetary rule with $\rho_r = \gamma_y = 0$ and no monetary shocks ($q_t = 0$).

In this case, the log-linearized model boils down to two stochastic difference equations which characterize the joint behavior of output and inflation in equilibrium:

$$\begin{aligned} y_t &= E_t\left[y_{t+1}\right] - \gamma_\pi \pi_t + E_t\left[\pi_{t+1}\right], \\ \pi_t &= \tilde{\kappa}(y_t - a_t) + \beta E_t\left[\pi_{t+1}\right], \end{aligned}$$

where $y_t = \hat{y}_t + a_t$. The first comes from (29), (39), (40), (49). The second comes from (39), (44), (50) and the fact that (51) requires $\hat{w}_t = \zeta n_t - \hat{\lambda}_t$ when $\theta_w \to 0$. The coefficient $\tilde{\kappa}$ is given by $\left(1 + \zeta / \left(1 - \alpha\right)\right)\kappa$. As long as $\gamma_\pi > 1$ this system has a unique locally stable solution where y_t and π_t are linear functions of the four exogenous state variables $a_t, x_{t|t}, x_{t-1|t}, z_{t|t}$,

$$\begin{pmatrix} y_t \\ \pi_t \end{pmatrix} = D \begin{pmatrix} a_t \\ x_{t|t} \\ x_{t-1|t} \\ z_{t|t} \end{pmatrix}.$$

The matrix D can be found using the method of undetermined coefficient as the solution to

$$\begin{bmatrix} 1 & \gamma_\pi \\ -\tilde{\kappa} & 1 \end{bmatrix} D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\tilde{\kappa} & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & \beta \end{bmatrix} D \begin{bmatrix} 0 & 1+\rho_x & -\rho_x & \rho_z \\ 0 & 1+\rho_x & -\rho_x & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \rho_z \end{bmatrix}.$$

The elements of D are a continuous non-linear function of $\tilde{\kappa}$ and a bit of algebra shows that

$$\lim_{\tilde{\kappa} \to 0} D = \frac{1}{1 - \rho_x} \begin{bmatrix} 0 & 1 & -\rho_x & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Since $\tilde{\kappa} \to 0$ when $\theta \to 1$, this shows that in the limit with very infrequent price adjustments we get (9).

1.3 Data

Our dataset spans the period 1954:3 to 2011:1. The sample starts in 1954:3 because the effective Federal Funds Rate quarterly series, published by the Federal Reserve Board, is not available before that date.

The series for Real GDP, Real Personal Consumption Expenditures, Real Personal Durable Consumption Expenditures, Real Gross Private Domestic Investment, Wages and the GDP Implicit Price Deflator are from the Bureau of Economic Analysis (available through the Federal Reserve Bank of Saint Louis online database). Population and employment series are from the Bureau of Labor Statistics online database (series IDs LNS10000000Q and LNS12000000Q respectively). The Federal Funds Rate series is from the Federal Reserve Board online database (series ID H15/H15/RIFSPFF N.M).

The GDP series is constructed by dividing Real GDP by population. The consumption series is constructed by subtracting Real Personal Durable Consumption from Real Personal Consumption and dividing by population. The investment series is constructed by dividing the sum of Real Gross Investment and Real Personal Durable Consumption by population. The labor input series is constructed by dividing Employment by Population. Inflation is constructed by computing the quarterly log difference of the Price Deflator. The real wage is constructed by dividing Real Wages by the Price Deflator. The nominal interest rate is the effective Federal Funds Rate.

1.4 More on recovering states and shocks

This section presents more results on the Kalman smoother for the baseline model, following up on Section 2.5. Figure 8 plots the RMSE of the smoothed estimates of x_t and z_t, when data up to $t+j$ are available, for $j = 0, 1, 2, ...$ The RMSE is the square root of $E_{t+j}[(x_t - E_{t+j}[x_t])^2]$, and can be computed using two different information sets: the econometrician's, which only includes observations of c_t and a_t, and the consumer's, which also includes s_t. For simplicity, we compute the RMSE at the steady state of the Kalman filter, that is, assuming the forecaster has access to data from $-\infty$ to $t+j$. In this case, the econometrician's information set coincides with the consumer's, that is, the econometrician can back up the current value of s_t perfectly from current and past ob-

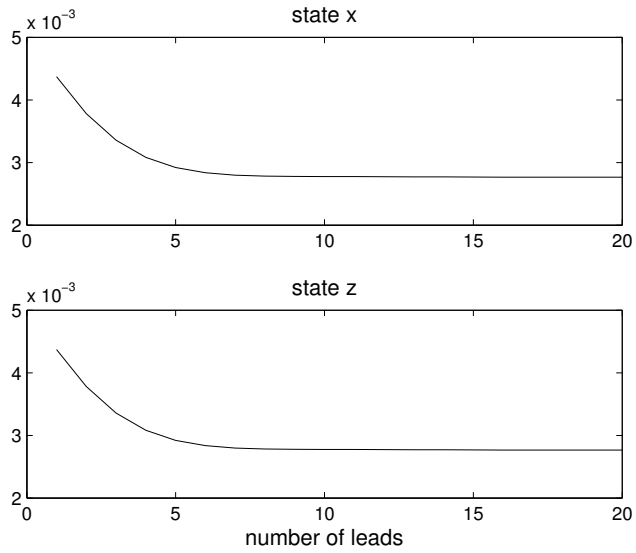


Figure 8: RMSE of the estimated states at time t using data up to $t + j$

servations of c_t and a_t. Although we have not established this result analytically, it holds numerically in all our examples: the computed RMSE of the econometrician's estimate of s_t is zero at $j = 0$. This implies that, in our model, with a sufficiently long data set, the direct observation of s_t does not add much to the econometrician's ability to recover the unobservable states or the shocks.

Figure 8 illustrates the results discussed in the text. Notice that most of the relevant information arrives in the first six quarters after t there are minimal gains after that.

Figure 9 we report the RMSE of the estimates of the shocks ϵ,η and ν. To help the interpretation, each RMSE is normalized dividing it by the ex ante standard deviation of the respective shock (e.g., by σ_ϵ for ϵ).

Notice that if the model was invertible, all RMSE would be zero at $j = 0$. The fact that all RMSE are bounded away from zero at all horizons shows that even an infinite data set would not allow us to recover the shocks exactly.

The transitory shock η_t is estimated with considerable precision already on impact and the precision of its estimate almost doubles in the long run. The noise shock ν_t is less precisely estimated, but the data still tell us a lot about it, giving us an RMSE which is about $1/3$ of the prior uncertainty in the long run. The shock that is least precisely estimated is the permanent shock ϵ_t. Even with an infinite series of future data, the residual variance is about 94% of the prior uncertainty on the shock.

How do we reconcile the imprecision of the estimate of ϵ_t with the fact that we have

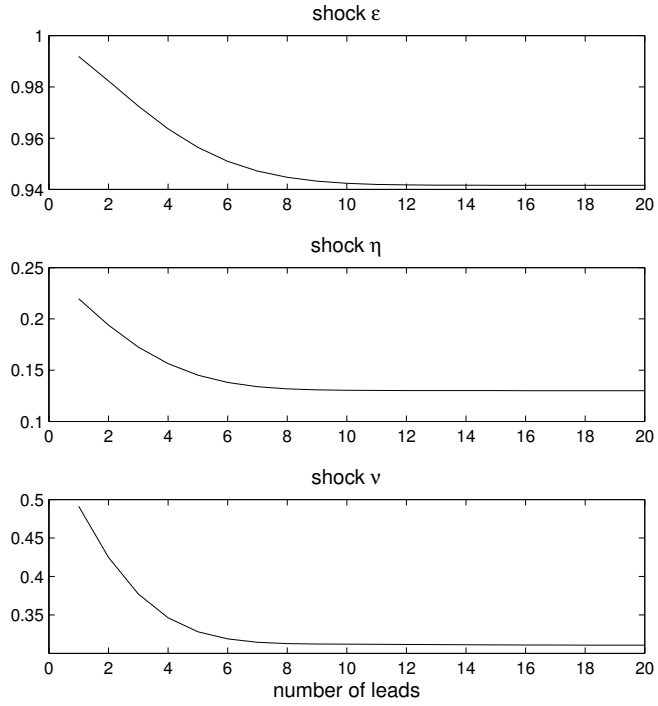


Figure 9: Normalized RMSE of the estimated shocks at time t using data up to $t + j$

relatively precise estimates of the state x_t, as seen in Figure 8? The explanation is that the econometrician can estimate the cumulated effect of permanent productivity changes by looking at productivity growth over longer horizons, but cannot pinpoint the precise quarter in which the change occurred. Therefore, it is possible to have imprecise estimates of past ϵ_t's, while having a relatively precise estimate of their cumulated effect on x_t.