Online Appendix for “Loss Leading as an Exploitative Practice”

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A Proof of Proposition 1

Suppose first that \( v_{AL} \geq w_S \), that is, \( r_{AL} \leq w_{AL} - w_S \) (“regime L”). We first show that, without loss of generality, we can focus on prices such that \( \tau \in [0, v_{AL}] \). If \( \tau > v_{AL} \) (i.e., \( w_S - w_L + r_L > w_{AL} - r_{AL} \), or \( r_L > r'_L \equiv (w_{AL} - r_A - (w_S - w_L))/2 \)), there are no one-stop shoppers: active consumers buy \( A \) from \( L \) and \( B_S \) from \( S \), and do so as long as \( 2s < v_A + w_S \); however, keeping \( r_A \) constant, decreasing \( r_L \) to \( r'_L \) such that \( \tau' = v'_L \) does not affect the number of active consumers (since \( v_A \) does not change), who still visit both stores as before. If instead \( \tau < 0 \) (i.e., \( r_L < -w_S - w_L \)), there are no multi-stop shoppers: active consumers only visit \( L \), and do so as long as \( s < v_{AL} = w_{AL} - r_{AL} \); however, keeping \( r_{AL} \) constant, increasing \( r_L \) to \( r'_L = -(w_S - w_L) \) yields \( \tau' = 0 \) without affecting consumer behavior. The condition \( \tau \geq 0 \) moreover ensures that prospective multi-stop shoppers are indeed willing to buy \( A \) on a stand-alone basis: \( w_S \leq v_{AL} = w_{AL} - r_A - r_L \) implies \( r_A \leq w_{AL} - w_S - r_L = w_{AL} - w_L - \tau < w_A \).

Thus, consumers whose shopping cost lies in \( [0, \tau] \) buy \( A \) from \( L \) (and \( B_S \) from \( S \)), whereas those with a shopping cost in \( [\tau, v_{AL}] \) buy both \( A \) and \( B_L \) from \( L \). Using \( v_{AL} = w_{AL} - r_{AL} \) and \( \tau = w_S - w_L + r_L \), \( L \)'s optimization program within regime \( L \) can thus be expressed as:

\[
\max_{r_{AL}, r_L} \Pi_L (r_{AL}, r_L) = r_{AL} F(w_{AL} - r_{AL}) - r_L F(w_S - w_L + r_L),
\]

subject to \( r_{AL} \leq w_{AL} - w_S \)

where \( \Pi_L (r_{AL}, r_L) \) is additively separable and moreover strictly quasi-concave\(^1\) in \( r_{AL} \) and \( r_L \). \( L \)'s optimization program can thus be decomposed into:

\[
\max_{r_{AL}} r_{AL} F(w_{AL} - r_{AL}),
\]

s. t. \( r_{AL} \leq w_{AL} - w_S \)

which leads to \( r_{AL} = \min \{r_{AL}^m, w_{AL} - w_S\} \) and \( v_{AL} = \max \{v_{AL}^m, w_S\} \), and

\[
\min_{r_L} r_L F(w_S - w_L + r_L),
\]

which yields the first-order condition:

\[
r^*_L = -h(w_S - w_L + r^*_L) = -h(\tau^*) < 0.
\]

\(^1\)The derivative w.r.t. \( r_{AL} \) is of the form \( f(w_{AL} - r_{AL}) \phi (r_{AL}) \), where \( \phi (r_{AL}) \equiv h(w_{AL} - r_{AL}) - r_{AL} \) is strictly decreasing. A similar reasoning applies below to the other profit functions of \( L \) and \( S \).
Using $r^*_L = \tau^* - (w_S - w_L) = -h(\tau^*)$, the optimal threshold $\tau^*$ is given by:

$$
\tau^* \equiv l^{-1}(w_S - w_L) > 0. \quad (3)
$$

Note that this threshold satisfies $\tau^* < v^m_{AL}$. To see this, take instead $v_{AL}$ and $\tau$ as control variables and rewrite $L$'s profit as:

$$
\Pi_L(v_{AL}, \tau) = r_{AL} F(v_{AL}) - r_L F(\tau)
= (w_{AL} - v_{AL}) F(v_{AL}) + (w_S - w_L - \tau) F(\tau).
$$

Then we have $v^m_{AL} = \arg \max_v (w_{AL} - v) F(v) > \arg \max_v (w_S - w_L - v) F(v) = \tau^*$, since $w_{AL} \geq l(w_S) (> w_S \geq w_S - w_L).

Suppose now that $v_{AL} < w_S$, that is, $r_{AL} > w_{AL} - w_S$ ("regime S"). $L$ then only attracts multi-stop shoppers, who buy $A$ from it as long as $s \leq v_A = w_A - r_A$. $L$ thus obtains:

$$
\Pi_L = r_A F(v_A) = r_A F(w_A - r_A),
$$

which is maximal for $r^m_A$ and $v^m_A = w_A - r^m_A$, characterized by:

$$
r^m_A = h(v^m_A), \quad v^m_A = l^{-1}(w_A).
$$

$L$'s profit in regime $S$ is thus at most:

$$
\Pi_A^m \equiv r^m_A F(v^m_A).
$$

As already noted, regime $L$ is clearly preferable when $v^m_{AL} \geq w_S$, since it then gives $L$ more profit than the monopolistic level $\Pi^m_{AL}$, which itself is greater than $\Pi_A^m$:

$$
\Pi^m_{AL} = \max_r r F(w_{AL} - r) > \max_r r F(w_A - r) = \Pi_A^m,
$$

since $w_{AL} > w_A$. We now show that regime $L$, and the associated loss-leading strategy, remains profitable when $w_{AL} \geq w_S > v^m_{AL}$, where it involves $r^*_L < 0$ and $\tilde{r}^*_A = w_{AL} - w_S$. To see this, fixing $\tilde{r}^*_A$ and using $r_A$ rather than $r_L$ as the optimization variable, the margin on $B_L$ and the shopping cost threshold can be expressed as:

$$
\tau_L = \tilde{r}^*_A - r_A = w_{AL} - w_S - r_A, \quad \tau = w_S - w_L + r_L = w_{AL} - w_L - r_A = w_A - r_A.
$$
The maximum profit achieved in regime $L$, $\hat{\Pi}_L^*$, can then be written as:

$$\hat{\Pi}_L^* = \tilde{r}_{AL}^* (F'(\tilde{v}_{AL}^*) - F'(\tau^*)) + r_A^* F(\tau^*)$$

$$= (w_{AL} - w_S) (F'(w_S) - F'(\tau^*)) + r_A^* F(\tau^*)$$

$$= \max_{r_A} \{(w_{AL} - w_S) (F'(w_S) - F'(w_A - r_A)) + r_A^* F(w_A - r_A)\}$$

$$\geq (w_{AL} - w_S) (F'(w_S) - F'(w_A - r_A^m)) + r_A^m F(w_A - r_A^m)$$

$$= (w_{AL} - w_S) (F'(w_S) - F'(v_{AL}^m)) + \Pi_A^m.$$

Since $w_S > v_{AL}^m = l^{-1}(w_{AL}) > l^{-1}(w_A) = v_A^m$, $\hat{\Pi}_L^* \geq \Pi_A^m$ whenever $w_{AL} \geq w_S$.

Conversely, when $w_{AL} < w_S$, then $L$ can indeed achieve $\Pi_A^m$ in regime $S$ (e.g., $r_L = 0$ and $r_A = r_A^m$ satisfy $r_{AL} = r_A^m > 0 > w_{AL} - w_S$, and thus $v_{AL} < w_S$), and we have:

$$\hat{\Pi}_L^* = (w_{AL} - w_S) (F'(w_S) - F'(w_A - \tilde{r}_A^*)) + \tilde{r}_A^* F(w_A - \tilde{r}_A^*)$$

$$< \tilde{r}_A^* F(w_A - \tilde{r}_A^*)$$

$$\leq \Pi_A^m,$$

where the first inequality stems from $w_S > w_{AL} (> w_A - \tilde{r}_A^*)$.

Finally, in the limit case where $w_{AL} = w_S$, using $B_L$ as a loss leader amounts to monopolizing product $A$. Offering $v_{AL} = w_S$ requires $r_{AL} = w_{AL} - v_{AL} = 0$, or $r_A = -r_L$, and the optimal subsidy thus maximizes $-r_L F'(\tau) = -r_L F(w_S - w_S + r_L) = r_A F(w_A - r_A)$.

Therefore, in both cases $L$ obtains (from multi-stop shoppers) the monopoly margin on $A$, and makes no profit (from one-stop shoppers) on the bundle $A - B_L$ (since either it charges them $r_{AL} = 0$, or they go to $S$). Finally, while the loss-leading strategy may yield a lower price for $B_L$ (in the monopolization scenario, $L$ may actually stop carrying $B_L$), this does not affect multi-stop shoppers (who do not buy $B_L$ from $L$), whereas one-stop shoppers are indifferent between buying $A$ and $B_L$ from $L$ or $B_S$ only from $S$.

**B Proof of Proposition 2**

We derive here the conditions under which the loss leading outcome ($\hat{\tau}_{AL}^* = r_{AL}^m$ and $\hat{\tau}_L^* = -\hat{\tau}_S^* = -h(\hat{\tau}^*)$, where $\hat{\tau}^* = j^{-1}(w_S - w_L)$) forms a Nash equilibrium, before checking the uniqueness of the equilibrium. To attract one-stop shoppers, $L$ must offer a better value
than $S$: \(^2\)
\[
v_{AL}^m \geq \hat{v}^*_S \equiv w_S - h(\hat{\tau}^*).
\]
This condition implies $v_{AL}^m \geq \hat{v}^*_S > \hat{v}^*_S - \hat{v}^*_L = \hat{\tau}^*$, which in turn implies $w_{AL} > w_S$:
\[
w_{AL} = l(v_{AL}^m) \geq l(\hat{v}^*_S) = \hat{v}^*_S + h(\hat{v}^*_S) = w_S - h(\hat{\tau}^*) + h(\hat{v}^*_S) > w_S.
\]

Moreover, while $L$ has no incentive to exclude its rival, since it earns more profit than a pure monopolist, $S$ may want to attract one-stop shoppers by reducing $r_S$ so as to offer $v_S \geq v_{AL}^m$. Such a deviation allows $S$ to attract all consumers (one-stop or multi-stop shoppers) with shopping costs $s \leq v_S$ and thus yields a profit $\Pi_S^d(v_S) \equiv r_S F(v_S) = (w_S - v_S) F(v_S)$. A simple revealed argument yields $\arg \max_v \Pi_S^d(v) \leq v_{AL}^m \equiv \arg \max_v (w_{AL} - v) F(v)$, since $w_S < w_{AL}$; as $\Pi_S^d(v_S)$ is quasi-concave in $v_S$, increasing $v_S$ further above $v_{AL}^m$ would thus reduce $S$’s profit. It is therefore optimal for $S$ to offer precisely $v_S^d = v_{AL}^m$ (or slightly above $v_{AL}^m$, if one-stop shoppers are indifferent between the two stores in this case), which gives $S$ a profit equal to $\Pi_S^d(v_{AL}^m) = (w_S - v_{AL}^m) F(v_{AL}^m)$.

The loss-winning outcome is immune to such a deviation if and only if
\[
\hat{\Pi}_S^* \equiv h(\hat{\tau}^*) F(\hat{\tau}^*) \geq \hat{\Pi}_S^d \equiv (w_S - v_{AL}^m) F(v_{AL}^m).
\]
This condition can be further written as:
\[
\Psi(w_{AL}; w_S) \equiv (w_S - v_{AL}^m) F(v_{AL}^m) \leq \hat{\Pi}_S^*,
\]
where $v_{AL}^m = l^{-1}(w_{AL})$ and thus satisfies $v_{AL}^m + h(v_{AL}^m) = w_{AL}$. Therefore:
\[
\frac{\partial \Psi}{\partial w_{AL}}(w_{AL}; w_S) = \left((w_S - v_{AL}^m) f(v_{AL}^m) - F(v_{AL}^m)\right) \frac{dv_{AL}^m}{dw_{AL}}
\]
\[
= \left(w_S - v_{AL}^m - h(v_{AL}^m)\right) \frac{f(v_{AL}^m)}{1 + h'(v_{AL}^m)}
\]
\[
= \left(w_S - w_{AL}\right) \frac{f(v_{AL}^m)}{1 + h'(v_{AL}^m)}.
\]
It follows that, in the range $w_{AL} \geq w_S$, $\Psi(w_{AL}; w_S)$ decreases with $w_{AL}$ (and strictly so for $w_{AL} > w_S$). Thus, condition (5) amounts to $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$, where $\hat{w}_{AL}(w_S, w_L)$ is the unique solution to $\Psi(w_{AL}; w_S) = \hat{\Pi}_S^*$. To show that this solution exists and

\(^2\)As before, this is equivalent to $w_{AL} - w_L - \hat{\tau}^*_A = v_{AL}^m - \hat{v}^*_L \geq \hat{v}^*_S - \hat{v}^*_L = \hat{\tau}^* (> 0)$, which implies that multi-stop shoppers are indeed willing to buy $A$ when visiting $L$. Moreover, this condition also implies $v_{AL}^m > \hat{v}^*_S - \hat{v}^*_L = \hat{\tau}^* (> 0)$. 

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lies above \( w_S \), note first that \( \Psi \) becomes negative for \( w_{AL} > l(w_S) \) (since then \( v_{AL}^m = l^{-1}(w_{AL}) > w_S \)), and that for \( w_{AL} = w_S \), \( \Psi(w_{AL}, w_S) = (w_{AL} - v_{AL}^m) F(v_{AL}^m) = \Pi_{AL}^m = \max_v (w_{AL} - v) F(v) \); since \( w_{AL} > w_S - w_L + \hat{\tau}_L^* \), this exceeds \( \Pi_S^* = \max_\tau (w_S - w_L + \hat{\tau}_L^* - \tau) F(\tau) \).

Finally, in the range \( w_{AL} > w_S (> w_S - \hat{\tau}_L^*) \), a simple revealed argument yields:

\[
\hat{\tau}^* = \arg \max_v (w_S - \hat{\tau}_L^* - \tau) F(\tau) < v_{AL}^m = \arg \max_v (w_{AL} - v) F(v).
\]

Therefore, (5), which is equivalent to:

\[
v_{AL}^m \geq w_S - \frac{h(\hat{\tau}^*) F(\hat{\tau}^*)}{F(v_{AL}^m)}, \tag{7}
\]

implies (4). The two conditions (4) and (5) thus boil down to \( w_{AL} \geq \hat{w}_{AL}(w_S, w_L) \).

It remains to show that \( \hat{w}_{AL}(w_S, w_L) \) increases with \( w_S \). Differentiating \( \hat{w}_{AL}(w_S, w_L) \) with respect to \( w_S \) yields:

\[
\frac{\partial \hat{w}_{AL}}{\partial w_S} = \frac{\frac{\partial \Psi}{\partial w_S} - \frac{\partial \Pi_S^*}{\partial w_S}}{\frac{\partial \Psi}{\partial w_{AL}}},
\]

where the denominator is positive in the relevant range, whereas the numerator is equal to:

\[
\frac{\partial \Psi}{\partial w_S} - \frac{\partial \Pi_S^*}{\partial w_S} = F(v_{AL}^m) - \frac{d \left(h(\hat{\tau}^*) F(\hat{\tau}^*)\right)}{d \hat{\tau}^*} \frac{\partial \hat{\tau}^*}{\partial w_S}
= F(v_{AL}^m) - \frac{1 + h'(\hat{\tau}^*)}{1 + 2h'(\hat{\tau}^*)} F(\hat{\tau}^*),
\]

which is positive since \( v_{AL}^m > \hat{\tau}^* \).

We now show that no other equilibrium exists when \( w_{AL} \geq \hat{w}_{AL}(w_S, w_L) \). First, we turn to regime \( S \), in which one-stop shoppers patronize \( S (v_{AL} < v_S) \), and show that there is no such equilibrium when \( w_{AL} > w_S \). In this regime, \( L \) faces only a demand \( F(v_A) \) for \( A \) from multi-stop shoppers, where \( v_A = w_A - r_A \), and thus makes a profit equal to \( r_A F(v_A) \). \( L \) could however deviate and attract one-stop shoppers by reducing \( r_L \) (keeping \( r_A \) and thus \( v_A \) constant) so as to offer \( v_{AL}^l = v_S \) (or slightly above \( v_S \)). Doing so would not change the number of multi-stop shoppers, since \( \tau' = v_S - v_L = v_{AL}^l - v_L = v_{AL}^l = v_A \), and \( L \) would obtain the same margin, \( r_A \), from those consumers. But it would now attract one-stop shoppers (those for which \( v_A \leq s \leq w_{AL} = v_S \), from which \( L \) could earn a total margin \( r_{AL}^l = w_{AL} - v_{AL}^l = w_{AL} - v_S = w_{AL} - w_S + r_S \). Since any candidate equilibrium requires \( r_S \geq 0 \), the deviation would be profitable when \( w_{AL} > w_S \).

Second, consider the boundary between the two regimes, in which one-stop shoppers are indifferent between visiting \( L \) or \( S (v_{AL} = v_S) \). Note that there must exist some
active consumers, since either retailer can profitably attract consumers by charging a small positive margin; therefore, we must have \( v_{AL} = v_S > 0 \). Suppose that all active consumers are multi-stop shoppers (in which case \( L \) only sells \( A \) while \( S \) sells \( B_S \) to all consumers), which requires \( v_{AL} = v_S \leq \tau \). Applying the same logic as in the beginning of Appendix A, we can without loss of generality focus on the case \( v_{AL} = v_S = \tau \). It is then profitable for \( L \) to transform some multi-stop shoppers into one-stop shoppers, by reducing its margin on \( B_L \) to \( r'_L = w_L - \varepsilon > 0 \) and increasing \( r_A \) by \( \varepsilon \), so as to keep \( v_{AL} \) constant: doing so does not affect the total number of active consumers, but transforms those whose shopping cost lies between \( \tau' = v_S - v'_L = \tau - \varepsilon \) and \( \tau \) into one-stop shoppers. While \( L \) obtains the same margin on them (since \( r'_{AL} = r_{AL} \)), it now obtains a higher margin \( r'_A > r_A \) on the remaining multi-stop shoppers.

Therefore, some consumers must visit a single store, and by assumption must be indifferent between visiting either store (\( v_{AL} = v_S \)). Suppose now some one-stop shoppers visit \( S \). Since \( S \) can avoid making losses, we must then have \( r_S \geq 0 \). But then, \( v_{AL} = v_S \) implies \( r_{AL} = r_S + w_{AL} - w_S > 0 \) and, thus, it would be profitable for \( L \) to reduce \( r_{AL} \) slightly, so as to attract all one-stop shoppers. Therefore, all one-stop shoppers must go to \( L \) if \( r_{AL} > 0 \). Conversely, we must have \( r_S \leq 0 \), otherwise \( S \) would benefit from slightly reducing its margin so as to attract all one-stop shoppers. Therefore, in any candidate equilibrium such that \( v_{AL} = v_S > 0 \), either:

- There are some multi-stop shoppers (i.e. \( \tau > 0 \)) and thus \( r_S = 0 \); but then, slightly increasing \( r_S \) would allow \( S \) to keep attracting some multi-stop shoppers and obtain a positive profit, a contradiction.

- Or, all consumers buy both products from \( L \), which requires \( r_L \leq r_S - (w_S - w_L) \leq -(w_S - w_L) < 0 \). But then, increasing \( r_L \) to \( r'_L = r_S - (w_S - w_L) + \varepsilon \) and reducing \( r_A \) by the same amount (so as to keep \( r_{AL} \) constant) would lead those consumers with \( s < \tau' = \varepsilon \) to buy \( B_S \) from \( S \), allowing \( L \) to avoid granting them the subsidy \( r_L \).

It follows that there is no equilibrium such that \( v_{AL} = v_S \).

Finally, loss leading (in which \( L \) not only offers, but actually sells below cost) can only arise when \( L \) sells to one-stop shoppers, which thus requires \( v_{AL} \geq v_S \). But this cannot be an equilibrium when \( w_{AL} < \tilde{w}_{AL}(w_S, w_L) \), since: (i) in the range \( v_{AL} > v_S \), the
only such candidate is the above described loss-leading outcome, which requires \( w_{AL} \geq \hat{w}_{AL} (w_S, w_L) \); and (ii) as just discussed, no equilibrium exists in the boundary case \( v_{AL} = v_S \).

## C Proof of Proposition 3

### Stackelberg leadership.

Suppose that \( L \) benefits from a first-mover advantage: it sets its prices first, and then, having observed these prices, \( S \) sets its own price. Retail prices are often strategic complements, and it is indeed the case here for \( S \) in the \( B \) segment: as noted before, \( S \)’s best response, \( \hat{r}_S (r_L) \), increases with \( r_L \). Thus, in the case of “normal competition” in market \( B \), \( L \) would exploit its first-mover advantage by increasing its price for \( B_L \), so as to encourage its rival to increase its own price and relax the competitive pressure. In contrast, here \( L \) has an incentive to decrease \( r_L \) even further. This leads \( S \) to decrease its own price, which allows \( L \) to raise the price for \( F \). To see this, note that \( L \)’s Stackelberg profit from a loss-leading strategy can be written as:

\[
\Pi^L_S (r_L) = \Pi^m_{AL} - r_L F (\hat{r} (r_L)) = \Pi^m_{AL} - r_L F (w_S - w_L + r_L - \hat{r}_S (r_L)).
\]

Denoting by \( r^*_L \) the optimal Stackelberg margin and using \( \hat{r} (r_L) = \hat{r}^*_L \), where \( \hat{r}^*_L \) and \( \hat{r}^*_S \) are the equilibrium margins when \( L \) moves simultaneously with \( S \), we have:

\[
-r^*_L F (w_S - w_L + r^*_L - \hat{r}_S (r^*_L)) \geq -\hat{r}^*_S F (w_S - w_L + \hat{r}_S (\hat{r}_L))
\]

\[
\geq -r^*_L F (w_S - w_L + r^*_L - \hat{r}^*_S),
\]

where the second inequality stems from the fact that \( \hat{r}^*_L \) constitutes \( L \)’s best response to \( r^*_S \). Since \( -r^*_L > 0 \) and \( F (\cdot) \) and \( \hat{r}_S (\cdot) \) are both increasing, this in turn implies \( r^*_L \leq \hat{r}^*_L \).

This inequality is moreover strict, since (using \( \hat{r} (\hat{r}^*_L) = \hat{r}^*_L \)):

\[
(\Pi^L_S)' (\hat{r}^*_L) = -F (\hat{r}^*_S) - \hat{r}^*_S f (\hat{r}^*_S) (1 - \hat{r}^*_S (\hat{r}^*_L)) = \hat{r}^*_S f (\hat{r}^*_S) \hat{r}_S (\hat{r}^*_L) < 0.
\]

Thus, \( L \) sells the competitive product \( B_L \) further below-cost, compared with what it would do in the absence of a first-mover advantage: \( r^*_L < \hat{r}^*_L \).

### Entry accommodation.

Suppose now that the presence of \( S \) is uncertain. To capture this possibility, assume that \( S \) incurs a fixed cost for entering the market, \( \gamma \), which is ex ante distributed according to a cumulative distribution function \( F_\gamma (\cdot) \), and consider the following timing:
• In stage 1, \( L \) chooses its prices.

• In stage 2, the entry cost is realized, and \( S \) chooses whether to enter; if it enters, it then sets its own price.

If entry were certain, maximizing its Stackelberg profit would lead \( L \) to adopt \( r_L^S \). But now, \( S \) enters only when its best response profit, \( \hat{\Pi}_S (r_L) \), exceeds the realized cost \( \gamma \), which occurs with probability \( \rho (r_L) \equiv F_\gamma (\hat{\Pi}_S (r_L)) \). \( L \)’s ex ante profit is therefore equal to

\[
\hat{\Pi}_L^S (r_L) = \Pi_{AL}^m + \rho (r_L) \Pi_L^S (r_L).
\]

The optimal margin, \( \hat{\rho}_L \), thus satisfies

\[
\rho (\hat{\rho}_L) \Pi_L^S (\hat{\rho}_L) \geq \rho (r_L) \Pi_L^S (r_L) \geq \rho (r_L) \Pi_L^S (\hat{\rho}_L),
\]

which implies

\[
\rho (\hat{\rho}_L) \geq \rho (r_L).
\]

Since \( F_\gamma \) and \( \hat{\Pi}_S \) are both increasing in \( r_L \), so is \( \rho \) and thus \( \hat{\rho}_L \geq r_L^S \). This inequality is moreover strict, since

\[
\left( \hat{\Pi}_L^S \right)' (r_L^S) = \rho' (r_L^S) \Pi_L^S (r_L^S) + \rho (r_L^S) \left( \Pi_L^S \right)' (r_L^S) = \rho' (r_L^S) \Pi_L^S (r_L^S) > 0.
\]

Therefore, when \( L \)’s comparative advantage leads it to adopt a loss-leading strategy, it limits the subsidy on \( B \) so as to increase the likelihood of entry: \( \hat{r}_L^S > r_L^S \).

\[\textbf{D Proof of Proposition 4}\]

In the equilibrium where \( L \) attracts one-stop shoppers in the absence of a ban, \( L \) must offer a higher value than \( S \): \( v_{AL} = v_{AL}^m > \hat{v}_S^* = w_S - \hat{r}_S^* \), and \( S \) must moreover not be tempted to deviate and attract one-stop shoppers, which boils down to \( \hat{\Pi}_S^d = h (\hat{\tau}^*) F (\hat{\tau}^*) \geq \hat{\Pi}_S^d = (w_S - v_{AL}^m) F (v_{AL}^m) \). If \( L \) keeps attracting one-stop shoppers (i.e., \( v_{AL} > v_S \)) when loss leading is banned, then the unique candidate equilibrium is \( r_{AL} = r_{AL}^m, r_L = 0 \) and \( r_S^d = h (\hat{\tau}^*) \), where \( \hat{\tau}^* = l^{-1} (w_S - w_L) \).

We show now this candidate equilibrium prevails when loss-leading would arise if below-cost pricing were allowed. Note that, since \( S \) increases its price (i.e., \( r_S^d = h (\hat{\tau}^*) > \hat{r}_S^* = h (\hat{\tau}^*) \)), it offers less value (\( v_S = v_S^d \equiv w_S - r_S^d < \hat{v}_S^* \)), and thus \( L \) indeed attracts
one-stop shoppers: \( v_{AL} = v_{AL}^m > (v_S^* > v_S^b) \). Furthermore, as \( S \) must again offer at least \( v_S = v_{AL} \) to attract one-stop shoppers, it still cannot obtain more than \( \hat{\Pi}_S^* \) by deviating in this way. Therefore, since \( S \) now obtains more profit \( \Pi_S^* \equiv h(\tau^*) F(\tau^*) > \hat{\Pi}_S^* = h(\hat{\tau}^*) F(\hat{\tau}^*) \), it is less tempted to deviate: \( \Pi_S^* > (\hat{\Pi}_S^*) \hat{\Pi}_S^* \). It follows that the conditions for sustaining the above equilibrium are less stringent than that for the loss-leading equilibrium.

E Product differentiation in the competitive market

We show that our main insights apply when consumers vary in their relative preferences over \( B_L \) and \( B_S \). For example, suppose \( B_L \) is a “standard” variety generating a homogeneous utility \( u_L \), whereas \( B_S \), a better variety supplied by specialist stores, yields a utility \( u_S + \theta q; \theta \in [0, 1] \) thus characterizes the consumer preference for quality and is distributed according to a c.d.f \( \Phi(\cdot) \) with density function \( \phi(\cdot) \), whereas \( q \) measures the degree of consumer heterogeneity. For the sake of exposition, we consider here the case where \( B_S \) is supplied by a competitive fringe and assume that:

- \( S \) provides better value for at least some quality-oriented consumers: \( w_S + q > w_L \); we allow however for \( w_L > w_S \), in which case \( L \) offers higher value than \( S \) for less quality-oriented consumers.
  - all one-stop shoppers favor \( L \): \( v_{AL} \geq \Pi_S^* = w_S + q \).

As before, consumers are willing to patronize \( L \) if \( s \leq v_{AL} \), and prefer multi-stop shopping to one-stop shopping if

\[
 s \leq w_S + \theta q - v_L = \tau + \theta q,
\]

where \( \tau = w_S - w_L + r_L \). \( L \) thus earns a profit

\[
 \Pi_L = r_AL D_AL(r_AL) - r_L D_AS(r_L)
\]

where \( D_AL(r_AL) = F(v_{AL}) \) and \( D_AS(r_L) = \int_0^1 F(\tau + \theta q) \phi(\theta) d\theta \). The loss leading logic of the baseline model applies again here: since \( v_{AL} = w_{AL} - r_AL \) and \( \tau = w_S - w_L + r_L \), \( L \)’s profit is separable in \( r_AL \) and \( r_L \), and still charges the price on \( B_L \) below-cost.
While we presented this example in terms of “vertical” quality differentiation, the same analysis applies to “horizontal” differentiation, with utilities for \( B_L \) and \( B_S \) of the form \( u_L + (1 - \theta) q \) and \( u_S + \theta q \); the only difference is that, since consumers have now heterogeneous valuations for \( B_L \) as well, the above demands become:

\[
D_{AL}(r_{AL}) = \int_0^1 F(v_{AL} + (1 - \theta) q) \phi(\theta) d\theta, \quad D_{AS}(r_L) = \int_0^1 F(\tau + (2\theta - 1) q) \phi(\theta) d\theta.
\]

### F Proof of Proposition 6

#### F.1 Local Monopolies with heterogeneous preferences on \( A \)

We show that introducing an elastic demand in market \( A \) does not preclude the large retailer from adopting a loss-leading strategy, so as to extract additional surplus from multi-stop shoppers. We focus on the large retailer’s best response to the strategies of the smaller retailer(s); thus, what follows applies equally to the case of a strategic rival and that of a competitive fringe.

\( L \)'s profit can be written as (see Figure 1):

\[
\Pi_L = r_{AL} D_{AL} + r_A D_{AS} = r_{AL} \int_0^{x_{AL}} G(x_A(s)) f(s) ds + r_A \int_0^\tau G(x_A(s)) f(s) ds.
\]

To characterize the equilibrium values of \( r_L \) and \( r_{AL} \), consider first a modification of \( r_A \) by \( dr \), adjusting \( r_L \) by \(-dr\) so as to keep \( r_{AL} \) constant. Such a change does not affect the behavior of one-stop shoppers (it has no impact on \( v_{AL} \) and \( x_{AL}(s) \)), but (see Figure 2):

- It affects multi-shop shoppers: for \( s < \tau \), the marginal consumer indifferent between buying \( A \) from \( L \) or patronizing \( S \) only becomes \( x = x_A(s) - dr \); therefore, \( L \) loses \( g(x_A(s)) dr \) consumers, on which it no longer earns the margin \( r_A \). \( L \) however increases its margin by \( dr \) on the mass \( G(x_A(s)) \) of consumers that buy \( A \). Thus, the overall impact of such an adjustment on multi-stop shoppers is equal to

  \[
  \int_0^\tau [G(x_A(s)) - r_A g(x_A(s))] f(s) ds dr.
  \]

- In addition, it alters the choice between one-stop and multi-stop shopping: those consumers for which \( s \in [\tau - dr, \tau] \) and \( x \leq x_A(s) \) turn to one-stop shopping and now buy \( B \) as well as \( A \) from \( L_1 \), which (noting that \( x_A(\tau) = \hat{x} \)) brings a gain \( r_\ell G(\hat{x}) f(\tau) dr \).
These effects must cancel out in equilibrium, which yields
\[
\int_0^\tau \left[ r_A - k(x_A(s)) \right] g(x_A(s)) f(s) \, ds = r_L G(\hat{x}) f(\tau).
\]
Likewise, adjusting slightly \( r_{AL} \) by \( dr \), keeping \( r_A \) constant (and thus changing \( r_L \) by \( dr \) as well) does not affect the behavior of multi-stop shoppers (it has no impact on \( v_{AS} \) and \( x_A(s) \)), but:

- It affects one-stop shoppers: for \( s > \tau \), the marginal shopper becomes \( x = x_{AL}(s) - dr \), and the resulting change in profit is
\[
\int_\tau^{v_{AL}} [G(x_{AL}(s)) - r_{AL} g(x_{AL}(s))] f(s) \, dsdr.
\]
- In addition, those consumers for which \( s \in [\tau, \tau + dr] \) and \( x \leq x_{AL}(s) \) become multi-stop shoppers and stop buying \( B \) from \( L \), which (noting that \( x_{AL}(\tau) = \hat{x} \)) brings a net effect \(-r_L G(\hat{x}) f(\tau) \, dr\).

In equilibrium, these effects must again cancel each other, which yields
\[
\int_\tau^{v_{AL}} [r_{AL} - k(x_{AL}(s))] g(x_{AL}(s)) f(s) \, ds = -r_L G(\hat{x}) f(\tau).
\]
Therefore, if in equilibrium \( r_L \) were non-negative, we would have
\[
\int_0^\tau \left[ r_A - k(x_A(s)) \right] g(x_A(s)) f(s) \, ds \geq 0 \geq \int_\tau^{v_{AL}} [r_{AL} - k(x_{AL}(s))] g(x_{AL}(s)) f(s) \, ds,
\]
that is, \( r_A \) would exceed a weighted average of \( k(x_A(s)) \) for \( s \in [0, \tau] \), whereas \( r_{AL} \) would be lower than a weighted average of \( k(x_{AL}(s)) \) for \( s \in [\tau, v_{AL}] \). But since \( k(x_A(s)) \) and \( k(x_{AL}(s)) \) decrease as \( s \) increases (\( k(.) \) increases by assumption, and both \( x_A(s) \) and \( x_{AL}(s) \) decrease by construction), this would imply \( r_A > r_{AL} \), a contradiction. Therefore, in equilibrium, \( r_L < 0 \).

If the shopping cost \( s \) is distributed over some interval \([0, \overline{s}]\), where \( \overline{s} > \tau \) to ensure that large retailers still attract some one-stop shoppers, the first-order conditions become:
\[
\int_0^\tau \left[ r_A - k(x_A(s)) \right] g(x_A(s)) f(s) \, ds = r_L G(\hat{x}) f(\tau),
\]
\[
\int_{\tau}^{\min\{v_{AL}, \overline{s}\}} \left[ r_{AL} - k(x_{AL}(s)) \right] g(x_{AL}(s)) f(s) \, ds = -r_L G(\hat{x}) f(\tau);
\]
it thus suffices to replace \( v_{AL} \) with \( \min\{v_{AL}, \overline{s}\} \) in the above reasoning.
F.2 Imperfect competition among large retailers

Suppose now that two large retailers, \( L_1 \) and \( L_2 \), facing the same costs in both markets and offering the same variety \( B_L \), are differentiated in market \( A \): they respectively offer \( A_1 \) and \( A_2 \), located at the two ends of a Hotelling line of length \( X \); a consumer with preference \( x \) thus obtains a utility \( u_A - x - p_{A_1} = w_A - r_{A_1} - x \) from buying \( A_1 \) and a utility \( w_A - r_{A_2} - (X - x) \) from buying \( A_2 \). We will restrict attention to symmetric distributions (that is, the density \( g(\cdot) \) satisfies \( g(x) = g(X - x) \)) and will focus on (symmetric) equilibria in which: (i) the large retailers compete against each other as well as against their smaller rivals; (ii) small retailers attract some multi-stop shoppers by offering a value \( v_S \) that exceeds the value \( v_L \) offered by large retailers on the \( B \) market; and (iii) large retailers attract some one-stop shoppers by offering them a value \( v_{AL} \) that exceeds \( v_S \), as well as the value \( v_A \) that they offer on the \( A \) market alone.

Large retailers may compete against each other for one-stop and/or for multi-stop shoppers. In the former case, in a symmetric equilibrium (of the form \( r_{A_1L_1} = r_{A_2L_2} = r_{AL} \) and \( r_{L_1} = r_{L_2} = r_L \)) some consumers (with \( x = X/2 \)) are indifferent between buying both goods from either \( L_1 \) or \( L_2 \), and prefer doing so to patronizing \( S \) only; this implies (using \( x = X/2 \), and dropping the subscripts 1 and 2 for ease of exposition):

\[
\hat{v}_{AL} = v_{AL} - \frac{X}{2} \geq v_S,
\]

which is equivalent to

\[
\hat{v}_A = v_A - \frac{X}{2} \geq \tau = v_S - v_L.
\]

Therefore, consumers with preference \( x = X/2 \) and shopping cost \( s < \tau \), who thus prefer multi-stop shopping (that is, buying \( B_S \) from \( S \) and \( A \) from either \( L_1 \) or \( L_2 \)) to visiting \( L_1 \) or \( L_2 \) only, also prefer multi-stop shopping to patronizing \( S \) only (since \( s < \tau \) then implies \( s < \hat{v}_A \)). In other words, if large retailers compete for one-stop shoppers, they will also compete for multi-stop shoppers. This observation allows us to classify the (symmetric) candidate equilibria into two types:

- **Type M**: large retailers compete only for **multi-stop** shoppers (see Figure 3a);
- **Type O**: large retailers compete for **one-stop** shoppers as well as for multi-stop shoppers.
shoppers (see Figure 3b).

In the first type of equilibria (Figure 3a), for \( x = X/2 \) some consumers with low shopping costs are indifferent between assortments \( A_1S \) and \( A_2S \), and prefer those assortments to any other option, whereas consumers with higher shopping costs patronize \( S \) only; the relevant threshold for the shopping cost satisfies

\[
\hat{v}_A + v_S - 2s = v_S - s,
\]

that is, \( s = \hat{v}_A \). Consumers with \( s < \hat{v}_A \) thus buy \( B \) from \( S \) and \( A \) from either \( L_1 \) or \( L_2 \) (depending on whether \( x \) is smaller or larger than \( X/2 \)). Conversely, consumers whose shopping costs exceed \( v_{AL} \) do not shop. As for consumers whose shopping costs lie between \( \hat{v}_A \) and \( v_{AL} \):

- when \( s < \tau \), consumers still buy \( B_S \) from \( S \); they also buy \( A \) from \( L_1 \) if \( x < x_A(s) = v_A - s \), or from \( L_2 \) if \( x > X - x_A(s) \);

- when \( s > \tau \):
  - if \( x < x_{AL}(s) \), consumers buy both goods from \( L_1 \);
  - if \( x > X - x_{AL}(s) \), consumers buy both goods from \( L_2 \);
  - if \( x_{AL}(s) < x < X - x_{AL}(s) \), consumers patronize \( S \) if \( s < v_S \), and buy nothing otherwise.
In the second type of equilibria (Figure 3b), all consumers with a shopping cost \( s < \tau \) buy \( B_S \) from \( S \) and \( A \) from either \( L_1 \) (if \( x < X/2 \)) or \( L_2 \) (if \( x > X/2 \)), while consumers with \( s > v_{AL} \) buy nothing. For consumers with \( \tau < s < v_{AL} \), then:

- if \( s < \hat{v}_{AL} \), consumers will buy both goods from either \( L_1 \) (if \( x < X/2 \)) or \( L_2 \) (if \( x > X/2 \));
- if \( \hat{v}_{AL} < s < v_{AL} \), consumers will buy both goods from \( L_1 \) if \( x < x_{AL} (s) \) or from \( L_2 \) if \( x > X - x_{AL} (s) \), and buy nothing otherwise.

A similar description applies when the shopping cost \( s \) is bounded, truncating as necessary the interval for \( s \).

We show now loss leading is still used as an exploitative device. Consider first (symmetric) equilibria of type \( M \), in which large retailers compete only for multi-stop shoppers. In the absence of any bound on shopping costs, the demands for assortments \( A_1L_1 \) and \( A_1S \) in such equilibrium, where \( r_{A_1L_1} = r_{A_2L_2} = r_{AL} \) and \( r_{L_1} = r_{L_2} = r_L \) (and thus \( r_{A_1} = r_{A_2} = r_A \)), can be expressed as:

\[
D_{AS} = \int_{0}^{\tau} G(\hat{x}_A (s)) f (s) \, ds \quad \text{and} \quad D_{AL} = \int_{\tau}^{v_{AL}} G(x_{AL} (s)) f (s) \, ds,
\]

where as before \( \tau = v_S - v_L \) and \( x_{AL} (s) = v_{AL} - \max \{ s, v_S \} \), and \( \hat{x}_A (s) \equiv v_A - \max \{ s, \hat{v}_A \} = \min \{ X/2, x_A (s) = v_A - s \} \).

Applying the same approach as above, starting from a candidate symmetric equilibrium, consider first a small change \( dr \) in \( r_{A_1} \), adjusting \( r_{L_1} \) by \(-dr\) so as to keep \( r_{A_1L_1} \) constant:

- For \( s < \hat{v}_A \), the marginal consumer who is indifferent between buying \( A \) from \( L_1 \) or \( L_2 \) is such that:
  \[
  w_A - (r_A + dr) - x = w_A - r_A - (X - x),
  \]
  or:
  \[
  x = \frac{X}{2} - \frac{dr}{2}.
  \]

The overall impact on \( L_1 \)'s profit is thus:

\[
\int_{0}^{\hat{v}_A} [G(\hat{x}_A (s)) - \frac{1}{2} r_{AG} (\hat{x}_A (s))] f (s) \, ds dr.
\]
• For $\hat{v}_A < s < \tau$, the marginal consumer indifferent between buying $A$ from $L_1$ or patronizing $S$ becomes $x = x_A (s) - dr$, and the resulting impact on profit is:

$$\int_{\hat{v}_A}^{\tau} [G(\hat{x}_A (s)) - r_A g (\hat{x}_A (s))] f (s) ds dr.$$

• In addition, those consumers for which $s \in [\tau - dr, \tau]$ and $x \leq \hat{x}_A (s)$ turn to one-stop shopping and now buy $B$ as well as $A$ from $L_1$, which brings an additional profit $r_L G (\hat{x}) f (\tau) dr$.

Therefore, in equilibrium, we must have:

$$\int_0^{\tau} [r_A - \eta_A (s)] \hat{g} (\hat{x}_A (s)) f (s) ds = r_L G (\hat{x}) f (\tau),$$

where (using $\hat{x}_A (s) = X/2$ for $s \leq \hat{v}_A$):

$$\eta_A (s) \equiv \begin{cases} 2k (\hat{x}_A (s)) & \text{for } s < \hat{v}_A \\ k (\hat{x}_A (s)) & \text{for } s > \hat{v}_A \end{cases} \quad \text{and } \hat{g} (x) \equiv \begin{cases} \frac{g (X/2)}{2} & \text{for } x = X/2 \\ g (x) & \text{for } x < X/2 \end{cases}.$$

Consider now a small change $dr$ in $r_{A_1 L_1}$, keeping $r_{A_1}$ constant (and thus adjusting $r_{L_1}$ by $dr$ as well):

• for $s > \tau$, the marginal (one-stop) shopper becomes $x = x_{AL} (s) - dr$ and the impact on the profit is

$$\int_{\tau}^{v_{AL}} [G (x_{AL} (s)) - r_{AL} g (x_{AL} (s))] f (s) ds dr;$$

• in addition, those consumers for which $s \in [\tau, \tau + dr]$ and $x \leq x_{AL} (s)$ become multi-stop shoppers and stop buying $B$ from $L_1$, which brings a net loss $-r_L G (\hat{x}) f (\tau) dr$.

In equilibrium, we must therefore have

$$\int_{\tau}^{v_{AL}} [r_{AL} - \eta_{AL} (s)] g (x_{AL} (s)) f (s) ds = -r_L G (\hat{x}) f (\tau),$$

where $\eta_{AL} (s) \equiv k (x_{AL} (s))$.

Thus, if $r_L$ were non-negative, the two conditions (8) and (9) would imply

$$\int_0^{\tau} [r_A - \eta_A (s)] \hat{g} (\hat{x}_A (s)) f (s) ds \geq 0 \geq \int_{\tau}^{v_{AL}} [r_{AL} - \eta_{AL} (s)] g (x_{AL} (s)) f (s) ds,$$
where $\eta_A$ and $\eta_{AL}$ decrease as $s$ increases, and coincide for $s = \tau$; this, in turn, would imply $r_A > r_{AL}$, a contradiction. A similar argument applies when the shopping cost $s$ is distributed over some interval $[0, \bar{s}]$.

The same approach can be used for (symmetric) equilibria of type $O$, in which large retailers compete as well for one-stop shoppers. In the absence of any bound on shopping costs, the demands for assortments $A_1L_1$ and $A_1S$ in such equilibrium can be expressed as

$$D_{AS} = \int_0^\tau G \left( \frac{X}{2} \right) f(s) \, ds \quad \text{and} \quad D_{AL} = \int_\tau^{\nu_{AL}} G (\hat{x}_{AL}(s)) f(s) \, ds,$$

where $\hat{x}_{AL}(s) \equiv v_A - \max \{s, \hat{v}_{AL}\} = \min \{X/2, x_{AL}(s) = v_{AL} - s\}$.

Following a small change $dr$ in $r_{A_1}$, adjusting $r_{L_1}$ by $-dr$ so as to keep $r_{A_1L_1}$ constant, we have:

- for $s < \tau$, the marginal consumer indifferent between buying $A$ from $L_1$ or $L_2$ becomes $X/2 - dr/2$;

- in addition, those consumers for which $s \in [\tau - dr, \tau]$ and $x \leq \hat{x}_{A}(s)$ become one-stop shoppers.

Therefore, in equilibrium we must have

$$\int_0^\tau [r_A - \hat{\eta}_A] \hat{g}(\frac{X}{2}) f(s) \, ds = r_{L_1} G \left( \frac{X}{2} \right) f(\tau),$$

where $\hat{\eta}_A \equiv 2k (X/2)$ and $\hat{g}(X/2) = g(X/2)/2$.

Likewise, following a small change $dr$ in $r_{A_1L_1}$, keeping $r_{A_1}$ constant (and thus changing $r_{L_1}$ by $dr$ as well), we have:

- for $\tau < s < \hat{v}_{AL}$, the marginal (one-stop) shopper becomes $x = x_{AL}(s) - dr/2$;

- for $\hat{v}_{AL} < s < v_{AL}$, the marginal (one-stop) shopper becomes $x = x_{AL}(s) - dr$;

- in addition, those consumers for which $s \in [\tau, \tau + dr]$ and $x \leq \hat{x}_{AL}(s)$ become multi-stop shoppers: they stop buying $B$ from $L_1$.

We must therefore have

$$\int_\tau^{\nu_{AL}} [r_{AL} - \hat{\eta}_{AL}(s)] \hat{g}(\hat{x}_{AL}(s)) f(s) \, ds = -r_{L_1} G (\hat{x}) f(\tau),$$

where $\hat{\eta}_{AL} \equiv 2k (X/2)$ and $\hat{g}(X/2) = g(X/2)/2$. 

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where
\[
\hat{\eta}_{AL}(s) \equiv \begin{cases} 2k(\hat{x}_{AL}(s)) & \text{for } s < \hat{v}_{AL} \\ k(\hat{x}_{AL}(s)) & \text{for } s > \hat{v}_{AL} \end{cases},
\]
and \(\hat{g}(x)\) is defined above with \(\hat{x}_{AL}(s) = X/2\) for \(\tau \leq s \leq \hat{v}_{AL}\). Thus, if \(r_L\) were non-negative, the above two conditions would imply:
\[
\int_0^\tau [r_A - \hat{\eta}_A] \hat{g}(X/2)f(s) \, ds \geq 0 \geq \int_{\tau}^{\hat{v}_{AL}} [r_{AL} - \hat{\eta}_{AL}(s)] \hat{g}(\hat{x}_{AL}(s)) \, f(s) \, ds,
\]
and a contradiction follows, since \(\hat{x}_{AL}(s) \leq X/2\), with a strict inequality for \(s > \hat{v}_{AL}\), and thus \(\hat{\eta}_{AL}(s) \leq 2k(\hat{x}_{AL}(s)) \leq \hat{\eta}_A\), with again a strict inequality for \(s > \hat{v}_{AL}\). A similar argument applies again when the shopping cost \(s\) is distributed over some interval \([0, \bar{s}]\).
If instead \(\bar{s} < \hat{v}_{AL}\), then all consumers buy both goods, in which case \(\hat{\eta}_{AL}(.) = \hat{\eta}_A\) and \(\hat{g}(\hat{x}_{AL}(s)) = \hat{g}(X/2)\), and \(r_L = 0\).