

Online Appendix for “Loss Leading as an Exploitative Practice”

Zhijun Chen

Patrick Rey

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A Proof of Proposition 1

Suppose first that $v_{AL} \geq w_S$, that is, $r_{AL} \leq w_{AL} - w_S$ (“regime L ”). We first show that, without loss of generality, we can focus on prices such that $\tau \in [0, v_{AL}]$. If $\tau > v_{AL}$ (i.e., $w_S - w_L + r_L > w_{AL} - r_{AL}$, or $r_L > r'_L \equiv (w_{AL} - r_A - (w_S - w_L))/2$), there are no one-stop shoppers: active consumers buy A from L and B_S from S , and do so as long as $2s < v_A + w_S$; however, keeping r_A constant, decreasing r_L to r'_L such that $\tau' = v'_{AL}$ does not affect the number of active consumers (since v_A does not change), who still visit both stores as before. If instead $\tau < 0$ (i.e., $r_L < -w_S - w_L$), there are no multi-stop shoppers: active consumers only visit L , and do so as long as $s < v_{AL} = w_{AL} - r_{AL}$; however, keeping r_{AL} constant, increasing r_L to $r'_L = -(w_S - w_L)$ yields $\tau' = 0$ without affecting consumer behavior. The condition $\tau \geq 0$ moreover ensures that prospective multi-stop shoppers are indeed willing to buy A on a stand-alone basis: $w_S \leq v_{AL} = w_{AL} - r_A - r_L$ implies $r_A \leq w_{AL} - w_S - r_L = w_{AL} - w_L - \tau < w_A$.

Thus, consumers whose shopping cost lies in $[0, \tau]$ buy A from L (and B_S from S), whereas those with a shopping cost in $[\tau, v_{AL}]$ buy both A and B_L from L . Using $v_{AL} = w_{AL} - r_{AL}$ and $\tau = w_S - w_L + r_L$, L 's optimization program within regime L can thus be expressed as:

$$\begin{aligned} \max_{r_{AL}, r_L} \quad & \Pi_L(r_{AL}, r_L) = r_{AL}F(w_{AL} - r_{AL}) - r_LF(w_S - w_L + r_L), \\ \text{subject to} \quad & r_{AL} \leq w_{AL} - w_S \end{aligned} \tag{1}$$

where $\Pi_L(r_{AL}, r_L)$ is additively separable and moreover strictly quasi-concave¹ in r_{AL} and r_L . L 's optimization program can thus be decomposed into:

$$\begin{aligned} \max_{r_{AL}} \quad & r_{AL}F(w_{AL} - r_{AL}), \\ \text{s. t.} \quad & r_{AL} \leq w_{AL} - w_S \end{aligned}$$

which leads to $r_{AL} = \min\{r_{AL}^m, w_{AL} - w_S\}$ and $v_{AL} = \max\{v_{AL}^m, w_S\}$, and

$$\min_{r_L} r_L F(w_S - w_L + r_L),$$

which yields the first-order condition:

$$r_L^* = -h(w_S - w_L + r_L^*) = -h(\tau^*) < 0. \tag{2}$$

¹The derivative w.r.t. r_{AL} is of the form $f(w_{AL} - r_{AL})\phi(r_{AL})$, where $\phi(r_{AL}) \equiv h(w_{AL} - r_{AL}) - r_{AL}$ is strictly decreasing. A similar reasoning applies below to the other profit functions of L and S .

Using $r_L^* = \tau^* - (w_S - w_L) = -h(\tau^*)$, the optimal threshold τ^* is given by:

$$\tau^* \equiv l^{-1}(w_S - w_L) > 0. \quad (3)$$

Note that this threshold satisfies $\tau^* < v_{AL}^m$. To see this, take instead v_{AL} and τ as control variables and rewrite L 's profit as:

$$\begin{aligned} \Pi_L(v_{AL}, \tau) &= r_{AL}F(v_{AL}) - r_LF(\tau) \\ &= (w_{AL} - v_{AL})F(v_{AL}) + (w_S - w_L - \tau)F(\tau). \end{aligned}$$

Then we have $v_{AL}^m = \arg \max_v (w_{AL} - v)F(v) > \arg \max_v (w_S - w_L - v)F(v) = \tau^*$, since $w_{AL} \geq l(w_S) (> w_S \geq w_S - w_L)$.

Suppose now that $v_{AL} < w_S$, that is, $r_{AL} > w_{AL} - w_S$ ("regime S "). L then only attracts multi-stop shoppers, who buy A from it as long as $s \leq v_A = w_A - r_A$. L thus obtains:

$$\Pi_L = r_A F(v_A) = r_A F(w_A - r_A),$$

which is maximal for r_A^m and $v_A^m = w_A - r_A^m$, characterized by:

$$r_A^m = h(v_A^m), \quad v_A^m = l^{-1}(w_A).$$

L 's profit in regime S is thus at most:

$$\Pi_A^m \equiv r_A^m F(v_A^m).$$

As already noted, regime L is clearly preferable when $v_{AL}^m \geq w_S$, since it then gives L more profit than the monopolistic level Π_{AL}^m , which itself is greater than Π_A^m :

$$\Pi_{AL}^m = \max_r rF(w_{AL} - r) > \max_r rF(w_A - r) = \Pi_A^m,$$

since $w_{AL} > w_A$. We now show that regime L , and the associated loss-leading strategy, remains profitable when $w_{AL} \geq w_S > v_{AL}^m$, where it involves $r_L^* < 0$ and $\tilde{r}_{AL}^* = w_{AL} - w_S$. To see this, fixing $\tilde{r}_{AL}^*$ and using r_A rather than r_L as the optimization variable, the margin on B_L and the shopping cost threshold can be expressed as:

$$r_L = \tilde{r}_{AL}^* - r_A = w_{AL} - w_S - r_A, \quad \tau = w_S - w_L + r_L = w_{AL} - w_L - r_A = w_A - r_A.$$

The maximum profit achieved in regime L , $\tilde{\Pi}_L^*$, can then be written as:

$$\begin{aligned}
\tilde{\Pi}_L^* &= \tilde{r}_{AL}^* (F(\tilde{v}_{AL}^*) - F(\tau^*)) + r_A^* F(\tau^*) \\
&= (w_{AL} - w_S) (F(w_S) - F(\tau^*)) + r_A^* F(\tau^*) \\
&= \max_{r_A} \{ (w_{AL} - w_S) (F(w_S) - F(w_A - r_A)) + r_A F(w_A - r_A) \} \\
&\geq (w_{AL} - w_S) (F(w_S) - F(w_A - r_A^m)) + r_A^m F(w_A - r_A^m) \\
&= (w_{AL} - w_S) (F(w_S) - F(v_A^m)) + \Pi_A^m.
\end{aligned}$$

Since $w_S > v_{AL}^m = l^{-1}(w_{AL}) > l^{-1}(w_A) = v_A^m$, $\tilde{\Pi}_L^* \geq \Pi_A^m$ whenever $w_{AL} \geq w_S$.

Conversely, when $w_{AL} < w_S$, then L can indeed achieve Π_A^m in regime S (e.g., $r_L = 0$ and $r_A = r_A^m$ satisfy $r_{AL} = r_A^m > 0 > w_{AL} - w_S$, and thus $v_{AL} < w_S$), and we have:

$$\begin{aligned}
\tilde{\Pi}_L^* &= (w_{AL} - w_S) (F(w_S) - F(w_A - \tilde{r}_A^*)) + \tilde{r}_A^* F(w_A - \tilde{r}_A^*) \\
&< \tilde{r}_A^* F(w_A - \tilde{r}_A^*) \\
&\leq \Pi_A^m,
\end{aligned}$$

where the first inequality stems from $w_S > w_{AL} (> w_A - \tilde{r}_A^*)$.

Finally, in the limit case where $w_{AL} = w_S$, using B_L as a loss leader amounts to monopolizing product A . Offering $v_{AL} = w_S$ requires $r_{AL} = w_{AL} - v_{AL} = 0$, or $r_A = -r_L$, and the optimal subsidy thus maximizes $-r_L F(\tau) = -r_L F(w_S - w_L + r_L) = r_A F(w_A - r_A)$. Therefore, in both cases L obtains (from multi-stop shoppers) the monopoly margin on A , and makes no profit (from one-stop shoppers) on the bundle $A - B_L$ (since either it charges them $r_{AL} = 0$, or they go to S). Finally, while the loss-leading strategy may yield a lower price for B_L (in the monopolization scenario, L may actually stop carrying B_L), this does not affect multi-stop shoppers (who do not buy B_L from L), whereas one-stop shoppers are indifferent between buying A and B_L from L or B_S only from S .

B Proof of Proposition 2

We derive here the conditions under which the loss leading outcome ($\hat{r}_{AL}^* = r_{AL}^m$ and $\hat{r}_L^* = -\hat{r}_S^* = -h(\hat{\tau}^*)$, where $\hat{\tau}^* = j^{-1}(w_S - w_L)$) forms a Nash equilibrium, before checking the uniqueness of the equilibrium. To attract one-stop shoppers, L must offer a better value

than S :²

$$v_{AL}^m \geq \hat{v}_S^* \equiv w_S - h(\hat{\tau}^*). \quad (4)$$

This condition implies $v_{AL}^m \geq \hat{v}_S^* > \hat{v}_S^* - \hat{v}_L^* = \hat{\tau}^*$, which in turn implies $w_{AL} > w_S$:

$$w_{AL} = l(v_{AL}^m) \geq l(\hat{v}_S^*) = \hat{v}_S^* + h(\hat{v}_S^*) = w_S - h(\hat{\tau}^*) + h(\hat{v}_S^*) > w_S.$$

Moreover, while L has no incentive to exclude its rival, since it earns more profit than a pure monopolist, S may want to attract one-stop shoppers by reducing r_S so as to offer $v_S \geq v_{AL}^m$. Such a deviation allows S to attract all consumers (one-stop or multi-stop shoppers) with shopping costs $s \leq v_S$ and thus yields a profit $\Pi_S^d(v_S) \equiv r_S F(v_S) = (w_S - v_S) F(v_S)$. A simple revealed argument yields $\arg \max_v \Pi_S^d(v) \leq v_{AL}^m \equiv \arg \max_v (w_{AL} - v) F(v)$, since $w_S < w_{AL}$; as $\Pi_S^d(v_S)$ is quasi-concave in v_S , increasing v_S further above v_{AL}^m would thus reduce S 's profit. It is therefore optimal for S to offer precisely $v_S^d = v_{AL}^m$ (or slightly above v_{AL}^m , if one-stop shoppers are indifferent between the two stores in this case), which gives S a profit equal to $\Pi_S^d(v_{AL}^m) = (w_S - v_{AL}^m) F(v_{AL}^m)$. The loss-leading outcome is immune to such a deviation if and only if

$$\hat{\Pi}_S^* \equiv h(\hat{\tau}^*) F(\hat{\tau}^*) \geq \hat{\Pi}_S^d \equiv (w_S - v_{AL}^m) F(v_{AL}^m). \quad (5)$$

This condition can be further written as:

$$\Psi(w_{AL}; w_S) \equiv (w_S - v_{AL}^m) F(v_{AL}^m) \leq \hat{\Pi}_S^*, \quad (6)$$

where $v_{AL}^m = l^{-1}(w_{AL})$ and thus satisfies $v_{AL}^m + h(v_{AL}^m) = w_{AL}$. Therefore:

$$\begin{aligned} \frac{\partial \Psi}{\partial w_{AL}}(w_{AL}; w_S) &= ((w_S - v_{AL}^m) f(v_{AL}^m) - F(v_{AL}^m)) \frac{dv_{AL}^m}{dw_{AL}} \\ &= (w_S - v_{AL}^m - h(v_{AL}^m)) \frac{f(v_{AL}^m)}{1 + h'(v_{AL}^m)} \\ &= (w_S - w_{AL}) \frac{f(v_{AL}^m)}{1 + h'(v_{AL}^m)}. \end{aligned}$$

It follows that, in the range $w_{AL} \geq w_S$, $\Psi(w_{AL}; w_S)$ decreases with w_{AL} (and strictly so for $w_{AL} > w_S$). Thus, condition (5) amounts to $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$, where $\hat{w}_{AL}(w_S, w_L)$ is the unique solution to $\Psi(w_{AL}; w_S) = \hat{\Pi}_S^*$. To show that this solution exists and

²As before, this is equivalent to $w_{AL} - w_L - \hat{r}_A^* = v_{AL}^m - \hat{v}_L^* \geq \hat{v}_S^* - \hat{v}_L^* = \hat{\tau}^* (> 0)$, which implies that multi-stop shoppers are indeed willing to buy A when visiting L . Moreover, this condition also implies $v_{AL}^m > \hat{v}_S^* - \hat{v}_L^* = \hat{\tau}^* (> 0)$.

lies above w_S , note first that Ψ becomes negative for $w_{AL} > l(w_S)$ (since then $v_{AL}^m = l^{-1}(w_{AL}) > w_S$), and that for $w_{AL} = w_S$, $\Psi(w_{AL}; w_S) = (w_{AL} - v_{AL}^m) F(v_{AL}^m) = \Pi_{AL}^m = \max_v (w_{AL} - v) F(v)$; since $w_{AL} > w_S - w_L + \hat{r}_L^*$, this exceeds $\hat{\Pi}_S^* = \max_\tau (w_S - w_L + \hat{r}_L^* - \tau) F(\tau)$.

Finally, in the range $w_{AL} > w_S (> w_S - \hat{v}_L^*)$, a simple revealed argument yields:

$$\hat{\tau}^* = \arg \max_v (w_S - \hat{v}_L^* - \tau) F(\tau) < v_{AL}^m = \arg \max_v (w_{AL} - v) F(v).$$

Therefore, (5), which is equivalent to:

$$v_{AL}^m \geq w_S - \frac{h(\hat{\tau}^*) F(\hat{\tau}^*)}{F(v_{AL}^m)}, \quad (7)$$

implies (4). The two conditions (4) and (5) thus boil down to $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$.

It remains to show that $\hat{w}_{AL}(w_S, w_L)$ increases with w_S . Differentiating $\hat{w}_{AL}(w_S, w_L)$ with respect to w_S yields:

$$\frac{\partial \hat{w}_{AL}}{\partial w_S} = \frac{\frac{\partial \Psi}{\partial w_S} - \frac{\partial \hat{\Pi}_S^*}{\partial w_S}}{-\frac{\partial \Psi}{\partial w_{AL}}},$$

where the denominator is positive in the relevant range, whereas the numerator is equal to:

$$\begin{aligned} \frac{\partial \Psi}{\partial w_S} - \frac{\partial \hat{\Pi}_S^*}{\partial w_S} &= F(v_{AL}^m) - \frac{d(h(\hat{\tau}^*) F(\hat{\tau}^*))}{d\hat{\tau}^*} \frac{\partial \hat{\tau}^*}{\partial w_S} \\ &= F(v_{AL}^m) - \frac{1 + h'(\hat{\tau}^*)}{1 + 2h'(\hat{\tau}^*)} F(\hat{\tau}^*), \end{aligned}$$

which is positive since $v_{AL}^m > \hat{\tau}^*$.

We now show that no other equilibrium exists when $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$. First, we turn to regime S , in which one-stop shoppers patronize S ($v_{AL} < v_S$), and show that there is no such equilibrium when $w_{AL} > w_S$. In this regime, L faces only a demand $F(v_A)$ for A from multi-stop shoppers, where $v_A = w_A - r_A$, and thus makes a profit equal to $r_A F(v_A)$. L could however deviate and attract one-stop shoppers by reducing r_L (keeping r_A and thus v_A constant) so as to offer $v'_{AL} = v_S$ (or slightly above v_S). Doing so would not change the number of multi-stop shoppers, since $\tau' = v_S - v'_L = v'_{AL} - v'_L = v'_A = v_A$, and L would obtain the same margin, r_A , from those consumers. But it would now attract one-stop shoppers (those for which $v_A \leq s \leq v_{AL} = v_S$), from which L could earn a total margin $r'_{AL} = w_{AL} - v'_{AL} = w_{AL} - v_S = w_{AL} - w_S + r_S$. Since any candidate equilibrium requires $r_S \geq 0$, the deviation would be profitable when $w_{AL} > w_S$.

Second, consider the boundary between the two regimes, in which one-stop shoppers are indifferent between visiting L or S ($v_{AL} = v_S$). Note that there must exist some

active consumers, since either retailer can profitably attract consumers by charging a small positive margin; therefore, we must have $v_{AL} = v_S > 0$. Suppose that all active consumers are multi-stop shoppers (in which case L only sells A while S sells B_S to all consumers), which requires $v_{AL} = v_S \leq \tau$. Applying the same logic as in the beginning of Appendix A, we can without loss of generality focus on the case $v_{AL} = v_S = \tau$. It is then profitable for L to transform some multi-stop shoppers into one-stop shoppers, by reducing its margin on B_L to $r'_L = w_L - \varepsilon > 0$ and increasing r_A by ε , so as to keep v_{AL} constant: doing so does not affect the total number of active consumers, but transforms those whose shopping cost lies between $\tau' = v_S - v'_L = \tau - \varepsilon$ and τ into one-stop shoppers. While L obtains the same margin on them (since $r'_{AL} = r_{AL}$), it now obtains a higher margin $r'_A > r_A$ on the remaining multi-stop shoppers.

Therefore, some consumers must visit a single store, and by assumption must be indifferent between visiting either store ($v_{AL} = v_S$). Suppose now some one-stop shoppers visit S . Since S can avoid making losses, we must then have $r_S \geq 0$. But then, $v_{AL} = v_S$ implies $r_{AL} = r_S + w_{AL} - w_S > 0$ and, thus, it would be profitable for L to reduce r_{AL} slightly, so as to attract all one-stop shoppers. Therefore, all one-stop shoppers must go to L if $r_{AL} > 0$. Conversely, we must have $r_S \leq 0$, otherwise S would benefit from slightly reducing its margin so as to attract all one-stop shoppers. Therefore, in any candidate equilibrium such that $v_{AL} = v_S > 0$, either:

- There are some multi-stop shoppers (i.e. $\tau > 0$) and thus $r_S = 0$; but then, slightly increasing r_S would allow S to keep attracting some multi-stop shoppers and obtain a positive profit, a contradiction.
- Or, all consumers buy both products from L , which requires $r_L \leq r_S - (w_S - w_L) \leq -(w_S - w_L) < 0$. But then, increasing r_L to $r'_L = r_S - (w_S - w_L) + \varepsilon$ and reducing r_A by the same amount (so as to keep r_{AL} constant) would lead those consumers with $s < \tau' = \varepsilon$ to buy B_S from S , allowing L to avoid granting them the subsidy r_L .

It follows that there is no equilibrium such that $v_{AL} = v_S$.

Finally, loss leading (in which L not only offers, but actually *sells* below cost) can only arise when L sells to one-stop shoppers, which thus requires $v_{AL} \geq v_S$. But this cannot be an equilibrium when $w_{AL} < \hat{w}_{AL}(w_S, w_L)$, since: (i) in the range $v_{AL} > v_S$, the

only such candidate is the above described loss-leading outcome, which requires $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$; and (ii) as just discussed, no equilibrium exists in the boundary case $v_{AL} = v_S$.

C Proof of Proposition 3

Stackelberg leadership. Suppose that L benefits from a first-mover advantage: it sets its prices first, and then, having observed these prices, S sets its own price. Retail prices are often strategic complements, and it is indeed the case here for S in the B segment: as noted before, S 's best response, $\hat{r}_S(r_L)$, increases with r_L . Thus, in the case of “normal competition” in market B , L would exploit its first-mover advantage by *increasing* its price for B_L , so as to encourage its rival to increase its own price and relax the competitive pressure. In contrast, here L has an incentive to *decrease* r_L even further. This leads S to decrease its own price, which allows L to raise the price for A . To see this, note that L 's Stackelberg profit from a loss-leading strategy can be written as:

$$\Pi_L^S(r_L) = \Pi_{AL}^m - r_L F(\hat{\tau}(r_L)) = \Pi_{AL}^m - r_L F(w_S - w_L + r_L - \hat{r}_S(r_L)).$$

Denoting by r_L^S the optimal Stackelberg margin and using $\hat{r}_S(\hat{r}_L^*) = \hat{r}_S^*$, where $\hat{r}_L^*$ and $\hat{r}_S^*$ are the equilibrium margins when L moves simultaneously with S , we have:

$$\begin{aligned} -r_L^S F(w_S - w_L + r_L^S - \hat{r}_S(r_L^S)) &\geq -\hat{r}_L^* F(w_S - w_L + \hat{r}_L^* - \hat{r}_S(\hat{r}_L^*)) \\ &\geq -r_L^S F(w_S - w_L + r_L^S - \hat{r}_S^*), \end{aligned}$$

where the second inequality stems from the fact that $\hat{r}_L^*$ constitutes L 's best response to r_S^* . Since $-r_L^S > 0$ and $F(\cdot)$ and $\hat{r}_S(\cdot)$ are both increasing, this in turn implies $r_L^S \leq \hat{r}_L^*$. This inequality is moreover strict, since (using $\hat{\tau}(\hat{r}_L^*) = \hat{\tau}^*$):

$$(\Pi_L^S)'(\hat{r}_L^*) = -F(\hat{\tau}^*) - \hat{r}_L^* f(\hat{\tau}^*)(1 - \hat{r}_S'(\hat{r}_L^*)) = \hat{r}_L^* f(\hat{\tau}^*) \hat{r}_S'(\hat{r}_L^*) < 0.$$

Thus, L sells the competitive product B_L further below-cost, compared with what it would do in the absence of a first-mover advantage: $r_L^S < \hat{r}_L^*$.

Entry accommodation. Suppose now that the presence of S is uncertain. To capture this possibility, assume that S incurs a fixed cost for entering the market, γ , which is ex ante distributed according to a cumulative distribution function $F_\gamma(\cdot)$, and consider the following timing:

- In stage 1, L chooses its prices.
- In stage 2, the entry cost is realized, and S chooses whether to enter; if it enters, it then sets its own price.

If entry were certain, maximizing its Stackelberg profit would lead L to adopt r_L^S . But now, S enters only when its best response profit, $\hat{\Pi}_S(r_L)$, exceeds the realized cost γ , which occurs with probability $\rho(r_L) \equiv F_\gamma(\hat{\Pi}_S(r_L))$. L 's ex ante profit is therefore equal to

$$\hat{\Pi}_L^S(r_L) = \Pi_{AL}^m + \rho(r_L) \Pi_L^S(r_L).$$

The optimal margin, $\hat{r}_L^S$, thus satisfies

$$\rho(\hat{r}_L^S) \Pi_L^S(\hat{r}_L^S) \geq \rho(r_L^S) \Pi_L^S(r_L^S) \geq \rho(r_L^S) \Pi_L^S(\hat{r}_L^S),$$

which implies

$$\rho(\hat{r}_L^S) \geq \rho(r_L^S).$$

Since F_γ and $\hat{\Pi}_S$ are both increasing in r_L , so is ρ and thus $\hat{r}_L^S \geq r_L^S$. This inequality is moreover strict, since

$$\left(\hat{\Pi}_L^S\right)'(r_L^S) = \rho'(r_L^S) \Pi_L^S(r_L^S) + \rho(r_L^S) (\Pi_L^S)'(r_L^S) = \rho'(r_L^S) \Pi_L^S(r_L^S) > 0.$$

Therefore, when L 's comparative advantage leads it to adopt a loss-leading strategy, it limits the subsidy on B so as to increase the likelihood of entry: $\hat{r}_L^S > r_L^S$.

D Proof of Proposition 4

In the equilibrium where L attracts one-stop shoppers in the absence of a ban, L must offer a higher value than S : $v_{AL} = v_{AL}^m > \hat{v}_S^* = w_S - \hat{r}_S^*$, and S must moreover not be tempted to deviate and attract one-stop shoppers, which boils down to $\hat{\Pi}_S^* = h(\hat{\tau}^*) F(\hat{\tau}^*) \geq \hat{\Pi}_S^d = (w_S - v_{AL}^m) F(v_{AL}^m)$. If L keeps attracting one-stop shoppers (i.e., $v_{AL} > v_S$) when loss leading is banned, then the unique candidate equilibrium is $r_{AL} = r_{AL}^m$, $r_L = 0$ and $r_S^b = h(\tau^*)$, where $\tau^* = l^{-1}(w_S - w_L)$.

We show now this candidate equilibrium prevails when loss-leading would arise if below-cost pricing were allowed. Note that, since S increases its price (i.e., $r_S^b = h(\tau^*) > \hat{r}_S^* = h(\hat{\tau}^*)$), it offers less value ($v_S = v_S^b \equiv w_S - r_S^b < \hat{v}_S^*$), and thus L indeed attracts

one-stop shoppers: $v_{AL} = v_{AL}^m > (\hat{v}_S^* >) v_S^b$. Furthermore, as S must again offer at least $v_S = v_{AL}$ to attract one-stop shoppers, it still cannot obtain more than $\hat{\Pi}_S^d$ by deviating in this way. Therefore, since S now obtains more profit ($\Pi_S^* \equiv h(\tau^*) F(\tau^*) > \hat{\Pi}_S^* = h(\hat{\tau}^*) F(\hat{\tau}^*)$), it is less tempted to deviate: $\Pi_S^* > (\hat{\Pi}_S^* >) \hat{\Pi}_S^d$. It follows that the conditions for sustaining the above equilibrium are less stringent than that for the loss-leading equilibrium.

E Product differentiation in the competitive market

We show that our main insights apply when consumers vary in their relative preferences over B_L and B_S . For example, suppose B_L is a “standard” variety generating a homogeneous utility u_L , whereas B_S , a better variety supplied by specialist stores, yields a utility $u_S + \theta q$; $\theta \in [0, 1]$ thus characterizes the consumer preference for quality and is distributed according to a c.d.f $\Phi(\cdot)$ with density function $\phi(\cdot)$, whereas q measures the degree of consumer heterogeneity. For the sake of exposition, we consider here the case where B_S is supplied by a competitive fringe and assume that:

- S provides better value for at least some quality-oriented consumers: $w_S + q > w_L$; we allow however for $w_L > w_S$, in which case L offers higher value than S for less quality-oriented consumers.
- all one-stop shoppers favor L : $v_{AL} \geq w_S + q$.

As before, consumers are willing to patronize L if $s \leq v_{AL}$, and prefer multi-stop shopping to one-stop shopping if

$$s \leq w_S + \theta q - v_L = \tau + \theta q,$$

where $\tau = w_S - w_L + r_L$. L thus earns a profit

$$\Pi_L = r_{AL} D_{AL}(r_{AL}) - r_L D_{AS}(r_L)$$

where $D_{AL}(r_{AL}) = F(v_{AL})$ and $D_{AS}(r_L) = \int_0^1 F(\tau + \theta q) \phi(\theta) d\theta$. The loss leading logic of the baseline model applies again here: since $v_{AL} = w_{AL} - r_{AL}$ and $\tau = w_S - w_L + r_L$, L 's profit is separable in r_{AL} and r_L , and still charges the price on B_L below-cost.

While we presented this example in terms of “vertical” quality differentiation, the same analysis applies to “horizontal” differentiation, with utilities for B_L and B_S of the form $u_L + (1 - \theta)q$ and $u_S + \theta q$; the only difference is that, since consumers have now heterogeneous valuations for B_L as well, the above demands become:

$$D_{AL}(r_{AL}) = \int_0^1 F(v_{AL} + (1 - \theta)q)\phi(\theta)d\theta, D_{AS}(r_L) = \int_0^1 F(\tau + (2\theta - 1)q)\phi(\theta)d\theta.$$

F Proof of Proposition 6

F.1 Local Monopolies with heterogeneous preferences on A

We show that introducing an elastic demand in market A does not preclude the large retailer from adopting a loss-leading strategy, so as to extract additional surplus from multi-stop shoppers. We focus on the large retailer’s best response to the strategies of the smaller retailer(s); thus, what follows applies equally to the case of a strategic rival and that of a competitive fringe.

L ’s profit can be written as (see Figure 1):

$$\Pi_L = r_{AL}D_{AL} + r_AD_{AS} = r_{AL} \int_{\tau}^{v_{AL}} G(x_{AL}(s)) f(s) ds + r_A \int_0^{\tau} G(x_A(s)) f(s) ds.$$

To characterize the equilibrium values of r_L and r_{AL} , consider first a modification of r_A by dr , adjusting r_L by $-dr$ so as to keep r_{AL} constant. Such a change does not affect the behavior of one-stop shoppers (it has no impact on v_{AL} and $x_{AL}(s)$), but (see Figure 2):

- It affects multi-shop shoppers: for $s < \tau$, the marginal consumer indifferent between buying A from L or patronizing S only becomes $x = x_A(s) - dr$; therefore, L loses $g(x_A(s))dr$ consumers, on which it no longer earns the margin r_A . L however increases its margin by dr on the mass $G(x_A(s))$ of consumers that buy A . Thus, the overall impact of such an adjustment on multi-stop shoppers is equal to

$$\int_0^{\tau} [G(x_A(s)) - r_A g(x_A(s))] f(s) ds dr.$$

- In addition, it alters the choice between one-stop and multi-stop shopping: those consumers for which $s \in [\tau - dr, \tau]$ and $x \leq x_A(s)$ turn to one-stop shopping and now buy B as well as A from L_1 , which (noting that $x_A(\tau) = \hat{x}$) brings a gain $r_L G(\hat{x}) f(\tau) dr$.

These effects must cancel out in equilibrium, which yields

$$\int_0^\tau [r_A - k(x_A(s))] g(x_A(s)) f(s) ds = r_L G(\hat{x}) f(\tau).$$

Likewise, adjusting slightly r_{AL} by dr , keeping r_A constant (and thus changing r_L by dr as well) does not affect the behavior of multi-stop shoppers (it has no impact on v_{AS} and $x_A(s)$), but:

- It affects one-stop shoppers: for $s > \tau$, the marginal shopper becomes $x = x_{AL}(s) - dr$, and the resulting change in profit is

$$\int_\tau^{v_{AL}} [G(x_{AL}(s)) - r_{AL}g(x_{AL}(s))] f(s) ds dr.$$

- In addition, those consumers for which $s \in [\tau, \tau + dr]$ and $x \leq x_{AL}(s)$ become multi-stop shoppers and stop buying B from L , which (noting that $x_{AL}(\tau) = \hat{x}$) brings a net effect $-r_L G(\hat{x}) f(\tau) dr$.

In equilibrium, these effects must again cancel each other, which yields

$$\int_\tau^{v_{AL}} [r_{AL} - k(x_{AL}(s))] g(x_{AL}(s)) f(s) ds = -r_L G(\hat{x}) f(\tau).$$

Therefore, if in equilibrium r_L were non-negative, we would have

$$\int_0^\tau [r_A - k(x_A(s))] g(x_A(s)) f(s) ds \geq 0 \geq \int_\tau^{v_{AL}} [r_{AL} - k(x_{AL}(s))] g(x_{AL}(s)) f(s) ds,$$

that is, r_A would exceed a weighted average of $k(x_A(s))$ for $s \in [0, \tau]$, whereas r_{AL} would be lower than a weighted average of $k(x_{AL}(s))$ for $s \in [\tau, v_{AL}]$. But since $k(x_A(s))$ and $k(x_{AL}(s))$ decrease as s increases ($k(\cdot)$ increases by assumption, and both $x_A(s)$ and $x_{AL}(s)$ decrease by construction), this would imply $r_A > r_{AL}$, a contradiction. Therefore, in equilibrium, $r_L < 0$.

If the shopping cost s is distributed over some interval $[0, \bar{s}]$, where $\bar{s} > \tau$ to ensure that large retailers still attract some one-stop shoppers, the first-order conditions become:

$$\begin{aligned} \int_0^\tau [r_A - k(x_A(s))] g(x_A(s)) f(s) ds &= r_L G(\hat{x}) f(\tau), \\ \int_\tau^{\min\{v_{AL}, \bar{s}\}} [r_{AL} - k(x_{AL}(s))] g(x_{AL}(s)) f(s) ds &= -r_L G(\hat{x}) f(\tau); \end{aligned}$$

it thus suffices to replace v_{AL} with $\min\{v_{AL}, \bar{s}\}$ in the above reasoning.

F.2 Imperfect competition among large retailers

Suppose now that two large retailers, L_1 and L_2 , facing the same costs in both markets and offering the same variety B_L , are differentiated in market A : they respectively offer A_1 and A_2 , located at the two ends of a Hotelling line of length X ; a consumer with preference x thus obtains a utility $u_A - x - p_{A_1} = w_A - r_{A_1} - x$ from buying A_1 and a utility $w_A - r_{A_2} - (X - x)$ from buying A_2 . We will restrict attention to symmetric distributions (that is, the density $g(\cdot)$ satisfies $g(x) = g(X - x)$) and will focus on (symmetric) equilibria in which: (i) the large retailers compete against each other as well as against their smaller rivals; (ii) small retailers attract some multi-stop shoppers by offering a value v_S that exceeds the value v_L offered by large retailers on the B market; and (iii) large retailers attract some one-stop shoppers by offering them a value v_{AL} that exceeds v_S , as well as the value v_A that they offer on the A market alone.

Large retailers may compete against each other for one-stop and/or for multi-stop shoppers. In the former case, in a symmetric equilibrium (of the form $r_{A_1 L_1} = r_{A_2 L_2} = r_{AL}$ and $r_{L_1} = r_{L_2} = r_L$) some consumers (with $x = X/2$) are indifferent between buying both goods from either L_1 or L_2 , and prefer doing so to patronizing S only; this implies (using $x = X/2$, and dropping the subscripts 1 and 2 for ease of exposition):

$$\hat{v}_{AL} \equiv v_{AL} - \frac{X}{2} \geq v_S,$$

which is equivalent to

$$\hat{v}_A \equiv v_A - \frac{X}{2} \geq \tau = v_S - v_L.$$

Therefore, consumers with preference $x = X/2$ and shopping cost $s < \tau$, who thus prefer multi-stop shopping (that is, buying B_S from S and A from either L_1 or L_2) to visiting L_1 or L_2 only, also prefer multi-stop shopping to patronizing S only (since $s < \tau$ then implies $s < \hat{v}_A$). In other words, if large retailers compete for one-stop shoppers, they will also compete for multi-stop shoppers. This observation allows us to classify the (symmetric) candidate equilibria into two types:

- Type M : large retailers compete only for *multi-stop* shoppers (see Figure 3a);
- Type O : large retailers compete for *one-stop* shoppers as well as for multi-stop

shoppers (see Figure 3b).

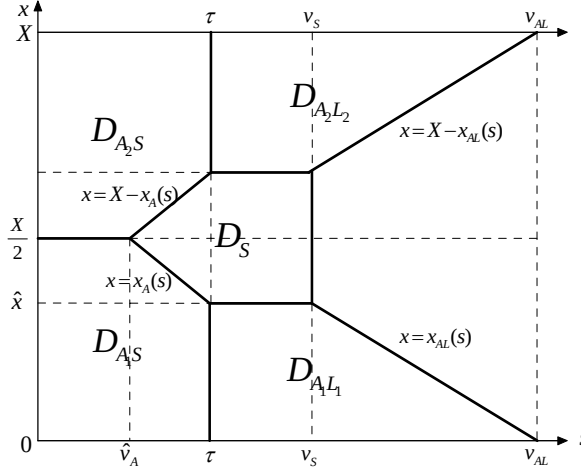


Figure 3a

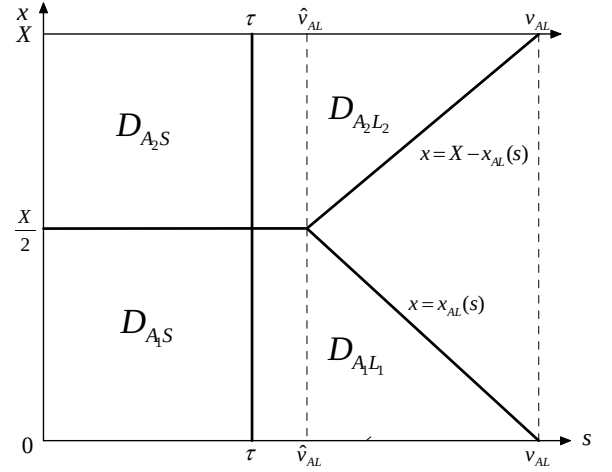


Figure 3b

In the first type of equilibria (Figure 3a), for $x = X/2$ some consumers with low shopping costs are indifferent between assortments A_1S and A_2S , and prefer those assortments to any other option, whereas consumers with higher shopping costs patronize S only; the relevant threshold for the shopping cost satisfies

$$\hat{v}_A + v_S - 2s = v_S - s,$$

that is, $s = \hat{v}_A$. Consumers with $s < \hat{v}_A$ thus buy B from S and A from either L_1 or L_2 (depending on whether x is smaller or larger than $X/2$). Conversely, consumers whose shopping costs exceed v_{AL} do not shop. As for consumers whose shopping costs lie between $\hat{v}_A$ and v_{AL} :

- when $s < \tau$, consumers still buy B_S from S ; they also buy A from L_1 if $x < x_A(s) = v_A - s$, or from L_2 if $x > X - x_A(s)$;
- when $s > \tau$:
 - if $x < x_{AL}(s)$, consumers buy both goods from L_1 ;
 - if $x > X - x_{AL}(s)$, consumers buy both goods from L_2 ;
 - if $x_{AL}(s) < x < X - x_{AL}(s)$, consumers patronize S if $s < v_S$, and buy nothing otherwise.

In the second type of equilibria (Figure 3b), all consumers with a shopping cost $s < \tau$ buy B_S from S and A from either L_1 (if $x < X/2$) or L_2 (if $x > X/2$), while consumers with $s > v_{AL}$ buy nothing. For consumers with $\tau < s < v_{AL}$, then:

- if $s < \hat{v}_{AL}$, consumers will buy both goods from either L_1 (if $x < X/2$) or L_2 (if $x > X/2$);
- if $\hat{v}_{AL} < s < v_{AL}$, consumers will buy both goods from L_1 if $x < x_{AL}(s)$ or from L_2 if $x > X - x_{AL}(s)$, and buy nothing otherwise.

A similar description applies when the shopping cost s is bounded, truncating as necessary the interval for s .

We show now loss leading is still used as an exploitative device. Consider first (symmetric) equilibria of type M , in which large retailers compete only for multi-stop shoppers. In the absence of any bound on shopping costs, the demands for assortments A_1L_1 and A_1S in such equilibrium, where $r_{A_1L_1} = r_{A_2L_2} = r_{AL}$ and $r_{L_1} = r_{L_2} = r_L$ (and thus $r_{A_1} = r_{A_2} = r_A$), can be expressed as:

$$D_{AS} = \int_0^\tau G(\hat{x}_A(s)) f(s) ds \text{ and } D_{AL} = \int_\tau^{v_{AL}} G(x_{AL}(s)) f(s) ds,$$

where as before $\tau = v_S - v_L$ and $x_{AL}(s) = v_{AL} - \max\{s, v_S\}$, and $\hat{x}_A(s) \equiv v_A - \max\{s, \hat{v}_A\} = \min\{X/2, x_A(s) = v_A - s\}$.

Applying the same approach as above, starting from a candidate symmetric equilibrium, consider first a small change dr in r_{A_1} , adjusting r_{L_1} by $-dr$ so as to keep $r_{A_1L_1}$ constant:

- For $s < \hat{v}_A$, the marginal consumer who is indifferent between buying A from L_1 or L_2 is such that:

$$w_A - (r_A + dr) - x = w_A - r_A - (X - x),$$

or:

$$x = \frac{X}{2} - \frac{dr}{2}.$$

The overall impact on L_1 's profit is thus:

$$\int_0^{\hat{v}_A} [G(\hat{x}_A(s)) - \frac{1}{2}r_{AG}(\hat{x}_A(s))] f(s) ds dr.$$

- For $\hat{v}_A < s < \tau$, the marginal consumer indifferent between buying A from L_1 or patronizing S becomes $x = x_A(s) - dr$, and the resulting impact on profit is:

$$\int_{\hat{v}_A}^{\tau} [G(\hat{x}_A(s)) - r_A g(\hat{x}_A(s))] f(s) ds dr.$$

- In addition, those consumers for which $s \in [\tau - dr, \tau]$ and $x \leq \hat{x}_A(s)$ turn to one-stop shopping and now buy B as well as A from L_1 , which brings an additional profit $r_L G(\hat{x}) f(\tau) dr$.

Therefore, in equilibrium, we must have:

$$\int_0^{\tau} [r_A - \eta_A(s)] \hat{g}(\hat{x}_A(s)) f(s) ds = r_L G(\hat{x}) f(\tau), \quad (8)$$

where (using $\hat{x}_A(s) = X/2$ for $s \leq \hat{v}_A$):

$$\eta_A(s) \equiv \begin{cases} 2k(\hat{x}_A(s)) & \text{for } s < \hat{v}_A \\ k(\hat{x}_A(s)) & \text{for } s > \hat{v}_A \end{cases} \quad \text{and } \hat{g}(x) \equiv \begin{cases} \frac{g(X/2)}{2} & \text{for } x = X/2 \\ g(x) & \text{for } x < X/2 \end{cases}.$$

Consider now a small change dr in $r_{A_1 L_1}$, keeping r_{A_1} constant (and thus adjusting r_{L_1} by dr as well):

- for $s > \tau$, the marginal (one-stop) shopper becomes $x = x_{AL}(s) - dr$ and the impact on the profit is

$$\int_{\tau}^{v_{AL}} [G(x_{AL}(s)) - r_{AL} g(x_{AL}(s))] f(s) ds dr;$$

- in addition, those consumers for which $s \in [\tau, \tau + dr]$ and $x \leq x_{AL}(s)$ become multi-stop shoppers and stop buying B from L_1 , which brings a net loss $-r_L G(\hat{x}) f(\tau) dr$.

In equilibrium, we must therefore have

$$\int_{\tau}^{v_{AL}} [r_{AL} - \eta_{AL}(s)] g(x_{AL}(s)) f(s) ds = -r_L G(\hat{x}) f(\tau), \quad (9)$$

where $\eta_{AL}(s) \equiv k(x_{AL}(s))$.

Thus, if r_L were non-negative, the two conditions (8) and (9) would imply

$$\int_0^{\tau} [r_A - \eta_A(s)] \hat{g}(\hat{x}_A(s)) f(s) ds \geq 0 \geq \int_{\tau}^{v_{AL}} [r_{AL} - \eta_{AL}(s)] g(x_{AL}(s)) f(s) ds,$$

where η_A and η_{AL} decrease as s increases, and coincide for $s = \tau$; this, in turn, would imply $r_A > r_{AL}$, a contradiction. A similar argument applies when the shopping cost s is distributed over some interval $[0, \bar{s}]$.

The same approach can be used for (symmetric) equilibria of type O , in which large retailers compete as well for one-stop shoppers. In the absence of any bound on shopping costs, the demands for assortments A_1L_1 and A_1S in such equilibrium can be expressed as

$$D_{AS} = \int_0^\tau G\left(\frac{X}{2}\right) f(s) ds \text{ and } D_{AL} = \int_\tau^{v_{AL}} G(\hat{x}_{AL}(s)) f(s) ds,$$

where $\hat{x}_{AL}(s) \equiv v_A - \max\{s, \hat{v}_{AL}\} = \min\{X/2, x_{AL}(s) = v_{AL} - s\}$.

Following a small change dr in r_{A_1} , adjusting r_{L_1} by $-dr$ so as to keep $r_{A_1L_1}$ constant, we have:

- for $s < \tau$, the marginal consumer indifferent between buying A from L_1 or L_2 becomes $X/2 - dr/2$;
- in addition, those consumers for which $s \in [\tau - dr, \tau]$ and $x \leq \hat{x}_A(s)$ become one-stop shoppers.

Therefore, in equilibrium we must have

$$\int_0^\tau [r_A - \hat{\eta}_A] \hat{g}\left(\frac{X}{2}\right) f(s) ds = r_L G\left(\frac{X}{2}\right) f(\tau),$$

where $\hat{\eta}_A \equiv 2k(X/2)$ and $\hat{g}(X/2) = g(X/2)/2$.

Likewise, following a small change dr in $r_{A_1L_1}$, keeping r_{A_1} constant (and thus changing r_{L_1} by dr as well), we have:

- for $\tau < s < \hat{v}_{AL}$, the marginal (one-stop) shopper becomes $x = x_{AL}(s) - dr/2$;
- for $\hat{v}_{AL} < s < v_{AL}$, the marginal (one-stop) shopper becomes $x = x_{AL}(s) - dr$;
- in addition, those consumers for which $s \in [\tau, \tau + dr]$ and $x \leq \hat{x}_{AL}(s)$ become multi-stop shoppers: they stop buying B from L_1 .

We must therefore have

$$\int_\tau^{v_{AL}} [r_{AL} - \hat{\eta}_{AL}(s)] \hat{g}(\hat{x}_{AL}(s)) f(s) ds = -r_L G(\hat{x}) f(\tau),$$

where

$$\hat{\eta}_{AL}(s) \equiv \begin{cases} 2k(\hat{x}_{AL}(s)) & \text{for } s < \hat{v}_{AL} \\ k(\hat{x}_{AL}(s)) & \text{for } s > \hat{v}_{AL} \end{cases},$$

and $\hat{g}(x)$ is defined above with $\hat{x}_{AL}(s) = X/2$ for $\tau \leq s \leq \hat{v}_{AL}$. Thus, if r_L were non-negative, the above two conditions would imply:

$$\int_0^\tau [r_A - \hat{\eta}_A] \hat{g}\left(\frac{X}{2}\right) f(s) ds \geq 0 \geq \int_\tau^{\hat{v}_{AL}} [r_{AL} - \hat{\eta}_{AL}(s)] \hat{g}(\hat{x}_{AL}(s)) f(s) ds,$$

and a contradiction follows, since $\hat{x}_{AL}(s) \leq X/2$, with a strict inequality for $s > \hat{v}_{AL}$, and thus $\hat{\eta}_{AL}(s) \leq 2k(\hat{x}_{AL}(s)) \leq \hat{\eta}_A$, with again a strict inequality for $s > \hat{v}_{AL}$. A similar argument applies again when the shopping cost s is distributed over some interval $[0, \bar{s}]$. If instead $\bar{s} < \hat{v}_{AL}$, then all consumers buy both goods, in which case $\hat{\eta}_{AL}(\cdot) = \hat{\eta}_A$ and $\hat{g}(\hat{x}_{AL}(s)) = \hat{g}(X/2)$, and $r_L = 0$.