

Social Preferences and Strategic Uncertainty: an Experiment on Markets and Contracts

Appendix A: The Mechanism Design Problem

NOT FOR PUBLICATION

1 Two mechanism design problems

1.1 Production technology

Technology closely follows Winter's (2004) model of moral hazard in teams. Let $G(b)$ define the game-form associated with a given benefit profile, $b = (b_1, b_2)$. The rules of the game-form are the following. Each agent $i = 1, 2$, has to decide, simultaneously and independently, whether to make a costly effort. We denote by $\delta_i \in \{0, 1\}$ agent i 's effort decision, where $\delta_i = 1(0)$ if agent i does (does not) make effort. Let also $\delta = (\delta_1, \delta_2) \in \{0, 1\}^2$ denote the agents' action profile. The cost of effort c is assumed to be constant across agents. Team activity results in either success or failure. Let $P(\delta)$ define production as the probability of success as a function of the number of agents in the team who have put effort:

$$P(\delta) = \begin{cases} 0 & \text{if } \delta_1 + \delta_2 = 0, \\ \gamma & \text{if } \delta_1 + \delta_2 = 1, \\ 1 & \text{if } \delta_1 + \delta_2 = 2, \end{cases} \quad (1)$$

with $\gamma \in (0, \frac{1}{2})$.¹

If the project fails, then all (principal and agents) receive a payoff of zero. If the project succeeds, then agent i receives a *benefit*, $b_i^k > 0$. Agent i 's expected monetary profit associated to contract k is given by

$$\pi_i^k(\delta) = P(\delta)b_i^k - \delta_i c. \quad (2)$$

The expected monetary payoff for the principal is the difference between expected revenues, for a given (randomly generated) value for the project $V \sim U[A, B]$, and expected costs:

$$\pi_0^k(\delta) = P(\delta)(V - b_1^k - b_2^k).$$

¹This is how Winter (2004) models moral hazard: agents' effort affect the overall probability of success of the project. However, since risk neutrality is assumed on agents' behalf, the fact that technology follows a random -as opposed to deterministic, as in our design- process has no impact in the solution of the mechanism design problem.

Assume a principal who wishes to design a mechanism that induces all agents to exert effort in (some) equilibrium of the game induced by $G(b)$, which we denote by $\Gamma(b)$. A mechanism is an allocation of benefits in case of success, i.e., a vector b that satisfies this property at the minimal cost for the principal. Following Winter (2004), the principal may consider mechanisms that *strongly* or *weakly* implement the desired solution, depending of how concerned he is about equilibrium multiplicity. More precisely:

Definition 1 (*sting contracts*) *The contract b is strongly effort-inducing (sting) if all Nash Equilibria (NE) of $\Gamma(b)$ entail effort by all agents with minimal benefit distribution, $b_1 + b_2$.*

Definition 2 (*wing contracts*) *The contract b is weakly effort-inducing (wing) if there exists at least one NE of $\Gamma(b)$ such that $\delta = (1, 1)$, with minimal benefit distribution.*

2 The solutions

By analogy with our experimental conditions (and without loss of generality), we assume $b_1 \geq b_2$.

There are four relevant subsets of parameters for the utility function, which we now describe. All these specifications do not consider reciprocal motives (i. e., it is always assumed $\theta_i = 0$), and, in this sense, define purely “distributional” preferences.

$$\text{Egoistic Preferences (EP): } \alpha_i = \beta_i = 0. \quad (3)$$

$$\text{Inequality Averse Preferences (IAP): } 0 \leq \beta_i < 1, \alpha_i \geq \beta_i. \quad (4)$$

$$\text{Status Seeking Preferences (SSP): } \alpha_i \in [0, 1), \beta_i \in (-1, 0], |\alpha_i| \geq |\beta_i| \quad (5)$$

$$\text{Efficiency Seeking Preferences (ESP): } \alpha_i \in (-\frac{1}{2}, 0], \beta_i \in [0, \frac{1}{2}), |\beta_i| \geq |\alpha_i| \quad (6)$$

In what follows, we shall assume that both agents hold either EP (as in Winter, 2004), or IAP, SSP and ESP, respectively. We allow for heterogeneous preferences, provided they belong to the same preference class.

2.1 Solution of the mechanism design problem under the *wing* program

In the case of *wing*, the search of the optimal mechanism corresponds to the following linear program:

$$b^* \equiv (b_1^*, b_2^*) \in \arg \min_{\{b_1, b_2\}} [b_1 + b_2] \text{ sub} \quad (7)$$

$$u_1(1, 1) \geq u_1(0, 1) \quad (8)$$

$$u_2(1, 1) \geq u_2(1, 0) \quad (9)$$

$$b_1 \geq b_2 \geq 0 \quad (10)$$

Assumption (10) is wlog. To solve the problem (7)-(10), we begin by partitioning the benefit space $B = \{(b_1, b_2) \in \mathbb{R}_+^2, b_1 \geq b_2\}$ in two regions, which specify the payoff ranking of each strategy profiles in $G(b)$. This partition is relevant for our problem, since it determines whether in (1,0) - player 1 exerts effort and player 2 does not - whether it is player 1 or 2 the one who experiences envy (guilt):

$$\begin{aligned} R_1 &= \left\{ b \in B : b_2 \leq b_1 - \frac{c}{\gamma} \right\}; \\ R_2 &= \left\{ b \in B : b_1 - \frac{c}{\gamma} \leq b_2 \leq b_1 \right\}. \end{aligned}$$

Let $g^1(b_1) = b_1$ ($g^2(b_1) = b_1 - \frac{c}{\gamma}$) define the two linear constraints upon which our partition is built. The strategy proof is as follows. We shall solve the linear program (7)-(10) in the two regions independently (since, within each region, social utility parameters are constant for each agent and strategy profile), checking which of the two solutions minimizes the overall benefit sum $b_1 + b_2$, and determining the constraints on preferences which determine the identity of the best-paid player 1.

2.1.1 *Wing* under EP

As for the solution of *wing* under EP (i.e. with $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0$), the linear program (7)-(10) simplifies to the following:

$$\min b_1 + b_2$$

subject to:

$$\begin{aligned} b_1 - c &\geq \gamma b_1 \\ b_2 - c &\geq \gamma b_2 \\ b_i &\geq 0; \quad \text{with } i = 1, 2 \end{aligned}$$

In this case, the solution of the problem is problem is trivial:

$$b_1^* = b_2^* = \frac{c}{1-\gamma}.$$

2.1.2 *Wing* under IAP

As for the solution of *wing* under IAP, we need to add to the basic linear program (7)-(10) the IAP constraint.

Proposition 3 (winiIAP) *The optimal wing mechanism under IAP is as follows:*

$$b_1^* = \left(\frac{c(-1+\alpha_2(-1+\beta_1)+2\beta_1+\gamma(-1+2\beta_1)(-1+\beta_2)-\beta_1\beta_2)}{\frac{(-1+\gamma)(1+\alpha_2-\beta_1+\gamma(-1+\beta_1+\beta_2))}{\frac{c(-1+\beta_1)(-1+\alpha_2-\beta_2+\gamma(-1+2\beta_2))}{(-1+\gamma)(1+\alpha_2-\beta_1+\gamma(-1+\beta_1+\beta_2))}}}, \right) \text{ if } \beta_1 < \frac{1}{2}; \quad (11)$$

$$b_2^* = \left(\frac{c(1-\beta_1)}{1-\gamma}, \frac{c(1-\beta_1)}{1-\gamma} \right) \text{ if } \beta_1 \geq \frac{1}{2}, \quad (12)$$

with $\beta_1 \leq \beta_2$.

To prove Proposition 3, some preliminary lemmas are required. Let $\hat{b}^k \equiv (\hat{b}_1^k, \hat{b}_2^k)$ define the solution of the linear program (7-10) in R_k .

Lemma 4

$$\hat{b}^1 = \left(\frac{c(1+\alpha_2)}{(1-\gamma)\gamma}, \frac{c(\gamma+\alpha_2)}{(1-\gamma)\gamma} \right) \quad (13)$$

Proof. In R_1 , agent 1's monetary payoff, as determined by $G(b)$, is always higher (i.e. $\pi_1(\delta) \geq \pi_2(\delta)$, $\forall \delta$). This, in turn, implies that constraints (8)-(9) correspond to

$$b_1 \geq f_1^1(b_1) \equiv \frac{c(1-\beta_1)}{(1-\gamma)\beta_1} - \frac{1-\beta_1}{\beta_1}b_1; \quad (14)$$

$$b_2 \geq f_2^1(b_1) \equiv \frac{c}{1-\gamma} + \frac{\alpha_2}{1+\alpha_2}b_1. \quad (15)$$

Let x_i^k define the value of b_1 such that $f_i^k(b_1) = 0$. By the same token, let y_i^k denote the intercept of $f_i^k(b_1)$, i.e. $f_i^k(0)$. Finally, let τ_i^k denote the slope of $f_i^k(b_1)$. We then have $x_1^1 = \frac{c}{1-\gamma}$ and $x_2^1 = -\frac{c(1+\alpha_2)}{(1-\gamma)\alpha_2}$. Also notice that $0 \leq \tau_2^1 = \frac{\alpha_2}{1+\alpha_2} < 1$ and $y_2^1 = \frac{c}{1-\gamma} > 0$. This implies that $f_2^1(b_1)$ and $g^2(b_1)$ intersect in the first quadrant of the $b_1 \times b_2$ space. On the other hand, $f_1^1(b_1)$ is never binding in this case, since $\tau_1^1 = -\frac{1-\beta_1}{\beta_1} < 0$ and $x_1^1 = \frac{c}{1-\gamma} < \frac{c}{\gamma}$ since $\gamma < \frac{1}{2}$. This implies that $b_1 + b_2$ is minimized where $f_2^1(b_1)$ and $g^2(b_1)$ intersect, i.e. when $\hat{b}_1^1 = \frac{c(1+\alpha_2)}{(1-\gamma)\gamma}$ and $\hat{b}_2^1 = \frac{c(\gamma+\alpha_2)}{(1-\gamma)\gamma}$. ■

Lemma 5 In R_2 , the optimal wing contract under IAP is (11) when $\beta_1 < \frac{1}{2}$, and (12) when $\beta_1 \geq \frac{1}{2}$, with $\beta_1 < \beta_2$.

Proof. In the case of R_2 , constraints (8)-(9) correspond to

$$b_1 \geq f_1^2(b_1) \equiv \frac{c(1-\beta_1)}{(1-\gamma)\beta_1} - \frac{1-\beta_1}{\beta_1}b_1; \quad (16)$$

$$b_2 \geq f_2^2(b_1) \equiv \frac{c(1-\beta_2)}{1+\alpha_2-\gamma(1-\beta_2)} + \frac{\alpha_2+\gamma\beta_2}{1+\alpha_2-\gamma(1-\beta_2)}b_1. \quad (17)$$

This implies that $f_1^1(b_1) = f_1^2(b_1)$ (i.e. the Nash equilibrium condition for player 1 remains unchanged in both R_1 and R_2), $\tau_1^2 = -\frac{1-\beta_1}{\beta_1} < 0$ (i.e. $|\tau_1^2| > 1$ if $\beta_1 < \frac{1}{2}$), and $0 \leq \tau_2^2 = \frac{\alpha_2+\gamma\beta_2}{1+\alpha_2-\gamma(1-\beta_2)} < 1$.

We first show that $\beta_1 \leq \beta_2$. Let $\check{\beta} = \min\{\beta_1, \beta_2\}$. If $\beta_1 > \beta_2$, then the optimal solution in R_2 would be $\hat{b}_i^1 = \hat{b}_i^2 = \frac{c(1-\check{\beta})}{1-\gamma}$ (i.e. $\hat{b}_i^1 + \hat{b}_i^2 = 2\frac{c(1-\check{\beta})}{1-\gamma}$). On the other hand, if $\beta_1 \leq \beta_2$, then $\hat{b}_i^1 + \hat{b}_i^2 \leq 2\frac{c(1-\check{\beta})}{1-\gamma}$. More precisely, if $\beta_1 < \frac{1}{2}$, the optimal solution is (11), that is, the intersection between $f_1^2(b_1)$ and $f_2^2(b_1)$; if $\beta_1 \geq \frac{1}{2}$, the solution is (12), that is, the intersection between $f_1^2(b_1)$ and $g^1(b_1)$. ■

We are in the position to prove Proposition 3.

Proof. [Proof of Proposition 1]. To prove the proposition, it is sufficient to show that $\hat{b}_i^1 > \hat{b}_i^2, i = 1, 2$. To see this, remember that $f_1^1(b_1) = f_1^2(b_1)$. Also remember that $f_1^k(b_1)$ is (not) binding for both $k = 1$ and $k = 2$. If x_i^{kl} solves $f_1^k(x) = g^l(x)$, then $x_2^{12} = x_2^{22} = \frac{c(1+\alpha_2)}{\gamma(1-\gamma)}$, which, in turn, implies

$$\begin{aligned}\hat{b}_1^1 &= \frac{c(1+\alpha_2)}{\gamma(1-\gamma)} > x_1^{21} = \frac{c(1-\beta_1)}{1-\gamma} \geq \hat{b}_1^2 \text{ and} \\ \hat{b}_2^1 &= \frac{c(\gamma+\alpha_2)}{\gamma(1-\gamma)} > x_1^{21} = \frac{c(1-\beta_1)}{1-\gamma} \geq \hat{b}_2^2.\end{aligned}$$

■

2.1.3 Wing with SSP

As for the solution of *wing* under SSP, we need to add to the basic linear program (7)-(10) the SSP constraint.

Proposition 6 (winiSSP) *The optimal wing mechanism under SSP is (11), with $\beta_1 \leq \beta_2$.*

Proof. We begin by showing that, as in the case of IAP, the optimal *wing* contract in R_1 is (13). This is because, also in this case, $f_1^1(b_1)$ is not binding, since $\tau_1^1 = -\frac{1-\beta_1}{\beta_1} > 1$ and $x_1^1 = \frac{c}{1-\gamma} < \frac{c}{\gamma}$.

On the other hand, the optimal *wing* contract in R_2 is (11), independently of the value of β_1 . This is because, given $-1 < \gamma_i < 0$, both τ_1^2 and τ_2^2 are positive. Since $\tau_1^2 = -\frac{1-\beta_1}{\beta_1}$; $|\tau_1^2| > 1$ (i.e., as before, $f_1^2(b_1)$ and $f_2^2(b_1)$ intersect in the first quadrant. Also notice that, given $\beta_i < 0, i = 1, 2$, $y_1^2 = \frac{c(-1+\beta_1)}{(1-\gamma)\beta_1} < 0$. Two are the relevant cases:

1. If $\beta_1 > \beta_2$, then $f_1^2(b_1)$ and $f_2^2(b_1)$ intersect outside R_2 , and the optimal solution would be $b_1 = b_2 = \frac{c(1-\check{\beta})}{(1-\gamma)}$.
2. If $\beta_1 < \beta_2$, then the solution is (11) which overall cost is never greater than $\frac{2c(1-\check{\beta})}{(1-\gamma)}$.

We complete the proof by noticing, by analogy with the Proof of Proposition 3, that the optimal solution lies in R_2 , rather than in R_1 . ■

2.1.4 Wing with ESP

In the case of *wing* with ESP, we need to add to the basic linear program (7)-(10) the ESP constraint.

Proposition 7 (winiESP) *The optimal wing mechanism under ESP is (11), with $\beta_1 \leq \beta_2$.*

Proof. We begin by showing that here the optimal *wing* contract in R_1 is (13) if $|\alpha_2| < \gamma$ and $\hat{b}^1 = \left\{ \frac{c}{\gamma}, 0 \right\}$ if $\beta_2 \geq \gamma$. This is because, like in the previous cases, $f_1^1(b_1)$ is never binding, since $x_1^1 = \frac{c}{1-\gamma} < \frac{c}{\gamma}$ and $\tau_1^1 = -\frac{1-\beta_1}{\beta_1} < 0$. On the other hand, given that $x_2^1 = -\frac{c(1+\alpha_2)}{\alpha_2(1-\gamma)}$ and $0 \leq \tau_2^1 \leq \frac{1}{2}$, $f_2^1(b_1)$ is binding if and only if $|\alpha_2| < \gamma$ (i.e. if $x_2^1 > \frac{c}{\gamma}$).

As for R_2 , we begin to notice that $\tau_1^2 = -\frac{1-\beta_1}{\beta_1} \geq -1$ (since $|\beta_1| < \frac{1}{2}$) and that $0 \leq \tau_2^2 = \frac{\alpha_2 + \gamma\beta_2}{1 + \alpha_2\gamma(1-\beta_2)} < 1$. This implies, like before, that $f_1^2(b_1)$ and $f_2^2(b_1)$ intersect in the first quadrant. The rest of the proof is identical of that of Proposition 6. ■

2.2 Solution of the mechanism design problem under the *sting* program

In the case of *sting*, the search of the optimal mechanism corresponds to the *wing* linear program (7)-(10) with an additional constraint (implementation with a unique equilibrium):

$$u_1(1, 0) \geq u_1(0, 0). \quad (18)$$

The constraint (18) makes, on behalf of player 1, the choice of putting effort a weakly dominant strategy.

2.2.1 Sting under EP

The solution of *sting* under EP is as follows (see Winter, 2004):

$$\begin{aligned} b_1^* &= \frac{c}{\gamma}, \\ b_2^* &= \frac{c}{1-\gamma}. \end{aligned}$$

2.2.2 Sting under IAP

Proposition 8 *The optimal sting mechanism under IAP is*

$$\begin{cases} b_1^* = \frac{c((1+\alpha_1)(1+\alpha_2)-\gamma(1-\beta_2))}{\gamma(1+\alpha_1+\alpha_2-\gamma(1+\alpha_1-\beta_2))}, \\ b_2^* = \frac{c(1+\alpha_1)(\gamma+\alpha_2)}{\gamma(1+\alpha_1+\alpha_2-\gamma(1+\alpha_1-\beta_2))}. \end{cases} \quad (19)$$

To prove Proposition 8, we follow the same strategy as before.

Lemma 9 $\hat{b}^1 = \left(\frac{c(1+\alpha_2)}{(1-\gamma)\gamma}, \frac{c(\gamma+\alpha_2)}{(1-\gamma)\gamma} \right).$

Proof. In R_1 , the constraints for agent 1 and 2 correspond to:

$$b_1 \geq f_1^1(b_1) \equiv \frac{c(1-\beta_1)}{(1-\gamma)\beta_1} - \frac{1-\beta_1}{\beta_1}b_1, \quad (20)$$

$$b_1 \geq f_3^1(b_1) \equiv \frac{c(1-\beta_1)}{\gamma(1-\gamma)\beta_1} - \frac{1-\beta_1}{\beta_1}b_1, \quad (21)$$

$$b_2 \geq f_2^1(b_1) \equiv \frac{c}{1-\gamma} + \frac{\alpha_2}{1+\alpha_2}b_1, \quad (22)$$

Let x_i^{kl} solves $f_i^k(x) = g^l(x)$. We first notice that (20) is not binding. This is because (20) defines a constraint which is parallel to (21), but with a smaller intercept ($y_1^1 < y_3^1$, since $\gamma < 1$). Also notice that, in this case, (21) is not binding either. This is because, $\tau_3^1 < 0$, $\tau_2^1 > 0$, and $x_3^{12} = \frac{c(1-\gamma\beta_1)}{\gamma(1-\gamma)} < x_2^{12} = \frac{c(1+\alpha_2)}{\gamma(1-\gamma)}$.

This implies that, in R_1 , $(b_1 + b_2)$ is minimized (like in *wing*) where $f_2^1(b_1)$ and $g^2(b_1)$ intersect, i.e. when $\hat{b}_1^1 = \frac{c(1+\alpha_2)}{(1-\gamma)\gamma}$ and $\hat{b}_2^1 = \frac{c(\gamma+\alpha_2)}{(1-\gamma)\gamma}$. ■

Lemma 10 *The optimal sting contract in R_2 is (19).*

Proof. R_2 , the relevant constraints are as follows:

$$b_1 \geq f_1^2(b_1) \equiv \frac{c(1-\beta_1)}{(1-\gamma)\beta_1} - \frac{1-\beta_1}{\beta_1}b_1 \quad (23)$$

$$b_1 \geq f_3^2(b_1) \equiv -\frac{c(1+\alpha_1)}{\gamma\alpha_1} + \frac{1+\alpha_1}{\alpha_1}b_1 \quad (24)$$

$$b_2 \geq f_2^2(b_1) \equiv \frac{c(1-\beta_2)}{1+\alpha_2-\gamma(1-\beta_2)} - \frac{\alpha_2+\gamma\beta_2}{1+\alpha_2-\gamma(1-\beta_2)}b_1. \quad (25)$$

Notice that, by analogy with R_1 , condition (23) is not binding since $\tau_1^2 < 0$, $\tau_3^2 > 0$ and $x_1^1 = \frac{c}{1-\gamma} < x_3^3 = \frac{c}{\gamma}$. Also notice that $0 < x_2^{21} = \frac{c(1-\beta_2)}{1-\gamma} < x_3^{21} = \frac{c(1+\alpha_1)}{\gamma}$ and $x_2^{22} = \frac{c(1+\alpha_2)}{\alpha(1-\gamma)} > x_3^{22} = \frac{c}{\gamma}$. This, in turn, implies that, $f_3^2(b_1)$ and $f_2^2(b_1)$ always intersect in the interior of R_2 , which implies the solution.² ■

We are in the position to prove Proposition 8.

Proof. To close the proposition, it is sufficient to show that $\hat{b}_i^1 \geq \hat{b}_i^2, i = 1, 2$. To see this, notice that $x_2^{12} = x_2^{22} = \frac{c(1+\alpha_2)}{\gamma(1-\gamma)}$ (i.e. $f_2^1(b_1)$ and $f_2^2(b_1)$ cross exactly at the intersection with $g^2(b_1)$). Since $\tau_2^2 = \frac{\alpha_2+\gamma\beta_2}{1+\alpha_2-\gamma(1-\beta_2)} > 0$ and $\hat{b}^2$ is interior to R_2 , the result follows. ■

2.2.3 Sting under SSP

Proposition 11 *The optimal sting mechanism under SSP is (19).*

Proof. By analogy with the IAP case, in R_1 , (20) is not binding. Also notice that $\tau_3^1 = -\frac{1-\beta_1}{\beta_1} > \tau_2^1 = \frac{\alpha_2}{1+\alpha_2} > 0$. Two are the relevant cases:

²As it turns out, unlike the *wini* case, the search for the appropriate conditions on preferences to identify player 1 has no (algebraically manageable) closed-form solution, but it has to be evaluated numerically (as we did in the calibration of our experimental conditions).

1. if $\alpha_2 \geq -\gamma\beta_1$, (i.e. if $x_3^{12} = \frac{c(1-\gamma\beta_1)}{\gamma(1-\gamma)} \leq x_2^{12} = \frac{c(1+\alpha_2)}{\gamma(1-\gamma)}$), then (24) is not binding, and the optimal solution is the intersection between $f_2^1(b_1)$ and $g_3(b_1)$, that is, $\hat{b}^1 = \left(\frac{c(1+\alpha_2)}{\gamma(1-\gamma)}, \frac{c(\gamma+\alpha_2)}{\gamma(1-\gamma)} \right)$;
2. if $\alpha_2 < -\gamma\beta_1$, then the optimal solution is the intersection between $f_2^1(b_1)$ and $f_3^1(b_1)$, that is, .

$$\hat{b}^1 = \left(\frac{c(1+\alpha_2)(1-\beta_1(1+\gamma))}{\gamma(1-\gamma)(1+\alpha_2-\beta_1)}, \frac{c(\alpha_2+\gamma(1+\alpha_2))(1-\beta_1)}{\gamma(1-\gamma)(1+\alpha_2-\beta_1)} \right).$$

As for R_2 , the optimal *sting* contract is, again, (19). This is because, by analogy with the IAP case, conditions (23) and $g^2(b_1)$ are not binding. Also notice that $x_2^{22} = \frac{c(1+\gamma_2)}{\gamma(1-\gamma)} > 0$ and $0 \leq \tau_2^2 = \frac{\alpha_2+\gamma\beta_2}{1+\alpha_2-\gamma(1-\beta_2)} < 1$. This, in turn, implies that, in R_2 , $(b_1 + b_2)$ is minimized where $f_3^2(b_1)$ and $f_2^2(b_1)$ intersect, which implies the solution. ■

2.2.4 Sting under ESP

Proposition 12 *The optimal sting mechanism under ESP is (19).*

Proof. By analogy with the previous cases, in R_1 , (20) is not binding. Also notice that, in this case, (23) is not binding either, since $\tau_2^1 < 0$ and $x_2^{12} = \frac{c(1+\alpha_2)}{\gamma(1-\gamma)} < \frac{c}{\gamma}$. Since ESP imply $\gamma_1 \leq \frac{1}{2}$, the unique solution in this case is $\hat{b}^1 = \left(\frac{c}{\gamma(1-\gamma)}, 0 \right)$. As for R_2 , we first notice that, given that $|\beta_1| \leq \frac{1}{2}$, $\tau_3^2 > 1$. Since ESP also imply $|\alpha_2| < \gamma$ (i.e. $x_2^2 > \frac{c}{\gamma}$), then the optimal solution is the intersection between $f_2^1(b_1)$ and $g_3(b_1)$, that is, (19). ■