

Bundling and Competition for Slots (Web Appendix)

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June 8, 2012

Proof of Proposition 2

(i) We need to verify that the mixed strategies described in Proposition 2(i) constitute a NE. We start with firm A , and the following lemma is a useful step.

Lemma 1 *Given an arbitrary (pure or mixed) strategy played by firm B , if A plays a pure strategy (p_x, p_y) such that $x - p_x < y - p_y$, then there exists (p'_x, p'_y) such that $x - p'_x > y - p'_y$ and A 's profit with (p'_x, p'_y) is at least as much as with (p_x, p_y) .*

Proof. Given (p_x, p_y) such that $x - p_x < y - p_y$, let δ denote the probability that C buys both products x and y and let η denote the probability that C buys only y among the products of A . The expected profit of A is then $\delta(p_x + p_y) + \eta p_y$. Now consider (p'_x, p'_y) such that $x - p'_x = y - p_y$ and $y - p'_y = x - p_x$, so that $x - p'_x > y - p'_y$. Then the expected profit of A is $\delta(p'_x + p'_y) + \eta p'_x = \delta(p_x + p_y) + \eta(x - y + p_y)$ and this is weakly higher than the expected profit of A with (p_x, p_y) .

Now we verify that A 's strategy in Proposition 2(i) is a best reply to B 's strategy. As it is well known, it suffices to consider pure strategies of A , and Lemma 2 allows to restrict to pure strategies satisfying $x - p_x \geq y - p_y$. Consider first $p_y \in [\frac{1}{2}(y - t), y - t]$. Then $x - p_x \geq y - p_y \geq t$ holds and thus C certainly purchases product x ; hence it is suboptimal for A to play $p_x < x - y + p_y$. Given $p_x = x - y + p_y$, we find

$$\begin{aligned}\pi_A &= x - y + p_y + p_y \Pr\{y - p_y > \beta\} \\ &= x - y + p_y + p_y G_B(y - p_y) = x - t\end{aligned}\tag{A1}$$

for any $p_y \in [\frac{1}{2}(y-t), y-t]$.¹ Playing $p_y < \frac{1}{2}(y-t)$ is not a profitable deviation for A since then $y - p_y > \frac{1}{2}(y+t) \geq \beta$ with probability one and $x - p_x \geq y - p_y$ implies that C buys both x and y ; therefore the profit of A is smaller than $x - \frac{1}{2}(y+t) + \frac{1}{2}(y-t) = x - t$. Playing $p_y > y - t$ is not a profitable deviation since then $y - p_y < t$ - thus C certainly does not buy y - and in order to induce C to buy product x , A cannot set p_x larger than $x - t$ given that $z - p_z \geq t$ with probability one and $p_t = 0$.

Now we verify that B 's strategy in Proposition 2(i) is a best reply to A 's strategy. Since $x - p_x = y - p_y \geq t$, C does not buy product t and B only needs to choose p_z . When he plays $p_z \in [z - \frac{1}{2}y - \frac{1}{2}t, z - t)$, his payoff is

$$\pi_B = p_z \Pr\{z - p_z \geq \alpha\} = p_z G_A(z - p_z) = z - \frac{1}{2}t - \frac{1}{2}y \quad (\text{A2})$$

for any $p_z \in [z - \frac{1}{2}y - \frac{1}{2}t, z - t)$.² Playing $p_z = z - t$ is not a profitable deviation since then C does not buy product z in case 2 and buys it with probability $G_A(t) = \frac{2z-t-y}{2z-2t}$ in case 1 (then, the profit of B is $\frac{2z-y-t}{2z-2t}(z-t) = z - \frac{1}{2}(y+t)$). Playing $p_z > z - t$ is not a profitable deviation as it implies $z - p_z < t$ and certainly C does not buy product z . Finally, playing $p_z < z - \frac{1}{2}t - \frac{1}{2}y$ is not a profitable deviation since C buys product z but B 's profit is smaller than $z - \frac{1}{2}y - \frac{1}{2}t$.

(ii) The payoffs of the two firms are obtained in (A1) and (A2). Let a denote the probability that C buys y given the mixed strategies; then

$$\begin{aligned} a &= \int_t^{\frac{1}{2}(y+t)} G_B(\alpha) G'_A(\alpha) d\alpha = \int_t^{\frac{1}{2}(y+t)} \frac{\alpha - t}{y - \alpha} \frac{2z - t - y}{2(z - \alpha)^2} d\alpha \\ &= \frac{y - t}{2(y - z)^2} [y - z + (2z - t - y) \ln \frac{2z - t - y}{z - t}]. \end{aligned}$$

For the example in Section I, with $(x, y, z, t) = (4, 3, 2, 0)$, we find $a = \frac{3}{2}(1 + \ln \frac{1}{2}) \simeq 0.46$. Suppose that $y \leq z + (z - t)/3$. The inequality $G_A(\alpha) > G_B(\alpha)$ is equivalent to $h(\alpha) \equiv 2\alpha^2 - (4z - y + t)\alpha + 2zy - ty + 2tz - y^2 > 0$, and $y \leq z + (z - t)/3$ implies that h is strictly decreasing with respect to $\alpha \in [t, \frac{1}{2}(y+t)]$, with $h(\frac{1}{2}(y+t)) = 0$.

¹The strict inequality in (A1) is correct in case 1, while in case 2 we should write $\Pr\{y - p_y \geq \beta\}$. But in fact there are no practical differences between the two cases, as G_B has no atoms.

²The weak inequality in (A2) is correct in case 1, while in case 2 we should write $\Pr\{z - p_z > \alpha\}$. But in fact there are no practical differences between the two cases, as $z - p_z$ belongs to $(t, \frac{1}{2}(y+t)]$ from $p_z \in [z - \frac{1}{2}(y+t), z - t)$ and G_A has no atoms in $(t, \frac{1}{2}(y+t)]$.

Proof of Proposition 3

Step 1 If $B_B^* = \emptyset$ and (4) is violated, then no pure-strategy NE exists and no efficient mixed-strategy NE exists.

When $B_B^* = \emptyset$ and (4) is violated, we know from Proposition 1(iii) that no pure-strategy NE exists. We now show that no efficient mixed-strategy NE exists.

Since $B_B^* = \emptyset$, efficiency requires that C buy all the products in B_A^* with probability one, and nothing else; this implies $\pi_B = 0$. Therefore $\Pr\{w_A(b_A) \geq v_B(b_B^1)\} = 1$ for each $b_A \in B_A^*$, otherwise firm B could play $p_B(b_B^1) = c_B(b_B^1) + \varepsilon$ and induce C to buy b_B^1 with positive probability, which would imply $\pi_B > 0$. From $\Pr\{w_A(b_A) \geq v_B(b_B^1)\} = 1$ for each $b_A \in B_A^*$ we infer that π_A is not larger than $\sum_{b_A \in B_A^*} [v(b_A) - v(b_B^1)]$. Since there exists no pure-strategy NE, we know from Proposition 1(iii) and (4) that there is some $\hat{j}$ between 1 and $k - 1$ such that $\sum_{b_A \in B_A^*} [v(b_A) - v(b_B^1)] < \sum_{b_A \in \{b_A^1, b_A^2, \dots, b_A^{\hat{j}}\}} [v(b_A) - \lambda(b_B^{k-\hat{j}+1})]$. Note however that if $\lambda(b_B^{k-\hat{j}+1}) = v(b_B^1)$, then selling $\{b_A^1, b_A^2, \dots, b_A^{\hat{j}}\}$ is definitely not a profitable deviation for A since the surplus he needs to leave to C for each of the $\hat{j}$ products in $\{b_A^1, b_A^2, \dots, b_A^{\hat{j}}\}$ is the same as he needs to leave on each of the k products in B_A^* . Thus, given that selling $\{b_A^1, b_A^2, \dots, b_A^{\hat{j}}\}$ is a profitable deviation for A , it is necessary that $\lambda(b_B^{k-\hat{j}+1}) = u(b_B^{k-\hat{j}+1}) < v(b_B^1)$. Hence

$$\sum_{b_A \in B_A^*} [v(b_A) - v(b_B^1)] < \sum_{b_A \in \{b_A^1, b_A^2, \dots, b_A^{\hat{j}}\}} [v(b_A) - u(b_B^{k-\hat{j}+1})] \quad (\text{A3})$$

holds for a certain $\hat{j} < k$. We now exhibit a profitable deviation for firm A : set $p_A(b_A) = u(b_A) - u(b_B^{k-\hat{j}+1})$ for $b_A \in \{b_A^1, b_A^2, \dots, b_A^{\hat{j}}\}$, and $p_A(b_A)$ large for $b_A \in \mathcal{B}_A \setminus \{b_A^1, b_A^2, \dots, b_A^{\hat{j}}\}$. In this way each product in $\{b_A^1, b_A^2, \dots, b_A^{\hat{j}}\}$ leaves C a net surplus equal to $u(b_B^{k-\hat{j}+1})$, and at most $k - \hat{j}$ products of firm B leave C a higher net surplus; therefore C buys all the products in $\{b_A^1, b_A^2, \dots, b_A^{\hat{j}}\}$. Firm A earns then $u(b_A) - u(b_B^{k-\hat{j}+1}) - c_A(b_A) = v(b_A) - u(b_B^{k-\hat{j}+1})$ on each $b_A \in \{b_A^1, b_A^2, \dots, b_A^{\hat{j}}\}$, and his total profit is $\sum_{b_A \in \{b_A^1, b_A^2, \dots, b_A^{\hat{j}}\}} [v(b_A) - u(b_B^{k-\hat{j}+1})]$, which is larger than his (supposed) equilibrium profit π_A , given (A3).

Step 2 If $B_B^* \neq \emptyset$ and (3) is satisfied, then no pure-strategy NE exists and no efficient mixed-strategy NE exists.

When $B_B^* \neq \emptyset$ and (3) holds, we know from Proposition 1(ii) that no pure-strategy NE exists. We now show that no efficient mixed-strategy NE exists.

We assume that an efficient mixed-strategy NE exists and we derive a contradiction. Efficiency requires that C buy all products in B_A^* and B_B^* with probability one. Therefore it is necessary that

$$\begin{aligned} \Pr\{w_A(b_A) \geq \max\{w_A(\tilde{b}_A), w_B(\tilde{b}_B)\}\} &= 1 \\ \Pr\{w_B(b_B) \geq \max\{w_A(\tilde{b}_A), w_B(\tilde{b}_B)\}\} &= 1 \end{aligned} \quad \text{for each} \quad \begin{aligned} b_A &\in B_A^*, \quad b_B \in B_B^* \quad \text{and} \\ \tilde{b}_A &\in \mathcal{B}_A \setminus B_A^*, \quad \tilde{b}_B \in \mathcal{B}_B \setminus B_B^* \end{aligned} \quad (\text{A4})$$

If we define $w_A^M \equiv \inf\{w : \Pr\{w_A(\tilde{b}_A) \leq w \text{ for each } \tilde{b}_A \in \mathcal{B}_A \setminus B_A^*\} = 1\}$ and $w_B^M \equiv \inf\{w : \Pr\{w_B(\tilde{b}_B) \leq w \text{ for each } \tilde{b}_B \in \mathcal{B}_B \setminus B_B^*\} = 1\}$, we can write (A4) as

$$\begin{aligned} \Pr\{w_A(b_A) \geq \max\{w_A^M, w_B^M\}\} &= 1 \\ \Pr\{w_B(b_B) \geq \max\{w_A^M, w_B^M\}\} &= 1 \end{aligned} \quad \text{for each} \quad b_A \in B_A^*, b_B \in B_B^*.$$

This suggests that unless $w_A(b_A) = \max\{w_A^M, w_B^M\}$ with probability one for each $b_A \in B_A^*$, a profitable deviation exists for firm A which consists in lowering $w_A(b_A)$. The same argument suggests that $w_B(b_B)$ needs to be equal to $\max\{w_A^M, w_B^M\}$ with probability one for each $b_B \in B_B^*$. Given that $w_A(b_A) = w_B(b_B) = \max\{w_A^M, w_B^M\}$ for each $b_A \in B_A^*$, $b_B \in B_B^*$, it follows that $w_A^M = w_B^M \equiv w^M$. Precisely, if $w_A^M > w_B^M$ then A can profitably reduce both w_A^M and $w_A(b_A)$ for each $b_A \in B_A^*$; if $w_A^M < w_B^M$, then a symmetric argument applies.

Finally, since $\bar{b}_A \in \mathcal{B}_A \setminus B_A^*$ is the product in $\mathcal{B} \setminus B^*$ with the highest net value, we exhibit a profitable deviation for A : set $w_A(\bar{b}_A) = w^M + \varepsilon$ and $w_A(b_A) = w^M + \varepsilon$ for each $b_A \in B_A^*$ with probability one. Then C buys all these products of A and A 's profit changes by $\Delta\pi_A > -\varepsilon k + p(\bar{b}_A) - c_A(\bar{b}_A) = v(\bar{b}_A) - w^M - \varepsilon(k+1)$. We find that $\Delta\pi_A > 0$ since $w^M = w_B^M \leq u(b_B)$ for some $b_B \in (\mathcal{B}_B \setminus B_B^*)$, and from (3) we know that $v(\bar{b}_A) > u(b_B)$ for any $b_B \in (\mathcal{B}_B \setminus B_B^*)$.

Proof of Lemma 1

Consider any arbitrary profile of strategies (s_i, s_j) and let π_i be the profit of firm i given (s_i, s_j) . We show that i can achieve the same profit π_i by playing the technology-renting strategy such that $F_i = \pi_i$. This fact is obvious if $\pi_i = 0$ and therefore we consider the case of $\pi_i > 0$. In order to prove this result, it suffices to show that the buyer buys at least one product from i when i plays tr_i . We find that, (i) given (tr_i, s_j) ,

the buyer can make the same payoff that she makes with (s_i, s_j) since she can buy the same products, with the same outlay; (ii) given (tr_i, s_j) , the buyer cannot realize a higher payoff than with (s_i, s_j) without buying at least one product of firm i , because otherwise she would not buy anything from i given (s_i, s_j) , and this contradicts $\pi_i > 0$.

Proof of Proposition 5

(i) First we note that if the firms play (tr_A^*, tr_B^*) , the buyer rents both A 's technology and B 's technology. Precisely, the buyer's payoff is (i) $V^* - F_A^* - F_B^* = V_A^{*S} + V_B^{*S} - V^* \geq 0$ if she rents both technologies; (ii) $V_A^{*S} - F_A^* = V_A^{*S} + V_B^{*S} - V^*$ if she rents only A 's technology; (iii) $V_B^{*S} - F_B^* = V_A^{*S} + V_B^{*S} - V^*$ if she rents only B 's technology. Since she maximizes the social surplus when she is indifferent among two or more alternatives, she rents both technologies and thus buys the products in B^* .

Now we prove that when firm B plays tr_B^* , firm A cannot make a profit larger than F_A^* (the same argument applies to firm B). From Lemma 1, it is enough to consider firm A 's deviations in the set of technology-renting strategies. Obviously, firm A has no incentive to decrease F_A below F_A^* . If instead A chooses $F_A = F_A^* + \varepsilon$ with $\varepsilon > 0$, then the buyer rents only B 's technology since she earns a payoff $V_A^{*S} + V_B^{*S} - V^* - \varepsilon$ by renting both technologies or by renting only A 's technology while she earns $V_A^{*S} + V_B^{*S} - V^*$ by renting only B 's technology.

(ii) The proof is by contradiction. Suppose there exists a NE such that A makes a profit higher than $F_A^* = V^* - V_B^{*S}$. Precisely, suppose that in the NE, the players' payoffs are $\pi_A(> F_A^*)$, π_B and π_C . Given that $\pi_A + \pi_B + \pi_C \leq V^*$ necessarily holds, we infer that $\pi_A > F_A^*$ implies $\pi_B + \pi_C < V_B^{*S}$. Then, let firm B deviate by using tr_B such that $F_B = \pi_B + \varepsilon$ with $\varepsilon > 0$ and such that $\pi_C < V_B^{*S} - \pi_B - \varepsilon$. This deviation of B is profitable if and only if the buyer buys at least one product from firm B . In order to prove that this is the case, we note first that if the buyer does not buy anything from B , then she buys only bundles offered by A and they cannot yield C a payoff larger than π_C ; otherwise we obtain a contradiction with the fact that the initial strategies generating payoffs (π_A, π_B, π_C) constitute a NE. We end the proof by observing that if the buyer pays F_B and trades only with firm B , a social surplus of V_B^{*S} is generated and the buyer obtains $V_B^{*S} - \pi_B - \varepsilon$, which is larger than π_C .

(iii) Suppose that C purchases an inefficient set of products. Then an inefficiently low social surplus $\bar{V}(< V^*)$ is generated, with payoffs $\pi_A \geq 0, \pi_B \geq 0, \pi_C \geq 0$ such that $\pi_A + \pi_B + \pi_C = \bar{V}$. Now suppose that A plays a technology-renting strategy with $F_A = \pi_A + \varepsilon$. By renting A 's technology and buying $B^*(= B_A^*)$, C earns $V^* - \pi_A - \varepsilon$ and $V^* - \pi_A - \varepsilon > \pi_C = \bar{V} - \pi_A - \pi_B$.