Coordination in the Presence of Asset Markets

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Web Appendix

I. Theoretical Analysis

Here, we present a theoretical analysis of the relationship between the market and the coordination game in our experiments. This analysis motivates the predictions that we present in the paper. As we describe in the main body, players participate in a two-stage game consisting of an asset market followed by an order-statistic coordination game. The values of the securities traded in the asset market depend upon the realization of the output (order statistic) in the coordination game.

We assume that the aggregate endowment for each asset is the same, or $\sum_{i=1}^{n} x_{mi} = k$ for all m. Under this assumption, the market is a constant-sum game, and therefore the efficiency properties of higher matched inputs is the same as it is in the original game. However, different ex post portfolios can affect individual players' incentives to select particular input levels. It is straightforward that all pure-strategy Nash equilibria of this game also involve identical input choices since the payoffs from the assets are only affected by the output. With this in mind, the following proposition characterizes the set of pure-strategy Nash equilibria of the market-modified coordination game.

PROPOSITION 1: *Consider the market-modified payoffs for different order statistics* $m_j(e)$.

1) *If* $m_1(e)$*, then the selection of input level* $e_i = m$ *for all* i *is a pure-strategy Nash equilibrium if and only if asset portfolios are such that for all* i *and for all* $\ell < m$*,*

$$x_{\ell i} - x_{mi} \leq \left[\frac{b}{\beta}\right](m - \ell). \tag{1}$$

2) *If* $m_j(e)$ *with* $1 < j < n$*, then the selection of input level* $e_i = m$ *for all* i *is a pure-strategy Nash equilibrium for all asset portfolios.*

PROOF:

Consider each part:

1) In order for $e_i = m$ for all i to be a Nash equilibrium it must be that

$$\overline{\pi}_i(m, m; x_i) \geq \overline{\pi}_i(\ell, m; x_i)$$

for all $\ell \neq m$. Consider $\ell > m$, then we have

$$\overline{\pi}_i(m, m; x_i) = a + bm - cm + \beta x_{mi}$$

and

$$\overline{\pi}_i(\ell, m; x_i) = a + bm - c\ell + \beta x_{mi}$$

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and obviously $\overline{\pi}_i(m, m; x_i) > \overline{\pi}_i(\ell, m; x_i)$ for all x_i. Now consider $\ell < m$. Nash equilibrium requires that

$$\begin{aligned} \overline{\pi}_i(m, m; x_i) &\geq \overline{\pi}_i(\ell, m; x_i) \\ a + bm - cm + \beta x_{mi} &\geq a + b\ell - c\ell + \beta x_{\ell i} \\ bm - cm + \beta x_{mi} &\geq b\ell - c\ell + \beta x_{\ell i} \\ \beta x_{\ell i} - \beta x_{mi} &\leq bm - cm - b\ell + c\ell \\ \beta(x_{\ell i} - x_{mi}) &\leq (b - c)(m - \ell) \\ x_{\ell i} - x_{mi} &\leq \left[\frac{b - c}{\beta}\right](m - \ell). \end{aligned}$$

2) Suppose that all other players $k \neq i$ are playing $e_k = m$ as described by the Nash equilibrium, then for all $1 < j < n$ and all e_i $m_j(e) = m$ or the order statistic is invariant in the single player's choice of input so player i's payoff function is given by:

$$\overline{\pi}_i(e_i; e_{-i} = m, x_i) = a + b_0 m - c\,|m - e_i| + \beta x_{mi} \tag{2}$$

which is obviously maximized by i selecting $e_i = m$ regardless of the asset allocation.

When output is determined by the minimum input $j = 1$, this proposition tells us that the asset market can directly alter the incentives and equilibrium outcomes of the game. When a particular player has enough of an asset that pays off under lower output (relative to higher output assets), he or she has a weakly dominant strategy to play the corresponding lower input, thus making the higher output equilibrium impossible to support. Thus, the lowest input choice, $e_i = 1$, by all players is the only Nash equilibrium that exists for *every* possible asset portfolio. For higher output outcomes, the relative payoff of the asset market (β) to the coordination game (b) determines the asset portfolios that are consistent with Nash equilibrium of the market-modified game. However, when the output is determined by *any* order statistic other than the minimum ($j > 1$), players' asset holdings do not affect the set of pure-strategy Nash equilibria nor their efficiency properties. Thus, any influence upon the outcome of the game produced by the asset markets must come from the markets' influence upon players' beliefs about the play of others or the addition of payoff asymmetries due to different asset holdings.

One factor that may significantly improve coordination on Pareto-efficient outcomes is communication through prices and other market activity. Given that there is no generally agreed upon model of asset market activity, and therefore no model of how such activity influences coordination, we explore the relationship between market communication and beliefs in our framework. We begin by positing a model that is consistent with the stylized details of strategic uncertainty in the coordination game. Suppose that each player has some beliefs about the output, $m_j(e)$, that will be observed, where $\mu_i(m|e_i)$ indicates player i's belief that $m_j(e) = m$ given her intended input choice of e_i. The usual interpretation of coordination failure arising due to strategic uncertainty in this game is that each player recognizes that high output is the Pareto-dominant equilibrium, but their beliefs about the choices of others players may make it rational for players to select $e_i < M$. Specifically, a player's expected payoff from his or her input choice given this strategic uncertainty is:

$$E\overline{\pi}(e_i, x_i) = \sum_{m=1}^{M} \mu_i(m|e_i)\overline{\pi}_i(e; x_i). \tag{3}$$

If players maximize their expected utility with respect to these preferences, then they may decide to play input levels other than the Pareto-dominant equilibrium. Let $\mu = (\mu_1, \ldots, \mu_n)$ be the set of beliefs for all agents. Then we say that $e^* = (e_1^*, \ldots, e_n^*)$ is an *equilibrium given beliefs* μ if e_i^* maximizes (3) for all i. Players maximizing with respect to these beliefs provide an explanation for inefficiency in coordination games. If a player's beliefs place sufficient likelihood on low input choices by other players, the player will prefer low input and the resulting outcome may yield an output level below the Pareto-optimal equilibrium.[2]

The market, like any communication device, may provide an opportunity for players to refine their beliefs prior to playing the game. Let p_m be the market price for the asset X_m that pays β in the event that the output is m. Rational expectations equilibrium represents the conclusion of a market communication process whereby by players' beliefs might be communicated and/or influenced by market prices. A rational expectation equilibrium in this context must also entail second-stage input choices that are consistent with expected payoff maximization given beliefs. We say that (x^*, p^*, e^*) is a *rational expectations equilibrium* if there exist beliefs μ such that: x_i^* and e_i^* maximizes $\sum_{m=1}^{M} [\mu_i(m|e_i^* \overline{\pi}_i(e^*; x_i) - p_m^* x_{mi}]$ for all i.

If there are no restrictions on the set of allowable beliefs, there are many potential rational expectations equilibria. As is typical in rational expectations type equilibria, we focus on prices that reveal information to the agents. A rational expectations equilibrium is *revealing* if the beliefs that support the equilibrium are given by $\mu_i(m|e_i) = p_m / \sum_{\ell=1}^{M} p_\ell$ for all i, or beliefs are simply given by the observed normalized prices. Further, the equilibrium is said to be *fully revealing* if there exists an output level m such that $p_m = \beta$ so $\mu_i(m|e_i) = 1$ and for all $\ell \neq m$, $p_\ell = 0$ so $\mu_i(\ell|e_i) = 0$ for all i. In the event of a fully revealing equilibrium, all strategic uncertainty is resolved and the resulting input choices must constitute a Nash equilibrium of the game. If markets are an effective communication tool for obtaining efficient coordination they should admit fully revealing rational expectations equilibria that result in high output. The next proposition characterizes the equilibria.

PROPOSITION 2: *1) If $m_1(e)$, then all fully revealing rational expectations equilibria involve $e_i^* = 1$ for all i.*

2) If $m_j(e)$ with $1 < j < n$, then $e_i^* = m$ for all i and for $m \in \{1, 2, \ldots, M\}$ is a fully revealing equilibrium.

PROOF:

Address each part in turn:

1) We need to show that $e_i = 1$ is an equilibrium and any other matched effort level cannot be an equilibrium.

   a) To show that $e_i = 1$ for all i is an equilibrium let $e_k = 1$ for all $k \neq i$ and note that $m_1(e) = 1, \delta_1(e) = 1$, and $\delta_\ell(e) = 0$ for all $\ell \neq 1$ and for all e_i.

[2]There is also another potential source of inefficiency, which involves mismatched choices, or "wasted input," by players. Of course, via repeated interaction players will refine their beliefs to be consistent with the observed history of play, resulting in convergence in beliefs and actions to a particular Nash equilibrium. Vincent Crawford (1995) proposes a model that formally interacts learning dynamics with strategic uncertainty in these kinds of games. Vincent Crawford and Bruno Broseta (1998) apply this kind of model to the experiment of John B. Van Huyck, Raymond C. Battalio and Richard O. Beil (1993).

Given fully revealing prices and beliefs player i's choice of input and assets must maximize:

$$\begin{aligned}
(4) \quad \overline{\pi}_i(e,x) &= a + b_0 m_1(e) - c\,|m_1(e) - e_i| + \sum_{\ell=1}^{M} \delta_\ell(e)\beta x_{\ell i} - p_\ell x_{\ell i} \\
(5) &= a + b_0 - c(e_i - 1) + \beta x_{1i} - \beta x_{1i} \\
(6) &= a + b_0 - c e_i - c
\end{aligned}$$

which is obviously minimized at any selection of x_i and $e_i = 1$ so any allocation such that $e_i = 1$ for all i can be supported as a fully revealing equilibrium.

b) To show that no higher output level can be achieved as a fully revealing equilibrium suppose that one exists and find a contradiction. Suppose there exists a fully revealing equilibrium with x^* and $e_i^* = m > 1$ for all i. Then e_i^* and x_i^* must joint maximize:

$$\begin{aligned}
(7)\ \overline{\pi}_i(e^*,x^*) &= a + b_0 m_1(e^*) - c\,|m_1(e^*) - e_i^*| + \sum_{\ell=1}^{M} \delta_\ell(e^*)\beta x_{\ell i}^* - p_\ell x_{\ell i}^* \\
(8) &= a + b_0 m + \beta x_{mi} - p_m^* x_{mi} \\
(9) &= a + b_0 m
\end{aligned}$$

Consider the alternative input for player i of $e_i = 1$ and any allocation such that $x_{\ell i} - x_{mi} > \frac{b_0}{\beta}(m-1)$, then we have the following:

$$\begin{aligned}
(10) \quad x_{\ell i} - x_{mi} &> \frac{b_0}{\beta}(m-1) \\
(11) \quad \beta\,(x_{\ell i} - x_{mi}) &> b_0(m-1) \\
(12) \quad a + b_0 + \beta x_{\ell i} - \beta x_{mi} &> a + b_0 m \\
(13) \quad \overline{\pi}_i(e_i = 1, e_{-i}^*, x_i, x_{-i}^*) &> \overline{\pi}_i(e^*, x^*)
\end{aligned}$$

or playing low input and buying sufficient excess amounts of the (free) asset provides greater utility than the proposed equilibrium. Thus we have a contradiction.

2) To show that $e_i = m$ for all i and for all m is an equilibrium let $e_k = m$ for all $k \neq i$ and note that $m_j(e) = m, \delta_m(e) = 1$, and $\delta_\ell(e) = 0$ for all $\ell \neq m$ and for all e_i. Given fully revealing prices and beliefs player i's choice of input and assets must maximize:

$$\begin{aligned}
(14) \quad \overline{\pi}_i(e,x) &= a + b_0 m_j(e) - c\,|m_j(e) - e_i| + \sum_{\ell=1}^{M} \delta_\ell(e)\beta x_{\ell i} - p_\ell x_{\ell i} \\
(15) &= a + b_0 m - c\,|m - e_i| + \beta x_{mi} - \beta x_{mi} \\
(16) &= a + b_0 m - c\,|m - e_i|
\end{aligned}$$

which is obviously minimized at any selection of x_i and $e_i = m$ so any allocation such that $e_i = m$ for all i can be supported as a fully revealing equilibrium.

In the minimum-input ($j = 1$) game, any player can unilaterally assure that the final

output is 1 by selecting $e_i = 1$, therefore the only prices that are consistent at *both* stages of the game must price the low output at β. If the price of the low output asset is lower than β then any player can profit by buying units of the low output asset; once she has a sufficient number of these low output assets it is a weakly dominant strategy to select the low input level $e_i = 1$. On the other hand, when $j > 1$, since the player cannot directly influence the order statistic and hence the output, any price that resolves uncertainty on a particular equilibrium output level can be consistent with equilibrium play in the second-stage coordination game.

Let (x^*, e^*) be the portfolio and input choices from a fully revealing rational expectations equilibrium identified above. While these equilibria suggest the types of outcomes that are consistent with both the information aggregation properties of the market and the strategic interaction properties of the game, they do not account for the fact that the information provided by the market might in reality be noisy or that some small "doubt" regarding other players' intended actions might still persist . We want to examine properties of these equilibria when players have some small doubt regarding the actual choices of other group members. In particular, if $m_j(e^*) = m$ then she believes there is an $\epsilon < .5$ probability that any individual independently selects either $m+1$ or $m-1$.[3] Specifically we define $\rho(\ell)$ to be the probability that any single individual selects input ℓ, thus we assume the following:

$$
\begin{aligned}
\rho(m-1) &= \begin{cases} \epsilon & m > 1 \\ 0 & m = 1 \end{cases} \\
\rho(m+1) &= \begin{cases} \epsilon & m < M \\ 0 & m = M \end{cases} \\
\rho(m) &= 1 - \rho(m-1) - \rho(m+1)
\end{aligned} \tag{17}
$$

Given this belief structure, $\mu_i(m_j|e_i)$ is generated by particular combinations of the binomial distribution and characterized in Table I where $\mu_i(m|e_i)$ is obviously defined as $1-\mu_i(m-1|e_i)-\mu_i(m+1|e_i)$. Importantly, note that $\mu_i(m-1|m) = \mu_i(m-1|m+1)$ and $\mu_i(m-1|m-1) = \mu_i(m-1|m)+\delta_1$ where $\delta_1 = \binom{n-1}{n-j} (\rho(m) + \rho(m+1))^{n-j} \rho(m-1)^{j-1}$. Similarly, $\mu_i(m+1|m) = \mu_i(m+1|m-1)$ and $\mu_i(m+1|m+1) = \mu_i(m+1|m)+\delta_2$ where $\delta_2 = \binom{n-1}{j-1} (\rho(m) + \rho(m-1))^{j-1} \rho(m+1)^{n-j}$. We assume all subjects have the same beliefs and examine the conditions under which the previously identified allocations and input choices that are fully revealing rational expectations remain rational expectations equilibrium when players have doubt as described above.

We can now define each agent's expected payoffs given these beliefs to be given by

[3] We examine only single step jumps in input for analytic tractability. The qualitative nature of these results should persist for more general types of beliefs.

TABLE 1—BELIEF STRUCTURE

e_i \ m_j	$m-1$	$m+1$
$m-1$	$\sum_{k=0}^{n-j}\binom{n-1}{k}(\rho(m)+\rho(m+1))^k\,\rho(m-1)^{n-1-k}$	$\sum_{k=0}^{j-2}\binom{n-1}{k}(\rho(m)+\rho(m-1))^k\,\rho(m+1)^{n-1-k}$
m	$\sum_{k=0}^{n-j-1}\binom{n-1}{k}(\rho(m)+\rho(m+1))^k\,\rho(m-1)^{n-1-k}$	$\sum_{k=0}^{j-2}\binom{n-1}{k}(\rho(m)+\rho(m-1))^k\,\rho(m+1)^{n-1-k}$
$m+1$	$\sum_{k=0}^{n-j-1}\binom{n-1}{k}(\rho(m)+\rho(m+1))^k\,\rho(m-1)^{n-1-k}$	$\sum_{k=0}^{j-1}\binom{n-1}{k}(\rho(m)+\rho(m-1))^k\,\rho(m+1)^{n-1-k}$

$E\overline{\pi}(e_i, x_i)$ for the following three relevant input level choices:[4]

$$E\overline{\pi}(m-1, x_i) =$$
$$\mu_i(m-1|m-1)\left[a+b_0(m-1)+\beta x_{im-1}\right]+\mu_i(m|m-1)\left[a+b_0 m-c+\beta x_{im}\right] \tag{18}$$
$$+\mu_i(m+1|m-1)\left[a+b_0(m+1)-2c+\beta x_{im+1}\right]$$
$$E\overline{\pi}(m, x_i) =$$
$$\mu_i(m-1|m)\left[a+b_0(m-1)-c+\beta x_{im-1}\right]+\mu_i(m|m)\left[a+b_0 m+\beta x_{im}\right] \tag{19}$$
$$+\mu_i(m+1|m)\left[a+b_0(m+1)-c+\beta x_{im+1}\right]$$
$$E\overline{\pi}(m+1, x_i) =$$
$$\mu_i(m-1|m+1)\left[a+b_0(m-1)-2c+\beta x_{im-1}\right]+\mu_i(m|m+1)\left[a+b_0 m-c+\beta x_{im}\right] \tag{20}$$
$$+\mu_i(m+1|m-1)\left[a+b_0(m+1)+\beta x_{im+1}\right]$$

It is clear that in the context of rational expectation equilibria defined earlier that each player will select e_i, x_i to maximize $E\overline{\pi}(e_i, x_i)-\sum_{\ell=1}^{M} p_\ell x_{i\ell}$ subject to the player's budget constraints.[5]

If even one agent assigns some small amount of strategic uncertainty to the choices of other players, and this uncertainty influences either market or game behavior to move the group away from the particular equilibrium outcome of the game, then this equilibrium is extremely fragile. Let (x^*, e^*, p^*) be a fully revealing rational expectations equilibrium where $p^*_m = \beta$ and 0 otherwise and $e^*_i = m$ for all i. Then allow one player to have a small amount of doubt in the form described above. We say that an equilibrium is *stable* if for all $\epsilon < \bar{\epsilon}$ there exists an allocation x' such that (x', e^*, p^*) remains a rational expectations equilibrium given these new beliefs. In other words, as long as the amount of doubt is sufficiently small, no player has an incentive to buy or sell assets such that she now also wants to select a different input level. An equilibrium is *unstable* if for any $\epsilon > 0$ the individual profit maximizing choice of x_i and e_i entails $e_i \neq e^*_i$. The following proposition characterizes the stability properties of these games:

PROPOSITION 3: *Let (x^*, e^*, p^*) be a fully revealing rational expectations equilibrium under $m_j(e)$. Then:*

1) *If $j = 1$, then all fully revealing rational expectations equilibria are stable.*
2) *If $1 < j < n$ then any fully revealing rational expectations equilibrium is unstable. Further,*
   a) *If $1 < j \leq \frac{n-1}{2}$ then the agent with doubt will always select a maximizing input and allocation combination that involves lower input or $e_i = m-1$.*

[4]Other input level choices are clearly dominated given the specification of beliefs we have used.
[5]Foor expositional ease we assume the budget constraint (or margin requirement) to be non-binding.

b) If $\frac{n+3}{2} \leq j < n$, then the agent with doubt will always select a maximizing input and allocation combination that involves higher input or $e_i = m+1$.

PROOF:

We begin by showing that any fully revealing rational expectations equilibrium is stable when $j = 1$. Note that by the earlier proposition all fully revealing equilibria when $j = 1$ involve $e_i^* = 1$ for all i. Given an allocation x^*, the relevant expected payoffs are given by:

$$E\overline{\pi}(1, x_i^*) = a + b_0 + \beta x_{i1}^* \tag{21}$$

$$E\overline{\pi}(2, x_i^*) = \mu_i(1|2)\left[a + b_0 - c + \beta x_{i1}^*\right] + \mu_i(2|2)\left[a + 2b_0 + \beta x_{i2}^*\right] \tag{22}$$

Player i will prefer to select $e_i = 2$ if and only if $E\overline{\pi}(2, x_i^*) \geq E\overline{\pi}(1, x_i^*)$ which is only true if

$$\mu_i(2|2) \geq \frac{c}{b_0 + c + \beta(x_{i2}^* - x_{i1}^*)} \tag{23}$$

or

$$\epsilon \geq \left(\frac{c}{b_0 + c + \beta(x_{i2}^* - x_{i1}^*)}\right)^{\frac{1}{n-1}}. \tag{24}$$

Therefore let $\bar{\epsilon}$ be less that this value. Then for any $\epsilon < \bar{\epsilon}$ player i will prefer to play the original input $e_i^* = 1$ and further the marginal value of holding additional units of x_2 is zero and the marginal value of holding additional units of x_1 is 1 so the agent is indifferent between the equilibrium asset allocation x_i^* and any other allocation.

Next, we show that for $1 < j < n$ any fully revealing equilibrium is unstable. Let $1 < j \leq \frac{n-1}{2}$. We demonstrate that the equilibrium is unstable via the following two steps. First, we show that for any ϵ the agent prefers to buy units of x_{m-1} regardless of his current utility maximizing plan of action. Second, we show that if the agents purchases enough x_{m-1} he will eventually prefer to play $e_i = m - 1$.

First, given that the market price of both x_{m-1} and x_{m+1} is $p_{m-1}^* = p_m^* = 0$, the agent should buy (infinitely) many units of whichever asset, given his plan of action e_i most greatly increases his expected utility. Note that $\frac{\partial E\overline{\pi}(k)}{\partial x_{i\ell}} = \mu_i(\ell|k)\beta$ so it is sufficient to show that $\mu_i(m-1|k) > \mu_i(m+1|k)$ for $k \in \{m-1, m, m+1\}$. If $1 < m < M$, then $j \leq \frac{n-1}{2}$ obviously implies that $j \leq n - j - 1$ and we have that

$$\sum_{k=j}^{n-j-1} \binom{n-1}{k}(1-\epsilon)^k \epsilon^{n-1-k} > 0 \tag{25}$$

$$\sum_{k=0}^{n-j-1} \binom{n-1}{k}(1-\epsilon)^k \epsilon^{n-1-k} - \sum_{k=0}^{j-1} \binom{n-1}{k}(1-\epsilon)^k \epsilon^{n-1-k} > 0 \tag{26}$$

$$\mu_i(m-1|m+1) - \mu_i(m+1|m+1) > 0 \tag{27}$$

$$\mu_i(m-1|m+1) > \mu_i(m+1|m+1) \tag{28}$$

It follows similarly that $\mu_i(m-1|m) > \mu_i(m+1|m)$ and $\mu_i(m-1|m-1) > \mu_i(m+1|m-1)$.

Second, since $\frac{\partial E\overline{\pi}(m-1)}{\partial x_{im-1}} = \mu_i(m-1|m-1)\beta$, $\frac{\partial E\overline{\pi}(m)}{\partial x_{im-1}} = \mu_i(m-1|m)\beta$, $\frac{\partial E\overline{\pi}(m+1)}{\partial x_{im-1}} = \mu_i(m-1|m+1)\beta$. Given that $\mu_i(m-1|m-1) > \mu_i(m-1|m) = \mu_i(m-1|m+1)$ we

know that eventually the expected payoff from playing $e_i = m-1$ will eventually surpass that of either m or $m+1$.

For $\frac{n+3}{2} \leq j < n$, show similarly that playing $e_i = m+1$ will eventually be preferred.

The first part of the proposition confirms the intuition that low input choices under the minimum order statistic output are indeed stable. This stems from the fact that, in equilibrium, all allocations must be such that either the agent has a weakly dominant strategy to play $e_i = 1$, or, if not, then playing $e_i = 1$ is still a Nash equilibrium. Moreover, given that $\mu_i(2|1) = 0$ in the minimum-input game, the agent has no incentive to purchase x_2 assets unless he or she actually plans on playing $e_i = 2$. This only happens when there is sufficient doubt $\epsilon > \bar{\epsilon}$.

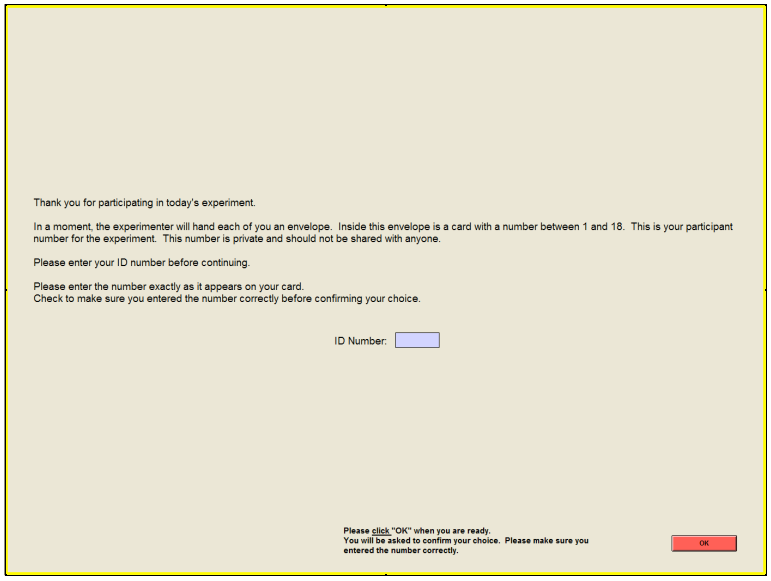
The second part of the proposition demonstrates that for higher order statistics the stability properties are quite different. Namely, any equilibrium is unstable to doubt as we have defined it since there is always a greater marginal value of purchasing assets associated with $m-1$ or $m+1$ than the zero prices on them. Since this marginal value is non-decreasing, the agent's demand is unbounded and she will always prefer some alternative input choice. Further we show that the expected deviations associated with low order statistics is always downward whereas the expected deviations associated with high order statistics is always upward. But the bounds provided in the proposition are not tight and there may exist j whereby the prediction of deviation is unclear. For example, if $j = 2$ then we know that for $n = 6$ if there is doubt we would expect players to prefer investment (and subsequent choice) in downward input from any equilibrium involving matched input greater than 1. However, for $n = 3$, neither of the conditions are satisfied and the effect of doubt is unclear. This is because at the median (as $j = 2$ is for $n = 3$) the deviations by others needed to either raise or lower the level of output are symmetric.

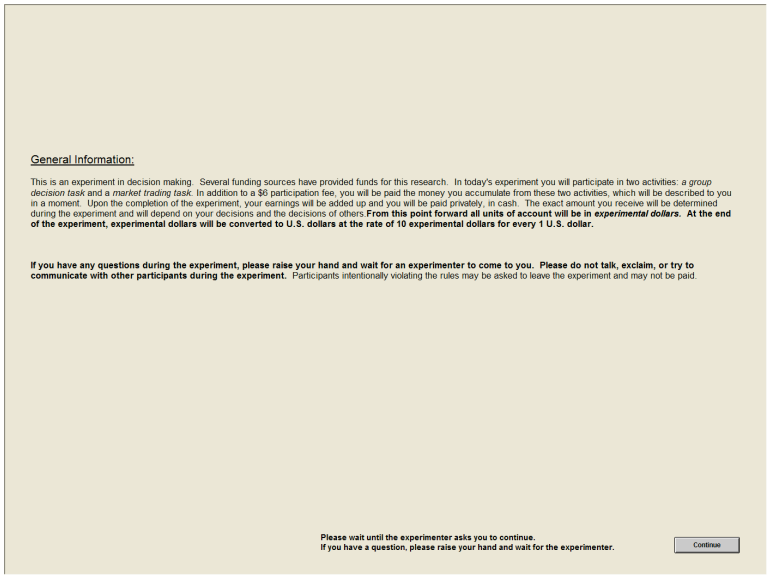
We can also examine the impact of similar doubt under the *Control* treatment where there is no market by forcing all trading portfolios $x_i = 0$. For any combination of order-statistic j and number of players n, there exists an $\bar{\epsilon}$ such that if all players are playing some matched input level m then if $\epsilon < \bar{\epsilon}$ the player with doubt will continue to prefer to play $e_i = m$ despite this doubt. However, if $\epsilon > \bar{\epsilon}$ the player will prefer to play either lower ($e_i = m-1$) input or higher ($e_i = m+1$) input. Table I contains a description of the cutoffs for the variants observed in our experiments. Importantly, in all cases, the amount of doubt that is needed to lead away from equilibrium is non-negligible. In contrast, the previous proposition tells us that *any* level of doubt with $j = 2$ will result in the player with doubt taking actions that will move the results away from the equilibrium and most likely in favor of lower input/output combinations.

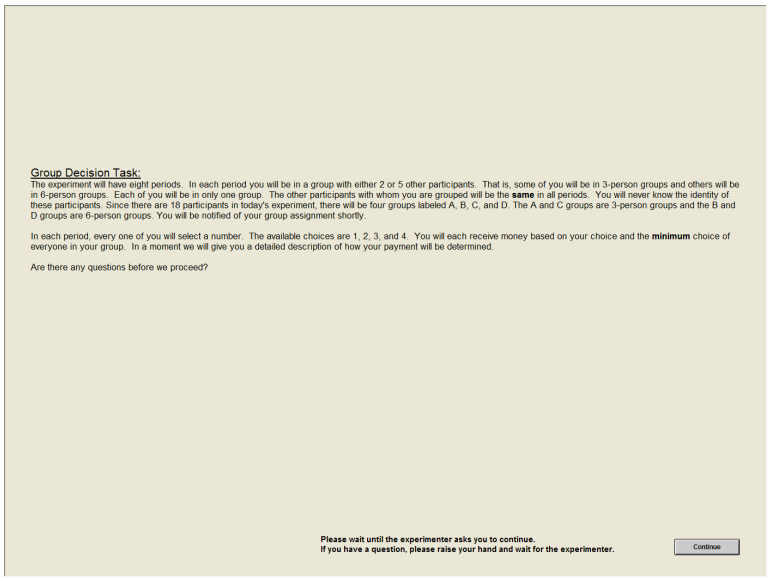
	$n = 3$	$n = 6$
$j = 1$	$\downarrow \epsilon > .293$	$\downarrow \epsilon > .129$
$j = 2$	$\uparrow \epsilon > .293$	$\downarrow \epsilon > .314$

TABLE 2—$\bar{\epsilon}$ UNDER THE *Control* TREATMENT SUCH THAT IF $\epsilon > \bar{\epsilon}$ THE PLAYER WILL PREFER TO DEVIATE FROM THE MATCHED EQUILIBRIUM STRATEGY OF $e_i = m$ FOR ALL PLAYERS. THE ARROW INDICATES THE DIRECTION OF DEVIATION BY THE PLAYER. WHEN THE DIRECTION OF DEVIATION IS NOT FEASIBLE ($m = 1$ OR $m = M$) THE OTHER DIRECTION OF DEVIATION IS ONLY FAVORED FOR UNFEASIBLE $\epsilon > .5$.

II. Experiment Instructions







Group Decision Task:

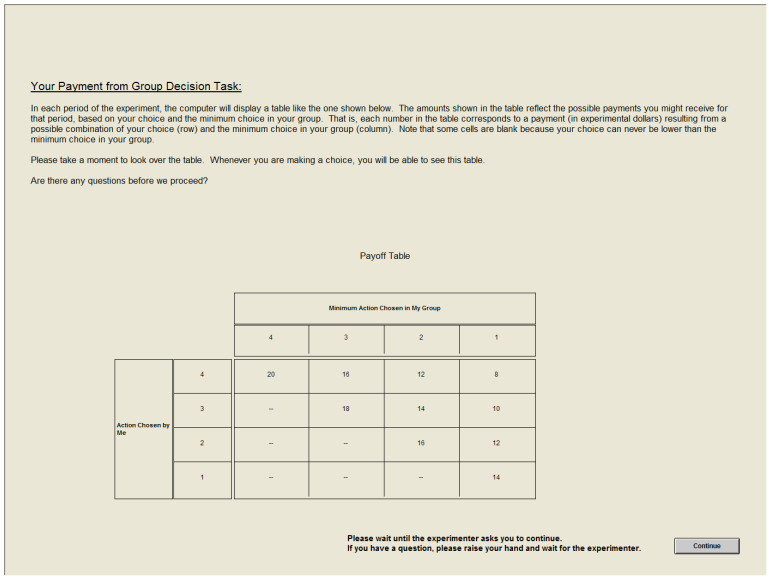
The experiment will have eight periods. In each period you will be in a group with either 2 or 5 other participants. That is, some of you will be in 3-person groups and others will be in 6-person groups. Each of you will be in only one group. The other participants with whom you are grouped will be the **same** in all periods. You will never know the identity of these participants. Since there are 18 participants in today's experiment, there will be four groups labeled A, B, C, and D. The A and C groups are 3-person groups and the B and D groups are 6-person groups. You will be notified of your group assignment shortly.

In each period, every one of you will select a number. The available choices are 1, 2, 3, and 4. You will each receive money based on your choice and the **minimum** choice of everyone in your group. In a moment we will give you a detailed description of how your payment will be determined.

Are there any questions before we proceed?

Please wait until the experimenter asks you to continue.
If you have a question, please raise your hand and wait for the experimenter.

Continue



Your Payment from Group Decision Task:

In each period of the experiment, the computer will display a table like the one shown below. The amounts shown in the table reflect the possible payments you might receive for that period, based on your choice and the minimum choice in your group. That is, each number in the table corresponds to a payment (in experimental dollars) resulting from a possible combination of your choice (row) and the minimum choice in your group (column). Note that some cells are blank because your choice can never be lower than the minimum choice in your group.

Please take a moment to look over the table. Whenever you are making a choice, you will be able to see this table.

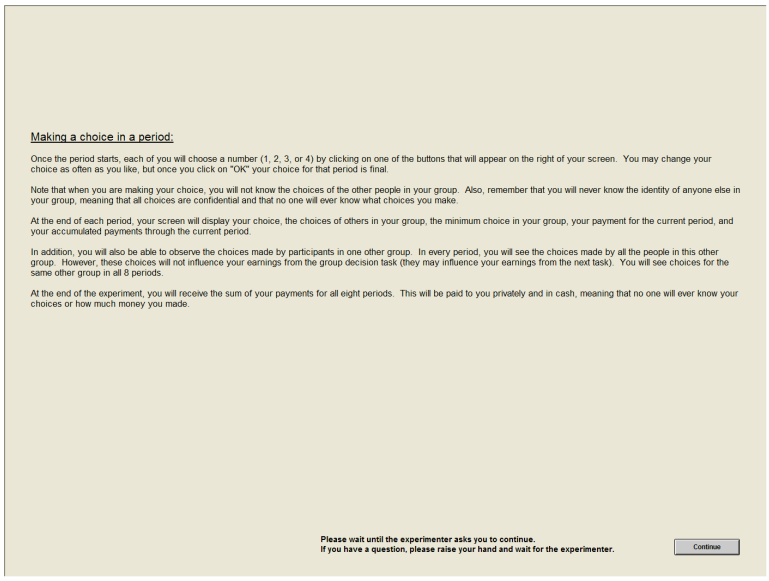
Are there any questions before we proceed?

Payoff Table

		Minimum Action Chosen in My Group			
		4	3	2	1
Action Chosen by Me	4	20	16	12	8
	3	--	18	14	10
	2	--	--	16	12
	1	--	--	--	14

Please wait until the experimenter asks you to continue.
If you have a question, please raise your hand and wait for the experimenter.

Continue



Making a choice in a period:

Once the period starts, each of you will choose a number (1, 2, 3, or 4) by clicking on one of the buttons that will appear on the right of your screen. You may change your choice as often as you like, but once you click on "OK" your choice for that period is final.

Note that when you are making your choice, you will not know the choices of the other people in your group. Also, remember that you will never know the identity of anyone else in your group, meaning that all choices are confidential and that no one will ever know what choices you make.

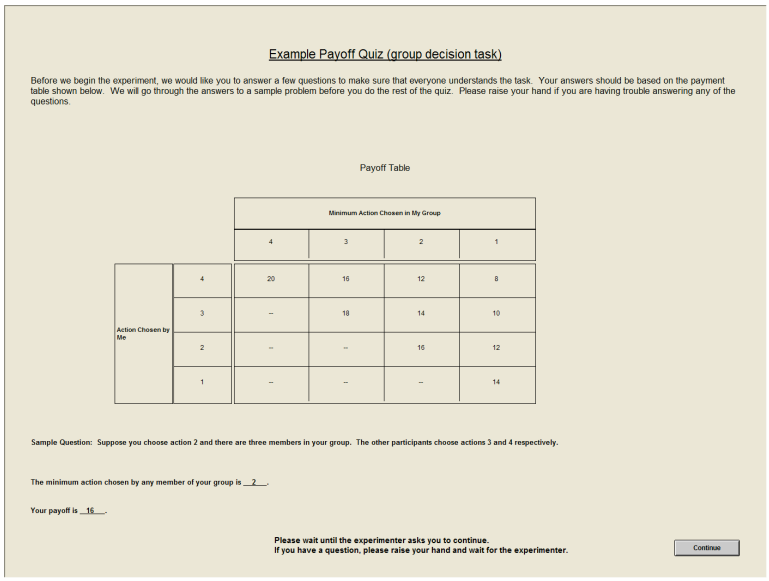
At the end of each period, your screen will display your choice, the choices of others in your group, the minimum choice in your group, your payment for the current period, and your accumulated payments through the current period.

In addition, you will also be able to observe the choices made by participants in one other group. In every period, you will see the choices made by all the people in this other group. However, these choices will not influence your earnings from the group decision task (they may influence your earnings from the next task). You will see choices for the same other group in all 8 periods.

At the end of the experiment, you will receive the sum of your payments for all eight periods. This will be paid to you privately and in cash, meaning that no one will ever know your choices or how much money you made.

Please wait until the experimenter asks you to continue.
If you have a question, please raise your hand and wait for the experimenter.

Continue



Example Payoff Quiz (group decision task)

Before we begin the experiment, we would like you to answer a few questions to make sure that everyone understands the task. Your answers should be based on the payment table shown below. We will go through the answers to a sample problem before you do the rest of the quiz. Please raise your hand if you are having trouble answering any of the questions.

Payoff Table

		Minimum Action Chosen in My Group			
		4	3	2	1
Action Chosen by Me	4	20	16	12	8
	3	--	18	14	10
	2	--	--	16	12
	1	--	--	--	14

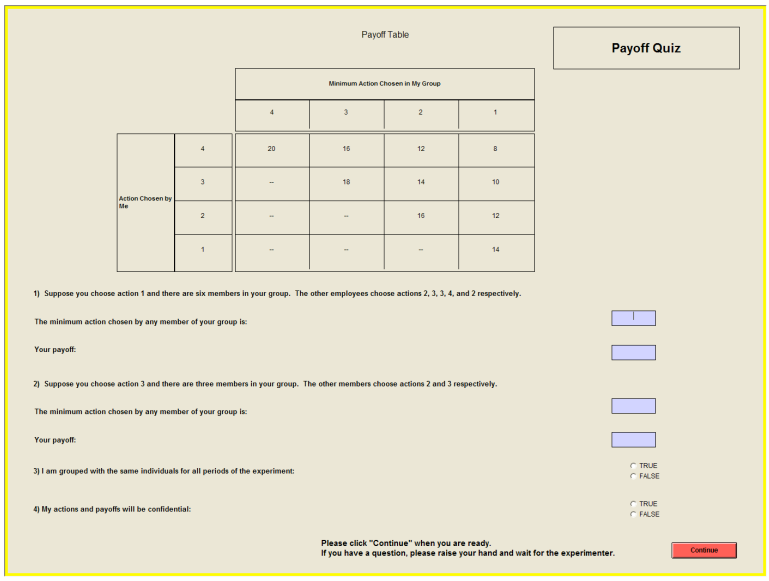
Sample Question: Suppose you choose action 2 and there are three members in your group. The other participants choose actions 3 and 4 respectively.

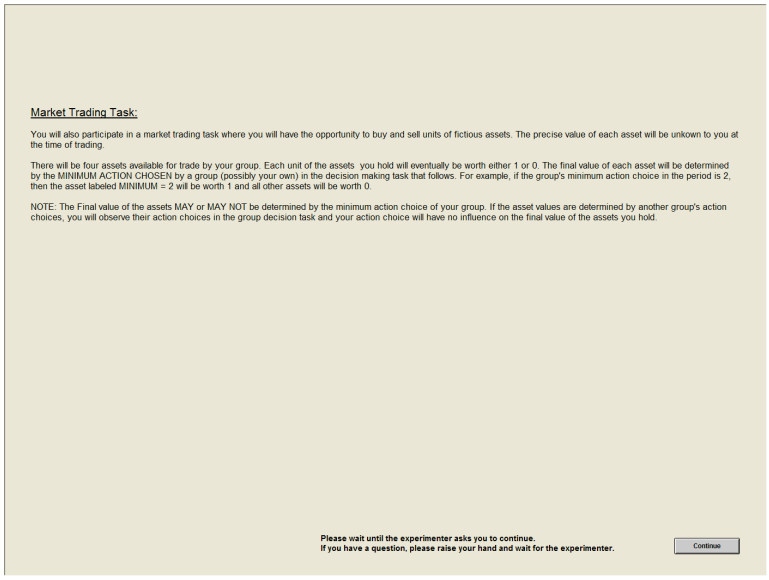
The minimum action chosen by any member of your group is __2__.

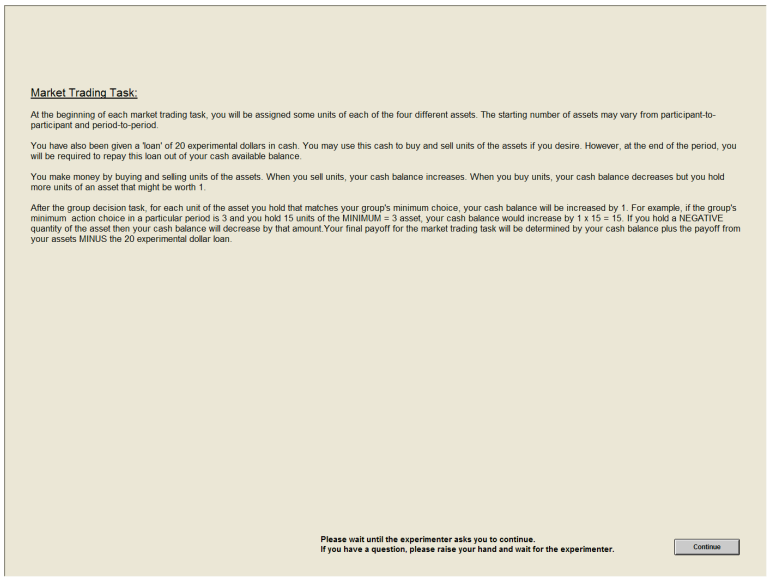
Your payoff is __16__.

Please wait until the experimenter asks you to continue.
If you have a question, please raise your hand and wait for the experimenter.

Continue


Payoff Table
Payoff Quiz
Minimum Action Chosen in My Group
4 3 2 1
Action Chosen by Me
4 20 16 12 8
3 -- 18 14 10
2 -- -- 16 12
1 -- -- -- 14
1) Suppose you choose action 1 and there are six members in your group. The other employees choose actions 2, 3, 3, 4, and 2 respectively.
The minimum action chosen by any member of your group is:
Your payoff:
2) Suppose you choose action 3 and there are three members in your group. The other members choose actions 2 and 3 respectively.
The minimum action chosen by any member of your group is:
Your payoff:
3) I am grouped with the same individuals for all periods of the experiment:
TRUE
FALSE
4) My actions and payoffs will be confidential:
TRUE
FALSE
Please click "Continue" when you are ready.
If you have a question, please raise your hand and wait for the experimenter.
Continue


Market Trading Task:
You will also participate in a market trading task where you will have the opportunity to buy and sell units of fictious assets. The precise value of each asset will be unkown to you at the time of trading.
There will be four assets available for trade by your group. Each unit of the assets you hold will eventually be worth either 1 or 0. The final value of each asset will be determined by the MINIMUM ACTION CHOSEN by a group (possibly your own) in the decision making task that follows. For example, if the group's minimum action choice in the period is 2, then the asset labeled MINIMUM = 2 will be worth 1 and all other assets will be worth 0.
NOTE: The Final value of the assets MAY or MAY NOT be determined by the minimum action choice of your group. If the asset values are determined by another group's action choices, you will observe their action choices in the group decision task and your action choice will have no influence on the final value of the assets you hold.
Please wait until the experimenter asks you to continue.
If you have a question, please raise your hand and wait for the experimenter.
Continue



Market Trading Task:

At the beginning of each market trading task, you will be assigned some units of each of the four different assets. The starting number of assets may vary from participant-to-participant and period-to-period.

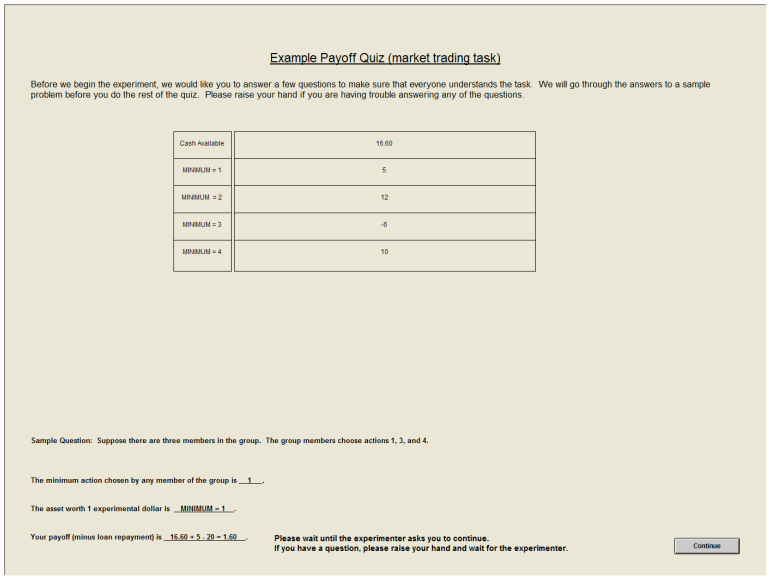
You have also been given a 'loan' of 20 experimental dollars in cash. You may use this cash to buy and sell units of the assets if you desire. However, at the end of the period, you will be required to repay this loan out of your cash available balance.

You make money by buying and selling units of the assets. When you sell units, your cash balance increases. When you buy units, your cash balance decreases but you hold more units of an asset that might be worth 1.

After the group decision task, for each unit of the asset you hold that matches your group's minimum choice, your cash balance will be increased by 1. For example, if the group's minimum action choice in a particular period is 3 and you hold 15 units of the MINIMUM = 3 asset, your cash balance would increase by 1 x 15 = 15. If you hold a NEGATIVE quantity of the asset then your cash balance will decrease by that amount.Your final payoff for the market trading task will be determined by your cash balance plus the payoff from your assets MINUS the 20 experimental dollar loan.

Please wait until the experimenter asks you to continue.
If you have a question, please raise your hand and wait for the experimenter.

Continue



Example Payoff Quiz (market trading task)

Before we begin the experiment, we would like you to answer a few questions to make sure that everyone understands the task. We will go through the answers to a sample problem before you do the rest of the quiz. Please raise your hand if you are having trouble answering any of the questions.

Cash Available	16.60
MINIMUM = 1	5
MINIMUM = 2	12
MINIMUM = 3	-5
MINIMUM = 4	10

Sample Question: Suppose there are three members in the group. The group members choose actions 1, 3, and 4.

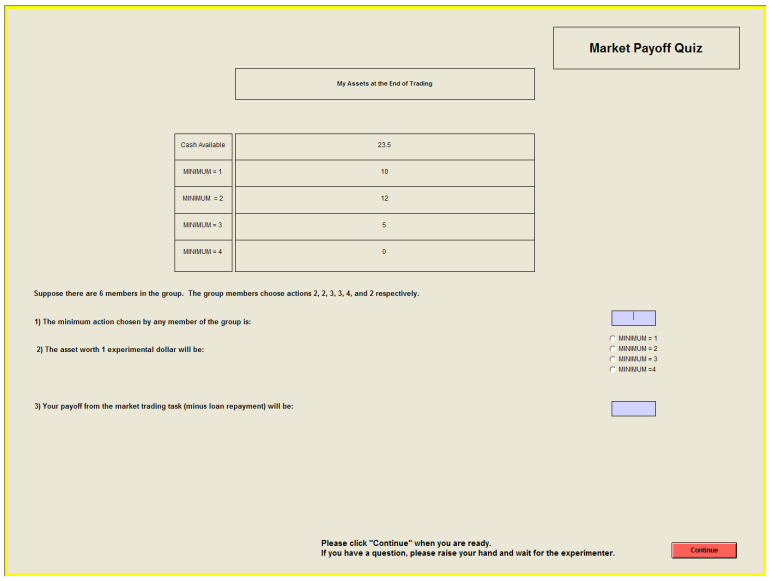
The minimum action chosen by any member of the group is __1__.

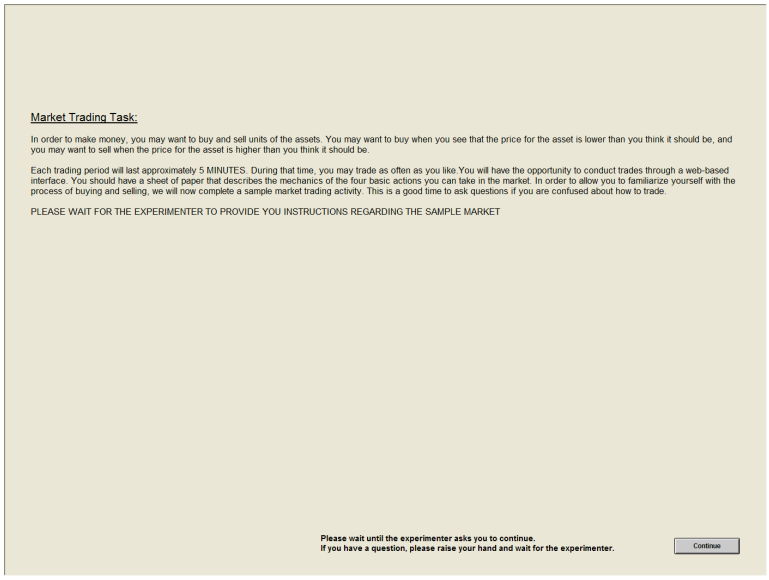
The asset worth 1 experimental dollar is __MINIMUM = 1__.

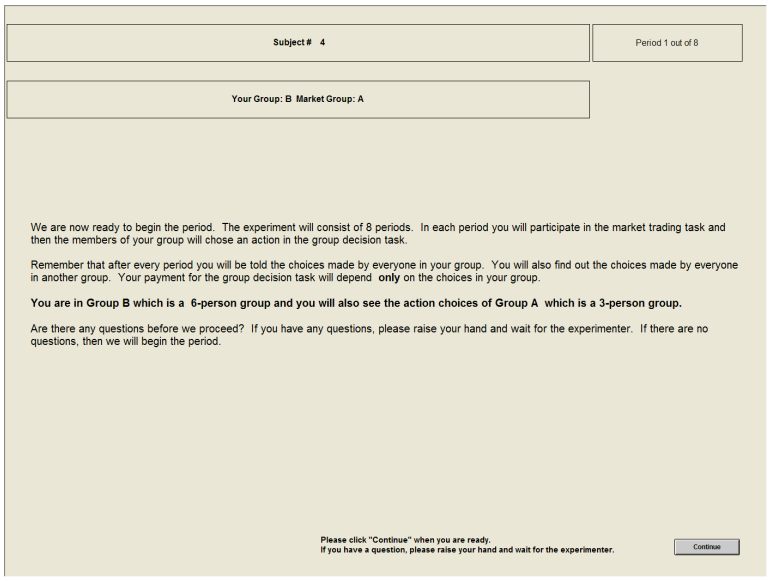
Your payoff (minus loan repayment) is __16.60 + 5 . 20 = 1.60__.

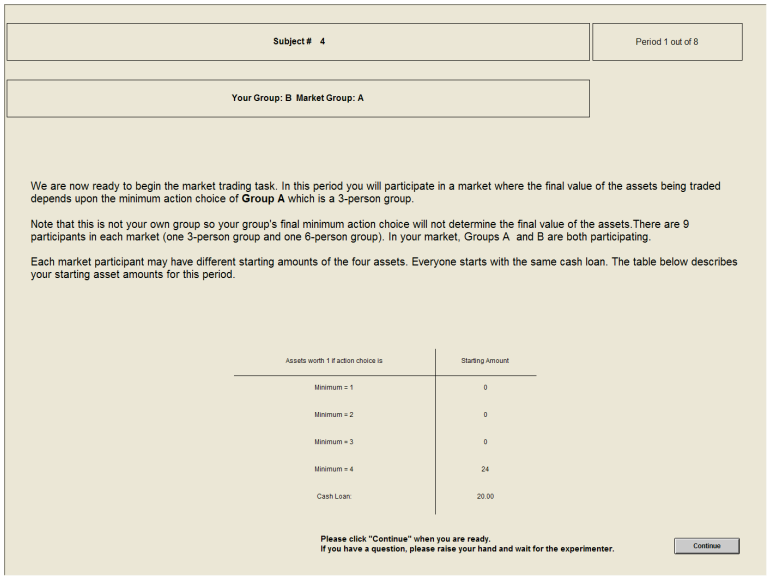
Please wait until the experimenter asks you to continue.
If you have a question, please raise your hand and wait for the experimenter.

Continue

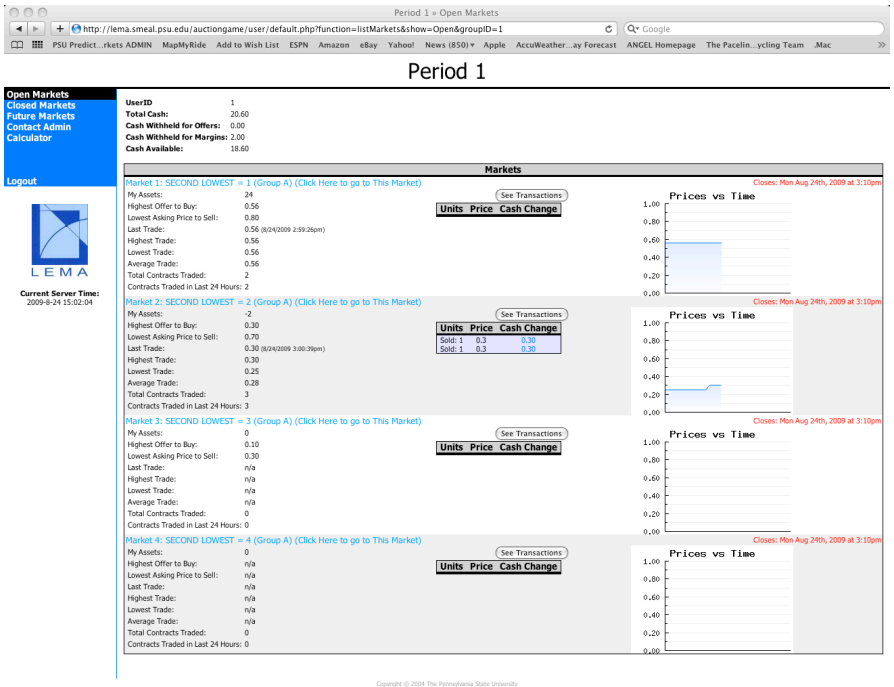

Market Payoff Quiz
My Assets at the End of Trading
Cash Available
23.5
MINIMUM = 1
10
MINIMUM = 2
12
MINIMUM = 3
5
MINIMUM = 4
0
Suppose there are 6 members in the group. The group members choose actions 2, 2, 3, 3, 4, and 2 respectively.
1) The minimum action chosen by any member of the group is:
2) The asset worth 1 experimental dollar will be:
MINIMUM = 1
MINIMUM = 2
MINIMUM = 3
MINIMUM =4
3) Your payoff from the market trading task (minus loan repayment) will be:
Please click "Continue" when you are ready.
If you have a question, please raise your hand and wait for the experimenter.
Continue

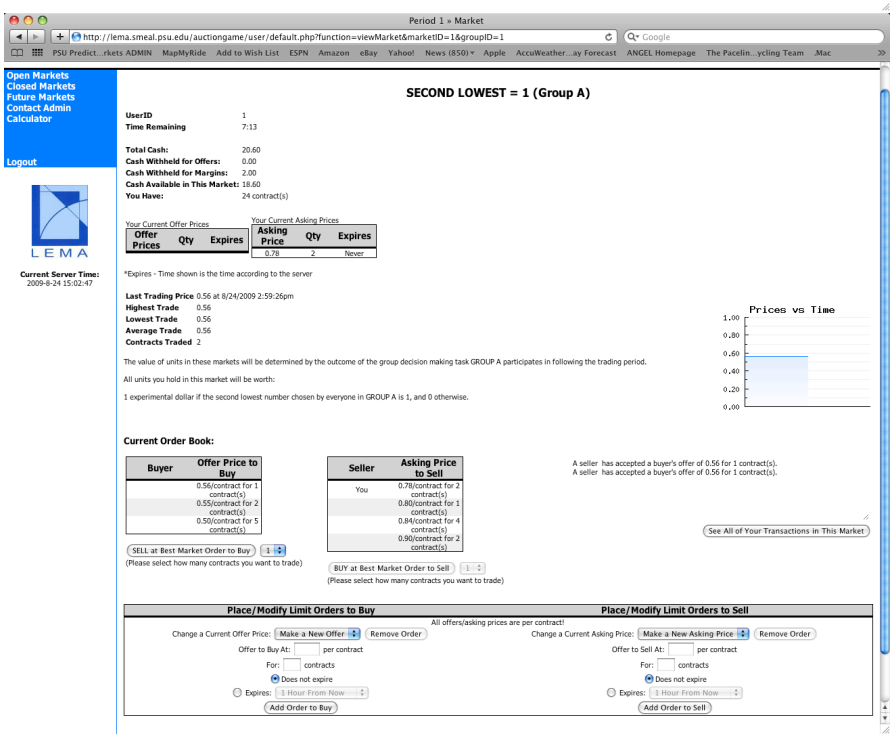

Market Trading Task:
In order to make money, you may want to buy and sell units of the assets. You may want to buy when you see that the price for the asset is lower than you think it should be, and you may want to sell when the price for the asset is higher than you think it should be.
Each trading period will last approximately 5 MINUTES. During that time, you may trade as often as you like.You will have the opportunity to conduct trades through a web-based interface. You should have a sheet of paper that describes the mechanics of the four basic actions you can take in the market. In order to allow you to familiarize yourself with the process of buying and selling, we will now complete a sample market trading activity. This is a good time to ask questions if you are confused about how to trade.
PLEASE WAIT FOR THE EXPERIMENTER TO PROVIDE YOU INSTRUCTIONS REGARDING THE SAMPLE MARKET
Please wait until the experimenter asks you to continue.
If you have a question, please raise your hand and wait for the experimenter.
Continue


Subject # 4
Period 1 out of 8
Your Group: B Market Group: A
We are now ready to begin the period. The experiment will consist of 8 periods. In each period you will participate in the market trading task and then the members of your group will chose an action in the group decision task.
Remember that after every period you will be told the choices made by everyone in your group. You will also find out the choices made by everyone in another group. Your payment for the group decision task will depend only on the choices in your group.
You are in Group B which is a 6-person group and you will also see the action choices of Group A which is a 3-person group.
Are there any questions before we proceed? If you have any questions, please raise your hand and wait for the experimenter. If there are no questions, then we will begin the period.
Please click "Continue" when you are ready.
If you have a question, please raise your hand and wait for the experimenter.
Continue


Subject # 4
Period 1 out of 8
Your Group: B Market Group: A
We are now ready to begin the market trading task. In this period you will participate in a market where the final value of the assets being traded depends upon the minimum action choice of Group A which is a 3-person group.
Note that this is not your own group so your group's final minimum action choice will not determine the final value of the assets.There are 9 participants in each market (one 3-person group and one 6-person group). In your market, Groups A and B are both participating.
Each market participant may have different starting amounts of the four assets. Everyone starts with the same cash loan. The table below describes your starting asset amounts for this period.
Assets worth 1 if action choice is
Starting Amount
Minimum = 1
0
Minimum = 2
0
Minimum = 3
0
Minimum = 4
24
Cash Loan:
20.00
Please click "Continue" when you are ready.
If you have a question, please raise your hand and wait for the experimenter.
Continue

III. Trading Interface





IV. Data Tables

TABLE 3—**Group-Level Data - Experiment 1**

Descriptors			Period							
Session	Group Size	Characteristics	1	2	3	4	5	6	7	8
111	3	Insider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Market-H	6	Outsider Output	2.00	2.00	1.00	1.00	1.00	1.00	1.00	1.00
		Expected Output	2.52	2.23		2.27		1.60		
114	6	Insider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Market-H	3	Outsider Output	2.00	2.00	3.00	3.00	2.00	3.00	3.00	3.00
		Expected Output	2.57	2.45	2.43	2.28	2.33	2.27		
121	3	Insider Output	2.00	3.00	3.00	3.00	1.00	2.00	3.00	3.00
Market-H	6	Outsider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Expected Output			2.40	2.37		2.24		
124	6	Insider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Market-H	3	Outsider Output	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
		Expected Output		2.62	2.25	2.34	2.53			
131	3	Insider Output	4.00	2.00	1.00	1.00	1.00	1.00	1.00	1.00
Market-H	6	Outsider Output	3.00	2.00	3.00	2.00	2.00	1.00	1.00	1.00
		Expected Output	2.53	2.34	2.49	2.43	2.02	1.86		
134	6	Insider Output	2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Market-H	3	Outsider Output	2.00	2.00	1.00	1.00	1.00	1.00	1.00	1.00
		Expected Output	2.42	2.39		1.61	1.47	1.54		1.25
141	3	Insider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Market-H	6	Outsider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Expected Output	1.69	1.69	1.66	1.08		1.36		
144	6	Insider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Market-H	3	Outsider Output	2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Expected Output			1.59				1.22	
151	3	Insider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Market-H	6	Outsider Output	4.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00
		Expected Output	2.07			1.36		1.35	1.11	1.07
154	6	Insider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Market-H	3	Outsider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Expected Output		2.10	2.08	1.60		1.60		
161	3	Insider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Market-H	6	Outsider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Expected Output	2.36		2.04	1.90	1.52	1.58	2.00	1.69
164	6	Insider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Market-H	3	Outsider Output	1.00	2.00	1.00	1.00	2.00	2.00	1.00	1.00
		Expected Output	2.38		1.69					
211	3	Insider Output	3.00	3.00	3.00	3.00	2.00	3.00	4.00	4.00
Market-L	6	Outsider Output	2.00	1.00	2.00	2.00	2.00	2.00	2.00	2.00
		Expected Output	2.40	2.44	2.52	2.63	2.57	2.50	2.47	2.63
221	3	Insider Output	3.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Market-L	6	Outsider Output	2.00	1.00	2.00	2.00	2.00	2.00	2.00	2.00
		Expected Output	2.58	2.46	2.58	2.79	2.69	2.60	2.66	2.70
234	6	Insider Output	2.00	2.00	2.00	2.00	2.00	2.00	1.00	2.00
Market-L	3	Outsider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Expected Output	2.44	2.30	2.20	2.01			2.02	1.78
244	6	Insider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Market-L	3	Outsider Output	1.00	2.00	1.00	1.00	2.00	2.00	1.00	1.00
		Expected Output	2.09	2.09	2.23	1.58		1.19		1.12
251	3	Insider Output	1.00	2.00	2.00	1.00	3.00	2.00	3.00	2.00
Market-L	6	Outsider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Expected Output			2.79		2.47		2.74	2.73
254	6	Insider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Market-L	3	Outsider Output	2.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
		Expected Output	2.28	2.39	2.31			1.70		
261	3	Insider Output	2.00	1.00	1.00	1.00	1.00	2.00	2.00	1.00
Market-L	6	Outsider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Expected Output	2.26	2.40	1.73		1.68	1.67	1.74	1.57
274	6	Insider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Market-L	3	Outsider Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Expected Output	2.35	1.76	1.70	1.76	2.26	1.54		
11	3	Group 1	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Control	6	Group 2	3.00	3.00	3.00	1.00	2.00	2.00	1.00	1.00
13	3	Group 1	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Control	6	Group 2	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
21	3	Group 1	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Control	6	Group 2	3.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00
23	3	Group 1	1.00	2.00	3.00	3.00	3.00	3.00	3.00	3.00
Control	6	Group 2	3.00	2.00	2.00	3.00	3.00	3.00	3.00	3.00
31	3	Group 1	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Control	6	Group 2	3.00	4.00	4.00	4.00	4.00	4.00	4.00	1.00
33	3	Group 1	3.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Control	6	Group 2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
41	3	Group 1	2.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
Control	6	Group 2	3.00	4.00	3.00	3.00	3.00	3.00	3.00	3.00
43	3	Group 1	3.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Control	6	Group 2	1.00	1.00	2.00	2.00	2.00	2.00	1.00	1.00

TABLE 4—**Group-Level Data - Experiment 2**

Descriptors			Period							
Session	Group Size	Characteristics	1	2	3	4	5	6	7	8
090423_1340_1	3	Insider Output	3.00	2.00	2.00	3.00	3.00	3.00	3.00	3.00
Market-L	6	Outsider Output	2.00	2.00	1.00	2.00	2.00	2.00	2.00	2.00
		Expected Output	2.27	2.27		2.13	2.32	2.21	2.42	2.91
090423_1340_4	6	Insider Output	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Market-L	3	Outsider Output	3.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
		Expected Output	2.90	2.72		2.09	2.98	2.92	3.64	3.41
090424_1223_1	3	Insider Output	2.00	4.00	3.00	4.00	2.00	3.00	3.00	3.00
Market-L	6	Outsider Output	2.00	3.00	3.00	3.00	3.00	3.00	3.00	2.00
		Expected Output	2.30	2.23	2.73	3.08	2.63	2.55	2.70	2.87
090424_1223_4	6	Insider Output	3.00	3.00	3.00	3.00	3.00	3.00	3.00	2.00
Market-L	3	Outsider Output	4.00	4.00	4.00	4.00	4.00	4.00	4.00	3.00
		Expected Output	2.25	2.43	2.50	2.67	2.53	2.55	2.71	2.62
090427_1559_1	3	Insider Output	3.00	4.00	3.00	4.00	3.00	3.00	4.00	3.00
Market-L	6	Outsider Output	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
		Expected Output	2.26	2.36	2.53	2.45	2.55	2.47	2.48	2.58
090427_1559_4	6	Insider Output	2.00	2.00	3.00	3.00	3.00	3.00	2.00	2.00
Market-L	3	Outsider Output	3.00	1.00	2.00	2.00	2.00	3.00	2.00	2.00
		Expected Output	2.42	2.46	2.50	2.43	2.47	2.56	2.69	2.54
090428_1326_1	3	Insider Output	4.00	3.00	4.00	4.00	4.00	4.00	4.00	4.00
Market-L	6	Outsider Output	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
		Expected Output	2.33	2.73	2.85	2.61	2.71	3.08	3.21	2.93
090428_1326_4	6	Insider Output	2.00	3.00	3.00	3.00	3.00	2.00	3.00	2.00
Market-L	3	Outsider Output	4.00	3.00	4.00	3.00	3.00	4.00	4.00	4.00
		Expected Output	2.61	2.80	2.82	2.61	2.75	2.61	2.53	2.70
090413_1131_1	3	Group 1	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Control	6	Group 2	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
090413_1131_3	3	Group 1	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Control	6	Group 2	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
090414_1409_1	3	Group 1	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Control	6	Group 2	3.00	3.00	3.00	4.00	4.00	4.00	4.00	4.00
090414_1409_3	3	Group 1	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Control	6	Group 2	3.00	3.00	4.00	4.00	4.00	4.00	4.00	4.00
090430_1432_1	3	Group 1	4.00	3.00	3.00	3.00	4.00	4.00	4.00	4.00
Control	6	Group 2	3.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
090430_1432_3	3	Group 1	3.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Control	6	Group 2	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00

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