Online Appendix for “Using Lagged Outcomes to Evaluate Bias in Value-Added Model”

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In this appendix, we provide further detail on how we estimate teacher value-added. The Stata code used to generate the simulated data, estimate VA, and produce the results is contained in two files: (1) simulations_table1_final.do, which generates the results in Table 1 and (2) simulations_fig1_final.do, which generates the results in Figure 1.

We estimate the value-added model in four steps:

1. Calculate student test-score gains in year \( t = 2 \) as \( \Delta A_{i,t=2} = A_{i,t=2} - A_{i,t=1} \) and classroom-specific average gains \( \Delta \bar{A}_{c,t=2} = \sum_{i \in c} \Delta A_{i,t=2} \). To simplify notation, we drop the \( t = 2 \) subscript for the remainder of this description.

2. Decompose the variance of test score gains into its four constituent pieces according to

\[
\text{Var}(\Delta A_{i,t=2}) = \text{Var}(\psi_{st} - \psi_{s,t-1}) + \text{Var}(\mu_j) + \text{Var}(\theta_t) + \text{Var}(\varepsilon_{it} - \tilde{\varepsilon}_{i,t-1})
\]

where \( \tilde{\varepsilon}_{i,t-1} = \varepsilon_{i,t-1} + \theta_{c(i,t-1),t-1} + \mu_{j(i,t-1)} \). We estimate these variance components as follows:

(a) Track-Level Variance: We estimate the track-level variance component of score gains as the covariance between classroom average scores in classrooms in the same track taught by different teachers:

\[
\widehat{\text{Var}}(\psi_{st} - \psi_{s,t-1}) = \text{Cov}(\Delta \bar{A}_c, \Delta \bar{A}_{c'}) |_{s(c)=s(c'), j(c) \neq j(c')}
\]

(b) Teacher-Level Variance: We estimate the sum of track-level and teacher-level variance as the covariance between average scores in classrooms taught by the same teacher. We then subtract the estimate of track-level variance from (a) to estimate teacher-level variance:

\[
\hat{\sigma}_\mu^2 = \text{Var}(\mu_j) = \text{Cov}(\Delta \bar{A}_c, \Delta \bar{A}_{c'}) |_{j(c)=j(c')} - \text{Var}(\psi_{st} - \psi_{s,t-1})
\]

(c) Individual-Level Variance: We estimate the individual level variance as the variance of test scores within classrooms, adjusted for the degrees of freedom. Note that the shock \( \varepsilon_{i,t-1} \) has greater variance than \( \varepsilon_{it} \) since it includes both the lagged teacher shock and lagged classroom shock (neither of which aggregate to the classroom level because students are reshuffled across classrooms in practice):

\[
\text{Var}(\varepsilon_{it} - \tilde{\varepsilon}_{i,t-1}) = \text{Var}(\Delta A_i - \Delta \bar{A}_c) * \frac{I}{I-1}
\]
(d) Class-Level Variance: We estimate the class-level variance as the residual variance present in the aggregate variance of test score gains after subtracting out the other three components:

\[
\hat{\sigma}_\theta^2 = \text{Var}(\Delta A_i) - \text{Var}(\psi_{st} - \psi_{s,t-1}) - \hat{\sigma}_\mu^2 - \text{Var}(\varepsilon_{it} - \tilde{\varepsilon}_{i,t-1})
\]

3. Calculate the shrinkage factor using the four component variances:

\[
\lambda = \frac{\hat{\sigma}_\mu^2}{\hat{\sigma}_\mu^2 + \text{Var}(\psi_{st} - \psi_{s,t-1}) + \frac{\hat{\sigma}_\theta^2}{C} + \frac{\text{Var}(\varepsilon_{it} - \tilde{\varepsilon}_{i,t-1})}{C*I}}
\]

4. Estimate value-added for each teacher:

\[
\hat{\mu}_j = \lambda \Delta \bar{A}_j
\]
Appendix Table 1. Baseline Parameters for Monte Carlo Simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Schools</td>
<td>2000</td>
</tr>
<tr>
<td>Number of Tracks per School</td>
<td>5</td>
</tr>
<tr>
<td>Number of Teachers per Track</td>
<td>4</td>
</tr>
<tr>
<td>Number of Classrooms per Teacher ($C$)</td>
<td>4</td>
</tr>
<tr>
<td>Number of Students per Classroom ($I$)</td>
<td>25</td>
</tr>
<tr>
<td>SD of Student Ability Levels ($\sigma_s$)</td>
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<tr>
<td>SD Of Student Ability Trends ($\sigma_\alpha$)</td>
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</tr>
<tr>
<td>SD Of Teacher Value-Added ($\sigma_\mu$)</td>
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</tr>
<tr>
<td>SD of Classroom Shocks ($\sigma_\theta$)</td>
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</tr>
<tr>
<td>SD of Track Level Shocks ($\sigma_\phi$)</td>
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</tr>
<tr>
<td>Correlation of Track Shocks Across Grades ($\rho$)</td>
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</tr>
<tr>
<td>Degree of Sorting on Levels: Corr($\delta,\mu$)</td>
<td>0.25</td>
</tr>
<tr>
<td>Degree of Sorting on Trends: Corr($\alpha,\mu$)</td>
<td>0.00</td>
</tr>
</tbody>
</table>