Online Appendices
for

Firm Dynamics, Job Turnover, and Wage Distributions in an Open Economy

by

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1 The Revenue Function

Value added net of exporting costs as a function of z and l (equation 12 in the paper) is given by

$$R(z,l) = \max_m \left\{ G(zl^{\alpha}m^{1-\alpha}) - Pm - c_x \mathcal{I}^x (zl^{\alpha}m^{1-\alpha}) \right\}. \tag{1}$$

Given $G(z,l,m) = \exp\left[d_H + d_F(\eta^0)\right] \left(zl^{\alpha}m^{(1-\alpha)}\right)^{\frac{\sigma-1}{\sigma}}$, the first order condition for m reads as

$$Pm = (1-\alpha)\frac{(\sigma-1)}{\sigma} \exp\left[d_H + \mathcal{I}^x d_F(\eta^0)\right] \left(zl^{\alpha}m^{(1-\alpha)}\right)^{\frac{\sigma-1}{\sigma}},$$

or rearranging,

$$m = \left(\frac{(1-\alpha)}{P} \frac{\sigma-1}{\sigma} \exp\left[d_H + \mathcal{I}^x d_F(\eta^0)\right] \right)^{\frac{\sigma}{\sigma-1}\Lambda} (zl^{\alpha})^{\Lambda}, \tag{2}$$

where $\Lambda = \frac{\sigma-1}{\sigma-(1-\alpha)(\sigma-1)} > 0$.

Using this expression to eliminate m from $G(z,l,m)$ yields:

$$\begin{aligned}
G(z,l) &= \exp\left[d_H + \mathcal{I}^x d_F(\eta^0)\right] (zl^{\alpha})^{\frac{\sigma-1}{\sigma}} \\
&\left\{ \left(\frac{(1-\alpha)}{P} \frac{\sigma-1}{\sigma} \exp\left[d_H + \mathcal{I}^x d_F(\eta^0)\right] \right)^{\frac{\sigma}{\sigma-1}\Lambda} (zl^{\alpha})^{\Lambda} \right\}^{\frac{(1-\alpha)(\sigma-1)}{\sigma}}, \\
&= \exp\left[d_H + \mathcal{I}^x d_F(\eta^0)\right] (zl^{\alpha})^{\frac{\sigma-1}{\sigma}} \\
&\left\{ \left(\frac{1-\alpha}{P} \right) \left(\frac{\sigma-1}{\sigma} \right) \exp\left[d_H + \mathcal{I}^x d_F(\eta^0)\right] \right\}^{(1-\alpha)\Lambda} (zl^{\alpha})^{\Lambda\frac{(1-\alpha)(\sigma-1)}{\sigma}}, \\
&= P^{-(1-\alpha)\Lambda} \left[(1-\alpha) \left(\frac{\sigma-1}{\sigma} \right) \right]^{(1-\alpha)\Lambda} \exp\left[d_H + \mathcal{I}^x d_F(\eta^0)\right]^{\frac{\sigma}{\sigma-1}\Lambda} (zl^{\alpha})^{\Lambda},
\end{aligned}$$

where the derivation uses the fact that

$$\frac{\sigma-1}{\sigma}+\Lambda\frac{(1-\alpha)(\sigma-1)}{\sigma}=\frac{(\sigma-1)}{\sigma}\left[1+(1-\alpha)\Lambda\right]=\Lambda.$$

We can now derive a parameterized version of the net revenue function (equation 13 in the paper). From (2), optimal expenditures on intermediate inputs are:

$$Pm=P^{-(1-\alpha)\Lambda}\left[\left((1-\alpha)\frac{\sigma-1}{\sigma}\right)\exp\left[d_H+\mathcal{I}^x d_F(\eta^0)\right]\right]^{\frac{\sigma}{\sigma-1}\Lambda}(zl^\alpha)^\Lambda.$$

Subtracting this expression and fixed exporting costs from gross revenues yields:

$$\begin{aligned}
R(z,l) &= G(z,l)-Pm-c_x\mathcal{I}^x(z,l)\\
&=\left[1-(1-\alpha)\frac{\sigma-1}{\sigma}\right]P^{-(1-\alpha)\Lambda}\left((1-\alpha)\frac{\sigma-1}{\sigma}\right)^{(1-\alpha)\Lambda}\\
&\exp\left[d_H+\mathcal{I}^x d_F(\eta^0)\right]^{\frac{\sigma}{\sigma-1}\Lambda}(zl^\alpha)^\Lambda-c_x\mathcal{I}^x(z,l)\\
&=\left[\frac{\sigma-(1-\alpha)(\sigma-1)}{\sigma}\right]P^{-(1-\alpha)\Lambda}\left((1-\alpha)\frac{\sigma-1}{\sigma}\right)^{(1-\alpha)\Lambda}\\
&\exp\left[d_H+\mathcal{I}^x d_F(\eta^0)\right]^{\frac{\sigma}{\sigma-1}\Lambda}(zl^\alpha)^\Lambda-c_x\mathcal{I}^x(z,l)\\
&=P^{-(1-\alpha)\Lambda}\left(\frac{\sigma-1}{\sigma\Lambda}\right)\left((1-\alpha)\frac{\sigma-1}{\sigma}\right)^{(1-\alpha)\Lambda}\\
&\exp\left[d_H+\mathcal{I}^x d_F(\eta^0)\right]^{\frac{\sigma}{\sigma-1}\Lambda}(zl^\alpha)^\Lambda-c_x\mathcal{I}^x(z,l)\\
&=\underbrace{\Theta P^{-(1-\alpha)\Lambda}\exp\left[d_H+\mathcal{I}^x d_F(\eta^0)\right]^{\frac{\sigma}{\sigma-1}\Lambda}}_{=\Delta(z,l)}(zl^\alpha)^\Lambda-c_x\mathcal{I}^x(z,l),
\end{aligned}$$

where $\Theta=\left(\frac{1}{(1-\alpha)\Lambda}\right)\left[\frac{(1-\alpha)(\sigma-1)}{\sigma}\right]^{\frac{\sigma}{\sigma-1}\Lambda}$. Note that $\alpha\Lambda<1$, so the net revenue function displays diminishing marginal returns to labor.

2 Wages and Markups

2.1 Hiring Wages

For a hiring firm in state (z',l'), the marginal worker generates surplus

$$\Pi^{firm}=\frac{1}{1+r}\left[\frac{\partial\tilde{\pi}(z',l')}{\partial l'}+\frac{\partial V(z',l')}{\partial l'}\right], \tag{3}$$

where $\tilde{\pi}(z', l') = R(z', l') - w(z', l')l'$ measures current period profits and $\frac{\partial V(z', l')}{\partial l'}$ measures the effect of the marginal worker on next period's expected hiring or firing costs.[1] Note that it is possible that a firm hires in the current period and exits at the beginning of the next period. In this case, the effect of the marginal worker on the firm's continuation value is $\frac{\partial V(z', l')}{\partial l'} = 0$.

Given the wage schedule $w(z', l')$, the marginal worker at this hiring firm enjoys surplus:

$$\Pi^{work} = \frac{1}{1+r}\left[w(z', l') + J^e(z', l') - (b + J^0)\right]. \tag{4}$$

Combined with the Nash bargaining condition,

$$(1-\beta)\,\Pi^{work} = \beta \Pi^{firm}, \tag{5}$$

these expressions allow us to derive the hiring wage schedule. The formulation of the bargaining problem follows Bertola and Caballero (1994), Bertola and Garibaldi (2001), and Koeniger and Prat (2007). If one assumes that a firm has to pay the firing costs in case the bargaining fails, then these costs should be subtracted from its threat point. Ljungqvist (2002) discusses alternative assumptions for the case of one-worker one-firm matching models. We assume that separations are costless when bargaining fails (which don't happen in equilibrium), so firing costs do not figure into (3).

The derivation first uses (3), (4) and (5) to express total worker surplus in terms of total firm surplus:

$$\Pi^{work} = \frac{\beta}{1-\beta}\frac{1}{1+r}\left[\frac{\partial \tilde{\pi}(z', l')}{\partial l'} + \frac{\partial V(z', l')}{\partial l'}\right]. \tag{6}$$

Assuming that continuation values are shared the same way as current flows, we can write:[2]

$$\beta \frac{\partial V(z', l')}{\partial l'} = (1-\beta)[J^e(z', l') - J^0]. \tag{7}$$

[1] To see the latter, let $I^f(z'', l')$ be an indicator function for whether a firm in interim state (z'', l') will fire workers. Also, let $I^c(z', l')$ be an indicator function for whether a firm beginning next period in state (z', l') will continue to operate. Then the envelope theorem implies the derivative of the continuation value of the marginal worker can be written as the probability-weighted average of the expected savings in hiring costs next period, given expansion, and the extra firing costs incurred, given contraction:

$$\begin{aligned}
\frac{\partial V(z', l')}{\partial l'} &= \frac{\partial E_{z''|z'} \max_{l''} \left[\pi(z'', l', l'') + V(z'', l'')\right]}{\partial l'}, \\
&= E_{z''|z'} \frac{\partial \pi(z'', l', l'')}{\partial l'}, \\
&= E_{z''|z'} I^c(z', l') \left[\left[1 - I^f(z'', l')\right] \frac{\partial C_h(l', l'')}{\partial l'} - I^f(z'', l') c_f\right].
\end{aligned}$$

[2] An alternative assumption would be to share the continuation values according to the bargaining rule *only when they are positive*, with firms absorbing all firing costs when they are incurred. An earlier version of our paper used this sharing rule.

Substituting these sharing rules into the worker's surplus equation (4) and the total surplus sharing rule (6), the wage function must solve:

$$\frac{\beta}{1-\beta}\left[\frac{\partial\tilde{\pi}(z',l')}{\partial l'}+\frac{\partial V(z',l')}{\partial l'}\right]=w(z',l')-b+\frac{\beta}{(1-\beta)}\frac{\partial V(z',l')}{\partial l'}. \tag{8}$$

Cancelling terms, re-arranging, and using $\frac{\partial\tilde{\pi}(z',l')}{\partial l'}=\frac{\partial R(z',l')}{\partial l'}-w(z',l')-\frac{\partial w(z',l')}{\partial l'}l'$, equation (8) implies:

$$w(z',l')=(1-\beta)b+\beta\left[\frac{\partial R(z',l')}{\partial l'}-\frac{\partial w(z',l')}{\partial l'}l'\right]. \tag{9}$$

Finally, following footnote 10 in Koeninger and Prat (2007), which in turn is based on Bertola and Garibaldi (2001), one can solve this differential equation. First re-write (9) in the form:

$$\frac{dx(l')}{dl'}+x(l')p(l')+q(l')=0, \tag{10}$$

where

$$x(l')=w(z',l'),$$

$$p(l')=1/(\beta l'),$$

and

$$q(l')=-\left[\frac{\partial R(z',l')}{\partial l'}+\frac{(1-\beta)}{\beta}b\right]/l'.$$

Plugging these expressions for $x(l')$, $p(l')$, and $q(l')$ into the solution to equation (10), and recalling $\frac{\partial R(z,l)}{\partial l}=\Delta\alpha\Lambda z^{\Lambda}l^{\alpha\Lambda-1}$, we can then express wages as[3]

$$w(z',l')=(l')^{-1/\beta}\int_0^{l'}\Delta\alpha\Lambda\left(z'\right)^{\Lambda}x^{\frac{1-\beta}{\beta}}x^{\alpha\Lambda-1}dx+(1-\beta)b.$$

Integration yields the wage expression in the text:

$$w_h(z',l')=(1-\beta)b+\frac{\beta}{1-\beta+\alpha\beta\Lambda}\underbrace{\Delta\alpha\Lambda\left(z'\right)^{\Lambda}\left(l'\right)^{\alpha\Lambda-1}}_{=\partial R(z',l')/\partial l'}. \tag{11}$$

2.2 Firing Wages

To derive the firing wage schedule, we begin by writing the value of employment at a firing firm in the interim stage as

[3]Here, we suppressed the dependence of $\Delta(\cdot)$ on l' since $\partial\Delta/\partial l'=0$ if the firm's exporting decision does not depend on the marginal worker. Since workers bargain individually and simultaneously with the firm, no single worker will be taken as the marginal worker for the export decision.

$$J_f^e(z',l) = \frac{1}{1+r}\big[p_f(z',l)(1+r)J^u + (1-p_f(z',l))\,(w_f(z',l') + J^e(z',l'))\,\big],$$

where $l' = L(z',l)$ and $p_f(z',l) = (l-l')/l'$ is the probability of being fired. Workers randomly laid off in the interim stage go back to the unemployment pool and can search for a job within the same period. Those who are retained do so for a wage level $w_f(z',l')$ that leaves them indifferent between staying and leaving. For this to be the case, the two terms inside the bracket must be equal, i.e.,

$$w_f(z',l') + J^e(z',l') = (1+r)J^u,$$

which yields the wage schedule according to which workers in firing firms are paid:

$$w_f(z',l') = rJ^u - [J^e(z',l') - J^u].$$

2.3 Price-Cost Mark-ups

Price-cost mark-ups vary across firms in our model because firms choose employment levels to maximize their expected net profit stream rather than their current operating profits. This subsection derives expressions for firm-level prices and marginal costs, then uses them to show how price-cost mark-ups vary across state space.

Consider prices first. Using equation (2) above, intermediate inputs m can be eliminated from the production function, $q = zl^\alpha m^{1-\alpha}$, obtaining each firm's output as a function of its state (l,z) and economywide aggregates:

$$q = (zl^\alpha)^{1+\Lambda(1-\alpha)}\left(\frac{(1-\alpha)}{P}\frac{\sigma-1}{\sigma}\exp\left[d_H + \mathcal{I}^x d_F(\eta^0)\right]\right)^{\frac{\sigma(1-\alpha)\Lambda}{\sigma-1}} \tag{12}$$

Plugging this expression for q into the demand function (equation 5 of the text), one can then calculate prices at each point in (l,z) space.

Marginal production costs are more tedious to derive. Inverting (12), one can write l as a monotonic function of q, given variables that are exogenous to firms: z, P, d_H, and $d_F(\eta^0)$. Accordingly, marginal production costs can be expressed as:

$$\frac{d\mathcal{C}}{dq} = \frac{\partial w}{\partial l}\frac{\partial l}{\partial q}l + w\frac{\partial l}{\partial q} + P\frac{\partial m}{\partial l}\frac{\partial l}{\partial q}$$

where $\mathcal{C} = wl + Pm$.

Each partial derivative in this expression can then be calculated. First, using equation (2) from the online appendix:

$$\frac{\partial m}{\partial l} = \alpha\Lambda\frac{m}{l}$$

Next, using equation (12) above, one finds:

$$\frac{\partial l}{\partial q} = \frac{l}{\alpha\left[1+\Lambda(1-\alpha)\right]q}$$

Finally, the slope of the wage function for hiring firms follows from equation (26) of the text:

$$\frac{\partial w_h}{\partial l} = \left[\alpha\Lambda - 1\right]\frac{w_h - (1-\beta)b}{l}$$

No closed-form expression exists for firing firms, but $\frac{\partial w_f}{\partial l}$ could be calculated numerically from equation (27).

Putting the pieces together, marginal costs at hiring firms are:[4]

$$\begin{aligned}
\left.\frac{d\mathcal{C}}{dq}\right|_{hiring} &= \frac{\partial w^h}{\partial l}\frac{\partial l}{\partial q}l + w^h\frac{\partial l}{\partial q} + P\frac{\partial m}{\partial l}\frac{\partial l}{\partial q} \\
&= \frac{\left[\alpha\Lambda-1\right]\left[w_h-(1-\beta)b\right]l}{\alpha\left[1+\Lambda(1-\alpha)\right]q} + \frac{w_h l}{\alpha\left[1+\Lambda(1-\alpha)\right]q} + \alpha\Lambda\frac{Pm}{l}\frac{l}{\alpha\left[1+\Lambda(1-\alpha)\right]q} \\
&= \frac{\left[\alpha\Lambda-1\right]\left[w_h-(1-\beta)b\right]+w_h}{\alpha\left[1+\Lambda(1-\alpha)\right]q}l + \frac{\alpha\Lambda}{\alpha\left[1+\Lambda(1-\alpha)\right]}\left(\frac{Pm}{q}\right) \\
&= \frac{\left(1-\alpha\Lambda\right)(1-\beta)b}{\alpha\left[1+\Lambda(1-\alpha)\right]}\frac{l}{q} + \frac{\sigma-1}{\sigma}\left(\frac{w_h l + Pm}{q}\right) \qquad (13)
\end{aligned}$$

This is the standard relationship between average cost and marginal cost with imperfectly competitive product markets (second term), adjusted for the fact that firms have monopsony power (first term). The first term is positive because $\alpha\Lambda < 1$.

Using the demand equations (equations 5 and 7 of the text) and equation (13) above, figures A1 plots hiring firms' mark-ups, $\left(p - \left.\frac{d\mathcal{C}}{dq}\right|_{hiring}\right) / \left.\frac{d\mathcal{C}}{dq}\right|_{hiring}$, in (z,l) space. Conditional on productivity level z, mark-ups are decreasing in firm size l but they jump up when a firm becomes an exporter by crossing the size threshold. Among hiring firms, exporters all lie on this ridge or to its northeast, and few have low productivity, so their mark-ups tend to be higher than those of non-exporters. The plot of firing firms' mark-ups (not pictured)

[4] To get the last equality, note that $\Lambda = \frac{\sigma-1}{\sigma-(1-\alpha)(\sigma-1)}$ implies:

$$1+\Lambda(1-\alpha) = 1 + \frac{(\sigma-1)(1-\alpha)}{\sigma-(1-\alpha)(\sigma-1)} = \frac{\sigma-(1-\alpha)(\sigma-1)+(\sigma-1)(1-\alpha)}{\sigma-(1-\alpha)(\sigma-1)} = \frac{\sigma}{\sigma-(1-\alpha)(\sigma-1)}.$$

Hence: $\frac{\Lambda}{[1+\Lambda(1-\alpha)]} = \frac{(\sigma-1)}{\sigma}$.

is similar except in that these firms face a different wage schedule.

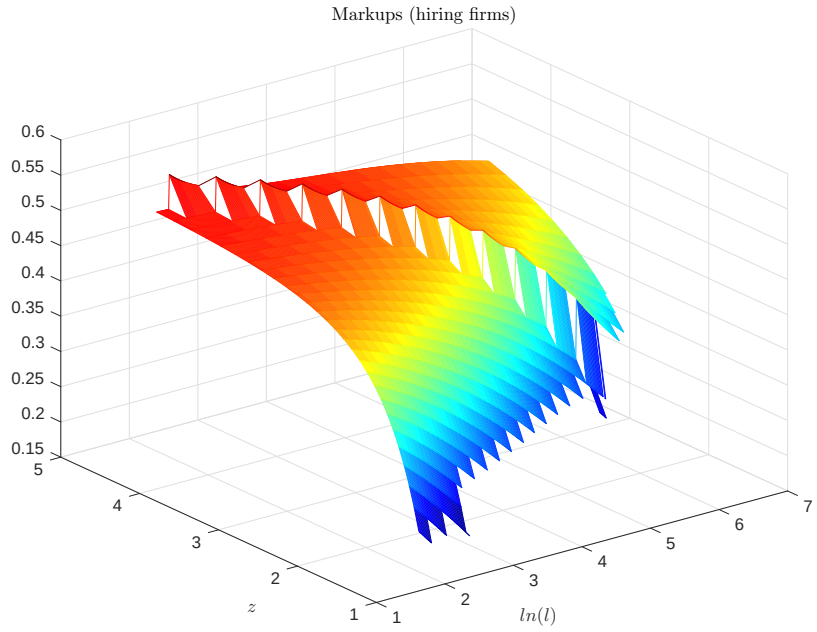


Figure A1: Mark-ups For Hiring Firms

3 Steady State Equilibrium

Let the transition density of the Markov process on z be denoted by $\Omega(z'|z)$. Given a measure of aggregate expenditure abroad denominated in foreign currency, D_F^*, a steady state equilibrium for a small open economy consists of a measure of domestic differentiated goods N_H; an exact price index for the composite good P; an aggregate domestic demand index for industrial goods D_H; aggregate income I; a measure of workforce in services L_s; a measure workers in differentiated goods sector L_q; a measure of workers searching for jobs in the industrial sector U; a measure of unemployed workers L_u; the job finding rate $\widetilde{\phi}$; the vacancy filling rate ϕ; the exit rate μ_{exit}; the fraction of firms exporting μ_x; the measure of entrants N_e; the value and associated policy functions $\mathcal{V}(z,l)$, $L(z,l)$, $\mathcal{I}^h(z,l)$, $\mathcal{I}^c(z,l)$, $\mathcal{I}^x(z,l)$, J^o, J^u, J^s, and J^e; the wage schedules $w_h(z,l)$ and $w_f(z,l)$; the exchange rate k; and end-of period and interim distributions $\psi(z,l)$ and $\widetilde{\psi}(z,l)$ satisfying the following conditions:

1. **Steady state distributions:** In equilibrium, $\psi(z,l)$ and $\widetilde{\psi}(z',l)$ reproduce themselves through the Markov processes on z, the policy functions, and the productivity draws upon entry. The interim distribution is composed of incumbent firms that did

not exit at the beginning of the period and entering firms. Since all entering firms start the interim period with l_e workers and a productivity draw from $\psi_e(z')$, we can measure the firms in state (z', l) as:

$$\widetilde{\psi}(z', l) = \begin{cases} (1-\delta)\int_z \Omega(z'|z)\psi(z,l)\mathcal{I}^c(z,l)dz & \text{if } l \neq l_e \\ \frac{N_e}{N_H}\psi_e(z') + (1-\delta)\int_z \Omega(z'|z)\psi(z,l)\mathcal{I}^c(z,l)dz & \text{if } l = l_e \end{cases},$$

where $\frac{N_e}{N_H}$ is the fraction of firms that turn over every period for exogenous or endogenous reasons:

$$\frac{N_e}{N_H} = 1 - (1-\delta)\int_l\int_z \psi(z,l)\mathcal{I}^c(z,l)dzdl.$$

Applying the policy function for employment choices to $\widetilde{\psi}(z', l)$, we obtain the end-of period distribution of firms across states:

$$\psi(z', l') = \frac{\int_l \widetilde{\psi}(z', l)\mathcal{I}_{L(z',l)=l'}dl}{\int_{z''}\int_l \widetilde{\psi}(z'', l)\mathcal{I}_{L(z'',l)=l'}dz''dl},$$

where $\mathcal{I}_{L(z',l)=l'} = 1$ if $L(z', l) = l'$ and $\mathcal{I}_{L(z',l)=l'} = 0$ otherwise.

2. **Market clearance in the service sector:** Demand for services comes from two sources: consumers spend a $(1-\gamma)$ fraction of aggregate income I on it, and firms demand it to pay their fixed operation and exporting costs, as well as labor adjustment and market entry costs. Aggregate income I itself is the sum of wage income earned by service and industrial sector workers, market services supplied by unemployed workers, tariff revenues rebated to worker-consumers, and aggregate profits in the industrial sector distributed to worker-consumers who own the firms, given by

$$\begin{aligned} I = & \underbrace{\int_l\int_z \psi(z,l)[\mathcal{I}^h(z,l)w_h(z,l) + (1-\mathcal{I}^h(z,l)w_f(z,)l]ldzdl + L_s + L_u b}_{\text{total wage income}} \\ & + \underbrace{D_H\tau_a^{-\sigma}(\tau_c k)^{1-\sigma}(\tau_a - 1)}_{\text{tariff revenue}} \\ & + \underbrace{N_H\int_l\int_z \psi(z,l)[\pi(z,l,L(z,l)) - \mu_{exit}c_e]dzdl}_{\text{aggregate profits}}. \end{aligned}$$

The average labor adjustment cost for firms in the interim with l is given by

$$\overline{c} = \int_z\int_l C(l, L(z,l))\widetilde{\psi}(z,l)dldz.$$

The market clearance condition is then given by

$$L_s + bL_u = (1-\gamma)I + N_H(\overline{c} + c_p + \mu_x c_x) + N_e c_e.$$

3. **Labor market clearing:** At the beginning of each period, the total number of industrial production jobs is

$$L_q = N_H \overline{l},$$

where

$$\overline{l} = \int_z \int_l l\psi(z,l)dldz, \tag{14}$$

is the sector's average employment. Some of these jobs are destroyed as firms exit—for exogenous or endogenous reasons—or downsize. Summing these sources of job destruction, we obtain our measure of industrial workers who are thrown into unemployment before the interim period:

$$\begin{aligned}\tilde{U} = {} & \delta N_H \int_z \int_l l\psi(z,l)dldz \\ & + (1-\delta)N_H \int_z \int_l [1-\mathcal{I}^c(z,l)]l\psi(z,l)dldz \\ & + N_H \int_{z'} \int_l \mathcal{I}^f(z',l)[l - L(z',l)]\tilde{\psi}(z',l)dldz'\end{aligned}$$

Recall that $\mathcal{I}^f(z',l) = 0$ for all entering firms, which must hire at least l_e workers. The associated rate of job destruction is:

$$\mu_l = \frac{\tilde{U}}{L_q}. \tag{15}$$

In the steady state equilibrium there are no net flows of workers out of the service sector. Accordingly, the total number of industrial job seekers each period includes those who just lost their jobs ($\tilde{U}$), and those who searched unsuccessfully for industrial jobs last period (L_u):

$$U = \tilde{U} + L_u, \tag{16}$$

Since $L_u = (1-\widetilde{\phi})U$, equations (15) and (16) imply:

$$U\widetilde{\phi} = L_q \mu_l.$$

That is, the number of workers flowing into industrial jobs, $U\widetilde{\phi}$, must match the number of industrial jobs that are turning over. Finally, at the end of each period, workers

either must have jobs in one of the sectors or be unsuccessful industrial job seekers:

$$1 = L_s + L_q + L_u.$$

On the vacancies side, the aggregate number of vacancies posted is given by

$$V = N_H \int_{z'} \int_l v(z', l) \widetilde{\psi}(z', l) dl dz' + N_e l_e / \phi,$$

which includes the l_e/ϕ vacancies that entrants post to hire their initial workforce. The total number of vacancies, V, together with U, determines matching probabilities $\phi(V, U)$ and $\widetilde{\phi}(V, U)$ that firms and workers take as given.

4. **Firm turnover:** In equilibrium, there is a positive mass of entry N_e every period so that the free entry condition

$$\mathcal{V}_e = \frac{1}{1+r} \int_z \max_{l'} \left[\pi(z, l_e, l') + \mathcal{V}(z, l')\right] \psi_e(z) dz \leq c_e, \tag{17}$$

holds with equality. The fraction of firms exiting is implied by the steady state distribution and the exit policy function,

$$\mu_{exit} = (1-\delta) \int_z \int_l [1 - \mathcal{I}^c(z, l)] \psi(z, l) dl dz + \delta,$$

and measure of exits equals that of entrants,

$$N_e = \mu_{exit} N_H.$$

5. **Trade balance:** Adding up final and intermediate demand, total domestic expenditures on imported varieties equals $D_H \left(\tau_a \tau_c k\right)^{1-\sigma}$. Taking the import tariff into account, domestic demand for foreign currency (expressed in domestic currency) is thus $\frac{D_H (\tau_a \tau_c k)^{1-\sigma}}{\tau_a} = D_H \tau_a^{-\sigma} (\tau_c k)^{1-\sigma}$. Tariff revenue is given by $D_H \tau_a^{-\sigma} (\tau_c k)^{1-\sigma} (\tau_a - 1)$, and is returned to worker-consumers in the form of lump-sum transfers. Total export revenues are $\frac{k D_F^* P_X^{*1-\sigma}}{\tau_c}$ with the foreign market price index for exported goods P_X^* as defined in Section I.C in the paper. Trade is balance given by

$$\underbrace{\frac{D_H \left(\tau_a \tau_c k\right)^{1-\sigma}}{\tau_a}}_{\text{domestic demand for foreign currency}} = \underbrace{\frac{k D_F^* P_X^{*1-\sigma}}{\tau_c}}_{\text{export revenue}}.$$

The exchange rate k moves to ensure that this condition holds. Balanced trade ensures

that national income matches national expenditure.

6. Workers are indifferent between taking a certain job in the undifferentiated sector and searching for a job in the industrial sector: $J^o = J^s = J^u$.

4 Numerical Solution Algorithm

To compute the value functions, we discretize the state space on a log scale using 550 grid points for employment and 60 grid points for productivity. We set the maximum firm size as 2000 workers and numerically check that this is not restrictive. In the steady state, a negligible fraction of firms reaches this size, which is also the case in the data. The algorithm works as follows:

1. Formulate guesses for D_H, $w_f(z,l)$, $w_h(z,l)$, d_F and ϕ. Given ϕ, calculate $\widetilde{\phi} = (1-\phi^\theta)^{1/\theta}$.

2. Given D_H, $w_f(z,l)$, d_F, ϕ and $w_h(z,l)$, calculate the value function for the firm, $\mathcal{V}(z,l)$ as
$$\mathcal{V}(z,l) = \max\left\{0, \quad \frac{1-\delta}{1+r} E_{z'|z} \max_{l'} \left[\pi(z',l,l') + \mathcal{V}(z',l')\right]\right\}, \tag{18}$$
and find the associated decision rules for exiting, hiring, and exporting. Calculate the expected value of entry, $\mathcal{V}_e$, using equation (17). Compare $\mathcal{V}_e$ with c_e. If $\mathcal{V}_e > c_e$, decrease D_H (to make entry less valuable) and if $\mathcal{V}_e < c_e$, increase D_H (to make entry more valuable). Go back to Step 1 with the updated value of D_H and repeat until D_H converges.

3. Given $w_f(z,l)$, d_F, ϕ and the converged value of D_H from Step 2, update $w_f(z,l)$. To do this, first calculate $J^e(z',l')$ using
$$J_h^e(z',l) = \frac{1}{1+r}\big[w_h(z',l') + J^e(z',l')\big], \tag{19}$$
and
$$\begin{aligned} J^e(z,l) = {} & \big[\delta + (1-\delta)\big(1-\mathcal{I}^c(z,l)\big)\big] J^u \\ & + (1-\delta)\mathcal{I}^c(z,l) \max\big\{J^u, E_{z'|z}\big[\mathcal{I}^h(z',l) J_h^e(z',l) + (1-\mathcal{I}^h(z',l)) J^u\big]\big\}, \end{aligned} \tag{20}$$
and imposing the equilibrium condition $J^u = J^o$. Given $J^e(z,l)$, update firing wage schedule using
$$w_f(z',l') = rJ^u - [J^e(z',l') - J^u]. \tag{21}$$

Compare the updated firing wage schedule with the initial guess. If they are not close enough go back to Step 1 with the new firing wage schedule and repeat Steps 1 to 3 until w_f converges. Note that if firing wages are too high, then $J^e(z,l)$—the value of being in a firm at the start of a period— is high, since the firm is less likely to fire workers. A high value of $J^e(z,l)$, however, lowers firing wages. Similarly, if firing wages are too low, then J^e is low, which pushes firing wages up.

4. Given d_F and ϕ, the converged value of D_H from step 2, and the converged value of $w_f(z,l)$ from Step 3, update $w_h(z,l)$ using

$$w_h(z',l') = (1-\beta)b + \frac{\beta}{1-\beta+\alpha\beta\Lambda}\Delta(z',l')\alpha\Lambda\left(z'\right)^{\Lambda}\left(l'\right)^{\alpha\Lambda-1}. \tag{22}$$

5. Given ϕ, the converged value of D_H from Step 2, the converged value of $w_f(z,l)$ from Step 3, and the converged value of $w_h(z,l)$ from step 4, calculate the trade balance. To do this:

   (a) Given firms' decisions, calculate $\psi(z,l)$ and $\widetilde{\psi}(z,l)$, the stationary probability distributions over (z,l) at the end and interim states, respectively.

   (b) Given $\widetilde{\psi}(z,l)$, calculate the average number of vacancies $\overline{v} = \int_{z'}\int_{l} v(z',l)\widetilde{\psi}(z',l)dldz'$ and the average employment in the industrial sector using equation 14.

   (c) Take a guess for N_H. Firms' decisions and the steady state distribution $\psi(z,l)$ pin down exit rate μ_{exit} as defined above, which implies a mass of entrants $N_e = \mu_{exit}N_H$. Given $N_H, N_e, \overline{v}$ and $v_e = l_e/\phi$, calculate the unique mass of unemployed U in the industrial sector solving

$$\phi(V,U) = \frac{M(V,U)}{V} = \frac{U}{\left[(N_H\overline{v}+N_e v_e)^{\theta}+U^{\theta}\right]^{1/\theta}}.$$

   Given U, calculate $L_u = (1-\widetilde{\phi})U$. Then, given $\overline{l}$, the size employment in the service sector is given by $L_s = 1 - L_u - N_H\overline{l}$. With N_H, L_s, L_u, N_e, and I (aggregate income) at hand, check if supply and demand are equal in the service sector:

$$\underbrace{L_s + bL_u}_{\text{supply}} = \underbrace{(1-\gamma)\,I + N_H(\overline{c}+c_p+\mu_x c_x) + N_e c_e}_{\text{demand}}.$$

   Update N_H until this market clearance condition holds.

   (d) Given the value of N_H from Step 5c, calculate exports and imports. If exports are larger than imports, lower d_F; if exports are less than imports, increase d_F.

Go back to Step 1 with the updated value of d_F, and repeat until convergence.

6. Given the converged value of D_H from Step 2, the converged value of $w_f(z,l)$ from Step 3, the converged value of $w_h(z,l)$ from Step 4, and the converged value of d_F from Step 5, update ϕ. In order to do that, first calculate EJ_h^e using (19). Given EJ_h^e and $\widetilde{\phi}$, calculate J^u using

$$J^u = \left[\widetilde{\phi} EJ_h^e + \frac{(1-\widetilde{\phi})}{1+r}\left(b + J^o\right)\right]. \tag{23}$$

If $J^o > J^u$, increase ϕ (to attract workers to the differentiated goods sector) and if $J^o < J^u$, we lower ϕ (to make the differentiated goods sector less attractive). Go back to Step 2, and repeat until ϕ converges.

4.1 Estimation Procedure

In the estimation of the model, we set D_H and d_F to their data counterparts, which allows us to skip Steps 2 and 5d above and considerably reduces the computation time. In order to compute D_H, we use the Olley-Pakes intercept $\tilde{d}_H$ estimated from

$$\ln G_{it} = \widetilde{d}_H + \mathcal{I}_{it}^x d_F(\eta_0) + \left[\frac{\sigma-1}{\sigma}(1-\alpha)\right]\ln(Pm_{it}) + \varphi(\ln l_{it-1}, \ln l_{it}) + \xi_{it}, \tag{24}$$

to calculate firms' net revenue schedule $R(\cdot)$.

Furthermore, in the estimation, we treat foreign market size D_F^* as a parameter to be estimated and d_F as a moment to be matched. Given $\tilde{d}_H$ and the estimated value of the foreign market size parameter D_F^*, we calculate η using

$$\eta^o = \arg\max_{0\leq\eta\leq 1} d_F(\eta) = \left(1 + \frac{\tau_c^{\sigma-1} D_H}{k^\sigma D_F^*}\right)^{-1}, \tag{25}$$

which allows to use the implied d_F directly in our solution algorithm. The price level P and exchange rate k can easily be solved in equilibrium so that trade balance holds and $\tilde{d}_H$ is consistent with D_H. Having used the empirical $\tilde{d}_H$ to construct the gross revenue function and solve for firms' problem in the estimation, we can calculate the value of entry $\mathcal{V}_e$. Assuming that the economy is in a steady state with positive entry, we back out c_e by $c_e = \mathcal{V}_e$. This approach to discipline the cost of entry c_e is in line with the quantitative literature (Hopenhayn and Rogerson 1993).

In our policy experiments, however, we use the complete algorithm to compute equilibrium outcomes for given a set of parameters, including the cost of entry c_e. In these experiments, both d_F and D_H are equilibrium objects that respond to changes in (τ_a, τ_c, c_f).

5 Further Results and Data Sources

5.1 Labor Units

Since workers are all identical in the model economy, we measure the labor input l in terms of "effective worker" units in our estimation. This allows us to control for the effects of worker heterogeneity on output. In the plant-level data we use to estimate our model for the pre-reform period, we observe five categories of workers: managerial, technical, skilled, unskilled, and apprentice. For a given plant-year, effective labor l is the sum of all workers in the plant, each weighted by the average wage (including fringe benefits) for workers in its category. For each category of worker, the average wage is based on the mean real wage in the entire 10-year panel and expressed as a ratio to the average real wage for unskilled workers during the same period. Thus wage weights are constant across plants and time, and the only source of variation in l is variation in the employment level of at least one category of worker.

After fitting the model to the pre-reform data, we simulate Colombian reforms and decreased trade costs in Section II of the paper. To evaluate the success of the model in explaining post-reform outcomes, we wish to compare the firm size distribution as predicted by the model to its empirical counterpart. Since we do not have access to the plant-level data from the post-reform period, we don't observe the above-described variables used to construct effective labor. While The Colombian Statistical Agency DANE publishes summary statistics on the size distribution of plants for the 2000-2006 period (`http://www.dane.gov.co/index.php/industria/encuesta-anual-manufacturera-eam`), these are based on the number of total employees.

The following procedure facilitates the comparison of model based and empirical size distributions in both periods (see Figure 4 in the paper). Using the pre-reform plant level, we first fit total number of workers to a polynomial of effective labor l. We then use the coefficients from this regression to convert model-generated effective labor l units to worker count for both the estimated pre-reform and simulated post-reform periods. The bars with patterns in Figure 4 in the paper—representing the model-based size distributions—are generated using this transformation. The bars with solid colors representing the empirical size distributions are generated directly from the data using total number of employees.

5.2 Sectoral Labor Flows in Colombia

The Colombian Statistical Agency DANE publishes monthly labor market indicators. We accessed the following link on August 11, 2015:
https://www.dane.gov.co/files/investigaciones/boletines/ech/ech/anexo_ech_may12.xls.

The file is in Spanish but variable names can be easily translated using online translators. In this file, the worksheets titled "*ocup ramas trim tnal*" indicates sectoral employment levels. The worksheet titled "*cesantes ramas trim tnal*" reports the sector of previous employment for separations. The ratio of the latter over the former gives sectoral separation rates. We exclude agriculture and mining, and find employment-weighted average for service industries. For the 2001-2008 period, average separation rates are 0.122 for manufacturing and 0.14 for services.

5.3 Size-Wage Relationship

Table A1 shows the effect of size (measured by the number of workers) and productivity on wages for hiring and non-hiring firms in the Colombian establishment survey data and the model.

Table A1: **Wage-Size Relationship Controlling for Productivity**

$\ln w = \alpha + \beta_l \ln l + \beta_z \ln z + \varepsilon$	Non-expanding firms		Expanding firms	
	Data	Model	Data	Model
β_l	0.075	-0.232	0.064	-0.231
	(0.002)	(0.003)	(0.002)	(0.004)
β_z	0.586	1.381	0.502	1.374
	(0.005)	(0.009)	(0.006)	(0.015)
R^2	0.46	0.55	0.51	0.54

Notes: l is effective workers defined in online Appendix 5.1, w is average wage per effective worker, and z is firm-level productivity implied by equation (29) *in the paper given the data and estimated parameter values.*

5.4 Additional Figures

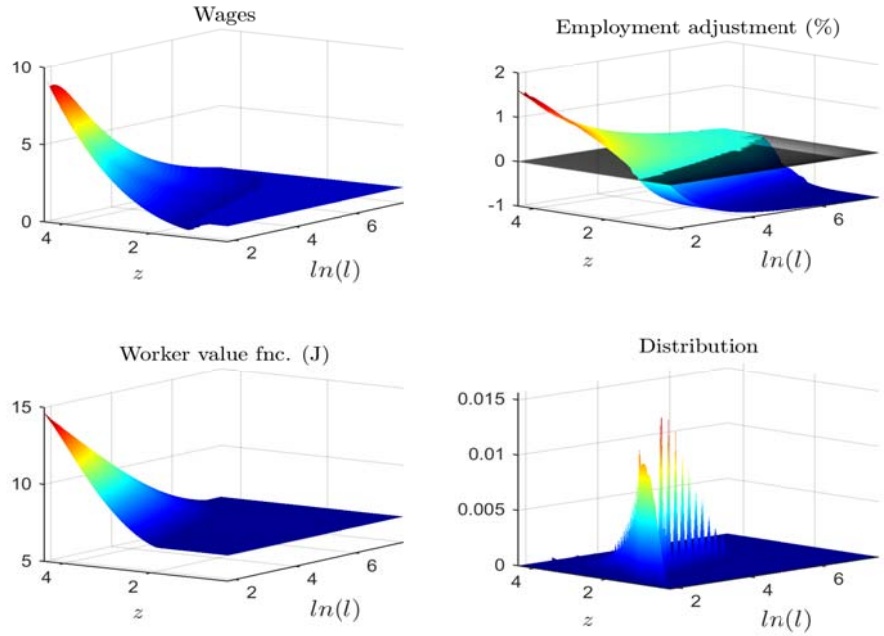


Figure A2: Baseline Outcomes

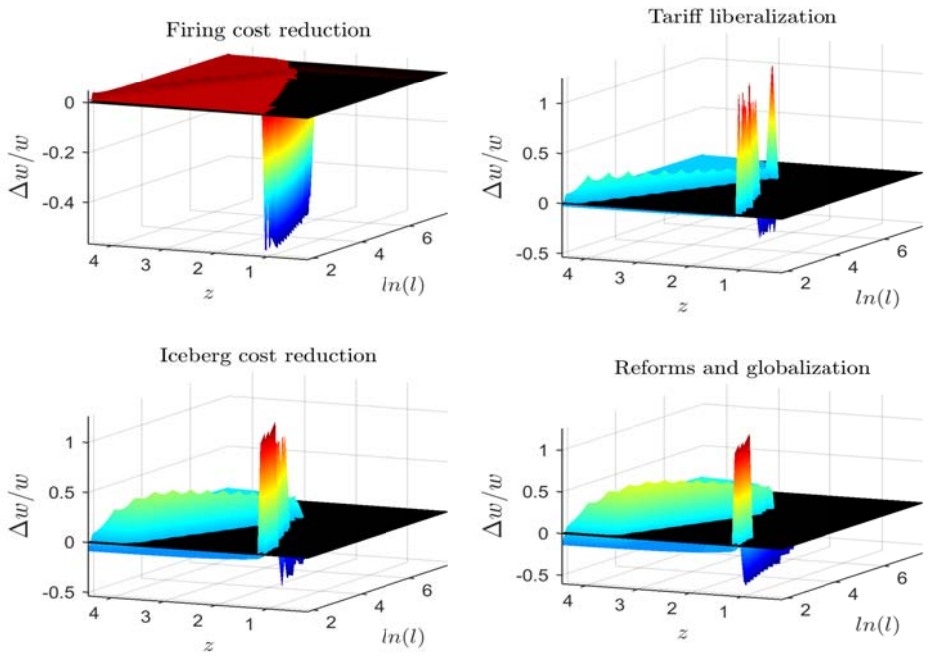


Figure A3: Change in Wage Schedules

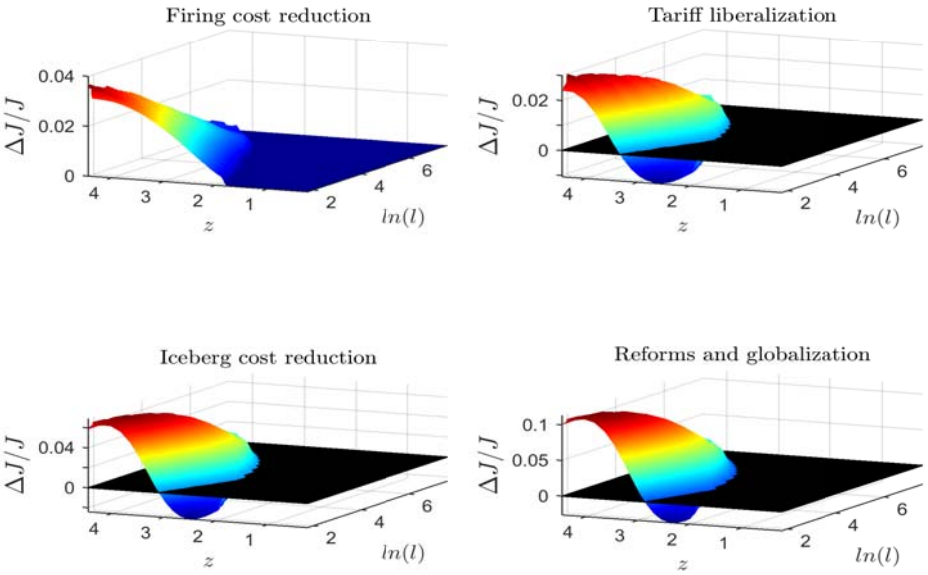


Figure A4: Change in Worker Value Functions

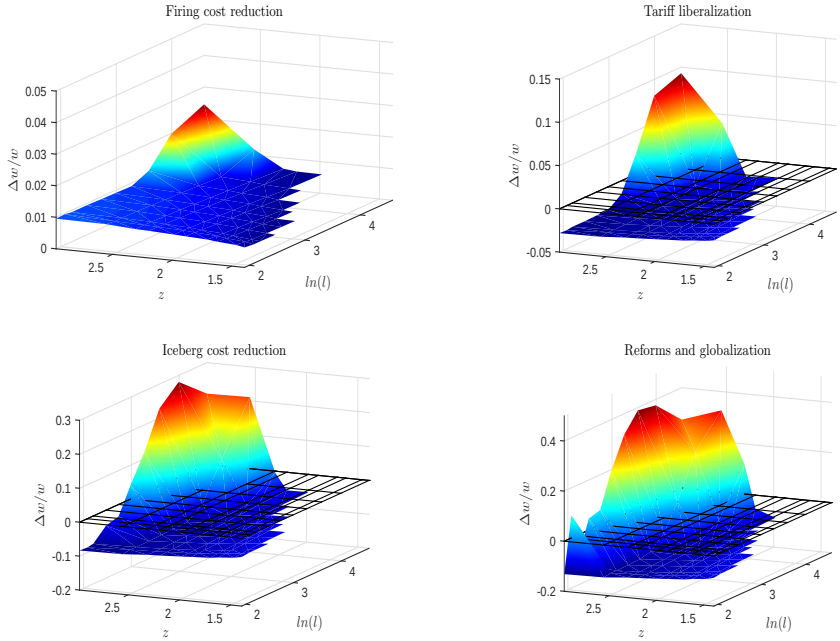


Figure A5: Change in Average Wages by Firm Type

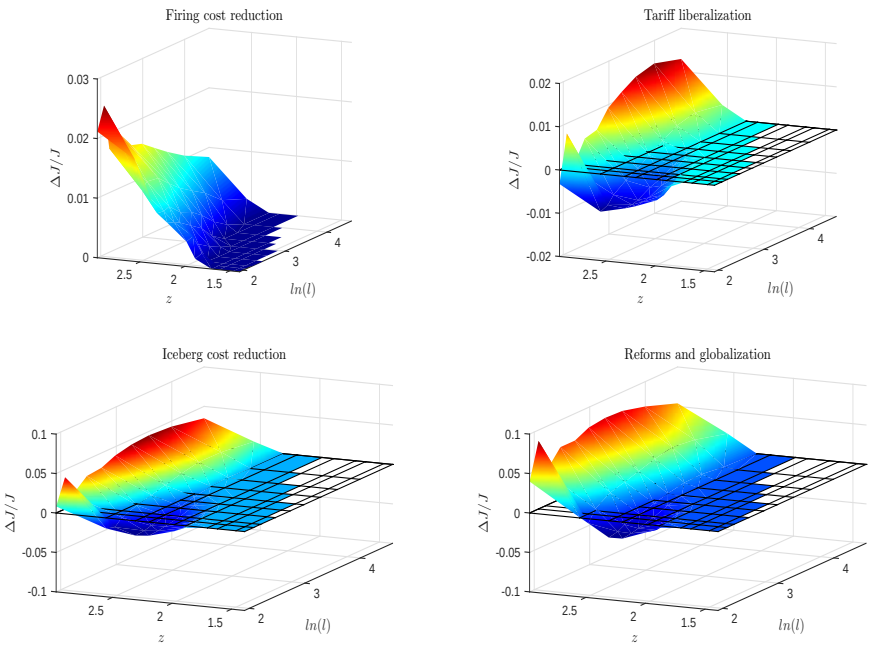


Figure A6: Change in Average Worker Value by Firm Type

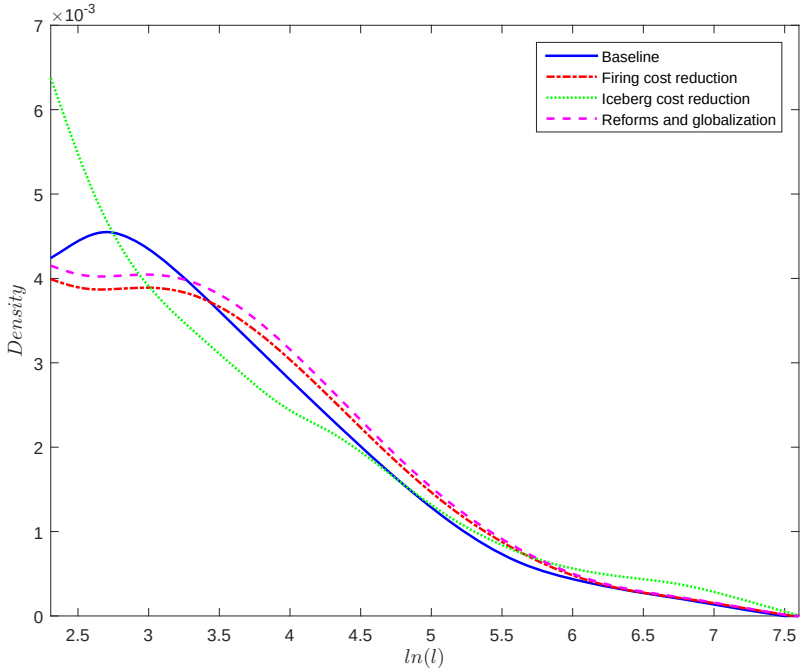


Figure A7: Shifts in the Size Distribution

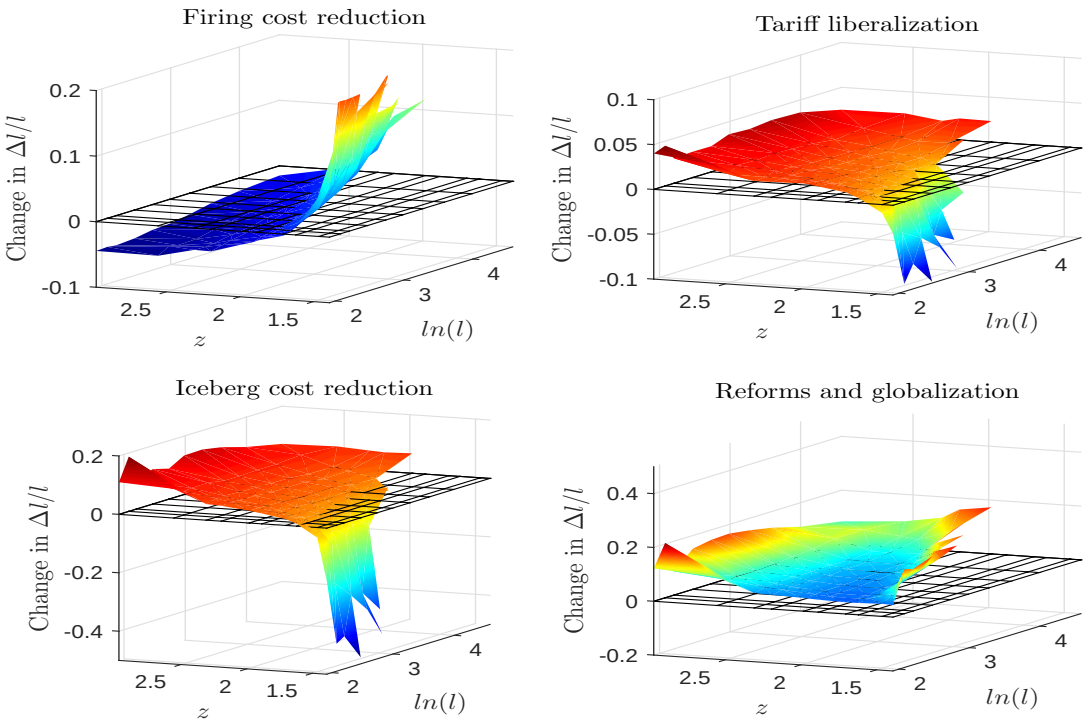


Figure A8: Employment Policy Changes

Figure A2 shows wage schedules (equation 26 or 27 in the main text depending on whether a firm is hiring or not), percentage employment adjustments $(L(z', l) - l)/l$, worker value functions J and the distribution $\psi(z, l)$.

In order to shed light on the mechanisms described in sections I.K and III.B of the paper, Figures A3 and A4 show the change with respect to the baseline in wage schedules and worker value functions, respectively, for any point in the state space $(z, \ln(l))$, regardless of whether it is populated by firms. In this and related figures, the first three panels feature the isolated policy changes (corresponding to columns 2-4 in table 4) while the fourth panel (bottom, right) features the case of reforms and globalization (column 6 in table 4).

Figures A5 and A6 are based on 100 grid points, each of which is the mid-point of a size and productivity decile in the baseline (z, l) distribution. Holding these grid points fixed, we plot the changes in average wages and worker value functions. These graphs show how wages and employment policies shift in the populated portions of the state space.

Figure A7 shows the size distribution of firms above $l > 10$ (mimicking the empirical sampling in the Colombian firm data), fit with a spline using a smoothing parameter of 0.6.

Figure A8 shows the change in employment policies, i.e., the percentage change in $(L(z', l) - l)/l$ with respect to the baseline.

6 Robustness to the Choice of Model Period

To isolate the role of model period in driving our results, we hold the estimation strategy fixed by using our estimated revenue function and productivity process to approximate their quarterly counterparts.[5] Then we re-estimated remaining parameters using the same moment vector as in the annual baseline, aggregating simulated quarterly outcomes on flow variables to their annual equivalents, and taking simulated fourth quarter realizations on stock variables to be representative of their annual counterparts (as is done in the annual manufacturing surveys).

Specifically, we kept our estimate of the elasticity of value added with respect to labor ($\alpha\Lambda$) based on annual data, and we chose the root of the quarterly productivity process to replicate our estimate of persistence in the annual process: $\rho_q = \rho_a^{1/4}$. Likewise, we adjusted the discount rate to $r_q = (1 + r_a)^{1/4} - 1$, and we divide the log revenue function intercept $\widetilde{d}_H$ by four to put revenue flows on a quarterly basis. Finally, since we saw no good way to approximate the relationship between the variance of the innovations in the annual data ($\sigma^2_{z,a}$) and the variance of the innovations in the quarterly data ($\sigma^2_{z,q}$), we included $\sigma^2_{z,q}$ in the set of parameters to be estimated.

Tables A2 and A3 present the resulting parameter estimates and the fit of the model. The quarterly version doesn't fit as well as the annual baseline, perhaps because of the way we have constrained our revenue function estimates. Nonetheless, the quarterly results do give us some insight into the effects of model period choice on parameter estimates and model performance.

The major differences in parameter estimates are in the elasticity of substitution σ, the elasticity of the matching function θ, and the value of home production b. The change in b can be explained by the effect of model frequency on wage inequality. Allowing workers to search more frequently increases their reservation wages, which in turn affects the entire wage schedule. Other things equal, this would lower wage dispersion in the model. So, in order to still match the dispersion of *log* wages, the quarterly calibration lowers the constant term $(1-\beta)b$ in the hiring wage schedule (22). It does so by reducing b from 0.433 to 0.302.[6] The other major change in parameter values is the decrease in matching function elasticity θ from 1.838 to 1.154. This compensates for the fact that, other things equal, switching to a quarterly frequency would have increased labor market tightness as workers enjoyed more opportunities to match with firms. In turn, this would have made it more difficult for firms

[5] We emphasize "approximate" here because there is no analytical relationship linking the parameters of the annual objects to their quarterly counterparts. The reason is that our revenue function characterizes logs of flows, and thus annual variables are not linear combinations of quarterly variables.

[6] Note that the unit of account is the service sector wage per period, so $b = 0.302$ from the quarterly estimation is directly comparable to the b from the annual baseline.

hire, and thus shifted the simulated firm size distribution leftward. Dropping θ improves the ability of firms to meet workers over the relevant range of (U, V) values, and thus prevents this from occurring. Other parameter values such as exogenous exit rate δ and the initial firm size l_e drop due to the increase in model frequency.

Table A4 addresses the main question of interest: how robust are the policy experiments in the paper to the change in model period? That is, it redoes Table 4 using the quarterly version of our model. Note that in order to facilitate comparison, we use the same $\tau_c = 2.19$. This number, calibrated to replicate the 150 percent increase in the revenue share of exports in the baseline model, generates a similar (143 percent) increase in the quarterly model.

The results in Table A4 show that the effects of policy experiments are robust. First, while "Reforms" and "Reforms and Globalization" experiments generate higher firm growth rates at the baseline size quantiles, aggregate job turnover declines in both experiments. The decline in job turnover is, however, slightly smaller with the quarterly model. This reflects the shifts in parameter estimates described above. On the one hand, with a smaller home production payoff, b, wages are more sensitive (percentagewise) to firm characteristics (z, l). On the other hand, a lower matching function elasticity makes job finding and fill rates less responsive to changes in aggregate labor market conditions. Second, "Reforms" and "Reforms and Globalization" experiments result in similar levels of inequality in worker values (J), measured either at the firm or worker levels. Finally, "Reforms and Globalization" generates a smaller increase in $Q-$sector unemployment compared to the baseline model (9.1 percent versus 19 percent) and a higher increase in real income (41.4 percent versus 12 percent).

Table A2: **Parameters Estimated with SMM - annual vs quarterly**

Parameter	Description	Annual	Quarterly
σ	Elasticity of substitution	6.667	10.358
α	Elasticity of output with respect to labor	0.195	0.240
β	Bargaining power of workers	0.441	0.411
θ	Elasticity of the matching function	1.838	1.154
δ	Exogenous exit hazard	0.064	0.019
c_h	Scalar, vacancy cost function	0.448	2.549
λ_1	Convexity, vacancy cost function	3.101	3.699
λ_2	Scale effect, vacancy cost function	0.385	0.332
b	Value of home production	0.433	0.302
l_e	Initial size of entering firms	5.906	3.338
c_p	Fixed cost of operating	7.839	12.291
c_x	Fixed exporting cost	112.943	71.378
c_e	Entry cost for new firms	15.794	146.816
σ_z	Standard deviation of the z process	0.137	0.061

Table A3: **Data-based versus Simulated Statistics - annual vs quarterly**

Moment	**Data**	**Annual**	**Quarterly**	**Size Distribution**	**Data**	**Annual**	**Quarterly**
$E(\ln G_t)$	5.442	5.274	5.919	20th percentile cutoff	14.617	15.087	35.043
$E(\ln l_t)$	3.622	3.638	4.632	40th percentile cutoff	24.010	24.736	74.043
$E(\mathcal{I}_t^x)$	0.118	0.108	0.320	60th percentile cutoff	41.502	42.559	148.92
$var(\ln G_t)$	2.807	3.334	67.94	80th percentile cutoff	90.108	87.137	334.54
$cov(\ln G_t, \ln l_t)$	1.573	1.888	11.535	**Firm Growth Rates**			
$var(\ln l_t)$	1.271	1.326	2.172	<20th percentile	1.425	1.287	1.321
$cov(\ln G_t, \mathcal{I}_t^x)$	0.230	0.264	2.788	20th-40th percentile	0.255	0.251	0.403
$cov(\ln l_t, \mathcal{I}_t^x)$	0.153	0.175	0.480	40th-60th percentile	0.209	0.191	0.242
$cov(\ln G_t, \ln G_{t+1})$	2.702	2.119	43.48	60th-80th percentile	0.184	0.155	0.168
$cov(\ln G_t, \ln l_{t+1})$	1.538	1.534	9.091	**Aggregate Turnover/**			
$cov(\ln G_t, \mathcal{I}_{t+1}^x)$	0.225	0.283	2.784	**Wage Dispersion**			
$cov(\ln l_t, \ln G_{t+1})$	1.543	1.409	-17.45	Firm exit rate	0.108	0.104	0.031
$cov(\ln l_t, \ln l_{t+1})$	1.214	1.192	2.696	Job turnover	0.198	0.222	0.237
$cov(\ln l_t, \mathcal{I}_{t+1}^x)$	0.152	0.195	0.292	Std. dev. of log wages	0.461	0.380	0.449
$cov(\mathcal{I}_t^x, \ln G_{t+1})$	0.220	0.273	2.292	**Olley-Pakes Statistics**			
$cov(\mathcal{I}_t^x, \ln l_{t+1})$	0.149	0.200	0.426	$(1-\alpha)\left(\frac{\sigma-1}{\sigma}\right)$	0.685	0.685	0.686
$cov(\mathcal{I}_t^x, \mathcal{I}_{t+1}^x)$	0.090	0.075	0.193	d_F	0.090	0.094	0.093

Table A4: **Simulation Results**

	Baseline	Reforms	Reforms and Globalization
c_f (firing cost)	0.60	0.30	0.30
τ_a (ad valorem tariff rate)	1.21	1.11	1.11
τ_c (iceberg trade cost)	2.50	2.50	2.19
Firm Growth Rates			
(at the baseline size quantiles)			
<20th percentile	1.321	1.323	1.439
20th-40th percentile	0.403	0.426	0.503
40th-60th percentile	0.242	0.246	0.313
60th-80th percentile	0.168	0.170	0.207
Aggregates			
Revenue share of exports	1	1.404	2.434
Exit rate	1	0.974	1.106
Job turnover	1	0.952	0.963
Mass of firms	1	0.924	0.730
Share of labor in Q sector	1	1.040	0.937
Vacancy filling rate (ϕ)	1	1	1.077
Unemployment rate in Q sector	1	0.951	1.091
Std. wages (firms)	1	1.054	1.147
Std. wages (workers)	1	1.037	1.019
Std. J (firms)	1	1.080	1.210
Std. J (workers)	1	1.079	1.215
Exchange rate	1	0.950	0.610
Real income	1	1.005	1.414