I. Proof of Propositions

A. Proof of proposition 1

Define total wealth (human plus financial) of a household of age \( j \), \( w_j \), the portfolio choice, \( \theta_j \), and the total investment return, \( r_j \) as in Section II of the paper. Using this notation, the sequential budget constraint is given in equation (8). For age \( j = 1, \ldots, J \), the Bellman equation associated with the household utility maximization problem reads:

\[
V_j(w_j, \theta_j, s_j) = \max_{c_j, w_{j+1}, \theta_{j+1}} \left\{ \gamma_0(s_j) + \gamma_1(s_j) \ln c_j + \beta \sum_{s_{j+1}} V_{j+1}(w_{j+1}, \theta_{j+1}, s_{j+1}) \pi_j(s_{j+1}|s_j) \right\}
\]

s.t. \[
w_{j+1} = (1 + r_j(\theta_j, s_j))w_j - c_j
\]
\[
1 = \theta_{h,j+1} + \sum_{s_{j+1}} q_j(s_{j+1}|s_j) \theta_{a,j+1}(s_{j+1})
\]
\[
c_j \geq 0 \quad w_{j+1} \geq 0 \quad \theta_{h,j+1} \geq 0
\]
\[
V_{j+1}(w_{j+1}, \theta_{j+1}, s_{j+1}) \geq V_{d,j+1}(w_{j+1}, \theta_{h,j+1}, s_{j+1})
\]

In default, a household who defaults at age \( j \) chooses a continuation plan, \( \{c_{j+n}, h_{j+n}\} \), so as to maximize

\[
\sum_{n=0}^{J-j} (p\beta)^n \sum_{s_{j+n}|s_j} \left[ \gamma_0(s_{1,j+n}) + \gamma_1(s_{2,j+n}) \ln c_{j+n}(s_{j+n}) \right] \pi(s_{j+n}|s_0)
\]
\[
+ \sum_{n=0}^{\infty} (p\beta)^{J-j+n} \sum_{s_{j+n+1}|s_{j+n}} V_{J+1}(h_{J+1+n}(s_{J+n}), a_{J+1+n}(s_{J+1+n}), s_{J+1+n}) \pi(s_{J+1+n}|s_0)
\]
\[
+ \sum_{n=0}^{J-j} ((1-p)\beta)^n \sum_{s_{j+n}|s_j} V_{j+n}^e(h_{j+n}(s_{j+n-1}), s_{j+n}) \pi(s_{j+n}|s_j)
\]
\[
+ \sum_{n=0}^{\infty} (p\beta)^{J-j+n} \sum_{s_{j+n+1}|s_{j+n}} V_{J+1+n}^e(h_{J+1+n}(s_{J+n}), s_{J+1+n}) \pi(s_{J+1+n}|s_0)
\]

where \( \{c_{j+n}, h_{j+n}\} \) has to solve the sequential budget constraint (3) with \( a_j = 0 \). Define the investment return of a household in default as \( r_d(\theta_{hj}, s_j) = (1 + \phi z(s_j)r_h - \delta_h + \eta_j(s_j) + \ldots + \phi z(s_{j+n-1})r_{h+n-1} - \delta_{h+n-1} + \eta_j(s_{j+n-1}) + \ldots + \phi z(s_{j+n})r_{h+n} - \delta_h + \eta_j(s_{j+n})) \).
\( \phi_j(s_j) \theta_{hj} \), which is simply the human capital return times the fraction of wealth invested in human capital. In the period of default, we have in general \( \theta_{hj} \neq 1 \), but in all periods subsequent to default we have \( \theta_{h,j+n} = 1 \). In the period of re-gaining access to financial markets, a household in default has no financial assets, and we still have \( \theta_{h,j+n} = 1 \). The Bellman equation of a household in default reads

\[
V_{dj}(w_j, \theta_{hj}, s_j) = \max_{c_j, w_{j+1}} \left\{ g_0(s_j) + g_1(s_j) \ln c_j + p \beta \sum_{s_{j+1}} V_{d,j+1}(w_{j+1}, 1, s_{j+1}) \pi_j(s_{j+1}|s_j) \right. \\
+ \left. (1 - p) \beta \sum_{s_{j+1}} V_{e,j+1}(w_{j+1}, 1, s_{j+1}) \pi_j(s_{j+1}|s_j) \right\}
\]

\[\text{s.t.} \quad w_{j+1} = (1 + r_{dj}(1, s_j))w_j - c_j \]
\[c_j \geq 0 \ , \ w_{j+1} \geq 0\]  

(A2)

The Bellman equation (A2) for the default value function together with the Bellman equation (A1) and the condition \( V^e = V \) define a Bellman equation determining simultaneously the value function \( V \) and \( V_d \). Suppose that the terminal value function \( V_{J+1} \) has the functional form (A7). Solving the problem backwards, guess-and-verify shows that the solution to this Bellman equation (A1) and (A2) for all \( j = 1, \ldots, J \) is

\[
V_j(w_j, \theta_j, s_j) = \bar{V}_{0j}(s_j) + \bar{V}_{1j}(s_j) [\ln w_j + \ln(1 + r_j(\theta_j, s_j))] \]

(A3)

\[
c_j(w_j, \theta_j, s_j) = \bar{c}_j (1 + r_j(\theta_j, s_j)) w_j \]

\[
V_{dj}(w_j, \theta_j, s_j) = \bar{V}_{d,0j}(s_j) + \bar{V}_{1j}(s_j) [\ln w_j + \ln(1 + r_{dj}(\theta_{hj}, s_j))] \]

\[
c_j(w_j, \theta_j, s_j) = \bar{c}_j (1 + r_{dj}(\theta_{hj}, s_j)) w_j \]

with

\[
\bar{c}_j(s_j) = \frac{g_1(s_j)}{\bar{V}_{1j}(s_j)}
\]

The coefficients \( \bar{V}_{1j} \) are determined recursively as the solution to

\[
\bar{V}_{1j}(s_j) = g_1(s_j) + \beta \sum_{s_{j+1}} \bar{V}_{1,j+1}(s_{j+1}) \pi_j(s_{j+1}|s_j)
\]

and the coefficients \( \bar{V}_{0j} \) and \( \bar{V}_{d,0j} \) together with the optimal portfolio choices \( \theta_{j+1}^* \) are the solutions to the equation

\[
\theta_{j+1}^*(s_j) = \arg \max_{\theta_{j+1} \in \Gamma_{j+1}} \sum_{s_{j+1}} \bar{V}_{1,j+1}(s_{j+1}) \ln (1 + r_{j+1}(\theta_{j+1}, s_{j+1})) \pi_j(s_{j+1}|s_j)
\]

(A4)
\[ \Gamma_{j+1}(s_j) = \left\{ \theta_{j+1} \left| \theta_{h,j+1} + \sum_{s_{j+1}} \frac{\theta_{a,j+1}(s_{j+1})\pi_j(s_{j+1}|s_j)}{1 + r} = 1 , \ \theta_{h,j+1} \geq 0 \right. \right\} \]

\[ \frac{\tilde{V}_{0,j+1}(s_{j+1}) - \tilde{V}_{0d,j+1}(s_{j+1})}{\tilde{V}_{1,j+1}(s_{j+1})} \geq [\ln(1 + r_{d,j+1}(\theta_{h,j+1}, s_{j+1})) - \ln(1 + r_{j+1}(\theta_{j+1}, s_{j+1}))]. \]

This completes the proof for the case \( j = 1, \ldots, J. \)

If \( j = J + 1, \) the household has entered a transition period from which retirement occurs stochastically at constant probability \( p_{ret}. \) In this case, the household problem is an infinite-horizon maximization problem with value function constraint, and the corresponding Bellman equation is a version of (A1) and (A2) in which the age-index is replaced by the constant \( J + 1 \) (i.e. the index can be dropped) and there is a constant probability \( p_{ret} \) that the continuation utility is equal to a given continuation utility \( V_{ret}: \)

\[ V_{J+1}(w, \theta, s) = \max_{c, w', \theta'} \left\{ \gamma_0(s) + \gamma_1(s)ln c + (1 - p_{ret})\beta \sum_{s'} V_{J+1}(w', \theta', s') \pi_{J+1}(s'|s) \right. \]

\[ \left. + p_{ret}\beta \sum_{s'} V_{ret}(w', \theta', s') \right\} \quad (A5) \]

s.t. \( w' = (1 + r_{J+1}(\theta, s))w - c. \)
\[ 1 = \theta' + \sum_{s'} q_{J+1}(s'|s) \theta_a(s') \]
\[ c \geq 0 \quad w' \geq 0 \quad \theta'_h \geq 0 \]
\[ V_{J+1}(w', \theta', s') \geq V_{d,J+1}(w', \theta'_h, s') , \]

and
\[ V_{d,J+1}(w, \theta_h, s) = \max_{c,w'} \left\{ \gamma_0(s) + \gamma_1(s) \ln c + p \beta \sum_{s'} V_{d,J+1}(w', 1, s') \pi_{J+1}(s'|s) \right\} \]
\[ s.t. \quad w' = (1 + r_{d,J+1}(1, s)) w - c \]
\[ c \geq 0 \quad w' \geq 0 \]

where we assumed that there is no retirement when the household is in default. We first discuss the retirement problem defining \( V_{ret} \) and then analyze the household problem in the pre-retirement phase (A5) determining \( V_{J+1} \) and \( V_{d,J+1} \).

A household in retirement can only invest in the risk-free asset and the only source of income is capital income. Thus, there is no portfolio choice. We assume that retired households die with probability \( p_{death} \) and normalize the continuation utility after death to zero. Thus, the retirement value function for a household who retires in the current period has the functional form
\[
V_{ret}(w, \theta, s) = \tilde{V}_{0,ret}(s) + \tilde{V}_{1,ret}(s) [\ln w + \ln(1 + r_{J+1}(\theta, s))] \quad (A6)
\]

where we assumed that the household still works in the period in which the transition into retirement occurs. The coefficients \( \tilde{V}_{0,ret} \) and \( \tilde{V}_{1,ret} \) are given by
\[
\tilde{V}_{1,ret}(s) = \gamma_1(s) + \beta(1 - p_{death}) \sum_{s'} \tilde{V}_{1,ret}(s') \pi_{ret}(s'|s)
\]

and
\[
\tilde{V}_{0,ret}(s) = \gamma_0(s) + \gamma_1(s) \ln(\tilde{c}_{ret}(s)) \]
\[ + (1 - p_{death}) \beta \sum_{s'} \tilde{V}_{0,ret}(s') \pi_{ret}(s'|s) \]
\[ + (1 - p_{death}) \beta \ln (1 + r_f) \sum_{s'} \tilde{V}_{1,ret}(s') \pi_{ret}(s'|s) \]
\[ + (1 - p_{death}) \beta \ln(1 - \tilde{c}_{ret}(s)) \sum_{s'} \tilde{V}_{1,ret}(s') \pi_{ret}(s'|s) \]
where $\tilde{c}_{ret}(s) = \frac{\gamma_1(s)}{V_{1,ret}(s)}$.

For the pre-retirement stage, we conjecture that the solution to (A5) is

\[
V_{J+1}(w, \theta, s) = \tilde{V}_{0,J+1}(s) + \tilde{V}_{1,J+1}(s) \left[ \ln w + \ln(1 + r_{J+1, \theta_{J+1}, s_{J+1}}) \right] 
\]

(A7)

\[
c_{J+1}(w, \theta, s) = \tilde{c}_{J+1} (1 + r_{J+1}(\theta, s)) w 
\]

\[
V_{d,J+1}(w, \theta, s) = \tilde{V}_{d,0,J+1}(s) + \tilde{V}_{1,J+1}(s) \left[ \ln w + \ln(1 + r_{d,J+1}(\theta_h, s)) \right] 
\]

\[
c_{d,J+1}(w, \theta, s) = \tilde{c}_{J+1} (1 + r_{d,J+1}(\theta_h, s)) w 
\]

where the coefficients $\tilde{V}_{J+1}$ are determined by the recursive equation

\[
\tilde{V}_{1,J+1}(s) = \gamma_1(s) + (1 - p_{ret}) \beta \sum_{s'} \tilde{V}_{1,J+1}(s') \pi_{J+1}(s'|s) + p_{ret} \beta \sum_{s'} \tilde{V}_{1,ret}(s') \pi_{J+1}(s'|s) 
\]

and the coefficients $\tilde{V}_{0,J+1}$ and $\tilde{V}_{d,0,J+1}$ together with the optimal portfolio choices $\theta^*_{J+1}$ are the solutions to the equation

\[
\theta^*_{J+1}(s) = \arg \max_{\theta_{J+1} \in \Gamma_{J+1}} \left\{ (1 - p_{ret}) \sum_{s'} \tilde{V}_{1,J+1}(s') \ln(1 + r_{J+1}(\theta_{J+1}, s')) \pi_{J+1}(s'|s) \right\} 
\]

(A8)

\[
\Gamma_{J+1}(s) = \left\{ \theta_{J+1} \left\vert \theta_{h,J+1} + \sum_{s'} \frac{\theta_{d,J+1}(s') \pi_{J+1}(s'|s)}{1 + r_f} = 1 \right, \theta_{h,J+1} \geq 0 \right, \right. 
\]

and

\[
\tilde{V}_{0,J+1}(s') - \tilde{V}_{d,0,J+1}(s') \geq [\ln(1 + r_{d,J+1}(\theta_{J+1}, s')) - \ln(1 + r_{J+1}(\theta_{J+1}, s'))] 
\]

\[
\frac{\tilde{V}_{0,ret}(s') - \tilde{V}_{d,ret}(s')}{\tilde{V}_{1,ret}(s')} \geq [\ln(1 + r_{d,ret}(\theta_{J+1}, s')) - \ln(1 + r_{ret}(\theta_{J+1}, s'))] 
\]

and

\[
\tilde{V}_{0,J+1}(s) = \gamma_0(s) + \gamma_1(s) \ln(\tilde{c}_{J+1}(s)) 
\]

\[
+ (1 - p_{ret}) \beta \sum_{s'} \tilde{V}_{0,J+1}(s') \pi_{J+1}(s'|s) 
\]

\[
+ (1 - p_{ret}) \beta \sum_{s'} \tilde{V}_{1,J+1}(s') \ln(1 + r_{J+1}(\theta_{J+1}, s')) \pi_{J+1}(s'|s) 
\]

\[
+ (1 - p_{ret}) \beta \ln(1 + \tilde{c}_{J+1}(s)) \sum_{s'} \tilde{V}_{1,J+1}(s') \pi_{J+1}(s'|s) 
\]

\[
+ p_{ret} \beta \sum_{s'} \tilde{V}_{0,ret}(s') \pi_{J+1}(s'|s) 
\]
\[ + p_{\text{ret}} \beta \sum_{s'} \tilde{V}_{1,\text{ret}}(s') \ln \left( 1 + r_{J+1}(\theta^*_{J+1}, s') \right) \pi_{J+1}(s'|s) \]
\[ + p_{\text{ret}} \beta \ln(1 - \tilde{c}_{J+1}(s)) \sum_{s'} \tilde{V}_{1,\text{ret}}(s') \pi_{J+1}(s'|s) \]

\[ \tilde{V}_{d_{0},J+1}(s) = \gamma_0(s) + \gamma_1(s) \log(\tilde{c}_{J+1}(s)) \]
\[ + p \beta \sum_{s'} \tilde{V}_{d_{0},J+1}(s') \pi_{J+1}(s'|s) \]
\[ + (1 - p) \beta \sum_{s'} \tilde{V}_{0,J+1}(s') \pi_{J+1}(s'|s) \]
\[ + \beta \sum_{s'} \tilde{V}_{1,J+1}(s') \log \left( 1 + r_{d,J+1}(1, s') \right) \pi_{J+1}(s'|s) \]
\[ + \beta \ln(1 - \tilde{c}_{J+1}(s)) \sum_{s'} \tilde{V}_{1,J+1}(s') \pi_{J+1}(s'|s) \]

We prove this conjecture as follows.

The sequential problem the household faces at the pre-retirement stage \( J + 1 \) is an infinite horizon problem with value function constraint, and the Bellman operator \( T \) associated with equation (A8) is monotone, but in general not a contraction mapping. However, adapting the argument made in Rusticchini (1998), the following result can be shown to hold in our setting:

**Lemma** Suppose that \( V_d \) and \( V^e \) are continuous functions. Suppose further that there is a unique continuous solution, \( V_0 \), to the Bellman equation without participation constraint. Let \( T \) stand for the operator associated with the Bellman equation. Consider the set of continuous functions \( B_W \) that are bounded in the weighted sup-norm \( ||V|| = \sup_{x} |V(x)|/W(x) \), where the weighting function \( W \) is given by \( W(x) = |L(x)| + |U(x)| \) with \( U \) an upper bound and \( L \) a lower bound, and endow this function space with the corresponding metric.\(^1\) Then

i) \( \lim_{n \to \infty} T^n V_0 = V_\infty \) exists and is the maximal solution to the Bellman equation (9)

ii) \( V_\infty \) is the value function, \( V \), of the sequential household maximization problem.

Notice first that a standard argument shows that the Bellman equation (A8) without participation constraint has a unique continuous solution, \( V_0 \). Guess-and-verify shows that this solution has the functional form (A7). Define \( V_n = T^n V_0 \). It is straightforward to show that

\[^1\text{Thus, } B_W \text{ is the set of all functions, } V, \text{ with } L(x) \leq V(x) \leq U(x) \text{ for all } x \in X. \text{ For each particular application of the lemma, it has to be shown that this definition of the set of candidate value functions is without loss of generality for certain lower bound, } L, \text{ and upper bound, } U. \text{ In our case, the construction of the lower and upper bound is straightforward.} \]
if $V_n$ has the functional form (A7), then the same is true for $V_{n+1} = TV_n$. From the lemma we know that $V_\infty = \lim_{n \to \infty} T^n V_0$ exists and that it is the maximal solution to the Bellman equation (A8) as well as the value function of the corresponding sequential maximization problem (principle of optimality). Since the set of functions with this functional form is a closed subset of the set of continuous functions, we know that $V_\infty$ has the functional form. This proves that the conjecture is correct.

Finally, suppose that the exogenous state can be decomposed into two components, $s = (s_1, s_2)$, where $s_1$ defines the family structure and $s_2$ labor market risk. Assume further that $s_2$ is i.i.d. It is straightforward to show from (A7) and (A8) that the i.i.d. component $s_2$ does not affect choices $\theta$ and $\tilde{c}$ or value function coefficients $\tilde{V}_0$ and $\tilde{V}_1$, that is, they are functions only of $s_1$. This completes the proof of proposition 1.

B. Proof of proposition 2

From proposition 1 we know that individual households maximize utility subject to the budget constraint and participation constraint. Thus, it remains to derive the intensive-form market clearing condition and the stationarity condition determining $\Omega$.

Let $\tilde{w}_j = (1 + r_j)w_j$ be the wealth of a household age $j$ after all assets have paid off. The aggregate stock of human capital is

$$
H = \sum_j E[\theta_{h,j+1}w_{j+1}]\pi_j \\
= \sum_j E[\theta_{h,j+1}(1 - \tilde{c}_j)(1 + r_j)w_j] \\
= \sum_j \sum_{s_1j} E[\theta_{h,j+1}(1 - \tilde{c}_j)\tilde{w}_j|s_1j]\pi_j(s_1j) \\
= \sum_j \sum_{s_1j} \theta_{h,j+1}(s_1j)(1 - \tilde{c}_j(s_1j))E[\tilde{w}_j|s_1j]\pi_j(s_1j) \\
= \tilde{W} \sum_{s_1j} \theta_{h,j+1}(s_1j)(1 - \tilde{c}_j(s_1j))\Omega_j(s_1j) .
$$

where $\tilde{W} = \sum_j E[\tilde{w}_j]\pi_j$ is aggregate total wealth after assets have paid off. The second line in (A9) uses the equilibrium law of motion for the individual state variable $w$, the third line is simply the law of iterated expectations, the fourth line follows from the fact that the portfolio choices only depend on $s_1$, and the last line is a direct implication of the definition of $\Omega$. A similar expression holds for the aggregate stock of physical capital, $K$. Dividing the two expressions yields the intensive-form market clearing condition

$$
\tilde{K} = \frac{\sum_{s_1j} (1 - \theta_{h,j+1}(s_1j))(1 - \tilde{c}_j(s_1j))\Omega_j(s_1j)}{\phi \sum_{s_1j} \theta_{h,j+1}(s_1j)(1 - \tilde{c}_j(s_1j))\Omega_j(s_1j)} .
$$

(A10)
Define by $\tilde{r}_{j+1}(s_{1j}, s_{1,j+1})$ the expected investment return conditional on age and $(s_{1j}, s_{1,j+1})$. In stationary equilibrium the wealth distribution, $\Omega$, has to satisfy

$$\Omega_{j+1}(s_{1,j+1}) = \frac{E[\tilde{w}_{j+1}|s_{1,j+1}]\pi_{j+1}(s_{1,j+1})}{\sum_j \sum_{s_{1,j+1}} E[\tilde{w}_{j+1}|s_{1,j+1}]\pi_{j+1}(s_{1,j+1})}$$

$$= \frac{E[(1 + r_{j+1})(1 - \tilde{c}_j)\tilde{w}_j|s_{1,j+1}]\pi_{j+1}(s_{1,j+1})}{\sum_j \sum_{s_{1,j+1}} E[(1 + r_{j+1})(1 - \tilde{c}_j)\tilde{w}_j|s_{1,j+1}]\pi_{j+1}(s_{1,j+1})}$$

$$= \frac{\sum_j \sum_{s_{1j}} E[(1 + r_{j+1})(1 - \tilde{c}_j)\tilde{w}_j|s_{1j}, s_{1,j+1}]\pi_{j}(s_{1j}|s_{1,j+1})\pi_{j+1}(s_{1,j+1})}{\sum_j \sum_{s_{1j}} \sum_{s_{1,j+1}} \sum_{s_{1j}} E[(1 + \tilde{r}_{j+1}(s_{1j}, s_{1,j+1}))\pi(s_{1,j+1}|s_{1j})(1 - \tilde{c}_j(s_{1j}))E[\tilde{w}_j|s_{1j}]\pi_j(s_{1j})]}$$

where the second line uses the equilibrium law of motion for the individual state variable $x$, the third line is simply the law of iterated expectations, the fourth line follows from the fact that portfolio choices only depend on $s_1$ in conjunction with the definition of $\tilde{r}$, and the last line is a direct implication of the definition of $\Omega$. This completes the proof of proposition 2.

**C. Proof of proposition 3**

For each household age $j$, the solution of the household maximization problem determines the optimal portfolio choice $\theta_j = (\theta_{hj}, \theta_{aj})$. Without loss of generality, assume that all households have some insurance in equilibrium, but not full insurance: $\theta_{aj}(d) \neq \theta_{aj}(n)$ and $\eta(d) \theta_{hj} \neq (\theta_{aj}(d) - E[\theta_{aj}])$. In this case, for all age groups $j$ the participation constraint binds if $s = n$ and does not bind if $s = d$. If the participation does not bind, the consumption growth rate must be equal to $1 + r_f$ with log-utility, which given the consumption rule (9) implies that the portfolio return in the bad state is equal to the risk-free rate. Adding the budget constraint, we find that the optimal portfolio choice, $\theta_j$, is determined by the following three equations:

$$\theta_{hj} (1 + \phi r_h - \delta_h + \varphi_j - \eta(d)) + \theta_{aj}(d) = 1 + r_f \quad (A12)$$

$$\theta_{hj} (1 + \phi r_h - \delta_h + \varphi_j - \eta(n)) + \theta_{aj}(n) = e^{-(1-\beta)(\tilde{V}_j - \tilde{V}_{dj})}\theta_{hj} (1 + \phi r_h - \delta_h + \varphi_j - \eta(n))$$

$$\theta_{hj} + \frac{\pi(d)\theta_{aj}(d)}{1 + r_f} + \frac{\pi(n)\theta_{aj}(n)}{1 + r_f} = 1.$$

Suppose now that defaulting households keep access to financial markets: $p = 0$. In this case, we have $\tilde{V}_j = \tilde{V}_{dj}$, and from the third equation in (A12) it follows that $\theta_{aj}(n) = 0$. 

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Further, solving for $\theta_{hj}$ using $\theta_{aj}(n) = 0$ yields:

$$\theta_{hj} = \frac{\pi(n)}{1 - \frac{\pi(d)}{1 + r_f}(1 + \phi r_h - \delta_h + \varphi_j - \eta(d))} \quad (A13)$$

Clearly, equation (A13) shows that $\theta_{hj} > \theta_{h,j+1}$ if $\varphi_{hj} > \varphi_{h,j+1}$. It further follows from equation (A12) that the insurance pay-out is given by:

$$\theta_{aj}(b) - E[\theta_{aj}] = \pi(n)(1 + r_f - \theta_{hj}(1 + \phi r_h - \delta_h + \varphi_j - \eta(d))) \quad (A14)$$

Using $\theta_{hj} > \theta_{h,j+1}$, it follows that $\theta_{aj}(d) - E[\theta_{aj}] < \theta_{a,j+1}(d) - E[\theta_{a,j+1}]$. This proves the first part of the proposition. A similar argument proves the second part of proposition 3.

**II. Computation**

For ages $j = 1, 2, \ldots, J$, we solve the household problem backwards starting at $j = J$. The solution procedure is as follows:

**Step 1:** Find $\tilde{V}_{1,j}(\cdot)$ and $\tilde{c}_j(\cdot)$ solving (A4)

**Step 2:** Find the optimal portfolio choice $\theta_j$ for given $\tilde{V}_{0,j+1}(\cdot)$ and $\tilde{V}_{0d,j+1}(\cdot)$ using (A5)

1. Pick a current family structure $s_{1j}$.
2. Pick a human capital choice, $\theta_{h,j+1}$.
3. Pick a future family structure $s_{1j+1}$.
4. Order the states $s_{2j+1}$ according to the size of the human capital shock $\eta$. Pick a partition $S = S_1 \cup S_2$, where $S_1 = \{1, \ldots, n\}$ and $S_2 = \{n + 1, \ldots, N\}$ with $N$ being the number of states $s_{2j+1}$.
5. For given $(s_{1j}, s_{1j+1})$, and human capital choice $\theta_{h,j+1}$, we find the asset portfolio, $\theta_{a,j+1}(\cdot)$, by

   (a) Use participation constraint for all $s_{2j+1} \in S_1$:

   $$\exp \left( \frac{1}{\tilde{V}_{1,j+1}(s_{1j+1})} \left( \tilde{V}_{0,j+1}(s_{1j+1}) - \tilde{V}_{0d,j+1}(s_{1j+1}) \right) \right) \left( (1 + r_{hj}(s_{1j}, s_{1j+1}, s_{2j+1}))\theta_{h,j+1} + \theta_{a,j+1}(s_{1j+1}, s_{2j+1}) \right)$$

   $$= (1 + r_{hj}(s_{1j}, s_{1j+1}, s_{2j+1}))\theta_{h,j+1}$$
(b) Equalization of IMRS for all $s_{2j+1} \in S_2$:

$$\frac{u'(c_j, s_j)}{u'(c_{j+1}, s_{j+1})} = \beta(1 + r_f).$$

Using our utility function this reads $\frac{c_{j+1}}{c_j} = \frac{\gamma_1(s_{j+1})}{\gamma_1(s_{j})} \beta(1 + r_f)$. Using our consumption policy function, we find $\frac{c_{j+1}}{c_j} = \frac{\tilde{c}_{j+1}}{\tilde{c}_j}(1 - \tilde{c}_j)(1 + r_{j+1})$. Further using $\tilde{c}_j = \frac{\gamma_1}{V_{1j}}$ we arrive at the following condition for all $s_{2j+1} \in S_2$:

$$\tilde{V}_{1j}(s_{1j}) - \gamma_{1j}(s_{1j}) \left( (1 + r_{hj}(s_{1j}, s_{1j+1}, s_{2j+1})) \theta_{h,j+1} + \theta_{a,j+1}(s_{1j+1}, s_{2j+1}) \right) = \beta(1 + r_f)$$

Thus, we have

$$\theta_{a,j+1}(s_{1j+1}, s_{2j+1}) = - (1 + r_{hj}(s_{1j}, s_{1j+1}, s_{2j+1})) \theta_{h,j+1} \left( 1 - \exp \left( \frac{\tilde{V}_{0d,j+1}(s_{1j+1}) - \tilde{V}_{0,j+1}(s_{1j+1})}{\tilde{V}_{1,j+1}(s_{1j+1})} \right) \right)$$

$$\forall s_{2j+1} \in S_1$$

$$\theta_{a,j+1}(s_{1j+1}, s_{2j+1}) = \frac{\tilde{V}_{1,j+1}(s_{1j+1})}{\tilde{V}_{1,j}(s_{1j}) - \gamma_1(s_{1j})} \beta(1 + r_f) - (1 + r_{hj}(s_{1j}, s_{1j+1}, s_{2j+1})) \theta_{h,j+1}$$

$$\forall s_{2j+1} \in S_2$$

6. Do this for all $s_{1j+1}$

For given current family structure $s_{1j}$, find the portfolio vector $(\theta_{h,j+1}, \theta_{a,j+1})$ that “solves” the portfolio constraint. This is our optimal portfolio for given $s_{1j}$.

7. Do this for all current family structures $s_{1j}$.

**Step 3:** Find $\tilde{V}_{0j}()$ and $\tilde{V}_{d0,j}()$ using (A5)

The household problem for $j = J + 1$ we solve as above, but now we drop the $j$-dependence and solve the corresponding fixed point problem.

### III. Survey of Consumer Finance Data

information on a number of economic and financial variables of individual families through triennial interviews, where the definition of a “family” in the SCF comes close to the concept of a “household” used by the U.S. Census Bureau. See Kennickell and Starr-McCluer (1994) for details about the SCF.

For the sample selection, we follow as closely as possible Heathcote, Perri, and Violante (2010). We restrict the sample to households where the household head is between 23 and 60 years of age. We drop the wealthiest 1.46% of households in each year. Heathcote, Perri, and Violante (2010) show that this step makes the sample more comparable to that of the Panel Study of Income Dynamics (PSID) and the Consumer Expenditure Survey (CEX). We drop all households that report negative labor income or that report positive labor income but have missing labor income or that report positive labor income but zero or negative hours worked. We compute the average wage by dividing labor income by total hours worked, and drop in each year households with a wage that is below half the minimum wage of the respective year. For the data on life-insurance, we restrict the sample further to households that are married or live with a partner.

For the definition of variables we follow Kennickell and Starr-McCluer (1994). We only depart from their variable definitions when considering labor income, where we follow Heathcote, Perri, and Violante (2010) and add two-thirds of the farm and business income as additional labor income. As common in the literature, we associate financial wealth in the model with net worth in the SCF. Households’ net worth includes the cash value of life-insurance as in Kennickell and Starr-McCluer (1994), but does not include the face value of insurance contracts. We associate life-insurance in the model with the face value of life-insurance from the data. All data has been deflated using the BLS consumer price index for urban consumers (CPI-U-RS). A detailed description of the relevant variables is as follows:

- **Assets** are the sum of financial and non-financial assets. The main categories of non-financial assets are cars, housing, real estate, and the net value of businesses where the household holds an active interest. Except for businesses all values are gross positions, i.e. before outstanding debt. The main categories of financial assets are liquid assets, CD, mutual funds, stocks, bonds, cash value of life-insurance, other managed investment, and assets in retirement accounts (e.g. IRAs, thrift accounts, and pensions accumulated in accounts.)

- **Debt** is the sum of housing debt (e.g. mortgages, home equity loans, home equity

---

2 We use their Sample B for our analysis.
lines of credit), credit card debt, installment loans (e.g. cars, education, others), other residential debt, and other debt (e.g. pension loans).

- **Net-worth** is the sum of all assets minus all debt.
- **Labor income** is wages and salaries plus $2/3$ of business and farm income.
- **Life-insurance** is the face value of all term life policies and the face value of all policies that build up a cash value. The cash value is not part of the life-insurance, but is part of the financial assets of an household.

### IV. Demographic Transitions

#### A. Construction of Family Transition Matrix

We construct the stochastic matrix describing the transition of households over family states $s_1$ as follows. We proceed in two steps. In the first step, we construct the transition function for marital states and in the second step we construct the transition matrix for the number of children for each marital state. Age subscripts are dropped for convenience.

1. **Marital States**

There are in total 5 marital states: Married (ma), female widowed (fw), female single and not widowed (fn), male widowed (mw), and male single and not widowed (mn). We stack family states in a vector $x = \{ma, fw, fn, mw, mn\}$ and construct transition matrix $\Pi$. The transition matrix follows the conventional structure with initial states in rows and terminal states in columns. The order of states is given by the order of $x$. We set all transition rates between sexes to zero.

$$
\Pi = \begin{pmatrix}
\pi(ma, ma) & \pi(ma, fw) & \pi(ma, fn) & \pi(ma, mw) & \pi(ma, mn) \\
\pi(fw, ma) & \pi(fw, fw) & 0 & 0 & 0 \\
\pi(fn, ma) & 0 & \pi(fn, fn) & 0 & 0 \\
\pi(mw, ma) & 0 & 0 & \pi(mw, mw) & 0 \\
\pi(mn, ma) & 0 & 0 & 0 & \pi(mn, mn) \\
\end{pmatrix}
$$

For a married household, the transition probabilities $\pi(ma, fw)$ and $\pi(ma, mw)$ are computed using the life tables for males, respectively females. We interpret the transition from married household to female single non-widowed, respectively male single non-widowed, as divorce. We assume that the female is the decision maker in a married household and that
after divorce the woman does not care about the well-being of the male, which is equiva-
lent to setting transition probability from married to single male non-widowed to zero in the
household decision problem: \( \pi(ma, mn) = 0 \).\(^3\) The probability to stay married is determined
as the residual \( \pi(ma, ma) = 1 - \pi(ma, fw) - \pi(ma, mw) - \pi(ma, fn) \).

For male and female widowed household, we assume that they either re-marry with proba-
bility \( \pi(mw, ma) \) and \( \pi(fw, ma) \), respectively, or stay widowed with probability \( \pi(mw, mw) \),
respectively \( \pi(fw, fw) \). Similarly, male and female single, non-widowed households can ei-
ther marry with probability \( \pi(mn, ma) \) and \( \pi(fn, ma) \), respectively, or stay single with
probabilities \( \pi(mn, mn) \) and \( \pi(fn, fn) \).

2. Children

We consider 4 different states for the number of children in the household: no children, 1
child, 2 children, 3 children (or more). The number of children increases by one in the case
of the birth of a child and decreases by one in the case that a child leaves the household
(moves out). The number of children also changes if households marry, in which case the
children of the two marrying households are added.

We distinguish between the fertility rate of a married woman and the fertility rate of a
single woman, but because of data scarcity assume that widowed woman and non-widowed
women have the same fertility rates. Similarly, we distinguish between moving-out rates
of children for married households and moving-out rates for single households. Denote the
probability that a married household increases/decreases the number of children by one by
\( \pi(ma, +1) \) and \( \pi(ma, -1) \) and the corresponding transition probability for a female single
household by \( \pi(f, +1) \) and \( \pi(f, -1) \). For married households, the transition rates for
the number of children are then summarized by the transition matrix

\[
T_{ma} = \begin{pmatrix}
1 - \pi(ma, +1) & \pi(ma, +1) & 0 & 0 \\
\pi(ma, -1) & 1 - \pi(ma, +1) - \pi(ma, -1) & \pi(ma, +1) & 0 \\
0 & \pi(ma, -1) & 1 - \pi(ma, +1) - \pi(ma, -1) & \pi(ma, +1) \\
0 & 0 & \pi(ma, -1) & 1 - \pi(ma, -1)
\end{pmatrix}
\]

Similarly, for single female households who do not re-marry the transition rates for the

\[^3\text{For the law of motion of the model distribution over family states, we adjust these transition probabilities to account for the fact that there are two new households, one } fn \text{ and one } mn.\]
number of children are summarized by the transition matrix

\[ T_f = \begin{pmatrix}
1 - \pi(f, +1) & \pi(f, +1) & 0 & 0 \\
\pi(f, -1) & 1 - \pi(f, +1) - \pi(f, -1) & \pi(f, +1) & 0 \\
0 & \pi(f, -1) & 1 - \pi(f, +1) - \pi(f, -1) & \pi(f, +1) \\
0 & 0 & \pi(f, -1) & 1 - \pi(f, -1)
\end{pmatrix} \]

For male single households who do not marry the number of children cannot increase, but can decrease by one due to moving out. If we denote the moving out rate by \( \pi(m, -1) \), the transition matrix for male single households who do not marry reads:

\[ T_m = \begin{pmatrix}
1 & 0 & 0 & 0 \\
\pi(m, -1) & 1 - \pi(m, -1) & 0 & 0 \\
0 & \pi(m, -1) & 1 - \pi(m, -1) & 0 \\
0 & 0 & \pi(m, -1) & 1 - \pi(m, -1)
\end{pmatrix} \]

Finally, there is the event that a single female household and a single male household get married and the number of children of the two single households is added. In this case, the transition matrix for female single households is

\[ T_{f,ma} = \begin{pmatrix}
\mu_0 & \mu_{1f} & \mu_{2f} & 1 - \mu_0 - \mu_{1f} - \mu_{2f} \\
0 & \mu_0 & \mu_1 & 1 - \mu_0 - \mu_1 \\
0 & 0 & \mu_{0f} & 1 - \mu_{0f} \\
0 & 0 & 0 & 1
\end{pmatrix} \]

where \( \mu_{if} \) denotes the probability that a female and a male marry and the total number of children in the new household is \( i \). A similar transition matrix describes the transition rates for male single households, which we denote by \( T_{m,ma} \).

Combining the transition matrices for marital status and the number of children results in the joint transition matrix for family states:

\[ \Pi \otimes T = \begin{pmatrix}
\pi(ma, ma)T_{ma} & \pi(ma, fw)T_{ma} & \pi(ma, fn)T_{ma} & \pi(ma, mw)T_{ma} & \pi(ma, mn)T_0 \\
\pi(fw, ma)T_{f,ma} & \pi(fw, fw)T_f & \pi(fw, fn)T_f & \pi(fw, mw)T_f & 0 \\
\pi(fn, ma)T_{f,ma} & \pi(fn, fw)T_f & \pi(fn, fn)T_f & \pi(fn, mw)T_f & 0 \\
\pi(mw, ma)T_{m,ma} & 0 & 0 & \pi(mw, mw)T_m & \pi(mw, mw)T_0 \\
\pi(mn, ma)T_{m,ma} & 0 & 0 & \pi(mn, mn)T_0 & \pi(mn, mn)T_0
\end{pmatrix} \]

where \( T_0 \) is the transition matrix to zero children in the next period independent of the current number of children.
B. Calibration of Family Transition Matrix

In this section, we describe how we estimate transition probabilities between family states from the data. The data are the core files of waves 1 to 9 and the wave 2 fertility history topical module from the 2001 panel of the Survey of Income and Program Participation (SIPP). The death probabilities for males and females are constructed using death probabilities from the life tables published by the Human Mortality Database (HMD).

1. SIPP Data

We use data from the 2001 Panel of the Survey of Income and Program Participation (SIPP) to get estimates of transition probabilities between family states, as well as for information on employer-provided life insurance and the within household split of life insurance between husband and wife. After the National Center for Health Statistics (NCHS) stopped publishing detailed data on marriage and divorce in 1990, the SIPP has become the primary data source for marital history information (See Kreider and Fields 2001 for details). Death probabilities for males and females are not derived using the SIPP but are taken from the life tables published by the Human Mortality Database (HMD 2011).

The SIPP is conducted by the Census Bureau. The 2001 Panel collects data on roughly 35,100 households that are representative of the U.S. non-institutionalized population. It collects information on demographic characteristics, marital status, household relationship, and education. It also collects data on labor market activity, income, and participation in benefit programs. In addition, there are topical modules that provide information on specific topics. We use data from interviews conducted between February 2001 and January 2004. A household in the panel is interviewed every 4 months and each household has 9 interviews in total over the survey period. At each interview, information for the 4 months preceding the interview is collected. If household members leave a sample household, they still stay in the sample and are followed over time. Each interview is referred to as a wave. Each wave is divided into 4 rotation groups so that each month roughly a fourth of the households are interviewed. There is a set of questions that is asked for each month covered by the survey. This is the information contained in the core modules. This data is supplemented by data from topical modules. We rely on wave 3 topical module for the data on life-insurance and on wave 2 fertility history topical module for transitions in the number of children in the household. We merge data from the core files of waves 1 to 9 to create a panel of marital status histories.

We restrict the sample to reference persons and their spouses to get a sample of household heads comparable to the Survey of Consumer Finances. We label persons as married that
report being married with the spouse present or absent.\textsuperscript{4} We label persons as widowed following the coding in the data, and label all other single persons as not widowed. This last status includes divorced, separated, and never married. For each individual, we assign an age-specific marital status using the marital status the person had for the longest period of each age. We derive age-specific transition rates by computing the share of individuals who change their marital status with age using the panel dimension of the data. The transition rates are computed for 5-year age bins. The first bin covers ages 21–25 and the last bin ages 58–62. The mid point of the bin is taken as point in the age profile to which the transition rate is assigned. We regress the raw data on a fourth order polynomial in age. We use the estimated profile as input to our model. If estimated transition rates are negative, we set them to zero.

2. Divorce Rates

Figure A1 shows the smoothed profile of divorce rates for all married households based on our SIPP data.

3. Remarriage Rates

We derive remarriage rates for divorced households and widowed households separately. The small sample size of widows at young ages leads to noisy estimates. We therefore impute remarriage rates for young widows by scaling the remarriage rates of divorced households. We use a pooled sample of widows from age 30 – 50 and compare it to a sample of divorcees. We find that remarriage rates for widows are 44\% of remarriage rates of divorcees at these ages. We use this scaling factor to impute remarriage rates for widows.\textsuperscript{5} This shifting factor is very close to a corresponding shifting factor found in the NCHS data for 1988 by Wilson and Clarke (1992). The NCHS data does not suffer from a small sample problem as they observe 77,000 widowed women who remarry out of a population of 12.3 million female widows.\textsuperscript{6} Using Table 2 from their paper, we find that female widows aged 25 – 54 have remarriage rates that are 47\% of that of divorcées of the same age.\textsuperscript{7} The difference in remarriage rates is not driven by a different age composition of the two samples. Wilson and

\textsuperscript{4}The SIPP does not have a marital state “living with partner” as in the Survey of Consumer Finances (SCF).

\textsuperscript{5}If we look over the age range 23 – 61 remarriage rates for widows are 42\% of the average rate of divorcées and for the a pooled sample from age 40 – 60 remarriage rates for widows are 63\% of the rates for divorcées.

\textsuperscript{6}They also report numbers for males but male widows are only a small fraction of all widows (17\%).

\textsuperscript{7}The number for male widows is 68\%.
Clarke (1992) report remarriage rates broken down to smaller age groups and the pattern is very stable across these groups. For age group 25 – 29 remarriage rates for widows are 44.9% of the remarriage rates of divorcée, for age group 30 – 34 the number is 46.5%, for age group 35 – 44 it is 44.4%, and for age group 45 – 54 it is 50.7%. Similar results can be found in the report by Norton and Miller (1990) that uses the 1985 marriage and fertility history supplement to the Current Population Survey (CPS). They report median duration completed time in divorce and widowhood for persons who remarry. Although this is a selected subsample of widows and divorcees, they report similar differences. The median duration of widows is almost twice as large as for divorcees for persons 45 years and younger. Hong and Ríos-Rull (2012) also find lower remarriage rates for widows using data from the Panel Study of Income Dynamics (PSID) but do not report specific figures in their online appendix.

Figure A2 shows remarriage rates calculated using data from the SIPP for both divorcees and widows (male and female). The remarriage rates of widows are depicted by blue diamonds. As shown in the figure, the rates for widows aged less than 30 are missing, due to their absence from the sample. Even after age 30, the rates fluctuate wildly, at around 4% at the beginning of the 30’s and rising higher than the levels observed for divorcees in their late 30s. This fluctuation is, however, driven by very few observations. The red dots show the rates for divorcees, and the red dotted line shows the smoothed version of these data that we use in the model. The blue solid line is our adjusted remarriage rate for widows. As can be seen, this line does a good job matching the remarriage rates of people in the 40s, for which we have more data.

4. Fertility Rates

We use the wave 2 fertility history topical module to derive fertility rates by age. This module has information on the year of birth of the last child. We assign a birth event to a women if there is at most one year difference between the current calendar year (2001 in our case) and the year of birth of the child. We adjust the age of the mother by one year if the

---

8 They also report data for 1980 and very similar pattern persist. However, comparing the remarriage rates from the two years shows that there are strong trends in remarriage rates over time so that the remarriage rates from their paper would substantially overstate remarriage rates in the period to which our model is calibrated to.

9 They only report the median time to remarriage for the pooled group of widows younger than 45 years (approximately 3.9 years). For divorcée of the different subgroups the duration below age 45 is very similar (approximately 2.3 years.)

10 The month information is suppressed for confidentiality in the public use files.
year of birth was in the previous calendar year. The age-specific fertility rate is the share of females at each age that had a birth event. Given that the period during which the child could have been born covers two calendar years, we adjust rates to a one-year time span. The fertility rates for each age are computed for centered 5-year age bins. We regress the fertility rate data on a fourth order polynomial in age and use the estimated profile as the input to our model. In line with observed fertility rates, we set fertility from age 45 onwards to zero. We derive separate fertility rates for single $\pi(f, +1)$ and married woman $\pi(ma, +1)$. We use marital status information from the fourth interview of the second wave when the question of the topical module are asked. The results are depicted in figure A3.

5. Moving Out Rates

For the calibration of the probability that children move out of the household, we restrict the sample to those households who have at least one household member who has information at all 9 waves. We do this to avoid the underestimation of moving out rates due to sample attrition. In line with our model, we consider as children those children of the reference person that are less than 23 years of age.\textsuperscript{11} A child “moves out” of the household if a person that has been a child of the reference person is no longer reported as residing at home, or if that child turns 23 of age. The moving out rates are computed for 5-year age bins using the age of the mother. We use the mother’s age to be consistent across married and single households. We regress the transition rate data on a fourth order polynomial in age. We use the estimated profile as input to our model.\textsuperscript{12} We derive separate moving out rates for single $\pi(m, -1)$ resp. $\pi(f, -1)$ and married households $\pi(ma, -1)$. As there are very few single fathers in the sample, moving out rates for single families are not distinguished by the sex of the household head, i.e. we set $\pi(m, -1) = \pi(f, -1)$. The results are shown in figure A4.

6. Death Probabilities

The probability that a household member dies is taken from the life tables of the Human Mortality Database (HMD 2011). We use averages of death probabilities separately for males and females for the period 1990 - 2007. Figure A5 shows the life-cycle profile of the death probabilities for males and females.

\textsuperscript{11}In contrast, the SIPP counts as children (variable RFNKIDS) all children in the household under age 18 including grandchildren or children of household members other than the reference person and its spouse.

\textsuperscript{12}If estimated transition rates are negative, we set them to zero.
7. Initial Distribution

To derive the initial distribution over family states, we use reference persons and their spouses. We assign each person the family status from the fourth interview in wave 2 (see fertility rates above). The definition for children is as in the case of the moving out rates. We consider all persons age 21 to 25 for the initial distribution (the 5-year bin around age 23).

8. Consistency

To check the consistency of the estimated family transition matrix with the observed cross-sectional distribution over family states, we compute various life-cycle profiles derived from the estimated transition matrix and initial distribution. Overall, the deviations between implied cross-sectional distributions and empirical distributions are small. This provides evidence in support for our calibration strategy.

V. Survivor Benefits and Taxes

A. Survivor Benefits

Suppose death of an adult household member occurs at age \( j \). For each age \( k > j \), we can compute a social security survivor benefit for a median-income widowed household, \( B_{j,k} \) that depends on the number of children, \( n \). We compute this benefit as follows:

- **Step 1**: For each \( j \) and \( n \), compute Average Indexed Monthly Earnings (AIME) as

\[
AIME_{j,n} = \frac{1}{j-20} \sum_{i=20}^{j} \nu_{i,n} y_{i,n}^m
\]

where \( y_{i,n}^m \) is the median labor income of a married household with \( n \) children age \( i \) and \( 0 < \nu_{i,n} < 1 \) is a weight that measures the fraction of household earnings that has been generated by the deceased household member. We use the difference for households with \( n \) children between married and a single households of the respective gender to proxy this fraction. We assume that households’ first year of full earnings is at age 20 and further assume \( y_{20,n}^m = y_{21,n}^m = y_{22,n}^m = y_{23,n}^m \).

- **Step 2**: Compute the Primary Insurance Amount (PIA) as

\[
PIA_{j,n} = 0.9*min\{b_1, AIME_{j,n}\} + 0.32*min\{b_2, max\{AIME_{j,n} - b_1, 0\}\} + 0.15*max\{AIME_{j,n} - b_2, 0\}
\]
For the “bend points” $b_1$, $b_2$, and $b_3$ we use the official bend points in the year 2000.

- **Step 3**: Compute the maximum family benefit, $B_{j,n,max}$

We have

$$B_{j,n,max} = 1.5 \times \min\{b_1^f, PIA_{j,n}\} + 2.72 \times \min\{b_2^f, \max\{PIA_{j,n} - b_1^f, 0\}\}$$

$$+ 1.34 \times \min\{b_3^f, \max\{PIA_{j,n} - b_2^f, 0\}\} + 1.75 \times \max\{PIA_{j,n} - b_3^f, 0\}$$

As bend points $b_1^f$, $b_2^f$, and $b_3^f$ we use again the official bend points in the year 2000.

- **Step 4**: Compute potential benefits, $\tilde{B}_{j,n,k}$:

The amount of benefits the surviving household members can potentially receive is

$$\tilde{B}_{j,n,k} = \max(0, I(n > 0) \times 0.75 \times PIA_{j,n} - \max\{0.5(y_{j,n,k}^s - \tau), 0\} + 0.75 \times n \times PIA_{j,n}$$

where $y_{j,n,k}^s$ is the labor income of the surviving spouse at age $k$, $n$ is the number of surviving children, and $\tau$ is fixed threshold for income from which on deduction lead to a phase out of benefits. $I(n > 0)$ denotes an indicator if there are children in the household. We set the value $\tau$ equal to the official threshold for the year 2000.

- **Step 5**: Compute the actual benefit $\hat{B}_{j,k}$

The actual benefit for the surviving family members are

$$\hat{B}_{j,n,k} = \min(\tilde{B}_{j,n,k}, B_{j,n,max})$$

- **Step 6**: To get the benefits $B_{j,n,k}$ paid out to households, we subtract income taxes that have to be paid on benefits $\tau_{j,n,k}^B$ by comparing income taxes of a household with benefits to a household without benefits. We include the income tax advantage of benefits, namely, that benefits are only taxed to 50%. We subtract the additional taxes that have to be paid on benefits from the benefits. The benefits paid out to the surviving family members are

$$B_{j,n,k} = \max(0, \hat{B}_{j,n,k} - \tau_{j,n,k}^B)$$

**B. Payroll and Social Security Taxes**

We compute the average tax rate for a median-income household using estimated earnings profiles for married households and single households. We compute federal taxes with standard deductions taking into account deductions and tax credits for children. We use nominal
Table A1: Tax rates for 2000

<table>
<thead>
<tr>
<th>Marginal Tax Rate</th>
<th>Married Filing Jointly</th>
<th>Single</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tax Brackets</td>
<td>Tax Brackets</td>
</tr>
<tr>
<td></td>
<td>Over Below</td>
<td>Over Below</td>
</tr>
<tr>
<td>15.0%</td>
<td>$0 $43,850</td>
<td>$0 $26,250</td>
</tr>
<tr>
<td>28.0%</td>
<td>$43,850 $105,950</td>
<td>$26,250 $63,550</td>
</tr>
<tr>
<td>31.0%</td>
<td>$105,950 $161,450</td>
<td>$63,550 $132,600</td>
</tr>
<tr>
<td>36.0%</td>
<td>$161,450 $288,350</td>
<td>$132,600 $288,350</td>
</tr>
<tr>
<td>39.6%</td>
<td>$288,350 –</td>
<td>$288,350 –</td>
</tr>
</tbody>
</table>

tax brackets for the year 2000 (which is consistent with using real data in year 2000 dollars) to compute average tax rates. The rates vary according to the filing status of the household. For 2000, the U.S. income tax brackets and marginal tax rates are given in Table A1.

The child tax credit was introduced in 1997 for tax year 1998. In 2000, it was $500 per qualifying child (under age 17). There is a means test for the credit. From the 1997 law, which was in force in 2000, the reduction was $50 per $1000 over the threshold of $110,000 for married couples filing jointly, and $75,000 for non married individuals.

The numbers for the personal exemption for married couples, single people, and per dependent for 2000 are 5600, 2800, and 2800. That is, in 2000, a married household filing jointly could claim $5600 plus an extra $2800 per dependent.

The social security tax and medicare tax paid by the employee was 6.2% and 1.45%, respectively. We add theses taxes to the federal income tax to arrive at a total average tax rate.

VI. Employer-Provided Life Insurance

Here we address the issue to what extent the existence of employer-provided group insurance has the potential to distort our results. If the amount of group insurance offered by the employer exceeds the amount households want to hold, then these households are “involuntarily” over-insured and the insurance holdings observed in the data are not the outcome of the optimal insurance choice by households. Clearly, the phenomenon of involuntary overinsurance can only occur for households who have not purchased any individual life insurance

from insurance companies. Although the SCF does not distinguish between group insurance and insurance purchased individually, we can use data on employer provided life insurance from the Survey of Income and Program Participation (SIPP) to analyze this issue. Figure A6 shows the median life insurance holding of married households with children who have purchased some life insurance, and also the holdings of employer-provided life insurance for the same group of households. The figure shows that for each age between 23 and 60, the median household with children holds substantially more life insurance than the amount of insurance provided by the employer. Further, for the median household the amount of employer-provided life insurance is roughly constant over the life-cycle and the shape of the life-cycle profile of total (group plus individual) life insurance holdings is therefore not much affected by the presence of group life insurance. Thus, we conclude that the consideration of insurance purchases as voluntary is appropriate to a first approximation. Hong and Rios-Rull (2012) come to a similar conclusion after analyzing data drawn from the International Survey of Consumer Financial Decisions.

VII. Model Extensions

A. Child- and Health-Dependent Preferences

Two recent papers (Koijen et al. 2012 and Hong and Rios-Rull 2012) have used data on life insurance holdings, and holdings of other assets, to estimate the evolution of household preferences as they age and decline in health, the strength of the bequest motive, as well as the effect of changes in household size on the cost of living. In both of these papers, patterns in life insurance data are assumed to be driven by variation in preferences and cost of living parameters, in contrast to our paper where under-insurance of young households is generated through borrowing constraints.\(^{14}\) Motivated by these papers, in this section we consider an extension of our model that incorporates household preferences that depend on the number of children and the health status of the household. To simplify the discussion, we focus here on a model without the additional mortality heterogeneity, that is, we focus on households with median level of mortality risk.

We introduce two changes to our baseline model. First, we parameterize the change in the marginal utility of consumption of a household following the death of a spouse, with the parameter varying with the number of children in the household. This may be interpreted

\(^{14}\)Koijen et al. (2012) study a complete-market model without financial frictions, which implies that under-insurance cannot occur. Hong and Rios-Rull (2012) use an incomplete-market model with ad-hoc borrowing constraint, but neither missing insurance markets nor binding borrowing constraints play an important role in their analysis.
as capturing changes in the cost of living (for example, if it is cheaper to live with a smaller household, the marginal utility should decline) beyond the simple insurance component accounted for in our baseline model, or as capturing the strength of the bequest motive for younger households. In terms of the model, we allow the marginal utility of consumption, $\gamma_1$, to change following the death of an adult household member or divorce, and assume that the size of the change may vary with the number of children. To reduce the number of free parameters, we assume that $\gamma_1$ is the same for all married households independently of the number of children and that for single households $\gamma_1$ is independent of sex (male/female) and marital status (divorcee/widow). We normalize $\gamma_1$ of married households to one and choose the value of the remaining parameters to match the life-cycle average of life insurance holdings of married households with different number of children separately.

The second change to the model specification is the introduction of a health state. Following Koijen et al. (2012), we assume that households can be either in good health or in bad health and that households in bad health have lower marginal utility of consumption, $\gamma_1$. Further, a married household in bad health who experiences the death of an adult household member becomes a healthy single household. This assumption captures the idea that it is the sick member of the household who dies, an assumption that seems plausible especially for older households. Finally, we assume that up to age 35 all households are in good health and that starting at age 35 the probability of becoming sick (moving into bad health) increases linearly. Thus, we have parameterized the health process by two parameters, and we calibrate these two parameters to match two targets taken from Koijen et al. (2012): (i) the relative number of households who move from self-reported good health into self-reported bad health in the age group 50 – 60; and, (ii) the difference in the demand for life insurance between bad health and good health households ages 50 – 60.\textsuperscript{15}

Our results can be summarized as follows. First, with these changes to the model, the basic facts about life insurance and other asset holdings over the life-cycle for all married households with children are unchanged. For example, the model’s prediction for the median life insurance holdings of households with children is barely affected by this change in model specification. In figure A7 we plot the life-cycle profile of life insurance holdings for all married household with children, and find that it is very close to the plot for our baseline model (Figure 9). Second, this extension improves the match between model and data in the sense

\textsuperscript{15}Koijen et al. 2012 report that 20 percent of all households age 50 – 60 move from good health to bad health. In a early working paper version, they also report the results of a regression that shows that moving from good health to poor health adds about 50,000 dollars to life insurance holdings controlling for age and other explanatory variables.
that the extended model replicates additional cross-sectional facts. Specifically, households in bad health demand more life insurance than households in good health and households with two and three children hold substantially more life insurance than households with one child. In particular, the extended model implies that, consistent with the data, moving from one child to two children increases the bequest motive by an amount that is equal to $25,000 of life insurance, while moving from two children to three children increases life insurance holdings by $10,000. Third, if we interpret the change in marginal utility following the death of a parent as reflecting the consequent change in the cost of living, the resulting changes are relatively modest and increase in the number of kids: the cost of living falls following the death of a spouse by roughly 4% for households with either no children or 1 child, rises 2% of households with 2 children, and rises by 3% for households with 3 (or more) children. Equivalently, this may be interpreted as the strength of the bequest motive for young households rising with the number of children.

B. Annuities, Life Insurance and Bequests

In our baseline model, prior to retirement all agents can buy a complete set of insurance products, including both life insurance and annuities. However, we constrain retirees to save in a risk free security with any wealth remaining at their death distributed to newborn households. In the absence of this constraint, and without a bequest motive, retirees would only purchase annuities. We briefly discuss a variant of our model in which retirees have a bequest motive and are able to buy annuities.

Suppose that retired households preferences are augmented with a bequest motive in the form of an additive utility term of the form $v(b, s) = \kappa * u(b, s)$ where $u(b, s)$ is the utility function of a household, $b$ are bequests and $\kappa$ governs the strength of the bequest motive. Note that under this assumption, the homotheticity properties of the model are preserved. If annuities are priced in an actuarially fair manner, and if $\kappa$ is chosen so that the marginal utility of a unit of bequests equals the marginal utility of a unit of annuity wealth, then retirees will choose a level of bequests that equals their annuity wealth. This may be implemented by a portfolio with equal holdings of annuities and life insurance, which is equivalent to the restriction imposed in our baseline model\(^{16}\).

This turns out to be a not unreasonable description of the data. Although Johnson, Burman and Kobes (2004) estimate that people aged 65 and older hold on average just

\(^{16}\)These decisions may also be implemented by holding equal amounts of life insurance and annuities with the remainder of their wealth in a risk free asset. Note that we are abstracting from the fact that life insurance can be used to avoid gift and inheritance taxes.
1% of their wealth in private annuities, Gustman, Mitchell, Sanwick and Steinmeier (1997) estimate that people aged 51-61 hold between one quarter and one half of their wealth in annuity-like pensions and social security. Thus, we conclude that our restriction on retirees portfolio choices is relatively innocuous.

C. Comparison with Incomplete Market Model

In this section, we consider an incomplete-market model in which households may borrow and save using a risk-free asset subject to an exogenously imposed borrowing constraint, and may purchase life insurance, but are exogenously prohibited from accessing other financial assets. In other words, we consider the standard incomplete market model augmented by a life insurance contract. This class of models has been used in Hong and Rios-Rull (2012) to analyze how age-dependent household preferences shape the pattern of life insurance holdings over the life cycle. In principle, this class of models can also provide an explanation of the observed positive correlation between age and insurance that is solely based on binding borrowing constraints. In this model, young households expect higher future earnings growth than older households and therefore want to borrow more than older households to smooth consumption, but might be prohibited from borrowing if the exogenous borrowing constraints are too tight. In this case, younger households also buy less life insurance than older households – a positive correlation between age and insurance emerges in equilibrium.

We now provide evidence that the incomplete-market model with two assets we described above cannot explain the empirical pattern of under-insurance without age-dependent preferences or some friction in addition to borrowing constraints. To see this, note that in this model the ad hoc borrowing constraints only generates a positive correlation between age and insurance for households with negative net financial wealth. By contrast, in our theory, households may hold positive net financial assets and yet still be constrained in their ability to borrow against some subset of the possible future states of nature. This stark prediction of the incomplete markets model is strongly rejected by the data. Figure A8 plots the life insurance coefficient $I$ using SCF data on only those married households with children that have positive networth, where the human capital loss is based on the present value of income losses as described in Section I of the paper. As in Figure 1, the under-insurance of young families with positive networth depicted in figure A8 is severe, while older households are almost fully insured. Indeed, there is almost no difference between the life-cycle profiles of insurance conditional and unconditional on positive financial wealth.

There are, of course, a continuum of incomplete markets models that differ in the restrictions on financial markets that are exogenously imposed. Indeed, by allowing a complete set of assets and carefully choosing exogenous borrowing constraints, it is possible to construct
a variant of the incomplete markets model that exactly replicates the equilibrium of our baseline model. More generally, our findings suggest that for any incomplete-market model to match the data on underinsurance it must allow agents to purchase a sufficiently rich array of financial assets so that they can be constrained in their borrowing against income earned in some state tomorrow, while still holding positive net financial assets on average. Of course, an incomplete-market model with sufficiently many assets is observationally very similar to our model, except that our modeling approach is more tractable and determines borrowing constraints endogenously.

VIII. Sensitivity Analysis

We have conducted an extensive sensitivity analysis varying the main parameters of interest within a range of empirically plausible values. Overall, the main quantitative results of this paper have shown to be quite robust to these variations in parameter values (targets). For the two most important parameter dimensions, namely human capital loss upon death and contract enforcement, the results are as follows.

The analysis conducted in the paper shows how variations in mortality risk (the size of the human capital loss in the case of death of a family member) affect our conclusions regarding the life-cycle profile of insurance and under-insurance. Figures 16 and 17 in the paper demonstrate that substantial variations in the level of mortality risk induce significant shifts in the life-cycle profile of life insurance holdings, but have only small effect on the general shape of the life-cycle profile. In addition, we show in figures A9 and A10 that the life-cycle profile of the insurance coefficient is very similar across a wide range of levels of mortality risk. Thus, the model’s main implication regarding the link between age and under-insurance holds across a wide range of parameter values, that is, regardless of the location of the life-cycle profile of human capital losses.

Our discussion of the reform of the Consumer Bankruptcy Code in Section IV of the paper provides an example of a substantial change in contract enforcement.\textsuperscript{17} The results in the paper show that even though the improvement in contract enforcement has a sizable effect on the relationship between age and under-insurance, the link between age and under-insurance is still very strong after the reform. Moreover, the life-cycle profile of under-insurance implied by the model provides a reasonable good fit of the data even after the reform – see figure

\textsuperscript{17}We conduct a policy experiment and therefore do not re-calibrate the model. We have also conducted the analysis re-adjusting all parameter values, and the results are almost identical to the ones shown in Section IV of the paper.
A11. Thus, we conclude that this paper’s main results is robust to substantial changes in the enforcement parameter. Note that this result does not rule out that very large changes in credit enforcement, as they occur in a cross-country comparison, have very large effects on the relationship between age and under-insurance.

IX. Empirical Robustness

A. Earnings losses

In this section we investigate to what extent our estimates of the earnings losses upon death or divorce could be biased. We consider two issues.

First, in the paper we estimate the earnings losses by comparing the household income of married couples with children to the earnings of single households (male, respectively female) with children. We use all single male/female households as a comparison group, instead of using only widows or divorcees, in order to obtain more precise estimates of the life-cycle profile of earnings (the SCF only contains a limited number of observations on divorcees and/or widows). Clearly, our estimated earnings losses are incorrect if the earnings of all single households substantially differ from the earnings of all divorcees/widows/widowers. In figures A12 and A13 we show that there is no evidence for this view. Specifically, figure A12 shows that the life-cycle profile of median earnings of single, female households with children is very similar to the corresponding life-cycle profile for single, female households with children who are divorcees or widows. Similarly, figure A13 shows that the life-cycle profile of median earnings of single, male households with children is very similar to the corresponding life-cycle profile for single, male households with children who are divorcees or widowers.

Second, our use of cross-sectional data (SCF) to estimate earnings losses upon divorce and/or death can potentially lead to selection bias. For example, if high-income people are more likely to stay married (i.e. less likely to divorce), then the pool of married households will have more high-income people than the group of single households. Further, single high-income people might have different re-marriage rates than all single households. To investigate these and related issues, we next use panel data on divorce rates and re-marriage rates drawn from the SIPP.

We first group married households into four bins divided by the quartiles of the married household earnings distribution and examined divorce rates for these groups. The quartiles vary with age. We compute quartiles by pooling all married households within a 5-year-window centered at each age. As the quartiles vary with age, married households
are “dynamically reclassified” according to their current place in the age specific earnings distribution. The result of this analysis is plotted by age in figure A14. As can be seen from the figure, the patterns are not monotonic in earnings. The lowest earning groups (denoted by the red circles) tend to have the lowest divorce rates, especially at younger ages. The highest earnings group (the pink asterisks) tended to have higher divorce rates at younger ages, while their divorce rates at older ages were broadly similar to the lowest earnings group. The second highest earnings group tends to have the highest divorce rates, while the second lowest group has higher divorce rates when young but when old has divorce rates roughly comparable to those for the highest and lowest earnings groups. Thus, we conclude that there is no clear relationship between earnings and divorce rates.

We next examine remarriage rates of divorcees and widows grouped similarly according to their individual earnings. Specifically, the groupings are done using age varying quartiles of the distribution of all single households calculated analogously to the divorce rate calculations above. These remarriage rates are plotted in Figure A15. The Figure shows that the remarriage rates for the lowest earnings group are indeed lower than for the higher earnings group and in some cases significantly so, although we should also note that these calculations are based on a relatively small number of observations (broken into four groups by earnings, there are roughly 20 observations on divorced and widowed individuals in each earnings class for each age in the early and mid 20’s). There is no stable relationship between the remarriage rates of the second lowest and second highest groups, with the second lowest group having higher remarriage rates than the second highest group in their late 20s and thirties, but roughly equal or lower rates the rest of the time.

Finally, we investigate if previously married individuals are more likely to remarry into lower earnings married households. Some evidence on this is collected in the table A2, which examines the transition rates (in percentage points) across four earnings groups associated with marriage of previously married single households. As can be seen in the table, there is a very strong tendency for the single households in the middle earnings categories to marry into lower earnings households. Specifically, almost 45% of previously married singles in the second lowest earnings category marry into the lowest earnings category of all married households, while 76% are in the lower half of the distribution. Likewise, almost 70% of previously married singles in the second highest earnings category marry into households in the lower half of the distribution of married household earnings. This pattern is also preserved after conditioning on age, although the data is very choppy due to the small number of observations: for almost all ages from 23 to 55, singles in the second lowest earnings grouping are most likely to transition to the lowest married earnings group, and
stay in the bottom of the distribution in excess of 70% of the time; the same is true for the second highest earnings grouping of previously married people, with the exception of ages 28 to 32 where they are most likely to stay in (that is, transition into) the second highest married earnings group.

Table A2: Earnings transitions at remarriage

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<td>14.8</td>
<td>39.1</td>
<td>42.7</td>
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B. Heterogeneity in Life Insurance Holding by Wealth

For tractability, the model embodies a number of assumptions designed to generate linear homogeneous policy functions. As a result, the model makes the strong prediction that, conditional on demographic type, all households make the same portfolio choices and hold, relative to wealth, the same amount of life insurance. Although this prediction is a result of assumptions made for tractability, and does not necessarily result from the main mechanism we emphasize, it is nonetheless of interest to examine how far the model strays from the data in this respect. In this section we construct measures of underinsurance by wealth level in order to investigate this phenomenon. Our general finding is that differences in underinsurance across different subsets of the wealth distribution are small for most ages, and that the remaining small differences are most likely due to unobserved differences in human capital returns or small sample size.

To investigate this question, we partition the sample at each age in four wealth (net worth) groups. We apply the same sample selection criteria as in the main part of the paper: In particular, we look at married households with children. For each age-wealth group, we first determine the group specific median life-insurance holdings conditional on having life-insurance. The data are in 2000 Dollars, and the profiles are shown in Figure A16.

Next, we turn to the heterogeneity of the human capital loss in case of death. To do this, we construct age-dependent wealth distributions. We do this separately for married and single households. We group single and married households according to their current wealth positions in four age groups using wealth quartiles as boundaries. We then derive life-cycle labor income profiles for each of these wealth groups. We use conditional median

\[18\] Specifically, to group households of age \(j\) in wealth groups, we look at all married households in a 5-year
labor income for each wealth group as our measure of labor income.\footnote{Using the group specific mean rather than the median does not change the results much. Human capital losses are slightly higher.} We use these income profiles by wealth groups to derive estimates of the human capital loss comparable to Figures 6 and 7.\footnote{In Figures 6 and 7, we report measures of the human capital loss separately for husband’s and wife’s death and decompose it for different assumptions on remarriage rates and availability of social insurance.} Recall, the human capital loss is the present value of the labor income loss over current labor income. To ease the exposition, we show here the sum of the human capital losses after transfers and taxes and use family transition rates as in the paper (see the discussion above on the dependence of transition rates on income). Note also that we are assuming that a widow with children from a second quartile married household by wealth transitions upon the death of their spouse to a second quartile single parent household and then, if and when they remarry, transition back to a second quartile married household.

Figure A17 shows the human capital loss for the four wealth quartiles. We also show the profile for the median household as used in the calibration of the benchmark model. Note that the median refers to the median of labor income and not wealth. The median corresponds to the sum of the human capital loss from Figures 6 and 7.

Finally, we combine the information from these figures to construct the measure of underinsurance separately for each wealth group of married households with children. The results are collected in Figure A18. The figure shows that for the second, third and fourth quartiles, the life insurance coefficients are surprisingly similar. In particular, for all three of these quartiles, the young appear to hold insurance against roughly 20% of the human capital loss in the event of the death of spouse, rising to roughly 80% or more in their late 50s. The fourth quartile does show somewhat more insurance at older ages, which may reflect the small sample, but the second and third quartiles are very similar throughout the entire life cycle.

The first quartile is also similar throughout the middle years of the life cycle, but is an exception at both the youngest and oldest ages. In fact, for the lowest quartile the measure of underinsurance is “U-shaped” starting at in excess of 80% before falling to 40% by age 30, and then rising back to 80% by their late 50s.

The differences exhibited by both the youngest and oldest lowest wealth quartile families
may also be the result of a small sample problem. However, an alternative explanation is that there is an additional factor correlated with low wealth that leads to low returns to human capital investment at young ages and hence results in less binding borrowing constraints. One obvious candidate is that there is an unmeasured and un-modeled ability difference, or alternatively that the difference reflects differences in their education before age 23. When education directly affects the return to human capital investment, or whether it is simply correlated with unmeasured differences in ability, this suggests that splitting the sample by education level would also be informative.

To assess this possibility, we also examine differences in the life insurance coefficient by wealth quartile conditional on a households level of education. After conditioning on demographic characteristics and wealth, further conditioning on education levels results in even smaller samples, and so we limit our analysis to two education groups: a “high” (at least some college education) level and a “low” (no more than high school education) level. The resulting underinsurance measures for the high education group are shown in Figure A19. As displayed in the Figure, the life insurance coefficients for all four wealth quartiles with a high level of education are very similar throughout almost all of the life cycle. This suggests that the difference in underinsurance for the youngest low wealth households may be due to differences in education or to unmeasured differences in ability.

In summary, we find that differences in underinsurance across different subsets of the wealth distribution are small for most ages, with any differences occurring at both ends of the age distribution where small sample sizes are a concern. We take this to be confirming evidence for our homogeneity generating assumptions. It is also possible that the high levels of insurance observed for the youngest low wealth households are driven by differences in their underlying return to human capital accumulation as a result of differences in ability and/or education, which is not a feature of the benchmark model.

C. Additional Insurance Through Inter Vivos Transfers

An alternative explanation for the underinsurance patterns we document in the data is that we have omitted some other form of insurance against the risk of loss of one’s spouse. One obvious candidate source for additional insurance are inter vivos transfers from family members: transfers that are not bequests (they occur during the giver’s lifetime). In this section, we present new data on the size of inter vivos transfers and review the secondary literature on the subject. In summary, looking across a broad range of studies and data sources, inter vivos transfers appear to be relatively uncommon and are typically small. More importantly, there is little or no evidence that inter vivos transfers to widows and widowers (or, in the absence of data on widows, inter vivos transfers to previously married
single parents) are more common or larger than transfers to “in tact” or “still married” families. Finally, there is little evidence to suggest that transfers to young single parents are larger than to older single parents. Hence, we conclude that any insurance provided by inter vivos transfers is small and likely negligible, and moreover that it is unable to explain the pattern of underinsurance of younger households found in the data.

In coming to this conclusion, we examined several sources. We first looked at data from surveys of US households. The first data source we considered was the Survey of Consumer Finance (SCF). It is important to stress that there are very few young widows and widowers in the SCF. Specifically, pooling across all waves from 1995 to 2007 there are 2012 widows in the data set, of which 751 have children living with them. However, of these, only 65 are under the age of 40, and none of them are under the age of 30. Hence, there is little we can say about young widows with children in general and so we must often look at the group of previously married—widowed or divorced—single parents as a proxy for widows.

There are two questions in the SCF that pertain to inter vivos gifts. First, there is a retrospective question about the reception of gifts or transfers that reads “Including any gifts or inheritances you may have already told me about, have you or your husband/wife/partner ever received an inheritance, or been given substantial assets in a trust or in some other form?” We further restrict attention to transfers, and further to those transfers received from parents, grandparents, or aunts and uncles (that is, we exclude inheritances and trusts as well as transfers from outside the family). A second question relates to recent income and asks whether the household has received an “inheritance/gift”, “other help/support from relatives”, or a “gift or support n.e.c.” Thus the second set of figures include bequests as well as transfers from outside the extended family. We refer to the first measure as asset transfers and the second as income transfers to capture the fact that the second includes support that is potentially ongoing.

Looking first at widows and widowers of all ages, we find that of the 2012 widows in the sample, only 15 report receiving income from these sources, and only 41 report receiving assets in this way. Hence, not only is the median transfer zero, transfers are also zero at the 90 percentile. If we restrict attention to widows with children, 20 receive an asset transfer but none report receipt of an income transfer.

Given the small number of widows we are unable to break these numbers down by age with any confidence and so we next turn to the sample of all previously married single parents (widows and divorcees). Once again, looking across all such households, we find that such

\[\text{21 We cannot use 1992 data because marital history information is only available from 1995 onwards.}\]
gifts and transfers are uncommon and small. If we look at all previously married households, less than one per-cent (0.9%) report receiving income from these sources while only 4% report receiving assets. Averaging across the entire sample, the mean income transfer was roughly $100 while the mean asset transfer was $1500 (all numbers have been converted to year 2000 dollars) reflecting the fact that a small number of households received very large transfers. If we further restrict attention to previously married households with children, the numbers are almost exactly the same, although the mean transfer of income rises slightly to $120. Strikingly, these numbers are quite similar to those for transfers to in tact families. Among in tact families with children, less than one per-cent (0.6%) report an income transfer and roughly 4% report an asset transfer while the mean amounts were roughly $80 of income and $1900 of assets. Thus, there is little evidence that transfers to single parents are greater than those given to in tact families.

There is also little evidence to suggest that these transfers are larger for younger families. Looking across all previously married single households with children, none of the families under age 30 report an income transfer while only 2% report an asset transfer with a mean value of $600. This compares to similarly aged in tact families of which 0.3% report an income transfer (a mean of $7) and 5% report an asset transfer (a mean of roughly $3000). If we turn to families under age 40, 0.6% of previously married single parents report an income transfer (mean of $167) while 3.5% report an asset transfer (mean of $1100). The numbers for married families with children are very similar: 0.6% report an income transfer (mean of $100) while 4% report an asset transfer (mean of $2000).

In summary, the SCF data does not provide support for the idea that widows receive significant inter vivos transfers from their extended families above and beyond what extended families provide for their children and grandchildren in general. Nor is there any sense that transfers are larger for young households. Hence, we do not believe they can explain the patterns of underinsurance observed in the data.

Next we examined the Panel Study of Income Dynamics (PSID) and, in particular, responses to the 1988 supplement on *Time and Money Transfers* which has been used previously to study inter vivos transfers by Altonji, Hayashi and Kotlikoff (1997) amongst others. Respondents were asked if they had received “money help” from a parent or someone else outside the family, and whether they had received “time help”. Answers to these questions were merged with demographic information from the PSID individual and family files.

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22 This literature argues that inter vivos transfers between parents and children are driven *not* by altruism or a desire to insure children against adverse outcomes, but rather in exchange for expected future transfers from the children (e.g. Cox 1987, Altonji, Hayashi, and Kotlikoff 1997, Cox and Rank 1992).
Once again, the relative scarcity of widows in the data—there are 152 of them in our sample—limits analysis, as does the fact that they are typically quite elderly (their median age is 54). Of the 152 widows, only 53 have children at home, 18 of which the parents are under age 40 and 3 of which the parents are under age 30. As a result we will again also look at the sample of previously married single parents as well as the sample of widows alone.

Looking across the sample of all widows, roughly 9% received monetary transfers (from either family members or from outside the extended family) with an average transfer of roughly $220. Likewise, 20% of widows received time transfers which averaged 47 hours per year. Restricting attention to widows with children (and bearing in mind the small sample size problem), the proportion receiving a monetary transfer rose to 15% with a mean of $276, although the fraction receiving time help fell slightly to 19% with the mean amount of help also falling to 34 hours.

In order to obtain more data, we turn to the subsample of all previously married single households of which there are 516 in the sample, and of which 214 have children. Of these 112 are under age 40 and 39 are under age 30. Thus there is still a relatively small sample of young households to work with. For this subsample as a whole, 15% received monetary transfers which averaged $460 per year while 27% received time transfers averaging 94 hours per year. Restricting attention to previously married single parents, 22% reported receiving a monetary transfer averaging $696 per year, while 32% report receiving time help in an amount averaging 151 hours per year. Looking at younger previously married single parents, and again bearing in mind the small sample problem, the receipt of transfers is more common but they tend to be no larger in size. Specifically, 25% of previously married single parents under age 40 receive money transfers averaging $450, while 31% of previously married single parents under 30 received money averaging $356 per year. As for time transfers, 37% of the under 40 sample received time help averaging 149 hours per week, while for the under 30 sample the corresponding numbers were 41% and 182 hours.

Finally, in order to assess whether these transfers are a form of insurance against losing a spouse, we look at the sample of intact households (that is, those households that are still married. We find that, if anything, transfers to intact households are no less common or smaller (and in some cases are larger) than to divorced and widowed households. Looking at all married households, 22% receive a money transfer averaging $901 per year while 32% receive a time transfer averaging 11 hours per year. Restricting attention to married households with children, 23% receive a money transfer averaging $683 while 37% receive a time transfer averaging 138 hours per year. For younger households with children, 26% of those under 40 receive a money transfer averaging $737 while 43% receive a time transfer
averaging 167 hours, while 32% of those under 30 receive a money transfer averaging $672 and 52% receive a time transfer averaging 212 hours per year.

To summarize, like the SCF data, the PSID data reveal that: transfers of either time or money are uncommon and small; provide no evidence that they are larger for widows and other single parents than for in intact families; and, are no larger for the youngest widows and single parents. Hence, we conclude that they cannot explain the pattern of underinsurance that we document in the paper.

We also reviewed the literature on inter vivos transfers and found that it comes to similar conclusions. For the USA, as noted above, there is very little evidence on widows and widowers due to the lack of data and most results are derived for previously married single parents including both widows and divorcees. The best study is probably Hao (1996) who uses 1980s data from the National Survey of Families and Households (NSFH 1987-88), a dataset which contains 68 widows and roughly 1000 previously married women with children living at home (no results for widows alone are presented). Hao also finds that inter vivos transfers are rare: only 15% of previously married single mothers have received a monetary transfer from family in the past five years (the number rises closer to 20% if we include transfers from non-family members such as neighbors and friends). Conditional on receiving a transfer, the mean transfer was about ten thousand dollars, although this is driven entirely by a small number of very large transfers (Hao does not report the median transfer conditional on receiving a transfer, or the unconditional mean transfer, but the standard deviation is reported to be roughly eighty (80) thousand dollars). By contrast, previously married single fathers were more likely to receive a transfer (27%) but the transfers were smaller (conditional on receiving a transfer, the mean was under five thousand dollars). Importantly, the frequency of transfers to single household heads was roughly similar to observed transfers to intact families (20% of all intact families received a transfer from family in the past 5 years, rising to 30% if we include transfers from non-family members), and while the mean transfer was lower (roughly seven thousand) the difference was not statistically significant. No results are presented by the age of the parent.

Another possibility is that parents provide other forms of non-financial transfers such as free child care above and beyond what would have been provided should both parents have survived. We know of little direct evidence on this question. One exception is Marks and McLanahan (1993) who also use the NSFH to look at the provision of instrumental support (child care, transport, and repairs to house or car) and emotional support of single and married parents with young children provided by their own parents. Respondents are asked only if they received support, and not about the quantity of support provided. They find that
previously married mothers are slightly more likely to receive instrumental support (42%) than married women (30%), as well as emotional support (40% to 31%) from their parents. However, single mothers were only slightly more likely to receive support from siblings and other family members, while single fathers were less likely to receive such support than their married counterparts. In addition, single mothers and fathers were no more likely to receive support from friends, with instrumental support for both mothers and fathers especially less likely.

Further evidence can be drawn from studies using foreign datasets, although it must be acknowledged that it can be difficult to interpret results from these data due to the presence of differences in tax and transfer systems that, amongst other things, favor bequests relative to inter vivos gifts or vice versa. Halvorsen and Thoresen (2011) study the inter vivos gifts of roughly two thousand Norwegian households from 2000 to 2001 where inter vivos gifts are defined to include “any money transfer, payment of regular or extraordinary expenses, payment of travels/holidays, interest on loans or down payments on loans, and financial support through transferring cars/housing or in other ways allowing the children to make free use of cars/housing.” They find that, even using this expansive definition, only 18% of parent-adult child pairs experienced an inter vivos gift, with a conditional average of less than five thousand dollars. They find that parental inter vivos transfers to unmarried children were somewhat more likely than to married children. This is somewhat surprising given that Norwegian institutions place strict limits on the ability of parents to leave unequal bequests to their children, and hence inter vivos gifts are the primary way through which unequal transfers can be made. They interpret their results as suggesting that households have a very strong desire to make equal bequests and gifts (in addition, between two thirds and three quarters of parents state that their aim is to give equal transfers) as opposed to making transfers and bequests based on the “needs” of their children, which includes presumably whether or not they have been widowed.

Similar evidence on the lack of inter vivos transfers to widows and widowers comes from French data. Arrondel and Masson (2001) use data from the INSEE “Actifs financiers 1992” survey and find that transfers from parents and grandparents to children appear unrelated to whether or not their child was a widow or widower. Specifically, whether or not an adult child is a widow or widower had no significant effect on the likelihood that a parent makes an inter vivos gift (and the point estimates were that it had a negative effect). By contrast, if one of the parents is a widow or widower the probability of inter vivos gifts to adult children increases significantly (possibly reflecting an early distribution of higher anticipated bequests). Likewise, a child that is a widow or widower is no more likely to receive free
housing, a regular monetary stipend, or a monetary loan from their parents nor were their parents more likely to act as cosigner on a mortgage.

Suggestive evidence may also be drawn from the literature looking at monetary transfers following other important life events such as the occurrence of disability of a primary wage earner. Gallipoli and Turner (2009) examine data from the 1999-2007 waves of the Canadian Survey of Labor and Income Dynamics following an occurrence of disability. They find that “there is a small, marginally significant difference in the amount of, and likelihood of receiving, transfers from individuals outside the household in the initial years following onset but that effect appears to peter out at longer durations” (they do not report numerical results).

Suggestive evidence on non-monetary transfers may also be drawn from the literature examining spousal labor supply responses following job displacement and disability shocks. If time transfers such as child care were large following such a shock this should allow spouses to significantly increase their labor supply following a shock. However, the literature on spousal labor supply in response to job displacement shocks find either no effect on labor supply (Layard, Barton and Zabalza 1980, Maloney 1987, 1991) or only small effects (Mincer 1962, Bowen and Finegan 1968, Heckman and Macurdy 1980, 1982, Lundberg 1985, Spletzer 1992, Gruber and Cullen 2000; for a contrary finding, see Stephens 2002). Likewise, Gallipoli and Turner (2011) find no evidence of increased participation or increased hours worked by spouses following disability of husband, while Coile (2004) finds that women decrease their labor supply when their husbands experience a health shock like a heart attack or a cancer diagnosis (contrast the findings of Charles 1999). There is also some evidence that labor supply responses are particularly small for families with children (see Juhn and Potter 2007, Gong 2010 and Lundberg 1981 on a wife’s labor supply following a husband’s job loss; and Reis 2010 following health shocks).

In summary, the evidence that we have obtained combined with the results from the literature leave us confident in asserting that inter vivos transfers do not provide much insurance against the risk of death of a spouse, and moreover are no greater for young families and hence cannot explain the pattern of underinsurance of the young that we document.

D. Additional Insurance Through Remarriage

It possible that young widows remarry into households that have higher than median married incomes, at least relative to older widows. If so, remarriage provides more insurance to young widows than to older widows and hence might explain the pattern of underinsurance for the young. To investigate this, we use our data from the SCF from 1995-2007. As noted
above, there are very few young widows in the SCF and so we must necessarily look at
data on divorced families for illumination on this question, under the assumption that the
remarriage process for divorcees is not too different than the process for widows. The SCF
contains information on a limited number of marital history items including whether or not
respondents have been married before or if they are currently in their first marriage. It does
not report the age of remarriage.23

Bearing in mind these limitations, we compute the median labor income of households in
which both spouses were in their first marriage with those of married households in which
one of the spouses has been previously married. In interpreting the results, it is important to
stress that even after adding divorcees to our sample, there are still very few young married
households in which one spouse has been previously married: there are only 30 married
households in our data in which the household head was aged between 23 and 25 and was
previously married, and only 55 married households in which the spouse of the household
head was aged between 23 and 25 and was previously married.

The results are plotted in Figures A20 and A21 The red circles correspond to first mar-
rriages while the blue squares correspond to married households in which one spouse was
previously married. The first figure compares the earnings of first time marriages plotted
against the age of the household head, against the earnings of married couples in which the
household head was previously married. The second figure compares the earnings of first time
marriages plotted against the age of the spouse of the household head, against the earnings
of married couples in which the spouse of the household head was previously married. If
both the household head and their spouse have been remarried, we include them in both
comparison groups. Two things stand out in the Figures. The first is that the household
earnings of 23 and 24 year old remarried people are somewhat larger than for similarly aged
households on their first marriage. Note that it is important to keep in mind the relatively
small number of data points generating this result. The second is that by ages 25-30, any
difference in earnings has roughly disappeared, while after age 30 the households of remarried
persons tend to have lower earnings than the households of persons in their first marriage.

We take three things away from these plots. First, over the lifecycle, divorced and widowed
spouses tend to marry into families that have slightly lower incomes than first time married
households. This means that our estimates of the degree of insurance provided by remarriage
are generous and hence that our measures of underinsurance are conservative. Second,
there is evidence that the very youngest (ages 23 and 24) divorced and widowed households

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23We cannot use 1992 data because marital history information is only available from 1995 onwards.
remarry into higher income households, although the difference in earnings is small (roughly between one and two years of earnings for these households) relative to the amount of underinsurance documented. However, we must be careful in pushing these results too hard given the relatively small samples involved. Third, any difference in remarried earnings has been eliminated by the time the household reaches their late 20s meaning that these forces cannot explain the underinsurance we observe across all younger households.

In summary, we believe that the evidence is inconsistent with remarriage providing substantially more insurance against the loss of a spouse than the figures we present in the paper, and that conceivably we are slightly underestimating the degree of underinsurance.
References


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Notes: Smoothed life-cycle profile of divorce rates (percentage points). Divorce rates are derived for all married households using 2001 SIPP data. Smoothed profiles come from a regression on a fourth order polynomial in age and a constant.

Notes: Life-cycle profile of remarriage rates (percentage points). Red dots show remarriage rates for divorced singles. Remarriage rates for non-widowed singles in the model correspond to the smoothed life-cycle profile (red dashed line). Blue diamonds show remarriage rates for widowed singles age 30-50 in the data. Remarriage rates for widowed singles in the model correspond to the (adjusted) life-cycle profile (blue solid line). The adjustment of the life-cycle profile of remarriage rates for widows is derived as the mean ratio of the blue diamonds to the red dots for ages 30-50. All remarriage rates are derived using 2001 SIPP data.
Figure A3: Fertility rates

Notes: Smoothed life-cycle profile of fertility rates (percentage points). Red dashed line shows singles and blue solid line married females. Fertility rates are derived using wave 2 topical module to the 2001 SIPP. Smoothed profiles come from a regression on a fourth order polynomial in age and a constant.

Figure A4: Moving out rates

Notes: Smoothed life-cycle profile of moving out rates for single and married households (percentage points). Red dashed line shows single parent households. Blue solid line married households. Moving out rates are derived using 2001 SIPP data. Smoothed profiles come from a regression on a fourth order polynomial in age and a constant.
Figure A5: Death probabilities

Notes: Death probabilities for males and females from the life tables of the Human Mortality Database (percentage points). Blue solid line shows death probability of males. Red dashed line shows death probability of females.

Figure A6: Employer-provided life insurance

Notes: Life-cycle profile of median life insurance holdings for married households with children who have purchased some life insurance. Red dots show all holdings. Blue diamonds show holdings of employer-provided insurance. All data are from wave 3 topical module to the 2001 SIPP (in thousands of year 2000 dollars).
Figure A7: Extended model with health- and child-dependent preferences

Notes: Life-cycle profile of median life insurance holdings for married households with children who have purchased some life insurance. Blue solid line shows model. Marginal utility from consumption is different for households in poor and good health and differs across single households with different number of kids. Red dots show SCF data (in thousands of year 2000 dollars).

Figure A8: Life Insurance Coefficient with Positive Networth

Notes: Life-cycle profile of life insurance coefficient for median married households with children who have purchased some life insurance. Red dots show data for all households. Blue squares show data for all households with positive networth. Data is from SCF.
Notes: Life-cycle profile of life insurance coefficient with different levels of life insurance for married households with children who have purchased some life insurance. The red solid line shows the coefficient for households with median levels of life insurance holdings from the model. The blue dashed line shows households with mean life insurance from the model. Red dots show the life insurance coefficient for median life-insurance holdings from the data. Blue squares show the mean life-insurance holdings from data.

Notes: Life-cycle profile of life insurance coefficient for married households with children who have purchased some life insurance. The red solid line shows the life insurance coefficient for households with median levels of life insurance holdings from the model. The blue dashed line shows households in the top decile of life insurance holdings from the model. Red dots show the life insurance coefficient for median life-insurance holdings from the data. Blue squares show households in the top decile of life insurance holdings from the data; to remove outliers, we calculate the mean life insurance holdings between the 90th and 99th percentile of life-insurance holdings; for comparability, model output has been truncated at the 99th percentile of the empirical distribution.
Figure A11: Life Insurance Coefficient

Notes: Life-cycle profile of life insurance coefficient before and after the reform of the bankruptcy code for median married households with children who have purchased some life insurance. Life insurance coefficient is the ratio of life-insurance holdings to present value income loss in case of death. Blue solid line shows benchmark model before the reform of the bankruptcy code. The red dashed line shows model with fixed human capital allocation after the reform of the bankruptcy code. Red dots show SCF data. See appendix for calculation of present value loss.

Figure A12: Labor income of single females with children

Notes: Life-cycle profile of median labor income for single female-headed households with children and different marital histories. Blue squares show all single households, red dots show all single households who are divorced or widowed. All data are from the SCF (in thousands of year 2000 dollars).
Figure A13: Labor income of single males with children

Notes: Life-cycle profile of median labor income for single male-headed households with children and different marital histories. Blue squares show all single households, red dots show all single households who are divorced or widowed. All data are from the SCF (in thousands of year 2000 dollars).

Figure A14: Divorce rates by income

Notes: Life-cycle profile of divorce rates for households conditional on household income quartile (percentage points). Red circles show first, blue squares second, green diamonds third, and pink stars fourth income quartile. Divorce rates are derived for all married households using 2001 SIPP data.
Figure A15: Remarriage rates by income

Notes: Life-cycle profile of remarriage rates for all single households conditional on individual income quartile (percentage points). Red circles show first, blue squares second, green diamonds third, and pink stars fourth income quartile. Remarriage rates are derived using 2001 SIPP data.

Figure A16: Face value of life-insurance by wealth quartile

Notes: Life-cycle profile of life insurance holdings conditional on wealth quartile for married households with children who have purchased some life insurance. Red dots show first, blue squares second, green diamonds third, and pink stars fourth wealth quartile. All data are from the SCF (in thousands of year 2000 dollars).
Notes: Life-cycle profile of sum of expected human capital loss in case of husband’s and wife’s death for all married households with children conditional on wealth quartile. Human capital loss is ratio of present value labor income loss over current labor income. Red solid line shows the median household as used in the calibration. Orange dots show data for first, pink squares second, green diamonds third, and blue stars fourth wealth quartile. All data are from the SCF. See appendix for further details.

Figure A18: Life Insurance Coefficient by wealth quartile

Notes: Life-cycle profile of life insurance coefficient by wealth quartiles for married households with children who have purchased some life insurance. Life insurance coefficient is the ratio of life-insurance holdings to present value income loss in case of death. Red dots show data for first, blue squares second, green diamonds third, and pink stars fourth wealth quartile. All data are from the SCF. See appendix for calculation of present value loss.
Figure A19: Life Insurance Coefficient for high education households by wealth quartile

Notes: Life-cycle profile of life insurance coefficient by wealth quartiles for married households with children who have purchased some life insurance. Life insurance coefficient is the ratio of life-insurance holdings to present value income loss in case of death. Red dots show data for first, blue squares second, green diamonds third, and pink stars fourth wealth quartile. All data are from the SCF and for households where the head has at least some college education. See appendix for calculation of present value loss.

Figure A20: Labor income of first- and second-time married households

Notes: Life-cycle profile of median labor income for married households. Age is age of head. Red circles show first-time married households, blue squares show households where one of the spouses is in his/her second marriage (remarried households). All data are from the SCF (in thousands of year 2000 dollars).
Figure A21: Labor income of first- and second-time married households

Notes: Life-cycle profile of median labor income for married households. Age is age of spouse. Red circles show first-time married households, blue squares show households where one of the spouses is in his/her second marriage (remarried households). All data are from the SCF (in thousands of year 2000 dollars).