Abstract

This document contains two supplementary appendices to the printed version of the paper. Appendix C provides a more detailed description of our data treatment. Appendix D provides additional details on the quantitative model (setup, solution method, and variable definitions).
C Data Treatment

C.1 Correction Methods for the U.S.

The extent of underreporting of income and consumption in CEX, and their variations over time, are depicted in Figure C.1. We describe two alternative methods to deal with underreporting in CEX. The first one makes adjustments using only aggregate data, while the second one makes adjustments to consumption at the sector level. In the main text, we only refer to the second method, which is in principle more accurate.

![Figure C.1: CEX Underreporting of Income and Consumption (CEX/NIPA ratios). Source: Aggregate CEX and NIPA data.](image)

C.1.1 Method 1: Corrections Using Aggregate Data

Let \( c_{j,t}^{CEX} \) and \( y_{j,t}^{CEX} \) denote average consumption and income reported in CEX for age \( j \) in year \( t \), and let \( C_t^D \) and \( Y_t^D \) denote aggregate consumption and income in dataset \( D \). We adjust consumption and income for all ages according to

\[
\tilde{c}_{j,t} = \frac{C_t^{NIPA}}{C_t^{CEX}} c_{j,t}^{CEX}, \quad \tilde{y}_{j,t} = \frac{Y_t^{NIPA}}{Y_t^{CEX}} y_{j,t}^{CEX}.
\]
By construction, consumption expenditures and income match NIPA in the aggregate. The corrected saving rate for age $j$ in period $t$ is $\hat{s}_{j,t} = (\hat{y}_{j,t} - \hat{c}_{j,t})/\hat{y}_{j,t}$.

### C.1.2 Method 2: Corrections Using Sectoral Expenditure Data

Since the degree of underreporting is likely to differ across types of goods, and since different age groups potentially have different consumption baskets, we implement sector-specific adjustments. Let $C^D_{kt}$ be the aggregate consumption expenditures of goods in sector $k$ at date $t$ from dataset $D$. Define the following sector-specific weight:

$$\chi_{kt} = \frac{C^NIPA_{kt}}{C^{CEX}_{kt}}.$$ (1)

For all goods, the weights are greater than one due to underreporting in CEX, and they increase over time as the bias gets larger. Consider consumption of good $k$ by age-group $j$ in CEX, denoted by $c^{CEX}_{jkt}$. Our corrected measure of consumption expenditures in sector $k$ for group $j$ is (up to the additional adjustment described below):

$$\hat{c}_{jkt} = \chi_{kt} c^{CEX}_{jkt}.$$ 

Total consumption expenditures of group $j$ is then $\hat{c}_{jt} = \sum_k \hat{c}_{jkt}$. The corrected income net of taxes $\hat{y}_{jt}$ of group $j$ is, as before: $\hat{y}_{jt} = \frac{Y^NIPA_{jt}}{\hat{y}_{jt}} \hat{y}^{CEX}_{jt}$. Finally, the corrected saving rate of group $j$ is $\hat{s}_{jt} = (\hat{y}_{jt} - \hat{c}_{jt})/\hat{y}_{jt}$.

The sector-specific factors need to be slightly modified to account for the fact that health expenditures are treated differently in CEX vs. NIPA. Indeed, health expenditures in CEX are restricted to ‘out-of-pocket’ expenses, but NIPA also includes health contributions (Medicare and Medicaid), leading to very large adjustment factor $\chi_{health} \approx 5$ — which primarily affects our consumption estimates for the old, for whom ‘out-of-pocket’ health expenditures constitute a large share of their consumption basket in CEX ($\approx 12\%$). Without additional correction, we

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1A small discrepancy remains since NIPA includes some expenditures (e.g., ‘Net foreign travel and expenditures abroad by U.S. residents’ and ‘Final consumption expenditures of nonprofit institutions serving households’) which cannot be matched with CEX categories.
would tend to under-estimate the saving rate of the old. To address this concern, we amend the computation of sectoral adjustment factors as follows. We set

$$\chi_{\text{health},t} = \frac{\sum_{k \neq \text{health}} C_{NIPA_{kt}}}{\sum_{k \neq \text{health}} C_{CEX_{kt}}},$$

and for other sectors $z \neq \text{health}$,

$$\chi_{z,t} = \frac{C_{NIPA_{zt}}}{C_{CEX_{zt}}} \left[1 + \frac{C_{NIPA_{\text{health},t}}}{\sum_{k \neq \text{health}} C_{NIPA_{kt}}} - \frac{C_{CEX_{\text{health},t}}}{\sum_{k \neq \text{health}} C_{CEX_{kt}}} \right].$$

Compared to the set of simple sector-specific weights given in (1), this amendment reduces the adjustment factor for health to its average across other sectors while slightly increasing the adjustment factor of other goods. Doing so, we still match NIPA aggregate consumption. Age-saving profiles with or without adjustment for health expenditures are very similar, except for the group of individuals above 65. We also find that Method 1 (using only aggregate data) and Method 2 (using disaggregated expenditures data) produce results that are very similar.

**C.2 Correction Methods for China**

In the text, we argue that in the presence of multi-generational households, age-saving profiles obtained from the ‘household approach’ are subject to an *aggregation bias*. Multigenerational households are very prevalent in China. Figure C.2, which mimics Figure 4 in Deaton and Paxson (2000), provides further evidence on Chinese household composition and its evolution over time. For the years 1992 and 2009, the figure plots, as a function of the age of individuals, the average age of the head in the households they live in. If everyone were a household head or lived with persons of the same age (i.e., in the absence of multi-generational households), the plot would be the 45-degree line. Instead, the plot lies above the 45-degree line for young people (many of whom live with their parents), then more or less runs along the line for those aged between 40-60, and then falls below the line for the elderly—many of whom live with their children. Comparing across years, the figure suggests that young individuals are leaving
their parents’ household on average later in 2009 than in 1992. Similarly, most likely as a result of an increase in life expectancy, the elderly join their children’s households at a later age in 2009 than in 1992. The fact that the degree of disconnect at various ages changes over time suggests that the household approach could lead to biases when estimating changes in saving rates across age groups.

Figure C.2: Average Age of Household Head By Age of Individual.

To correct for the biases inherent to the household approach, we provide two alternative ‘individual’ methods — one based on the sub-sample of uni-generational households, and another based on a projection technique proposed by Chesher (1997). In the main text, we describe only the second method as we believe it is more accurate for the early years of our sample, for which we have fewer observations. However we show below that the two methods yield similar age-saving profiles (even more so towards the end of the sample period), which is quite noteworthy as they rely on different sub-samples and different identification strategies.
C.2.1 Individual Method 1: Projection Method (Chesher (1997))

In order to identify individual consumption from household consumption, we estimate the following model on the cross-section of households for every year

\[ C_h = \exp(\gamma \cdot Z_h) \left( \sum_{j \in J} c_j N_{h,j} \right) + \epsilon_h, \]

where \( C_h \) is the aggregate consumption of household \( h \), \( N_{h,j} \) is the number of members of household \( h \) in age bracket \( j \), and \( Z_h \) denotes a set of household-specific controls. In our implementation, we use 33 age brackets in \( J \) — two-year brackets from 19-20 up to 81-82, and one bracket above 83. Following Chesher (1997), multiplicative separability is assumed to limit the number of degrees of freedom, and control variables enter in an exponential term.

The control variables include:

- Household composition: number of children aged 0-10, number of children 10-18, number of adults, and depending on the specification, the number of old and young dependents. The coefficient associated with the number of children is positive, as children-related expenses are attributed to the parents.

- Household income group: households are grouped into income quintiles. The sign of the control variable (a discrete variable 1-5) is positive: individuals living in richer households consume more.

In the estimation, a roughness penalization term is introduced to guarantee smoothness of the estimated function \( c_j = c(j) \). This term is of the form:

\[ P = \kappa^2 \int [c''(j)]^2 \, dj, \]

where \( \kappa \) is a constant that controls the amount of smoothing (no smoothing when \( \kappa = 0 \) and forced linearity as \( \kappa \to \infty \)). The discretized version of \( P \) can be written \( \kappa^2(Mc_j)'(Mc_j), \)
where $M$ is the $31 \times 33$ band matrix

$$
M = \begin{bmatrix}
1 & -2 & 1 & 0 & \ldots & 0 & 0 & 0 \\
0 & 1 & -2 & 1 & \ldots & 0 & 0 & 0 \\
0 & 0 & 1 & -2 & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \ldots & -2 & 1 & 0 \\
0 & 0 & 0 & 0 & \ldots & 1 & -2 & 1
\end{bmatrix},
$$

and $c_j = [c_j]_{j \in J}$ is a $33 \times 1$ vector. Pre-multiplying $c_j$ by $M$ produces a vector of second differences. We set $\kappa = 10$.

As a robustness check, we use the projection method to estimate individual income distributions by age from household income data, and then confront the estimated distributions to the actual ones—which we observe after 1992. The estimated income distributions are indeed very close to the observed ones.\(^2\)

### C.2.2 Individual Method 2: Re-Sampling of Uni-Generational Households

Our second approach to deal with the aggregation bias consists in restricting our attention to the sub-sample of individuals living in uni-generational households, which constitute more than 40$\%$ of the entire sample.\(^3\) Individual consumption is inferred from household consumption by applying an equal-sharing rule among members of the households.\(^4\) The main issue that arises with this approach is that individuals of a certain age who live in a uni-generational household may differ systematically, along a number of attributes, from indi-

\(^2\)For the year 1986, information on income is available only at the household level. For that year, we therefore use the projection method to estimate both individual income and individual consumption. The estimated age-saving profile for 1986 is then used in the construction of the average profile over the first three years of observations (1986, 1992, and 1993).

\(^3\)Any household with one adult or several adults belonging to the same generation (i.e., with a maximum age difference less than 18 years), possibly living with a child, is treated as uni-generational. Another benefit of restricting the analysis to uni-generational households is to minimize concerns related to intrahousehold transfers, which could potentially obscure actual saving behavior.

\(^4\)Some aggregation bias remains if the equal-consumption rule does not apply to husband and wife, for example, but it is reasonable to believe that consumption sharing is more equal within a generation than across generations.
viduals of the same age living in multi-generational households. We find that individuals in uni-generational households indeed differ from the whole sample in terms of income, gender, and marital status.\textsuperscript{5} In particular, (i) individuals who live in uni-generational households tend to be richer than average, and (ii) women tend to be over-represented among the young and under-represented among the old.\textsuperscript{6} To address potential selection biases, we re-weight the observations to match the distribution of these attributes in the whole sample for each age, as described below. Given the limited number of uni-generational household observations for the youngest and oldest age groups, it is difficult to re-sample the data to match the distribution of all three attributes simultaneously for these groups. Since income and gender appear to be the variables having the greater impact on saving rates, we focus on these two variables to control for selection issues.\textsuperscript{7}

**Re-sampling of the restricted sample to match the income distribution**

Young and old individuals who live alone tend to be richer than average. To address this issue, observations are re-weighted so that the distribution of individual income for each age in the restricted sample matches the aggregate income distribution. We group individuals into 2-year age bins, and then assign weights to match the income decile distribution for each of the 2-year bins. When the number of observations is insufficient (especially at the ends of the age distribution), we use income quintiles. One potential problem with the approach is that very high weights are assigned to individuals in the lowest income quantile for the young, and that these young individuals may not be representative of the low-income youth who live with their parents. Another potential concern comes from the fact that the elderly living alone are more likely to receive monetary transfers from their children than those living in their children’s household. Hence by focusing on uni-generational households, the income

\textsuperscript{5}We find no difference between the two samples along other characteristics (e.g., the number of children).

\textsuperscript{6}In terms of marital status, young and old individuals who live in uni-generational households are more likely to be married, the reason being that young people tend to move out of their parents’ household when they get married, and the elderly are more likely to move back to their offsprings’ household when they lose their spouse. The observed gender bias may come from the fact that young women marry and leave their parents at an earlier age than men, and that widows are more likely to live with their children than widowers.

\textsuperscript{7}Re-weighting observations to match the income distribution only, the income & gender distribution, or the income & marital status distribution yields similar age-saving profiles. The only notable difference is that estimated saving rates for the youngest individuals are lower when gender is not taken into account.
of the old could be overestimated.\(^8\) Using CHIP survey data for the year 2002, for which more detailed information on inter-household transfers is available, we find that this bias exists but is small.

\textit{Re-sampling of the restricted sample to match the income & gender distribution}

To correct for the gender bias among uni-generational households, we re-sample observations to match the income distribution separately for men and women, and then combine the two distributions with weights reflecting the gender composition in the whole sample.\(^9\)

\section*{C.2.3 Estimated Profiles: Individual Methods vs. Household Approach}

Figure C.3 shows the age-saving profiles estimated by the two individual methods for the years 1992 and 2009. Although the two methods use different samples of households and different identification strategies, they yield very similar age-saving profiles.\(^10\) The discrepancy is larger at the beginning of the sample period. For the 1992 survey, there are only about 5,000 households in our sample (compared to about 16,000 after 2001), 44\% of which are uni-generational households. This makes it difficult to re-sample observations to match the aggregate distributions of attributes as for some combinations of age and income level, there are very few observations. The larger size of the sample in more recent years makes the problem less severe and the profiles produced by the two methods become even more similar. Figure C.4 displays the age-saving profiles obtained by applying the commonly used household approach based on the age of the household head. As expected, this approach generates flatter profiles (aggregation bias) and much higher saving rates for the youngest individuals (largely driven by the selection bias).

\(^8\)The information available in UHS data does not allow us to identify the component of individual income coming from inter-household transfers.

\(^9\)We proceed in the same way when controlling for income & marital status.

\(^10\)This suggests that our re-sampling procedure to control for income and gender characteristics in the first method (using the sub-sample of uni-generational) takes care of selection issues quite well. The first method does give slightly lower saving rates, indicating that some unobservable characteristics correlated with the household composition are also correlated with saving behavior.
Figure C.3: Estimated Age-Saving Profile for China in 1992 and 2009, Individual Methods. 
Notes: The uni-generational method resamples the data to match gender and income distributions by age group in the full sample. The youngest age bracket consists of individuals aged 23-26 due to lack of observation for individuals younger than 24 in the sample of uni-generational households. Chesher method controls for household characteristics as described in Appendix B.3. UHS data (1992 and 2009).

Figure C.4: Estimated Age-Saving Profile for China in 1992 and 2009, Household Method. 
Notes: The saving rate for a given age is obtained as the average household saving rate for households whose head is of that age. UHS data (1992 and 2009).
C.2.4 Robustness Checks

*Treatment of transfers.* Intra-household transfers are not directly observable but should affect the estimates to a lesser extent when only the sub-sample of uni-generational households are considered. Regarding inter-household transfers, the UHS data on individual income include information on received transfers (gifts from relatives, alimony, pensions and grants), but without detailed information on the source of the transfer (e.g., other households or government). Our measure of individual income includes all received transfers. The UHS survey also gives information on household transfer expenditures, i.e., transfer payments to other households (gifts to relatives, alimony, family support), but only at the household level. In the aggregate, we find that transfer expenditures are an order of magnitude larger than (received) income transfers, possibly due to underreporting of the latter. Furthermore, including transfer expenditures implies an estimate of the aggregate household saving rate in China that is significantly lower than estimates typically reported in the literature. As a result transfer expenditures are ignored in the estimated saving rates that we report. When implementing individual methods 1 and 2 on a measure of expenditures obtained as the sum of household consumption and transfer expenditures, we find age-saving profiles for the years 1992 and 2009 similar to those depicted in Figure C.3, but shifted downward by about 3%. However, estimates of changes in saving rates across age groups over the sample period are very similar whether transfers are included or not.

The “Three Cities” survey, available for the year 1999, provides detailed information on transfers between parents and children.\(^{11}\) In this dataset, we find that the net transfers received by young adults were on average positive only until age 22. This suggests that the young’s borrowing from their parents is mostly for education, and that there is no significant amount of borrowing by young adults within the household after they start working.

*Another potential selection bias: Selection into family arrangements.* Chesher’s projection method is not subject to selection into headship but does not control for selection into family

\(^{11}\)The study of Family Life in Urban China, referred to as the “Three Cities” survey, was conducted in Shanghai, Wuhan and Xian in 1999. The survey provides information on financial transfers between each respondent and his/her offspring.
arrangements. It could be that, for a given income, individuals living alone have a low consumption rate (and that identification in Chesher’s method relies mostly on uni-generational vs. multi-generational households). We check whether this type of selection issue leads to a potential bias in our baseline estimation procedure by performing a series of robustness checks.

First, we investigate whether the bias caused by selection into family living arrangements applies to the estimation of age-income profiles. Indeed this bias could potentially be even more relevant for income than it is for consumption: controlling for other characteristics, people living alone may not have the same earnings as those who live in a multigenerational household. One should thus expect that performing the Chesher method on household income to derive individual income for a given age would lead to a bias compared to the true age-income profile. However, applying the same Chesher method to income, we obtain an age-income profile that is very similar to the one observed in the UHS data (UHS provides data on individual income starting 1992). This indicates that the selection issue does not prevent the Chesher method from producing accurate estimates — at least when it comes to income.

Second, we apply Chesher’s projection method to estimate individual consumption and savings per age on a sample of households excluding unigenerational households with at least one individual under 30 or above 65. Here, identification of the savings of the young and old derives only from household composition within multigenerational households (where the selection bias is arguably much weaker). Age-saving profiles are then obtained by aggregating, for each age, the savings of this truncated sample and the savings of individuals in excluded households, the latter being accurately measured. The estimated age-saving profile is very much the same as the one obtained in our benchmark estimation. Similarly, using directly the whole sample but using dummies to control for unigenerational households below 30 and unigenerational households above 65, we also obtain a very similar age savings profile.

Last, we did provide an alternative methodology based on uni-generational households only (described above in Appendix C.2.2). Suppose that for a given income, individuals living alone have a low consumption tendency and identification in the Chesher method relies mostly on uni-generational vs. multi-generational households. If that were the case, then one
should see large differences in the estimated age-saving profiles for certain age groups obtained from the Chesher method and the one obtained from this alternative methodology. This other methodology estimates the age-saving profiles on the subsample of only uni-generational households, controlling for selection along some selected observables. However, we found that the difference in estimated age-saving profiles between the two methods is reasonably small, including for young households. We believe this indicates that apart from income differences, no important differences for savings decisions exist between young people living alone from young people living with their parents.

*Other (non-reported) robustness checks.* We also investigate alternative sets of controls in the Chesher and uni-generational methods, and alternative treatments of very low-income observations in the Chesher method. We also try dropping the top 1% and top 5% income earners from the sample. Estimated age-saving profiles are similar across all procedures — with the exception of the saving rates of individuals under 25, which vary between $-5\%$ and $+5\%$ of our baseline estimates depending on the method and the controls. The estimate for the younger age bracket is particularly sensitive at the beginning of the sample period for the method based on uni-generational households due to the small number of observations in this bracket. Finally, we use the two individual methods to estimate age-saving profiles with an alternative Chinese survey (Chinese Household Income Project, CHIP) for the two years where these data are available (1995 and 2002). Results are very similar both across methods and across surveys.
D Quantitative Model: Technical Appendix

This appendix provides further details on the setup, solution method, and variable definitions for the quantitative model of Section 4, which embeds the three-period version of Section 2. The definitions given in Section D.6 apply straightforwardly to the three-period case.

We consider agents whose economic life runs for \( J + 1 \) periods. Age is indexed by \( j = 0, ..., J \). We let \( c_{j,t} \) denote the consumption of an agent of age \( j \) in period \( t \) and country \( i \). In order to obtain a more realistic savings behavior for the old, we introduce a bequest motive along the lines of Abel (2001). The lifetime utility of an agent born in period \( t \) in country \( i \) is

\[
U_{t}^{i} = \sum_{j=0}^{J} \beta^{j} u(c_{j,t+j}^{i}) + \phi \beta^{J} u(R_{t+j+1}^{i}b_{t+j+1}^{i}), \tag{D-1}
\]

where \( b_{t}^{i} \) denotes the amount of bequest left in period \( t \) by an agent born in period \( t - J \), and \( \phi \) captures the strength of the bequest motive. Agents of age \( j < J \) receive a fraction \( \vartheta_{j} \) of the bequests left in every period. Thus the amount of bequests received by an agent of age \( j \) in period \( t \), denoted by \( q_{j,t}^{i} \), is related to \( b_{t}^{i} \) as follows

\[
q_{j,t}^{i} = \vartheta_{j} \frac{L_{t-j}^{i}}{L_{t}^{i}} b_{t}^{i}, \tag{D-2}
\]

where \( L_{t}^{i} \) denotes the size of the generation born in period \( t \). Gross output in country \( i \) is

\[
Y_{t}^{i} = (K_{t}^{i})^{\alpha} \left[ A_{t}^{i} \sum_{j=0}^{J} e_{j,t}^{i} L_{t-j}^{i} \right]^{1-\alpha} = A_{t}^{i} \bar{L}_{t}^{i} (k_{t}^{i})^{\alpha}, \tag{D-3}
\]

where \( \bar{L}_{t}^{i} = \sum_{j=0}^{J} e_{j,t}^{i} L_{t-j}^{i} \) denotes the total efficiency-weighted population, and \( k_{t}^{i} = K_{t}^{i}/(A_{t}^{i} \bar{L}_{t}^{i}) \) denotes the capital-effective-labor ratio. The set of efficiency weights \( \{e_{j,t}^{i}\}_{j=0}^{J} \) captures the shape of the age-income profile in period \( t \) and country \( i \). Indeed, the labor income received by agent of age \( j \) in country \( i \) in period \( t \) is \( w_{j,t}^{i} = c_{j,t}(1-\alpha)A_{t}^{i} (k_{t}^{i})^{\alpha} \equiv c_{j,t} w_{t}^{i} \). Finally the gross
rate of return between $t - 1$ and $t$ is

$$R^i_t = 1 - \delta + r^i_{K,t} = 1 - \delta + \alpha (k^i_t)^{\alpha - 1}. \tag{D-4}$$

### D.1 Individual Optimization

Consider the consumption-saving problem of an agent born in period $t$ and country $i$. This agent faces a sequence of gross rates of return \{\(R^i_{t+j+1}\)\}_{j=0}^J and labor income \{\(w^i_{j,t+j}\)\}_{j=0}^J, and receives bequest \{\(q^i_{j,t+j}\)\}_{j=0}^{J-1}. Let \(a^i_{j,t+j}\) denote his end-of-period net asset holdings at age $j$. Flow budget constraints are

$$c^i_{j,t+j} + a^i_{j,t+j} = R^i_{t+j} a^i_{j-1,t+j-1} + w^i_{j,t} + q^i_{j,t}, \quad 0 \leq j \leq J - 1, \tag{D-5}$$

$$c^i_{j,t+J} + b^i_{t+J} = R^i_{t+J} a^i_{j-1,t+J-1} + w^i_{j,t+J}, \tag{D-6}$$

with \(a^i_{-1,t-1} = 0\). Define the discounted present value of current and future labor income

$$H^i_{j,t} = w^i_{j,t} + \sum_{\tau=1}^{J-j} \frac{w^i_{j+\tau,t+j+\tau}}{\prod_{s=1}^{\tau} R^i_{t+s}}, \quad 0 \leq j \leq J - 1, \tag{D-7}$$

$$H^i_{j,t} = w^i_{j,t}. \tag{D-8}$$

The credit constraint faced by the agent at age \(j \leq J - 1\) is

$$a^i_{j,t+j} \geq -g^i_t H^i_{j+1,t+j+1} \frac{R^i_{t+j+1}}{R^i_{t+j+1}}. \tag{D-9}$$

### D.2 Autarky Steady State

Consider a steady state for country $i$ where \(c^i_{j,t} = c^i_{j}\) in every period $t$, productivity grows at constant rate \(g^i_A\), and \(L^i_{t+1} = (1 + g^i_L) L^i_t\). Let \(k^i\) denote the autarky steady state level of $k$ in country $i$,

$$\frac{K^i_t}{A^i_t L^i_t} = k^i.$$
The age-income profile is given by $w_{ij,t} = e_j^i w_t^i$, where $w_t^i = (1 - \alpha)A_t^i (k^i)^\alpha$. Thus an agent born in period $t$ faces the wage sequence $\{w_{ij,t+j}\}_{j=0}^J$ with

$$w_{ij,t+j} = e_j^i w_{i,t+j} = e_j^i (1 + g_A^i)^j w_t^i = \tilde{w}_j^i w_t^i,$$

where $\tilde{w}_j^i = e_j^i (1 + g_A^i)^j$. Taking the steady state value of the gross rate of return $R_i^t$ as given, consider the stationary individual optimization problem with normalized labor income sequence $\{\tilde{w}_j^i\}_{j=0}^J$ and a path of received bequests $\{\tilde{q}_j^i\}_{j=0}^{J-1}$ that satisfies

$$\tilde{q}_j^i = \frac{\partial j}{\partial \ell} \left[ (1 + g_L^i)(1 + g_A^i)^j \right]^{-\ell} \tilde{q}_\ell^i, \quad j = 0, \ldots, J - 1,$$

where $\ell \leq J - 1$ is an integer such that $\tilde{q}_\ell \neq 0$. Let $\{\tilde{a}_j^i(\tilde{q}_\ell^i)\}_{j=0}^{J-1}$ denote the optimal path of wealth for an agent facing this problem, and $\tilde{b}^j(\tilde{q}_\ell^i)$ the amount of bequest left by this agent. Define $\tilde{q}_\ell^i(R_i^t)$ the value of $\tilde{q}_\ell$ such that

$$\tilde{q}_\ell^i = \frac{\partial j}{\partial \ell} \left[ (1 + g_L^i)(1 + g_A^i)^j \right]^{-\ell} \tilde{b}^j(\tilde{q}_\ell^i),$$

and let $\tilde{a}_j^i \equiv \tilde{a}_j^i[\tilde{q}_\ell^i(R_i^t)]$. Stationarity and homogeneity imply that, at steady state in country $i$, the wealth at age $j$ of an agent born in period $t$ is $a_{j,t+j}^i = \tilde{a}_j^i w_t^i$.

The market clearing condition at the end of period $t$ is

$$K_{t+1}^i \equiv A_{t+1}^i \bar{L}_{t+1}^i k^i = \sum_{j=0}^{J-1} L_t^i a_{j,t}^i.$$

Using the fact that $a_{j,t}^i = \tilde{a}_j^i w_t^i/(1 + g_A^i)^j$ along with $L_{t+1}^i = (1 + g_L^i)L_t^i$, the market clearing condition can be rewritten as

$$\left( \sum_{j=0}^J \frac{e_j^i}{(1 + g_L^i)^j} \right) (k^i)^{1-\alpha} = (1 - \alpha) \sum_{j=0}^{J-1} \frac{\tilde{a}_j^i(k^i)}{[(1 + g_L^i)(1 + g_A^i)]^{j+1}},$$

where the notation $\tilde{a}_j^i(k^i)$ makes explicit the dependence of the path of net asset positions on
the steady state rate of return. Equation (D-14) implicitly defines the steady-state level of the capital-effective-labor ratio.

### D.3 Integrated Steady State

Consider an integrated steady state where $e_{j,t}^i = e_j$ for all $i$ and $t$, productivity grows at constant rate $g_A$, and $L_{t+1}^i = (1 + g_L)L_t^i$, implying $n_t^i = (1 + g_L)^A$ for all $i$ and $t$. At steady state,

$$\frac{K_t^i}{A_t^i L_t^i} = k.$$

Now the income profile by age is given by $w_{j,t}^i = e_j w_t^i$, where $w_t^i = (1 - \alpha)A_t^i k^{\alpha}$. Hence an agent born in period $t$ faces the wage sequence $\{w_{j,t+j}^i\}_{j=0}^J$ with

$$w_{j,t+j}^i = e_j w_{t+j}^i = e_j (1 + g_A)^j w_t^i \equiv \tilde{w}_j w_t^i.$$

(D-15)

The integrated steady state can be determined along the same logic as for the autarky steady state. First, taking the steady state value of the gross rate of return $R$ as given, we consider the stationary individual optimization problem with normalized labor income sequence $\{\tilde{w}_j\}_{j=0}^J$. For a given bequest sequence $\{\tilde{q}_j = \tilde{q}_j(\tilde{q}_\ell)\}_{j=0}^{J-1}$ that satisfies (D-11), let $\{\tilde{a}_j(\tilde{q}_\ell)\}_{j=0}^J$ denote the optimal path of wealth for an agent in country $i$, and $\tilde{b}(\tilde{q}_\ell)$ the amount of bequest left by this agent. For each country $i$, define $\tilde{q}_\ell^i(R)$ the value of $\tilde{q}_\ell$ such that

$$\tilde{q}_\ell = \frac{g_\ell}{[(1 + g_L)(1 + g_A)]^{J-\ell}} \tilde{b}(\tilde{q}_\ell),$$

(D-16)

and let $\tilde{a}_j^i \equiv \tilde{a}_j^i[\tilde{q}_\ell^i(R)]$. Stationarity and homogeneity imply that, at the integrated steady state, the wealth at age $j$ of an agent born in period $t$ in country $i$ is $a_{j,t+j}^i = \tilde{a}_j^i w_t^i$.

The market clearing condition at the end of period $t$ is

$$\sum_i K_{t+1}^i = k \sum_i A_{t+1}^i \bar{L}_{t+1}^i = \sum_i \sum_{j=0}^{J-1} L_{t+j}^i a_{j,t}^i.$$

(D-17)
Let $\lambda^i \equiv A^i_t L_t / \left( \sum_h A^h_t L^h_t \right)$ denote the constant share of country $i$ in world effective labor. The market clearing condition can be rewritten as

$$\left( \sum_{j=0}^{J} \frac{e_j}{(1 + g_L)^j} \right) k^{1-\alpha} = (1 - \alpha) \sum_{j=0}^{J-1} \sum_i \lambda^i \bar{a}^i_j(k) \left[ (1 + g_L)(1 + g_A) \right]^{j+1}. \quad (D-18)$$

**D.4 Dynamics**

The law of motion for $k_t \equiv (k^i_t)_{i=1}^N$ depends on whether countries are financially integrated or in financial autarky. If countries are closed financially in period $t$, the market clearing condition in country $i$ is

$$A^i_{t+1} \bar{L}^i_{t+1} k^i_{t+1} = \sum_{j=0}^{J-1} L^i_{t-j} a^i_{j,t}. \quad (D-19)$$

The generations who matter in period $t$ are those born in periods $t - J + 1$ to $t$. Thus market clearing in period $t$ pins down $k^i_{t+1}$ given

- lagged values $K^i_{L,t+1} \equiv \{ k^i_{t-J+1}, \ldots, k^i_t \}$ and future values $K^i_{F,t+1} \equiv \{ k^i_{t+2}, \ldots, k^i_{t+J+1} \}$,
- bequests $B^i_{t+1} \equiv \{ b^i_{\tau} \}_{\tau=t-J+1}$,
- productivity sequence $\{ A^i_{\tau} \}_{\tau=t-J+1}$,
- evolution of demographics, i.e., $\{ L^i_{\tau} \}_{\tau=t-J+1}$,
- evolution of age-income profile, i.e., $\{ e^i_{j,\tau+j} \}_{j=0}^{J}$ for $\tau = t - J + 1, \ldots, t$.

Note that bequests $\{ b^i_{\tau} \}_{\tau=t-J+1}$ are determined along with $k^i_{t+1}$.

If instead countries are financially integrated in period $t$, then rates of return are equalized across countries

$$R^i_{t+1} = R_{t+1}, \quad \text{for all } i,$$

and so are their capital-effective-labor ratios

$$k^i_{t+1} \equiv K^i_{t+1} / (A^i_{t+1} \bar{L}^i_{t+1}) = k_{t+1}, \quad \text{for all } i.$$
The market clearing condition in period $t$ is

$$\sum_i K^i_{t+1} = k_{t+1} + \sum_i A^i_{t+1} L^i_{t+1} = \sum_i \sum_{j=0}^{J-1} L^i_{t-j} a^i_{j,t}. \quad (D-20)$$

Thus market clearing in period $t$ pins down $k_{t+1}$ given

- lagged and future values, $K_{L,t+1} \equiv \{k^t\}_{t=-J+1}$ and $K_{F,t+1} \equiv \{k^t\}_{t=t+2}$,
- bequests $B_{t+1} \equiv \{b^t\}_{t=-J+1}$, where $b^t = (b^t_i)_{i=1}$,
- productivity sequence $\{A^i_t\}_{t=-J+1}$ for $i = 1, \ldots, N$,
- evolution of demographics, i.e., $\{L^i_t\}_{t=-J+1}$ for $i = 1, \ldots, N$,
- evolution of age-income profile, i.e., $\{e^i_{j,t}\}_{j=0}$ for $\tau = t - J + 1, \ldots, t$ and $i = 1, \ldots, N$.

### D.5 Simulations

In our main experiment, we consider a situation where countries start in financial autarky and integrate in period $X$ (we set $X = 0$ in the paper). Hence for $t \geq X + 1$, $k^i_t = k^*_t$, for all $i$. Around the integration period, we also feed the model with “shocks” to productivity, demography, and age-income profiles. We allow for shocks over the window $[X - \tau, X + \tau]$.

All shocks are perfectly anticipated. In order to determine how the global economy responds to financial integration and other contemporaneous shocks, we use the following algorithm.

1. We assume each country starts at its autarkic steady state, and that the economy does not react to future shocks before period $X - T$, for $T > \tau$ large. That is, $k^i_t = k^i*$ for $t \leq X - T - 1$. The initial steady state for country $i$ is determined by the country-specific parameter $\theta^i$, along with $(g^i_t, g^i_A)$ and $\{e^i_{j,t}\}_{j=0}$, as described in Section D.2. From the initial steady state, we obtain bequests $b^i_t$ for $t \leq X - T - 1$, which gives us $B^i_{X-T}$.

2. We assume the global economy has converged to its integrated steady state $k^*$ in period $X + T + 1$. That is, $k_t = k^*$ for $t \geq X + T + 1$. The final steady state is determined
3. We then determine the transition path \( \{k_i\}_{t=T}^{X+T} \) iteratively as follows.

- We start with a guess \( \{k_i^{(0)}\}_{t=T}^{X+T} \). We set \( k_i^{(0)} = k^* \) for \( t \leq X \) and \( k_i^{(0)} = k^* \) for all \( i \) for \( t \geq X + 1 \).

- For \( n \geq 0 \), given the path \( \{k_i^{(n)}\}_{t=T}^{X+T} \), the updated path \( \{k_i^{(n+1)}\}_{t=T}^{X+T} \) is obtained as follows. First, we obtain \( \{k_i^{(n+1)}\}_{t=T}^{X} \) by iterating on the autarkic forward-backward difference equation (FBDE) in each country (see Section D.4).

Specifically, for each country \( i \),

- We compute \( k_i^{(n+1)} \) as the solution to the autarkic FBDE, given \( K_i^{(n)} \),

\[ K_{F,X,T}^{i} \text{ and } B_{X,T}^{i}. \]

Along with \( k_i^{(n+1)} \), we get \( q_i^{(n+1)} \).

- We compute \( k_i^{(n+1)} \) as the solution to the autarkic FBDE, given \( K_i^{(n)} \),

\[ K_{F,X,T+1}^{i} \text{ and given past bequests } B_{X,T+1}^{i} = \left( b_{X,T-J+1}^{(n)}, \ldots, b_{X,T-1}^{(n)}, b_{X,T}^{(n+1)} \right). \]

Along with \( k_i^{(n+1)} \), we get \( u_i^{(n+1)} \) which is later used for computing \( k_i^{(n+1)} \).

- We repeat the previous steps until we have determined \( k_i^{(n+1)} \).

Then, we determine the common path \( \{k_i^{(n+1)}\}_{t=T}^{X+T} \) by iterating on the integrated FBDE. Specifically:

- We compute \( k_{X+T}^{(n+1)} \) as the solution to the integrated FBDE given \( K_{L,X+1}^{(n)} \),

\[ K_{F,X+1}^{(n)} \text{ and past bequests } \left( b_{X+J+1}^{(n+1)}, \ldots, b_{X+1}^{(n+1)} \right). \]

Along with \( k_{X+1}^{(n+1)} \), we get \( b_{X+1}^{(n+1)} \) which is used for computing \( k_{X+2}^{(n+1)} \).

- We proceed until we have determined \( k_{X+T}^{(n+1)} \).

- We iterate on \( n \) until convergence, based on the distance between two consecutive paths \( \{k_i^{(n)}\}_{t=T}^{X+T} \) and \( \{k_i^{(n+1)}\}_{t=T}^{X+T} \).

4. We set \( T \) large enough for the distances \( |k_{X-T}^{(n)} - k^*| \) and \( |k_{X+T}^{(n)} - k^*| \) to fall below some convergence threshold.
D.6 Definitions

Investment in country \(i\) in period \(t\) is

\[
I_t^i \equiv K_{t+1}^i - (1 - \delta)K_t^i = A_{t+1}^i \tilde{L}_{t+1}^i k_{t+1}^i - (1 - \delta)A_t^i \tilde{L}_t^i k_t^i.
\]

Let \(W_{t-1}^i\) denote aggregate wealth in country \(i\) at the end of period \(t - 1\):

\[
W_{t-1}^i \equiv \sum_{j=0}^{t-1} L_{t-j}^i \alpha_{j,t-j}^i.
\]

The net foreign asset position of country \(i\) at the end of period \(t - 1\) is defined as

\[
NFA_{t-1}^i \equiv W_{t-1}^i - K_t^i.
\]

Aggregate savings are defined as GNP minus aggregate consumption, i.e.,

\[
S_t^i \equiv Y_t^i + (R_t - 1)NFA_{t-1}^i - C_t^i,
\]

where \(C_t^i = \sum_{j=0}^{J} L_{t-j}^i c_{j,t-j}^i\). One can easily show that:

\[
S_t^i = \Delta W_t^i + \delta K_t.
\]

Savings net of capital depreciation correspond to the change in country wealth \(\Delta W_t^i\).

The current account position of country \(i\) in period \(t\) is

\[
CA_t^i \equiv NFA_t^i - NFA_{t-1}^i = \Delta NFA_t^i.
\]

From the definitions above, it follows that \(CA_t^i = S_t^i - I_t^i\). Finally, let \(S_{j,t}^i\) denote the individual level of savings for an agent of age \(j\) in country \(i\) and period \(t\), defined as

\[
S_{j,t}^i \equiv NDI_{j,t}^i - c_{j,t}^i.
\]
where the first term denotes the agent’s net disposable income

\[ NDI_{j,t}^i \equiv (R_t^i - 1) a_{j-1,t-1}^i + w_{j,t}^i + q_{j,t}^i. \]  
(D-27)

The saving rate for age \( j \) is computed as

\[ s_{j,t}^i \equiv \frac{S_{j,t}^i}{NDI_{j,t}^i} = 1 - \frac{c_{j,t}^i}{NDI_{j,t}^i}. \]  
(D-28)

### D.7 Calibration

Section 4.2 summarizes the calibration of the quantitative model. The ratio of credit constraint parameters \( \theta^{US}/\theta^{CH} \) is set to match household debt-to-GDP data. The vector of unobservable parameters \( \psi \equiv [\sigma, \beta, \phi, \theta^{US}] \) is calibrated to savings and bequest data. Let \( s_0^i \) and \( s_{j,o}^i \) denote the model-implied aggregate saving rate and the saving rate of agents of age \( j \) in country \( i \) in the integration period, and let \( s_{0,d}^i \) and \( s_{j,d}^i \) denote their counterparts in the data.\(^{12}\) Also let \( b_{0}^{US} \) and \( b_{0,d}^{US} \) denote the U.S. bequest-to-GDP ratio in the model and in the data, respectively. We search over a large grid \( \Psi \) the vector \( \psi^* \) such that

\[
\psi^* = \arg\min_{\psi \in \Psi} \sum_i \left| s_{0}^i (\psi) - s_{0,d}^i \right| + \sum_i \sum_j \omega_j^i \left( s_{j,o}^i (\psi) - s_{j,d}^i \right)^2
\]

subject to \( \left| b_{0}^{US} (\psi) - b_{0,d}^{US} \right| < \epsilon, \)

with \( \sum_j \omega_j^i = 1 \) in each country.\(^{13}\) We set \( \epsilon = 10^{-5} \) and search over a wide range of parameter values — involving values for \( 1/\sigma \) in \([1.5, 3.5]\), values for \( \beta \) (annualized) in \([0.88, 0.99]\), values for \( \phi \) in \([0, 0.3]\), and values for \( \theta^{US} \) in \([0.05, 0.3]\). In practice, we start the search with a coarse grid to identify the region of the parameter space where the solution lies, and then refine the grid gradually.

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\(^{12}\)In the micro term of the distance metric, we use age-saving profiles with 10-year age brackets, as micro data aggregated by finer age groups were not available for the U.S. at the beginning of the sample period.

\(^{13}\)The optimal set of parameter values \( \psi^* \) does not depend on whether age groups are equally weighted or weighted to reflect their shares in each country’s population (or effective population) at the time of opening.