Estimating a War of Attrition: The Case of the U.S. Movie Theater Industry
Online Appendix

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Appendix A: Proofs (Online Appendix, Not for Publication)

Claim *Consider a two-player game. Pick θ arbitrarily such that $\theta > \overline{\Pi}_2$. Let $T(\boldsymbol{\omega}, \cdot)$ be the policy function if the two-player subgame starts at $t = 0$; i.e., $\boldsymbol{\omega} = (2, 0, \mathbf{h}^0)$. Let $\Phi(t; \boldsymbol{\omega})$ be its inverse. For any t_0 such that $0 < t_0 < T(\boldsymbol{\omega}, \theta)$, let $\widetilde{T}(\boldsymbol{\omega}', \cdot)$ with $\boldsymbol{\omega}' = (2, t_0, \mathbf{h}^{t_0})$ denote another policy function. Also let $\widetilde{\Phi}(t; \boldsymbol{\omega}')$ be its inverse. Then, $T(\boldsymbol{\omega}, \theta) = \widetilde{T}(\boldsymbol{\omega}', \theta)$ for all $\theta \leq \widetilde{\theta}$ where $\widetilde{\theta} = \Phi(t_0; \boldsymbol{\omega})$.*

In other words, the optimal exit time that is planned at time zero equals the one that is planned at some later time, conditional on nobody having exited until then.

Proof of Claim In equilibrium, at time t_0, theater i knows that θ_j is equal to or smaller than $\Phi(t_0; \boldsymbol{\omega})$. If not, theater j would have been better off by exiting at $t_0 - \varepsilon$, which contradicts the construction of $\Phi(t_0; \boldsymbol{\omega})$. Thus, $\widetilde{\theta} = \Phi(t_0; \boldsymbol{\omega})$ is the highest possible value of θ of surviving opponents and is a sufficient statistic for the history of the game up until t_0. If theater i chooses stopping time $\tau \geq t_0$, the present discounted sum of its expected profits at

t_0 is

$$V(\tau, \widetilde{T}_{-i}, \boldsymbol{\omega}', \theta_i) = \Pr(\widetilde{T}(\boldsymbol{\omega}', \theta_j) \geq \tau) \left[\int_{t_0}^{\tau} \Pi_n(t) e^{-r(t-t_0)} dt + \frac{\theta_i}{r} e^{-r(\tau - t_0)} \right] + \int_{\left\{\theta_j | \widetilde{T}(\boldsymbol{\omega}', \theta_j) < \tau\right\}} \left[\int_{t_0}^{\widetilde{T}(\boldsymbol{\omega}', \theta_j)} \Pi_n(t) e^{-r(t-t_0)} dt + e^{-r(\widetilde{T}(\boldsymbol{\omega}', \theta_j) - t_0)} V(\widetilde{T}, \boldsymbol{\omega}'', \theta_i) \right] g(\theta_j | \theta_j \leq \widetilde{\theta}) d\theta_j,$$

where $\boldsymbol{\omega}'' = (1, \widetilde{T}(\boldsymbol{\omega}', \theta_j), \mathbf{h}^{\widetilde{T}(\boldsymbol{\omega}', \theta_j)})$. Taking the first-order condition and rearranging gives

$$\begin{aligned} \widetilde{\Phi}'(t; \boldsymbol{\omega}') &= -\frac{G(\widetilde{\Phi}(t; \boldsymbol{\omega}') | \theta_j \leq \widetilde{\theta})}{g(\widetilde{\Phi}(t; \boldsymbol{\omega}') | \theta_j \leq \widetilde{\theta})} \left[\frac{\widetilde{\Phi}(t; \boldsymbol{\omega}') - \Pi_2(t)}{V(\widetilde{T}, \boldsymbol{\omega}'', \widetilde{\Phi}(t; \boldsymbol{\omega}'')) - \widetilde{\Phi}(t; \boldsymbol{\omega}')/r} \right] \\ &= -\frac{G(\widetilde{\Phi}(t; \boldsymbol{\omega}'))}{g(\widetilde{\Phi}(t; \boldsymbol{\omega}'))} \left[\frac{\widetilde{\Phi}(t; \boldsymbol{\omega}') - \Pi_2(t)}{V(\widetilde{T}, \boldsymbol{\omega}'', \widetilde{\Phi}(t; \boldsymbol{\omega}'')) - \widetilde{\Phi}(t; \boldsymbol{\omega}')/r} \right]. \end{aligned} \tag{A1}$$

where the second equality follows from $\frac{G(\widetilde{\Phi}(t;\boldsymbol{\omega}') | \theta_j \leq \widetilde{\theta})}{g(\widetilde{\Phi}(t;\boldsymbol{\omega}') | \theta_j \leq \widetilde{\theta})} = \frac{G(\widetilde{\Phi}(t;\boldsymbol{\omega}'))}{g(\widetilde{\Phi}(t;\boldsymbol{\omega}'))}$. The boundary condition is

$$\widetilde{\Phi}(t_0; \boldsymbol{\omega}') = \Phi(t_0; \boldsymbol{\omega}). \tag{A2}$$

Since (1) and (A1) are the same, (A2) implies

$$\widetilde{\Phi}(t; \boldsymbol{\omega}') = \Phi(t; \boldsymbol{\omega}) \; \forall t \geq t_0.$$

Equivalently, $T(\boldsymbol{\omega}, \theta) = \widetilde{T}(\boldsymbol{\omega}', \theta)$ for all $\theta \leq \Phi(t_0; \boldsymbol{\omega})$.■

Value Function To characterize the value of theaters, define $t^*(\boldsymbol{\theta}_{-i}) = \min_{j \neq i} \{T(\boldsymbol{\omega}, \theta_j)\}$. That is, $t^*(\boldsymbol{\theta}_{-i})$ gives the earliest exit time of i's competitors. Note that I make the dependence of t^* on state variables and strategy implicit. If theater i chooses stopping time τ when the state variables are given by $(\boldsymbol{\omega}, \theta_i)$ and the other theaters follow strategy $\mathbf{T}_{-i}$, the present discounted value of i's expected payoff is

$$V_i(\tau, \mathbf{T}_{-i}, \boldsymbol{\omega}, \theta_i) = \Pr(t^*(\boldsymbol{\theta}_{-i}) \geq \tau) \left[\int_{s}^{\tau} \Pi_n(t) e^{-r(t-s)} dt + \frac{\theta_i}{r} e^{-r(\tau - s)} \right] + \int_{\{\boldsymbol{\theta}_{-i} | t^*(\boldsymbol{\theta}_{-i}) < \tau\}} \left[\int_{s}^{t^*(\boldsymbol{\theta}_{-i})} \Pi_n(t) e^{-r(t-s)} dt + e^{-r(t^*(\boldsymbol{\theta}_{-i}) - s)} V(\mathbf{T}, \boldsymbol{\omega}', \theta_i) \right] g(\boldsymbol{\theta}_{-i}) d\boldsymbol{\theta}_{-i}.$$

where $\boldsymbol{\omega}' = (n-1, t^*(\boldsymbol{\theta}_{-i}), \mathbf{h}^{t^*(\boldsymbol{\theta}_{-i})})$ and $g(\boldsymbol{\theta}_{-i}) = \prod_{j \neq i} g(\theta_j)$.

Proof of Lemma 2 (i) Arbitrarily pick $\theta \in \left[\Pi_{N-1}(0), \overline{\theta}\right]$ and let $\boldsymbol{\omega} = (N, 0, \mathbf{h}^0)$. Suppose $T(\boldsymbol{\omega}, \theta) > 0$. Then, θ is the marginal type at $t = T(\boldsymbol{\omega}, \theta)$ that is indifferent between exiting at t and exiting at $(t + dt)$. The value of waiting until $(t + dt)$ is the probability that one of the opponents drops out in $[t, t + dt]$ conditional on it having survived until t, times the value of entering the $N - 1$ player subgame. On the other hand, the cost of waiting until $(t + dt)$ is $\{(\theta - \Pi_N(t))\, dt\}$, which is positive because $\theta \geq \Pi_{N-1}(0) > \Pi_N(0) > \Pi_N(t)$. Since the value should be equal to the cost of waiting by assumption of the marginal type, the value of entering the $N - 1$ player subgame should be positive as well.

If one of the competitors, say j, drops out in the interval, theater θ's value of entering the subgame depends on the exit values of other surviving competitors. Since θ is the marginal type at $T(\boldsymbol{\omega}, \theta)$, however, it follows that $\Pr(\theta < \theta_k) = 0$ for any competitor k. Otherwise, player θ_k should have dropped out earlier. This implies that player θ drops out immediately after the $(N - 1)$-player subgame starts. Therefore, the value of staying for theater θ should be zero. This is a contradiction.

(ii), (iii) By the same argument as Lemma 1 of Fudenberg and Tirole (1986).■

Proof of Lemma 3 (i') The value of exit of surviving theaters is θ^* or lower, since the equilibrium strategy $T(\boldsymbol{\omega}, \theta)$ is strictly decreasing in θ, as shown in Lemma 2. Therefore, $\Pi_n(t)$ is higher than the value of exit for any surviving theater until t^*, and thus no theater will exit between t' and t^*.

(ii'), (iii') By the same argument as Lemma 2.■

Expression for Beliefs Lemma 2 (ii) and Lemma 3 (ii') allow me to describe $g(\theta_j|\mathbf{h}^t)$ explicitly:

$$g(\theta_j|\mathbf{h}^t) = \begin{cases} \frac{g(\theta_j)}{\Pr(T(\boldsymbol{\omega},\theta_j)\geq t)} & T(\boldsymbol{\omega}, \theta_j) \geq t, \\ 0 & T(\boldsymbol{\omega}, \theta_j) < t. \end{cases}$$

Proof of Proposition 5 In the symmetric case, a slight modification of the proof for Lemma 3 in Fudenberg and Tirole (1986) suffices. In particular, letting $\Phi(t; \boldsymbol{\omega})$ and $\widetilde{\Phi}(t; \boldsymbol{\omega})$ be distinct solutions of (1), (2), and (3), I can obtain a contradiction.■

Proof of Proposition 6 A theater with $\theta_i = \Phi(t; \boldsymbol{\omega})$ does not drop out if $\Pi_n(t)$ is strictly larger than its exit value. If $\Pi_n(t) = \Phi(t; \boldsymbol{\omega})$, the theater does not drop out either, because there is

a positive probability that some of its competitors exit in the next instant; i.e.,

$$\Pr(\Phi(t;\boldsymbol{\omega}) < \max_{j\neq i}\{\theta_j\} < \Phi(t+s;\boldsymbol{\omega})) > 0$$

for all t and $s > 0$. Thus, $\Pi_n(t) < \Phi(t;\boldsymbol{\omega})$. Next, to show that $\Phi(t;\boldsymbol{\omega}) < \Pi_{n-1}(t)$, suppose $\Phi(t;\boldsymbol{\omega}) = \Pi_{n-1}(t)$. A theater with $\theta_i = \Pi_{n-1}(t)$ is indifferent between staying in and exiting. By Assumption 1, however, there is always a positive probability that every theater stays in; i.e., $\Pr(\max\{\theta_j\}_{j\neq i} < \overline{\Pi}_n) > 0$. Therefore, theater $\theta_i = \Pi_{n-1}(t)$ would be better off by dropping out at $t-\varepsilon$. This contradicts the construction of $\Phi(t;\boldsymbol{\omega})$. Thus, $\Phi(t;\boldsymbol{\omega}) < \Pi_{n-1}(t)$.■

Appendix B: Initial Conditions Problem (Online Appendix, Not for Publication)

Formally, letting $f_Y(y|z)$ be the joint density of $Y \in \mathbb{R}^K$ conditional on $Z = z$, the initial conditions problem suggests that

$$f_{\boldsymbol{\theta},\alpha}(\boldsymbol{\theta},\alpha|\mathbf{X},n) \neq f_{\boldsymbol{\theta},\alpha}(\boldsymbol{\theta},\alpha|\mathbf{X}),$$

where n is the observed number of theaters in 1949 and $\mathbf{X}$ is a set of observable market covariates. In addition, by assumption, we have

$$f_{\boldsymbol{\theta},\alpha}(\boldsymbol{\theta},\alpha|\mathbf{X}) = f_{\boldsymbol{\theta}}(\boldsymbol{\theta}|\mathbf{X})\, f_{\alpha}(\alpha|\mathbf{X})$$

but

$$f_{\boldsymbol{\theta},\alpha}(\boldsymbol{\theta},\alpha|\mathbf{X},n) \neq f_{\boldsymbol{\theta}}(\boldsymbol{\theta}|\mathbf{X},n)\, f_{\alpha}(\alpha|\mathbf{X},n).$$

Thus, I need to obtain $f_{\boldsymbol{\theta},\alpha}(\boldsymbol{\theta},\alpha|\mathbf{X},n)$, which is generated endogenously from the game before 1949 in order to calculate the likelihood function or to simulate moments of the dynamic game of exit. One way to address this issue is to simulate the model starting from the time when the industry was born, and infer the distributions of $\boldsymbol{\theta}$ and α that are consistent with the industry structure in 1949. The movie theater industry, however, had a non-stationary structure in the sense that it experienced a boom in the 1920's and 1930's, and afterwards faced declining demand in the 1950's. Therefore, simulating the entire history of the industry requires one to model the life cycle of the industry, which is beyond the scope of the paper.

Instead, I approximate $f_{\boldsymbol{\theta},\alpha}(\boldsymbol{\theta}, \alpha|\mathbf{X}, n)$ by simulation as follows. I assume that $\bar{N}$ potential players play an entry game at time zero (in 1949 in my model), and entrants play the exit game afterwards. Specifically, each potential entrant draws its value of exit, and by comparing it with the expected value of entry, it chooses whether or not to enter the market. For a given value of market heterogeneity and set of exit values of all potential entrants, the entry game predicts the number of entrants in equilibrium. Since the initial number of theaters is observed, I use the entry game to restrict the support of market heterogeneity and exit values so that, in equilibrium, the entry game predicts the same number of entrants as is observed. Since it is hard to characterize such a restriction analytically, I use a simulation to approximate the joint distribution of market heterogeneity and exit values, conditional on the observed number of entrants. This simulated distribution can then be used as an input to simulate and solve the dynamic stage of the game.

My approach can be regarded as a reduced form of a full simulation of the entire history of the industry. Heckman (1981) proposes introduction of an additional reduced-form equation to account for the potential correlation between the initial state of a sample and unobservable heterogeneity, and jointly estimates the model with the additional equation. Thus, the current approach can also be regarded as an extended version of Heckman (1981)'s model in the context of games with serially correlated private information. Aguirregabiria and Mira (2007) also account for the initial conditions problem associated with market-level unobservables, but in their context private information is iid over time and thus players are not selected samples in that aspect.

To begin, I make the following assumption:

Assumption *All potential players make an entry decision simultaneously at time zero without knowing that a war of attrition will immediately follow the entry game. That is, every player assumes that, upon entry, it earns the same per-period payoff forever.*

I use $\pi_n^e(m)$ as the payoff from entering the game when n theaters do so. The above assumption justifies this. If theaters expect that a war of attrition will start or payoffs will change sometime in the future, then computing the value of entry is a lot more complicated.

Remember that theater i's optimal choice D_i^* is given by

$$\begin{aligned} D_i^* &= \mathbf{1}\left\{(\alpha_m + \boldsymbol{\beta}'\mathbf{X}_m)\, E\left[n_m^{-\delta_e}\right] \geq \theta_i\right\} \\ &= \mathbf{1}\left\{(\alpha_m + \boldsymbol{\beta}'\mathbf{X}_m)\left[\sum_{k=0}^{\bar{N}-1} \Pr\left(n_{m,-i} = k\right)(k+1)^{-\delta}\right] \geq \theta_i\right\} \end{aligned} \tag{1}$$

where $\bar{N}$ is the number of potential entrants and $n_{m,-i}$ is the number of entrants aside from i. Let P denote a theater's subjective probability that a competitor enters the market. By the binomial theorem, we have

$$\Pr\left(n_{m,-i}=k\right)=\binom{\bar{N}-1}{k}P^{k}\left(1-P\right)^{\bar{N}-k-1}.$$

Finally, (1) implies

$$\Pr\left(D_{i}^{*}=1\right)=G\left(\left(\alpha_{m}+\boldsymbol{\beta}'\mathbf{X}_{m}\right)\left[\sum\nolimits_{k=0}^{\bar{N}-1}\binom{\bar{N}-1}{k}P^{k}\left(1-P\right)^{\bar{N}-k-1}\left(k+1\right)^{-\delta}\right]\right).$$

In equilibrium, the beliefs are consistent with the strategies, meaning that

$$P=G\left(\left(\alpha_{m}+\boldsymbol{\beta}'\mathbf{X}_{m}\right)K\left(P\right)\right) \tag{2}$$

where

$$K(P)=\sum\nolimits_{k=0}^{\bar{N}-1}\binom{\bar{N}-1}{k}P^{k}\left(1-P\right)^{\bar{N}-k-1}\left(k+1\right)^{-\delta}.$$

Letting $P^{*}\left(\mathbf{X}_{m},\alpha_{m}\right)$ denote the unique fixed point of (2), the number of entrants predicted by this entry model equals the number of $\theta_{i}s$ in $\left\{\theta_{1},...,\theta_{\bar{N}}\right\}$ that satisfy

$$\left(\alpha_{m}+\boldsymbol{\beta}'\mathbf{X}_{m}\right)K\left(P^{*}\left(\mathbf{X}_{m},\alpha_{m}\right)\right)-\theta_{i}\geq 0. \tag{3}$$

B.1: Simulating the Joint Distributions of α and θs

This subsection explains how the entry model is used to approximate the joint distributions of $\boldsymbol{\theta}$ and α conditional on $\mathbf{X}$ and n. Let $F_{\boldsymbol{\theta},\alpha}\left(\boldsymbol{\theta},\alpha|\mathbf{X},n\right)$ be the CDF of the density $f_{\boldsymbol{\theta},\alpha}\left(\boldsymbol{\theta},\alpha|\mathbf{X},n\right)$. Based on the entry model, I simulate the joint distribution of $\boldsymbol{\theta}$ and α conditional on $\left(\mathbf{X}_{m},n_{m}\right)$ in the following way:

Step 1: Draw α^{0} from $N\left(0,\sigma_{\alpha}^{2}\right)$.

Step 2: Calculate $P^{*}\left(\mathbf{X}_{m},\alpha^{0}\right)$ using (2). Then, define θ^{*} as

$$\theta^{*}=\left(\alpha^{0}+\boldsymbol{\beta}'\mathbf{X}_{m}\right)K\left(P^{*}\left(\mathbf{X}_{m},\alpha^{0}\right)\right).$$

That is, θ^{*} is the threshold of exit values below which a theater finds it profitable to enter the game.

Step 3: Draw a value of exit n_m times from $TN\left(0,\sigma_\theta^2; l_\theta, \theta^*\right)$ and $\bar{N}-n_m$ times from $TN\left(0,\sigma_\theta^2; \theta^*, h_\theta\right)$, where $TN(\mu,\sigma^2;a,b)$ denotes the truncated normal distribution with mean μ, variance σ^2, and lower and upper bounds of a and b. Sort these values in an ascending order. Call them $\boldsymbol{\theta}^0 = \left(\theta_1^0, ..., \theta_{n_m}^0, \theta_{n_m+1}^0, ..., \theta_{\bar{N}}^0\right)$.

Step 4: Define α_l and α_h such that

$$\theta_{n_m}^0 = \left(\alpha_l + \boldsymbol{\beta}'\mathbf{X}_m\right) K\left(P^*\left(\mathbf{X}_m, \alpha_l\right)\right)$$
$$\theta_{n_m+1}^0 = \left(\alpha_h + \boldsymbol{\beta}'\mathbf{X}_m\right) K\left(P^*\left(\mathbf{X}_m, \alpha_h\right)\right).$$

Step 5: Draw α^1 from $TN\left(0,\sigma_\alpha^2;\alpha_l,\alpha_h\right)$.

That is, α^1 is large enough to support entry of n_m theaters but not enough to support entry of n_m+1 theaters.

Step 6: Return to Step 2 and repeat these steps J times to get $\left(\boldsymbol{\theta}^j, \alpha^j\right)_{j=1}^J$.[1] Call this $\hat{F}_{\boldsymbol{\theta},\alpha|\mathbf{X}_m,n_m}$.

Note that this procedure is completed for each market. Any random draw $(\boldsymbol{\theta},\alpha)$ from $\hat{F}_{\boldsymbol{\theta},\alpha|\mathbf{X}_m,n_m}$ supports exactly n_m entrants in a symmetric equilibrium of the entry game.

B.2: Role of Initial Conditions

If a market has a high unobservable demand shifter, we would observe more firms in the market than otherwise. If one were to ignore the unobservables in such a case, one would underestimate the negative effect of competition, since many firms appear to be able to operate profitably. On the other hand, if the unobservable demand shifter is low, the number of competitors would be small. In such a case, one would infer that the effect of competition is strong. Furthermore, ignoring the initial conditions problem affects estimates of other parameters as well.

To see the role of the initial conditions problem, I estimate the model using the *ex-ante* distributions of market level heterogeneity and exit values; i.e., I use $F_{\boldsymbol{\theta},\alpha|n_m}$ instead of $\hat{F}_{\boldsymbol{\theta},\alpha|n_m}$. Note that the entry stage is used only for identifying the parameters in the payoff function. I have 27 moments to estimate 15 parameters.[2] Table B1 shows the parameter estimates. The effect of com-

[1] I discard the first 30 sets of $\left(\boldsymbol{\theta}^j,\alpha^j\right)$ before storing.

[2] Compared to the full model, the average number of entrants $E(n_m)$ is excluded from the set of moments for the following reason. Since the mean of α_m is *ex-ante* zero, the exit game predicts too many exits at $t=0$ for markets with many competitors, unless the mean profit is very high. If the mean profit is high, however, the entry stage predicts too many entrants.

petition in the dynamic game, δ, is estimated to be smaller than the full model, while that in the entry game, δ_e, is larger. Thus, ignoring the initial conditions problem significantly changes estimates of competition effects. The estimates of the variance of market-level heterogeneity are also different between the two specifications.

To investigate the model fit, I simulate the model ten times and for each simulation calculate the rate of exit during the sample period. Then, I take average over simulation draws for different market structures. The rate of exit in monopoly markets is 9.5% in the data, while the prediction by the model is 9.6%. In duopoly or triopoly markets, the empirical and predicted exit rates are 14.4% and 12.4%, respectively. The corresponding numbers for markets with four or five theaters are 19.7% and 17.1%, respectively. Finally, in markets with more than five theaters, the rate of exit in the data is 21.6%, while the prediction of the model is 20.6%.

It is difficult, however, to evaluate the importance of initial conditions, since parameters in the payoff function are reduced-form parameters.[3] To better understand the role of initial conditions, I first simulate the model starting from 1949 until 1955, using the *ex-ante* distribution of unobservables.[4] To fix observable variables, I only use Steuben county in Indiana for the simulation, as the TV penetration rate in 1955 and the total population in 1950 are close to the median values in the sample.[5] Then, I assume 1955 is the "initial" period of my hypothetical sample. Put differently, I assume that the industry was born in 1949 and the data are available from 1955. Panel (a) in Figure B1 plots the distribution of market-level heterogeneity (α) conditional on the number of surviving theaters in 1955. This result clearly shows that $f_\alpha(\alpha|\mathbf{X}, n) \neq f_\alpha(\alpha|\mathbf{X})$. Next, I simulate $\hat{F}_{\boldsymbol{\theta},\alpha|\mathbf{X}_m,n_m}$ using the simulation method proposed above, and integrate over θ to calculate $\hat{f}_\alpha(\alpha|\mathbf{X}, n)$ for $n = 3$. Panel (b) in Figure B1 plots this and compares it with the true distribution of α conditional on $n = 3$, which I already showed in panel (a), along with the *ex-ante* distribution of α. The "true" distribution of α is closer to the simulated distribution than to the *ex-ante*

[3]Another reason is that I use the entry model not only for addressing the initial condition problem but also for identifying parameters in the base demand. One alternative would be to identify all the parameters from "dynamic" moments only, and to use the entry model only for addressing the initial condition problem. Then, I would be able to compare the estimation results with and without the entry model. I did not use this strategy because the correlations between exit rates and $\mathbf{X}_m$ are weak in the data, and thus I would not identify $\boldsymbol{\beta}$ in the base demand well.

[4]Throughout this exercise, I fix $\sigma_\alpha^2 = 0.6$ to facilitate comparisons. Other parameters are set at the estimated values.

[5]The TV penetration rate in 1955 and the total population in 1950 of Steuben county are 0.54 and 17,087, respectively. The median value for each variable is 0.54 and 17,150, respectively. The number of theaters in 1949 in this county was three in the data, but I start from four theaters to have more variations in the number of theaters in 1955.

distribution.

Since unobservable exit values are time-invariant, selection based on exit values may be substantial too. In panel (c) in Figure B1, I plot the distribution of exit values when the industry was born in this exercise (i.e., *ex-ante* distribution) and the distribution of exit values of surviving theaters in 1955. As expected, these two distributions are significantly different from one another. In particular, the distribution of survivors' exit values has a high density at the low end compared to the unconditional distribution. I also calculated $\hat{f}_{\boldsymbol{\theta}}\left(\boldsymbol{\theta}|\mathbf{X}, n=3\right)$ for surviving theaters in 1955, using the proposed simulation method. Panel (c) plots this distribution. Again, the simulated distribution is closer to the true distribution of exit values of surviving theaters. Although these two distributions are still different, it can be seen that the proposed method, to some extent, alleviates the initial conditions problem.

One caveat is that in this exercise the game played by the theaters before the "initial" period (i.e., 1955) is the exit game in which the payoff function is very similar to that of the entry game. As I discussed above, in the current application, the type of game that was played before 1949 may have been different from the exit game. If the game before the initial period were, however, different, the parameter estimates in the entry game would change accordingly, and hence be able to capture the correlation between n, $\boldsymbol{\theta}$, and α.

References

[1] **Aguirregabiria, Victor, and Pedro Mira.** 2007. "Sequential Estimation of Dynamic Discrete Games." *Econometrica*, 75 (1): 1-53.

[2] **Heckman, James J.** 1981. "The Incidental Parameters Problem and the Problem of Initial Conditions in Estimating a Discrete Time-Discrete Data Stochastic Process and Some Monte Carlo Evidence." In *Structural Analysis of Discrete Data With Econometric Applications*, edited by Charles F. Manski and Daniel L. McFadden, 179-195. Cambridge: The M.I.T. Press.

Appendix C: Interpolation of TV Penetration Rates (Online Appendix, Not for Publication)

The TV penetration rate is reported once in 1950 and afterwards every year starting from 1953. I use data from all available years up to 1960 to approximate the TV diffusion process by the CDF of the Weibull distribution. For market m, I observe

$$(TV_{m1950}, TV_{m1953}, TV_{m1954}, ..., TV_{m1960}) .$$

Using 1949 as the reference year, let

$$\widehat{TV}_{mt}(k_1, k_2) = 1 - e^{-((t-1949)/k_1)^{k_2}}.$$

Then, I choose (k_1, k_2) to minimize

$$\sum_{t=1950,1953,...,1960} (TV_{mt} - \widehat{TV}_{mt}(k_1, k_2))^2$$

for each market. Once the minimizers are obtained, I can calculate $\widehat{TV}_{mt}(\hat{k}_1, \hat{k}_2)$ for all $t \in [1949, 1955]$.

This approach has several advantages. First, the interpolated TV penetration rate is continuous and smooth. Linear interpolation would generate many kinks, which may not be natural in the continuous-time model. Second, this method ensures that the theater's payoff is monotonic, which satisfies Assumption 1 (i). In 84 markets (about 3% of all markets), the TV penetration rate decreases slightly from one year to another. This is most likely due to mis-measurement. I do not need to discard these observations. Third, this method enables me to exploit information after 1955. Given that the TV penetration rate is missing in 1951 and 1952, such additional information is valuable.

Appendix D: Importance of Movie Theater Chains (Online Appendix, Not for Publication)

Movie theater chains existed in the sample period, although their influence and market power were much more limited compared to later periods. Unfortunately, comprehensive information on movie theater chains in the time period of this study is not available. The *Film Daily Yearbook of Motion*

Pictures, however, lists all theaters owned by big theater chains (circuits); i.e., theater chains that own at least four theaters in the U.S. I collect markets in which at least two movie theaters are owned by the same theater chain and call them "market with circuit". The remaining markets are called "market without circuit". In other words, in these markets, any two theaters in the market are not owned by the same theater chain. It could still be the case that two theaters are owned by two different chains. Table D1 shows the frequency of these markets. The majority of markets do not have a circuit that owns more than one theater in the same market.

To further investigate if this may distort my results, I construct a dummy variable defined as whether the market is a "market with circuit" or not, and regress the market-level exit rate on the dummy variable as well as on the initial number of competitors, its squared term, the change in the TV rate, the change in the population, and the number of new entrants in the market. The sign of the coefficient of the dummy variable is positive, but not statistically significant. If theaters under the same ownership coordinate, there would be no strategic delay and the exit rate would be higher. In this sense, the sign of the coefficient is consistent with the argument that competition delays exits. There is no evidence, however, that "market with circuit" significantly distort the result. This should not be taken as evidence that chains are not important, as (i) this additional data contain only big theater chains, and (ii) the explanatory power of the linear regression is limited. Rather, the delay and cost of asymmetric information in this paper can be interpreted as their lower bounds, as coordination would hasten exits.

Appendix E: Numerical Solution (Online Appendix, Not for Publication)

To solve this differential equation, the method of forward shooting cannot be used because the problem is ill-defined at zero. The standard backward-shooting method does not work either, as a finite end point does not exist. To deal with this issue, I apply the following algorithm. I start from an arbitrary point $t' > 0$ and use backward shooting from t' to 0. Then, for $\boldsymbol{\omega} = (n, 0, \mathbf{h}^0)$, I choose $\Phi(t'; \boldsymbol{\omega})$ such that $\Phi(0; \boldsymbol{\omega}) = \Pi_{n-1}(0)$. Then, for an arbitrary $\varepsilon > 0$, I confirm that there exists t'' such that $|\Phi(t''; \boldsymbol{\omega}) - \Pi_n(t'')| < \varepsilon$.

Parameters	Coef.	Std. Err
δ (competition in dynamic game)	0.2009	0.1050
δ_e (competition in entry game)	2.1173	0.2285
β_0 (constant)	8.7248	1.8764
β_1 (population)	0.0687	4.3860
β_2 (median age)	2.5228	1.5975
β_3 (median income)	0.0206	4.0410
β_4 (urban share)	-2.5481	1.1484
β_5 (employment share)	2.7038	1.0338
β_6 (log of land area)	-2.7572	1.3948
λ_1 (TV rate)	0.5487	0.0596
λ_2 (change in population)	-3.3260	0.8608
λ_3 (new entrants)	0.4326	0.1257
σ_θ (std. of exit value)	2.7545	0.5959
σ_α (std. of demand shifter)	1.8622	0.5057
$\rho_{\alpha\lambda}$ (corr coef. b/w α_m and λ_m)	0.3320	0.0861

Table B1: **Estimates of Structural Parameters without Correcting the Initial Conditions Problem**.

Number of Theaters in 1949	Market with Circuit	Market w/o Circuit	Total
1	0	451	451
2	124	396	520
3	160	285	445
4	174	195	369
5	138	107	245
6	123	86	209
7	111	32	143
8	72	15	87
9	64	16	80
10	46	11	57
Total	1,012	1,594	2,606

Table D1: **Number of Markets With and Without Circuit.**

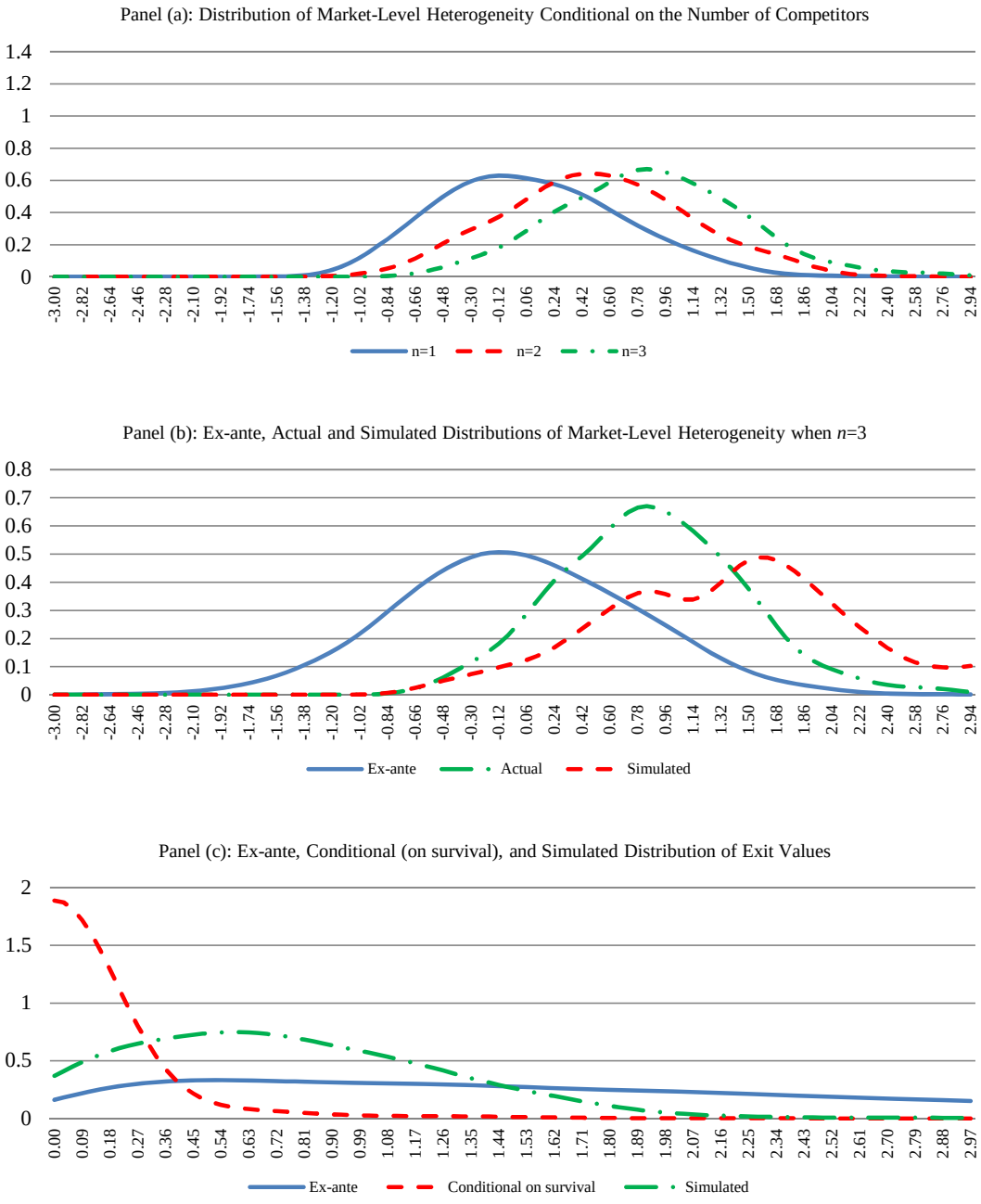


Figure B1 Importance of Initial Condition Problem

Note: I simulate the model many times starting from 1949 until 1955, using the ex-ante distributions of unobservables. I fix observable covariates at the level of the median county. Panel (a) plots the distribution of market-level heterogeneity, splitting the simulated outcomes according to the number of surviving theaters in 1955. Then, assuming that 1955 is the initial period of my hypothetical sample, I simulate the distribution of market-level heterogeneity for $n=3$, using the method I discribed in Section B.1. Penel (b) plots the resulting distribution and the distribution conditional on $n=3$ from panel (a), along with its *ex-ante* distribution. Finally, panel (c) plots the *ex-ante* , conditional (on survival in 1955), and simulated distributions of exit values, which I obtained from the above simulation.