Wages and Informality in Developing Countries

Online Appendix*

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I. Equilibrium Offer and Accepted Contract Distributions

In this section, we derive $G_1$ and $G_2$ from $F_1$ and $F_2$.

From the main paper recall the equilibrium flow condition for the formal sector

$$\left[\lambda_{10} + \lambda_{11}F_1(W)\right] m_1 G_1(W) + \lambda_{12} m_1 \int_{W_1}^{W} F_2(x) dG_1(x) = \lambda_{01} uF_1(W) + \lambda_{21} m_2 \hat{W} W_1 F_2(x) dG_1(x).$$

By equation (.1), for any $W \in [W_1, W_1]$,

$$\left[\lambda_{10} + \lambda_{11}F_1(W)\right] m_1 G_1(W) + \lambda_{12} m_1 \int_{W_1}^{W} F_2(x) dG_1(x) = \lambda_{01} uF_1(W) + \lambda_{21} m_2 \hat{W} W_1 F_2(x) dG_1(x).$$

Making use of the identities (integration by parts):

$$\int_{W_1}^{W} F_2(x) dG_1(x) = F_2(W) G_1(W) + \int_{W_1}^{W} G_1(x) dF_2(x),$$

$$\int_{W_2}^{W} [F_1(W) - F_1(x)] dG_2(x) = \int_{W_2}^{W} G_2(x) dF_1(x),$$

we can rewrite this equation as

$$d_1(W) \frac{m_1}{u} G_1(W) = \lambda_{01} F_1(W) - \Phi(W),$$

where $d_1(W) = \lambda_{10} + \lambda_{11}F_1(W) + \lambda_{12} F_2(W)$, and

$$\Phi(W) = \lambda_{12} \int_{W_1}^{W} \frac{m_1}{u} G_1(x) dF_2(x) - \lambda_{21} \int_{W_2}^{W} \frac{m_2}{u} G_2(x) dF_1(x).$$

Turning to the informal sector, the equivalent equation

$$\left[\lambda_{20} + \lambda_{22}F_2(W)\right] m_2 G_2(W) + \lambda_{21} m_2 \int_{W_2}^{W} F_1(x) dG_2(x) = \lambda_{02} uF_2(W) + \lambda_{12} m_1 \hat{W} W_1 [F_2(W) - F_2(x)] dG_1(x).$$
indicates that for \( W \in [W_2, W_2] \),
\[
\left[ \lambda_{20} + \lambda_{22} F_2(W) \right] m_2 G_2(W) + \lambda_{21} m_2 \int_{W_2}^{W} F_1(x) dG_2(x) = \lambda_{02} u F_2(W) + \lambda_{12} m_1 \int_{W_2}^{W} [F_2(W) - F_2(x)] dG_1(x).
\]

Using the same integrations by part, we obtain that
\[
d_2(W) \frac{m_2}{u} G_2(W) = \lambda_{02} F_2(W) + \Phi(W),
\]
where \( d_2(W) = \lambda_{20} + \lambda_{21} F_1(W) + \lambda_{22} F_2(W) \).

Next, multiplying equation (.2) by \( \frac{\lambda_{12} f_2(W)}{d_2(W)} \) (with \( f_2 = F'_2 \)) and equation (.5) by \( -\frac{\lambda_{21} f_1(W)}{d_2(W)} \), and adding the two resulting equations, we obtain the first-order differential equation
\[
\Phi' = A - B \Phi,
\]
where
\[
A = \lambda_{01} F_1 \frac{\lambda_{12} f_2}{d_1} - \lambda_{02} F_2 \frac{\lambda_{21} f_1}{d_2},
B = \frac{\lambda_{12} f_2}{d_1} + \frac{\lambda_{21} f_1}{d_2},
\]
with boundary condition \( \Phi(U) = 0 \) (in fact \( \Phi(W) = 0, \forall W \leq \max\{W_1, W_2\} \)).

The solution of differential equation (.6) is given by
\[
\Phi(W) = \int_{U}^{W} \frac{e^{\int_{U}^{x} B(x') dx'} A(x) dx}{e^{\int_{U}^{W} B(x) dx}}.
\]
Substituting this solution back into equations (.2) and (.5) we obtain the equilibrium relationship between the distribution of offered \( (F) \) and accepted \( (G) \).

**II. Computing the Equilibrium**

In this section we describe the computation of the equilibrium. We assume known the number of workers \( M \), the potential number of firms \( N \) and the potential distribution of productivity \( \Gamma \).

**A. Exogenous Transition Rates**

Assume that rates \( \lambda_{ij} \) are fixed.

1. Define contract value offer distribution \( F_1 \) and \( F_2 \), with supports bounds \( W_2 = U < W_1 < W_2 < W_1 \). Recall that
\[
rU = b + \lambda_{01}(\mu_1 - U) + \lambda_{02}(\mu_2 - U),
\]
where \( \mu_1 = \int_{W_1}^{W_2} x dF_1(x), \mu_2 = \int_{W_2}^{W_1} x dF_2(x) \) denote the mean contract values offered in the formal and the informal sector respectively, and \( b \) is the flow-value of leisure. Hence
\[
W_2 = U = \frac{b + \lambda_{01}\mu_1 + \lambda_{02}\mu_2}{r + \lambda_{01} + \lambda_{02}}.
\]
Define the shares of firms in each sector \( n_i, i = 1, 2 \), with \( n_1 + n_2 \leq 1 \).
2. Use steady-state flow conditions in Appendix () to derive $m_1, m_2, u$ and $G_1, G_2$ from $F_1, F_2.$

3. Profit maximization then implies that optimal decision rules satisfy

$$p = K_1^{-1}(W) = (1 + \tau + \lambda_{10}s)w_1(W) + w'_1(W)\frac{\ell_1(W)}{\ell'_1(W)},$$

$$p = K_2^{-1}(W) = w_2(W) + w'_2(W)\frac{\ell_2(W)}{\ell'_2(W)} + c\gamma\ell_2(W)^{\gamma-1},$$

with

$$(1 + \lambda_{10}s)w_1(W) = (r + \lambda_{10})W - \lambda_{10}(U + UI) - \lambda_{11}\int_W \overline{W}F_1(x)\,dx - \lambda_{12}\int_W \overline{W}F_2(x)\,dx,$$

$$(1 + \lambda_{10}s)w'_1(W) = r + \lambda_{10} + \lambda_{11}F_1(W_1(w)) + \lambda_{12}F_2(W_1(w)),$$

$$\ell_i(W) = \frac{M}{N} \frac{h_i(W)}{n_i d_i(W)}, \quad i = 1, 2,$$

$$d_i(W) = \lambda_{i0} + \lambda_{ii}F_1(W) + \lambda_{i2}F_2(W),$$

$$h_i(W) = \lambda_{0i}u + \lambda_{1i}m_1G_1(W) + \lambda_{2i}m_2G_2(W),$$

$$G'_i(W) = \frac{1}{m_i d_i(W)} F'_i(W).$$

4. Then calculate productivity distributions

$$\Gamma_i(K_i^{-1}(W)) = n_i F_i(W), \quad i = 1, 2.$$

5. Consistency with the predetermined distribution of productivity $\Gamma$ requires that

$$\Gamma(p) = \begin{cases} 
\Gamma(p_2) + \Gamma_2(p), & \forall p \in [p_2, p_1], \\
\Gamma(p_2) + \Gamma_1(p) + \Gamma_2(p), & \forall p \in [p_2, p_2], \\
\Gamma(p_2) + n_2 + \Gamma_1(p), & \forall p \in [p_2, p_1], 
\end{cases}$$

with $\Gamma(p_2) + n_1 + n_2 = 1.$

6. If this consistency restriction is not satisfied, reiterate that sequence with another guess of $F_1, F_2$ and $n_1, n_2.$

In practice we discretize functions and approximate integrals as described in the estimation section, and we search for discrete approximations of $F_1$ and $F_2,$ as well as shares $n_1, n_1$ so as to minimize a distance between $\Gamma$ and its prediction. The dimensionality of the optimization problem can be reduced by using simple parametric approximations for $F_1, F_2$ such as the beta distribution used in the estimation section.

### B. Endogenous Transition Rates

If the transition rates are endogenous then

$$\lambda_{i1} = \frac{n_1}{(n_1 + \alpha_{n_2})} s_i f(\xi); \quad \lambda_{i2} = \frac{\alpha_{n_2}}{(n_1 + \alpha_{n_2})} s_i f(\xi); \quad \xi = \frac{n_1 + \alpha_{n_2}}{u + s_1 m_1 + s_2 m_2}. $$

So step 2 cannot be calculated because tightness $\xi$ is a function of $m_1, m_2.$ Let $m_i(\xi)$ and $u(\xi)$ denote the shares of workers in formal/informal sectors and unemployed that can be calculated in step 2 given $\xi.$
Then $\xi$ is the fixed point

$$\xi = \frac{n_1 + \alpha n_2}{u(\xi) + s_1 m_1(\xi) + s_2 m_2(x)}.$$ 

Solving for $\xi$ must be embedded in step 2 if rates are endogenous.

### III. Estimation

Let $F_1$ and $F_2$ be two candidate offer distributions, defined on the spaces of contract present values. Although we could implement this procedure nonparametrically, we use non standard beta distributions as approximations:

$$F_i(x) = \text{betacdf}\left(\frac{x - W_i}{\bar{W}_i - W_i}; \alpha_i, \beta_i\right) \quad i = 1, 2; \quad W_i \leq x \leq \bar{W}_i,$$

where $\text{betacdf}(\cdot; \alpha, \beta)$ is the CDF of a beta distribution with parameters $\alpha$ and $\beta$. An important practical reason why a (flexible) parametric specification is useful is that, in order to calculate the function $\Phi$ and other transition rates (see below) we need to calculate offer densities $f_1 = F'_1$ and $f_2 = F'_2$. Assuming a parametric specification guarantees the smoothness of both the distribution function and its derivative.

To estimate the parameters we use the method of moments. We match the distribution of wages for each sector and the transition rates implied by the model to those observed in the data. Given the above specification, we need to estimate the six arrival rates and the two job destruction rates all denoted by $\lambda = (\lambda_{ij})_{i,j=0,1,2}$ and six further parameters $\theta = (W_1^1, W_1^2, W_2^1, W_2^2, \alpha_1, \beta_1, \alpha_2, \beta_2)$ characterizing the offer distribution. Our algorithm estimates $\theta$ given the $\lambda$, then $\lambda$ given $\theta$, and iterates until convergence. Although we could estimate all parameters at the same time, this turned out to be a very quick procedure in practice.

### A. Contract Offer Distributions

We start by taking the arrival rates $\lambda$ as given to estimate $\theta$ as follows. Let $z_k = \cos(k\pi/K), k = 0, ..., K$, be $K + 1$ Chebychev nodes on $[-1, 1]$. These nodes allow to define grids on $[W_1^1, \bar{W}_1^1]$ and $[W_2^1, \bar{W}_2^1]$ as

$$W_{ik} = \frac{W_i + \bar{W}_i}{2} + \frac{W_i - \bar{W}_i}{2} z_k, \quad i = 1, 2, \quad k = 0, ..., K.$$

Recall that

$$(1 + \lambda_{10} s) w_1(W) = (r + \lambda_{10}) W - \lambda_{10} (U + UI) - \lambda_{11} \int_W^{\bar{W}_1} F_1(x) \, dx - \lambda_{12} \int_W^{\bar{W}_2} F_2(x) \, dx, \quad (9)$$

and

$$w_2(W) = (r + \lambda_{20}) W - \lambda_{20} U - \lambda_{21} \int_W^{\bar{W}_1} F_1(x) \, dx - \lambda_{22} \int_W^{\bar{W}_2} F_2(x) \, dx. \quad (10)$$

For each point on the grids, one can calculate a corresponding wage $w_{ik}$ using equations (9) and (10), and replacing integrals by quadrature approximations. The appropriate quadrature for Chebychev nodes is the Clenshaw-Curtis (CC) quadrature, whose weights $\omega_k$ can be easily calculated using Fast Fourier
Transform (FFT) (see Waldvogel, 2006). For example, we have

\[(1 + \lambda_{10}s)w_{1k} = (r + \lambda_{10})W_{1k} - \lambda_{10}(W_2 + U)I
- \lambda_{11}\frac{W_1 - W_1}{2}\sum_{\ell=0}^{K} \omega_{\ell}1_{(W_{1\ell} > W_{1\ell})}\hat{F}_1(W_{1\ell})
- \lambda_{12}\frac{W_2 - W_2}{2}\sum_{\ell=0}^{K} \omega_{\ell}1_{(W_{2\ell} > W_{2\ell})}\hat{F}_2(W_{2\ell}),\]

where \(1_{(\cdot)}\) is the indicator function. A similar expression can be obtained to determine wage nodes for the informal sector, \(w_{2k}\).

Then we search for \(\theta\) minimizing

\[Q_1(\theta|\lambda) = \sum_{i=1,2}\sum_{k=0}^{K}\left(\hat{G}_i^*(w_{ik}) - G_i(W_{ik})\right)^2,\]

where \(G_i(W_{ik})\) is calculated using equation (.11),

\[m_iG_i(W) = \frac{\lambda_{iW}F_i(W) - \Phi(W)}{d_i(W)}u, \quad i = 1, 2, \quad (.11)\]

and replacing integrals by CC-quadrature approximations, and \(\hat{G}_i^*\) is an estimate of wage distribution functions.

Note that, assuming that \(U = W_2 \leq W_1\) and \(\bar{W}_2 \leq \bar{W}_1\), we have

\[(1 + \lambda_{10}s)w_1 = (r + \lambda_{10})W_1 - \lambda_{10}(W_2 + U)I - \lambda_{11}(\mu_1 - W_1) - \lambda_{12}\int_{W_1}^{W_2} \hat{F}_2(x) dx, \quad (.12)\]

\[(1 + \lambda_{10}s)\bar{w}_1 = (r + \lambda_{10})\bar{W}_1 - \lambda_{10}(\bar{W}_2 + U)I, \quad (.13)\]

\[w_2 = rW_2 - \lambda_{21}(\mu_1 - W_2) - \lambda_{22}(\mu_2 - W_2), \quad (.14)\]

\[\bar{w}_2 = (r + \lambda_{20})\bar{W}_2 - \lambda_{20}\bar{W}_2 - \lambda_{21}\int_{\bar{W}_2}^{\bar{W}_1} \hat{F}_1(x) dx, \quad (.15)\]

where \([w_1, \bar{w}_1]\) and \([\bar{w}_2, \bar{w}_2]\) are the observed wage supports in the formal and informal sectors, respectively, and with

\[\mu_i = \frac{W_i}{\alpha_i} + \frac{W_i}{\alpha_i + \beta_i}, \quad i = 1, 2.\]

Hence, we can simplify the estimation problem slightly by using equations (.13) and (.14) to substitute observed wage bounds \(\bar{w}_2\) and \(\bar{w}_1\) for \(\bar{W}_2 = U\) and \(\bar{W}_1\) (given the \(\alpha, \beta\) and \(W_1, W_2\)).

**B. Transition Rates**

In a similar way as we estimate \(\theta\) given \(\lambda\), we can estimate \(\lambda\) given \(\theta\). Natural counterparts to the theoretical transition rates can be calculated from observed flows between states (0: unemployment; 1: working in the formal sector; and 2: working in the informal sector).

From the labor force survey, we calculate the intensity of transitions from unemployment to job (\(\hat{D}_{0j}; j = 1, 2\)), from a formal sector job to unemployment, to another job in the same sector or to the informal sector (\(\hat{D}_{1j}; j = 0, 1, 2\)) and similar ones for a workers initially in the informal sector (\(\hat{D}_{2j}; j = 0, 1, 2\)).
We then estimate our transition parameters using the method of moments. We choose the parameters so as to match the observed transition rates between sectors.

Consider first the workers who are unemployed at the date of the first interview, that we follow over $T$ periods. Workers are not heterogeneous in this model and hence the remaining unemployment duration is exponentially distributed. Thus the implied proportion of those who move out of unemployment and into a job in sector $j$ over the time period of observation $T$ is

$$D_{0j} = \frac{\lambda_{0j}}{\lambda_{01} + \lambda_{02}} (1 - e^{-(\lambda_{01} + \lambda_{02})T}), \quad j = 1, 2$$

(16)

Now consider workers in the formal sector. Over $T$ periods the proportion making a transition to an alternative job in the same sector, to a job in the informal sector or to unemployment is, respectively, for $i = 1, 2$,

$$D_{ij} = \int_{W_i}^{W_i} \frac{\lambda_{ij} \bar{F}_i(x)}{d_i(x)} (1 - e^{-d_i(x)T}) dG_i(x), \quad j = 1, 2,$$

$$D_{i0} = \int_{W_i}^{W_i} \frac{\lambda_{i0}}{d_i(x)} (1 - e^{-d_i(x)T}) dG_i(x).$$

where $d_i(W) = \lambda_{i0} + \lambda_{i1} \bar{F}_1(W) + \lambda_{i2} \bar{F}_2(W)$. Now, in equilibrium,

$$\frac{dG_i(x)}{dF_i(x)} = \frac{1}{m_i} \frac{h_i(x)}{d_i(x)}.$$

(18)

with $h_i(W) = \lambda_{i0}u + \lambda_{i1}m_1G_1(W) + \lambda_{i2}m_2G_2(W)$. This allows to replace the derivative of $G_i$ by that of $F_i$ inside the integral. This is useful as we have parameterized $F_1, F_2$ using a continuous distribution. Then CC-quadrature can be used to approximate the integral.

These are the model counterparts for these empirical moments as functions of the arrival rates, the job destruction rates, the offers distributions $F_i$ and as a function of the equilibrium contract values distributions $G_i$ ($i = 1, 2$). Contract offers and equilibrium distributions are related by a complex function as explained in Appendix.

We can thus estimate $\lambda$ given $\theta$ by minimizing the criterion

$$Q_2(\lambda|\theta) = \sum_{i,j=0,1,2} \left( \hat{D}_{ij} - D_{ij} \right)^2,$$

where $\hat{D}_{ij}$ is the empirical counterpart of $D_{ij}$.

Alternating between the minimization of $Q_1$ and $Q_2$, given respectively the latest update on the parameters is just a standard and simple numerical optimization algorithm for minimizing both functions jointly, taking into account that they both depend on common parameters. Since the model is just identified (for nonparametric specification of $F_1$ and $F_2$) weighing the moments is irrelevant for efficiency. Although we could implement this procedure nonparametrically (using sieve estimators), we use instead the non standard beta distribution as an approximation.

C. Productivity Distribution

Up to this point, there has been no need to use the firm profit functions, or indeed the distribution of productivities. To complete estimation we need to estimate the cost function of informality. This will allow us to characterize the choice of firms to locate in either sector and ultimately to carry out counterfactual simulations.
We specify the cost function as $C = c\ell_2(W)^\gamma$, with $c$ and $\gamma$ being the parameters to be estimated. Given values for $c$ and $\gamma$, and $n_2 \leq 1$, we solve for the labor force size in the formal sector ($\ell_1(W) = M/n_1 h_1(W)$) and in the informal sector ($\ell_2(W) = M/n_2 h_2(W)$). From the firm’s maximization problem in each sector, we next derive the support of the distribution of formal and informal productivities, i.e. $p_1 = K_1^{-1}(W)$ and $p_2 = K_2^{-1}(W)$ respectively.

Recall the firms optimization problem:

\[
\pi_1(p) = \max_{W \geq \max\{U, W_1(w_{\text{min}})\}} (1-t)[p - (1+\tau + \lambda_{10}s)w_1(W)] \ell_1(W), \tag{.19}
\]

\[
\pi_2(p) = \max_{W \geq U} [p - w_2(W)] \ell_2(W) - C(\ell_2(W)), \tag{.20}
\]

The first order conditions gives

\[
p_1 = K_1^{-1}(W) = (1+\tau + \lambda_{10}s) \left[ w_1(W) + w_1'(W) \frac{\ell_1(W)}{\ell_1'(W)} \right], \tag{.21}
\]

\[
p_2 = K_2^{-1}(W) = w_2(W) + w_2'(W) \frac{\ell_2(W)}{\ell_2'(W)} + c_\gamma \ell_2(W)^{-1}, \tag{.22}
\]

where the expressions for $w_i'(W)$, $i = 1, 2$, are given by

\[
w_1'(W) = \frac{r + \lambda_{10} + \lambda_{11}F_1(W_1(w)) + \lambda_{12}F_2(W_1(w))}{1 + \lambda_{10}s},
\]

\[
w_2'(W) = \frac{r + \lambda_{20} + \lambda_{21}F_1(W_2(w)) + \lambda_{22}F_2(W_2(w))}{1 + \lambda_{20}s},
\]

and where firm sizes can be differentiated using

\[
\frac{\ell_i}{\ell_i'} = \frac{h_i/d_i}{(h_i/d_i)'} = \frac{h_i d_i}{h_i' d_i - h_i d_i'}, \quad h_i' = \lambda_{0i} u + \lambda_{11} \frac{h_1}{d_1} F_1' + \lambda_{21} \frac{h_2}{d_2} F_2', \quad d_i' = \lambda_{11} F_1' + \lambda_{21} F_2'.
\]

For each point of the contract grids, $W_{ik}$, one can thus calculate a point $p_{ik}$ on a productivity grid, with $p_2 = p_{20}$, $p_1 = p_{10}$, $p_2 = p_{2N}$ and $p_1 = p_{1N}$:

\[
p_{1k} = (1+\tau + \lambda_{10}s) \left[ w_1(W_{1k}) + w_1'(W_{1k}) \frac{h_1^2}{h_1'd_1 - h_1'd_1'} \bigg|_{W=W_{1k}} \right], \quad k = 0, \ldots, K,
\]

and

\[
p_{2\ell} = w_2(W_{2\ell}) + w_2'(W_{2\ell}) \frac{h_2 d_2}{h_2'd_2 - h_2'd_2'} \bigg|_{W=W_{2\ell}} + \tilde{c} \times \gamma \left( \frac{h_2(W_{2\ell})}{\tilde{n}_1 d_2(W_{2\ell})} \right)^{-1}, \quad \ell = 0, \ldots, K,
\]

for $\tilde{n}_2 = \frac{n_2}{n_1 + n_2}$ and $\tilde{c} = c \left( \frac{M}{(n_1 + n_2)N} \right)^{-1} = c \left( \frac{M}{N_1 + N_2} \right)^{-1}$ (where $N_1, N_2$ are the total numbers of formal and informal firms operating in the economy at the equilibrium).

Moreover, we have

\[
\pi_1(p_{1k}) = (1-t)(1+\tau + \lambda_{10}s)w_1'(W_{1k}) \frac{\ell_1(W_{1k})^2}{\ell_1'(W_{1k})} \frac{h_1^2}{h_1'd_1 - h_1'd_1'} \bigg|_{W=W_{1k}},
\]

\[
= \frac{M}{N} \frac{1}{n_1 + n_2} \left[ \frac{1}{n_1} \times (1-t)(1+\tau + \lambda_{10}s)w_1'(W_{1k}) \frac{h_1^2}{h_1'd_1 - h_1'd_1'} \bigg|_{W=W_{1k}} \right],
\]

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for $\tilde{n}_1 = \frac{n_1}{n_1 + n_2}$, and

$$\pi_2(p_{2\ell}) = \frac{w'_{2}(W_{2\ell})^2}{\ell_2(W_{2\ell})} + c(\gamma - 1)\ell_2(W_{2\ell})$$

$$= \frac{M}{N_1 + N_2} \left[ \frac{1}{\tilde{n}_2} \times w'_{2}(W_{2\ell}) \right] \frac{h_2^2}{h_2 d_2 - h_2 d_2^p} \left[ W = W_{2\ell} \right] + \tilde{c} \times (\gamma - 1) \left( \frac{1}{\tilde{n}_2} d_2(W_{2\ell}) \right)^\gamma.]$$

Note that both profit functions are proportional to $\frac{M}{N_1 + N_2}$, which is not necessarily known at this stage if the number of informal firms in the economy is not directly observed.

Equilibrium conditions require that $\pi_2(p_{i}) = 0$, and $\pi_1(p) = \pi_2(p) > 0$ for $p \in [\bar{p}_1, \bar{p}_2]$. We thus estimate $\tilde{c}$ and $\gamma$, and $\tilde{n}_1 \geq 0$ and $\tilde{n}_2 \geq 0$ such that $\tilde{n}_1 + \tilde{n}_2 = 1$, so as to minimize

$$\pi_2(p_{20}) + \sum_{k, \ell = 0}^K K(p_{1k} - p_{2\ell})[\pi_1(p_{1k}) - \pi_2(p_{2\ell})]$$

where $K$ is a kernel density. Notice that a direct estimate of $\tilde{n}_1, \tilde{n}_2$ immediately follows if estimates of the numbers of formal and informal firms ($N_1$ and $N_2$) are available:

$$\tilde{n}_i = \frac{N_i}{N_1 + N_2}.$$

Then define

$$\tilde{\Gamma}_i(p_{ik}) = \tilde{n}_i F_i(W_{ik}), \quad k = 0, ..., K,$$

and interpolate for all $p \in [p_{i0}, p_{iK}]$, and set $\tilde{\Gamma}(p) = 0$ for $p < p_{i0}$ and $\tilde{\Gamma}(p) = \tilde{n}_i$ for $p > p_{iK}$. The unconditional productivity distribution $\Gamma$ satisfies

$$\Gamma(p) = \begin{cases} 
\Gamma(p_{20}) + (n_1 + n_2)\tilde{\Gamma}_2(p), & \forall p \in [\bar{p}_2, \bar{p}_1], \\
\Gamma(p_{20}) + (n_1 + n_2)\tilde{\Gamma}_1(p) + \tilde{\Gamma}_2(p), & \forall p \in [\bar{p}_1, \bar{p}_2], \\
\Gamma(p_{20}) + (n_1 + n_2)\tilde{n}_2 + \tilde{\Gamma}_1(p) & \forall p \in [\bar{p}_2, \bar{p}_1].
\end{cases}$$

By postulating a parametric form for $\Gamma$, say a log-normal with parameters $\mu$ and $\sigma^2$, one can then estimate $n_1 + n_2, \mu, \sigma^2$ by maximizing the fit on a grid for $[p_{20}, p_{1K}]$.

Finally, $\frac{M}{N_1 + N_2}$ can be estimated by fitting the average firm size in the informal sector. The cost parameter $c$ is then $c = \tilde{c} \left( \frac{M}{N_1 + N_2} \right)^{1-\gamma}$.

**IV. Comparing transitions estimated from the PME data to those implied from the RAIS data**

Our data set, the PME (2008), is the only source including workers in both the formal and the informal sector. An interesting question is how the transitions we compute over the short window of observation offered by the PME compare to those we observe if we had a longer window of observation. Table C.1 below shows annualised transition rates estimated based on the quarterly transitions rates we obtain from the PME and compares them to annual transitions computed from RAIS, an administrative matched employer employee data set. Since the latter includes only formal workers and firms, the comparisons that can be made are limited. Moreover, as we explain below sampling issues make the comparison imperfect.

Most obviously we can compare the annualised transition rates between formal jobs ($D_{11}$). These are
### Table 1
**Comparing some transitions between PME and RAIS data**

<table>
<thead>
<tr>
<th></th>
<th>Sao Paulo</th>
<th>Salvador</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Males</td>
<td>Females</td>
<td>Males</td>
</tr>
<tr>
<td>$D_{11}$</td>
<td>PME</td>
<td>6.2</td>
<td>4.7</td>
</tr>
<tr>
<td></td>
<td>RAIS</td>
<td>6.3</td>
<td>7.6</td>
</tr>
<tr>
<td>$D_{10} + D_{12} + D_{10}'$</td>
<td>PME</td>
<td>24.0</td>
<td>36.7</td>
</tr>
<tr>
<td></td>
<td>RAIS</td>
<td>33.7</td>
<td>31.6</td>
</tr>
</tbody>
</table>

Note: Transitions are percentages per year. The transitions from RAIS data (Census of registered firms) are averages obtained from month j in year t to month j in year t+1, over the period 2002-2007. $D_{11}$: annualised formal to formal transition rates. $D_{10} + D_{12} + D_{10}'$ for the PME is exit from the formal sector to unemployment, informal jobs and inactivity. For RAIS it is the rate of exit from the data base, which includes only formal employment. Standard errors for the PME annualied transition rates in parentheses.

of the same order of magnitude and there is no systematic difference between the data sets: PME implies sometimes larger and sometimes smaller transitions, reflecting some noise. When we average across regions the numbers are much closer. Once sampling variation is accounted for there is not much difference left between the two.

We can also compare the annualized exit from the formal sector (to unemployment, informal and inactivity) estimated from the PME to the exit rate from the RAIS (2014) data base, which should correspond, assuming all formal workers are correctly reported by firms once they stay within the formal sector. Note that the inactive are not part of the model and have been removed from the data we use for estimation. However, to make the transitions more comparable between the two data sets we add them back in for the purposes of this exercise. We compare this to the rate at which people disappear from the RAIS data. On average for females the numbers are almost the same, and once we allow for sampling error in the PME they are indistinguishable. For males PME implies a lower overall exit rate from the formal sector, consistent with the effects of stock sampling.

Thus, overall the two data sets give similar albeit not identical transition rates. The main difference is for males. Nevertheless one needs to remember that the sampling structure of the two data sets is different. Formally the two sets of transitions would be the same if there was no unobserved heterogeneity across jobs, which is both unlikely and inconsistent with our model. In particular, the PME stock sampling (which is fully accounted for by the estimation of the model) will skew the sample towards individuals employed in high productivity firms from which exit rates will be lower. On the other hand the RAIS includes the universe of formal firms.

**References**

