Online Appendix for “Inequality, Leverage and Crises”

A. Calibration Appendix

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I. Introduction

The objective of this appendix is to provide a detailed explanation of how the preference for wealth specification is calibrated in our model. The procedure is based on equating the model-based marginal propensity to save (MPS) of top earners with the data-based MPS of top earners, using micro-level panel data on individual saving behavior. The use of empirical estimates to independently calibrate the preference for wealth parameters of top earners implies that matching the post-1983 evolution of the debt-to-income ratio of bottom earners is not a criterion or target of the calibration. Therefore, empirical support for the model can be assessed by comparing the post-1983 evolution of the model-generated debt-to-income ratio of bottom earners to its empirical counterpart.

The rest of appendix A is divided into three parts. Section II uses our theoretical model to derive an approximate formula for the response of top earners’ savings to a permanent change in their income, in other words for their MPS out of permanent income shocks. Section III uses the approach of Dynan, Skinner and Zeldes (2004) to estimate the empirical counterpart of this formula, by establishing a nonlinear empirical relationship between the MPS and the level of permanent income of different income brackets, and then identifying the portion of that relationship which identifies the MPS of the mean household within the top 5% income bracket. Section IV explains how the preference for wealth specification is calibrated by equating the model-based MPS to its empirical counterpart.

II. Model-Based MPS

A. Linearization of the Investor’s Problem

In this section we compute the MPS of top earners in the model to a first-order approximation. To do so, we solve their optimization problem in partial equilibrium. Specifically, for our derivation we assume that total income in the economy is at its steady state level \( y_t = \bar{y} = 1 \), that the real interest rate is at the steady state level of the complete general equilibrium model, \( p_t = \bar{p} \), that there is no uncertainty and no risk of default.

\(^1\)Recall that in the paper shocks to income shares are permanent.
The budget constraint of top earners is then given by

$$c_t^r = z_t \frac{1}{\chi} + (l_t - b_t \bar{p}) \frac{1 - \chi}{\chi}$$  \hspace{1cm} (1)

where $\frac{z_t}{\chi}$ is the level of income per top earner. At date $t$ each top earner lends an amount $\frac{1 - \chi}{\chi} b_t \bar{p}$ and receives a repayment of $\frac{1 - \chi}{\chi} l_t$, with $l_t = b_{t-1}$. The income process $z_t$ is assumed to be deterministic and perfectly persistent. The optimal saving decision satisfies the Euler equation:

$$\bar{p} = \beta_r E_t \left[ \left( \frac{c_{t+1}^r}{c_t^r} \right)^{-\frac{\eta}{\sigma}} \right] + \varphi \left( \frac{1 + b_t \frac{1 - \chi}{\chi}}{(c_t^r)^{-\frac{1}{\sigma}}} \right)^{-\frac{1}{\eta}}$$  \hspace{1cm} (2)

For a given permanent level of income of investors $\bar{z}$, and given the initial value of debt $\bar{b}$, we choose the parameter $\beta_r$ so that the combination of $\bar{b}$ and $c^r$ is a steady state that satisfies equations (1) and (2).

Our goal here is to characterize, to a first-order approximation, the law of motion of $b_t$ following an initial permanent increase in inequality. We denote the increase in top earners’ income by $dz$ so that we have, at all times $t > 1$, $dz = z_t - \bar{z}$. We also denote by $db_t$ the deviation of debt from its initial level, i.e. $db_t = b_t - \bar{b}$.

The Euler equation (2) can be differentiated at the steady-state to get

$$0 = \beta_r \frac{1}{\sigma} \left( dc_t^r - dc_{t+1}^r \right) + \varphi \left( \frac{1 + \bar{b} \frac{1 - \chi}{\chi}}{(c^r)^{-\frac{1}{\sigma}}} \right)^{-\frac{1}{\eta}} \left[ - \frac{1}{\eta} + \frac{1}{\chi} \frac{1 - \chi}{\chi} db_t + \frac{1}{\sigma} \frac{1}{\varphi} dc_t^r \right],$$

or

$$0 = \beta_r \left( dc_t^r - dc_{t+1}^r \right) - \left( \bar{p} - \beta_r \right) \left[ \frac{\sigma}{\eta} \frac{c^r}{\bar{z} + \frac{1 - \chi}{\chi} db_t - dc_t^r} \right].$$  \hspace{1cm} (3)

From the budget constraint (1) we get

$$dc_t^r = \frac{1 - \chi}{\chi} \frac{1}{1 - \chi} dz + \frac{1 - \chi}{\chi} db_{t-1} - \bar{p} \frac{1 - \chi}{\chi} db_t,$$

$$dc_{t+1}^r = \frac{1 - \chi}{\chi} \frac{1}{1 - \chi} dz + \frac{1 - \chi}{\chi} db_t - \bar{p} \frac{1 - \chi}{\chi} db_{t+1}.$$  

By substituting these equations into (3), dividing by $\frac{1 - \chi}{\chi} (\beta_r \bar{p})$, and reordering terms, we end up with the equality

$$db_{t+1} - 2A db_t + B db_{t-1} + C dz = 0,$$  \hspace{1cm} (4)
where

\[
A = \frac{1}{2} \left( \frac{1}{\beta_r} + \frac{\bar{p}}{\beta_r} \right) + \frac{1}{2} \left( \frac{1}{\bar{p}} - \frac{1}{\beta_r} \right) \left( 1 - \frac{\sigma}{\eta} \left( \frac{\bar{c}}{1 + \frac{1 - \chi}{\chi}} \right) \right),
\]

\[
B = \frac{1}{\beta_r},
\]

\[
C = \frac{1}{1 - \chi} \left( \frac{1}{\beta_r} - \frac{1}{\bar{p}} \right).
\]

We look for a linear solution of the form

\[
\text{db}_t = X \text{db}_{t-1} + Y \text{dz}.
\] (5)

When we substitute (5) into (4) we obtain

\[
\left( X^2 - 2AX + B \right) \text{db}_{t-1} + (XY + Y - 2AY + C) \text{dz} = 0.
\]

Since (4) is valid for any small deviations \( \text{db}_{t-1} \) and \( \text{dz} \), the first coefficient \( X \) must be a root of the polynomial \( P = X^2 - 2AX + B \). When there is no preference for wealth (i.e. \( \frac{1}{\bar{p}} = \frac{1}{\beta_r} \)), this equation has a unit root and an unstable root of \( 1/\beta_r > 1 \). In general the product of the two roots is \( B > 1 \). Thus, there is at most one stable root. If there exists a real stable root, it is given by

\[
X = A - \sqrt{A^2 - B}.
\] (6)

The value of \( Y \) satisfies the equation \( XY + Y - 2AY + C = 0 \) and is given by

\[
Y = \frac{-C}{1 + X - 2A}.
\] (7)

**B. Computation of the Marginal Propensity to Save**

By iterating on (5), we can characterize the time profile of savings at any horizon. We compute the response, after \( T \) periods, of debt per top-earner \( \frac{(1-\chi)}{\chi} \text{db}_T \) to an initial increase in top-earner’s income \( \frac{1}{\chi} \text{dz} \) as

\[
(1 - \chi) \frac{\text{db}_T}{\text{dz}} = (1 - \chi) \left( \sum_{t=0}^{T-1} X^t \right) Y
\]

\[
= (1 - \chi) \left( \frac{1 - X^T}{1 - X} \right) Y.
\] (8)
When there is no preference for wealth, we have $C = 0$ implying $Y = 0$, so that debt does not respond to permanent income shocks. With preferences for wealth, the immediate response to an income shock is given by $Y$ while the long run effect is

$$
(1 - \chi) \frac{db_\infty}{dz} = (1 - \chi) \left( \frac{Y}{1 - X} \right).
$$

In Section III we will compute a MPS for top earners from the data, using a saving rate computed that is computed between two dates that are six years apart. The model counterpart of that measure is given by

$$
MPS_{top 5\%} = \frac{1}{6} (1 - \chi) \frac{db_6}{dz}.
$$

This metric will be used to match model-based and data-based MPS.

### III. Data-Based MPS

In order to compute the MPS of top earners from the data, we use two different empirical methodologies that both closely follow Dynan, Skinner and Zeldes (2004), henceforth DSZ. The first uses data from the 1983 and 1989 editions of the Survey of Consumer Finances (SCF), and the second uses data from the 1984, 1989 and 1994 editions of the Panel Study of Income Dynamics (PSID).

#### A. Empirical Methodologies

##### A.1 Two-Step Methodology Using SCF Data

In the first step of this methodology, saving rates are estimated for different percentiles of permanent income, after controlling for age-group variables that are known to affect saving rates independently of income levels. Saving rates are defined as the level of saving divided by the respective level of income, where in turn saving is defined as the change in net worth between two reference dates. The estimation results can be converted into a piecewise linear

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2The replication files for the MPS estimations are available at http://www.mosphere.fr/files/krw2014/
relationship between the level of permanent income and the level of savings.\textsuperscript{3} In the second step, the MPS is computed as the increase in savings divided by the increase in permanent income as we move from one percentile of the income distribution to the next, or as the slope of the estimated relationship between the level of permanent income and the level of savings.

**Step 1** DSZ estimate a quantile regression of the conditional median saving behavior in each income group, with the saving rate as the dependent variable and dummies for income quantiles and age categories as independent variables.\textsuperscript{4} The saving rate of individual $i$ in quantile $q$ is computed as the difference in net worth between 1983 and 1989 divided by $6$ times the average income between 1982 and 1988:\textsuperscript{5}

$$\text{Saving Rate}_{i,q} = \frac{\Delta_{1983-1989}NW_{i,q}}{6 * \bar{y}_{i,q}}$$

with $\bar{y}_{i,q} = \frac{y_{i,q,1982} + y_{i,q,1988}}{2}$. \hfill (11)

Income dummies include dummies for the five income quintiles, and dummies for the top 5\% and top 1\% income quantiles.\textsuperscript{6} The DSZ median regression specification is given by

$$\text{Saving Rate}_{i,q} = \alpha_q * 1_{i \in q} + \beta_1 * 1_{\text{age},i \in (30,39)} + \beta_2 * 1_{\text{age},i \in (50,59)} + \epsilon_{i,q}$$ \hfill (12)

where $1_{i \in q}$ is equal to 1 if individual $i$ belongs to quantile $q$, and where $q$ denotes either a quintile of the income distribution, or the top 5\% and top 1\% of the income distribution.\textsuperscript{7}

The dummies for the 30-39 and 50-59 age groups refer to the age of household heads in 1986. They imply that the estimated coefficient $\hat{\alpha}_q$ corresponds to the saving rate for households in the quantile $q$ with household heads between 40 and 49 years old in 1986.\textsuperscript{8}

\textsuperscript{3}An alternative procedure would be to directly estimate the level of savings, rather than to first estimate saving rates and then predict saving levels based on saving rate estimates. The main problem with this approach is that saving level regressions tend to be much more heavily influenced by outliers than saving rate regressions. Therefore we follow DSZ who, for the same reason, first estimate saving rates.

\textsuperscript{4}In the literature, this is known as a median regression.

\textsuperscript{5}In the SCF panel data, assets and liabilities are measured at the time of the survey interview, which occurs during the first semester of the year of the survey, that is 1983 (1989), while income refers to income in the previous year, that is 1982 (1988).

\textsuperscript{6}The Survey of Consumer Finances (1983-1989) is the only data source with good coverage of top incomes. Between 1983 and 2007, 1989 is the only year in which the 1983 SCF respondents were re-surveyed, thereby producing an individual household panel. The other SCF surveys between 1983 and 2007 do not have a panel dimension and therefore cannot be used to compute saving rates. The timing of this SCF panel is very fortuitous for our purposes, because 1983 is also the year for which we calibrate the initial steady state of our model.

\textsuperscript{7}Note that the top quantile includes the top 5\% percent, and the top 5\% percent includes the top 1\% percent.

\textsuperscript{8}The sample includes only households for which the head is between 30 and 60 years old in 1986.
The median regression estimates $\hat{\alpha}_q$, as shown by DSZ, can be interpreted as the estimated saving rates of individuals at the median of their income quantile. For example, $\hat{\alpha}_5$ and $\hat{\alpha}_{top5}$ give the estimated saving rates for individuals at the $90^{th}$ and $97.5^{th}$ percentiles.\footnote{We follow DSZ in using the median regression estimates for the 40-49 age group, $\hat{\alpha}_q$, as the basis for our computation of the MPS. However, the estimated dummies for the other two age groups, $\beta_1$ and $\beta_2$, are small compared to the estimated saving rate for the top income group, $\hat{\alpha}_q$. Computing MPS on the basis of an average coefficient across all age groups therefore yields very similar results to MPS computed for the 40-49 age group, with a difference of only around 2 percentage points.}

A challenge for this estimation procedure is to insure that households are allocated to quantiles according to their permanent income, meaning their income after removing transitory income shocks. In order to deal with this issue, DSZ use an IV approach. Income is first regressed on lagged income, which serves as an instrument for predicting permanent income, and on age dummies:

$$y_{i,1988} = \rho * y_{i,1982} + \gamma_1 * 1_{age,i(30,39)} + \gamma_2 * 1_{age,i(50,59)} + \eta_i .$$

(13)

The fitted values from this regression are used to place households into predicted permanent income quantiles, with associated indicator dummies. Equation (12) is then estimated with the saving rate as the dependent variable and the predicted permanent income quantile dummies and age dummies as independent variables. An alternative is to use average income $\bar{y}_i$, on the assumption that the long averaging period (1982-1988) insures that the effects of transitory income shocks are removed. We will include this non-IV approach in our results. It typically implies MPS that are higher.

**Step 2** From these median regression estimates $\hat{\alpha}_q$ we can compute a MPS between quantiles $q$ and $q+1$ as

$$\overline{MPS}_{q,q+1} = \frac{\hat{\alpha}_{q+1} * \text{Median}(y_{q+1}) - \hat{\alpha}_q * \text{Median}(y_q)}{\text{Median}(y_{q+1}) - \text{Median}(y_q)} ,$$

(14)

where $\hat{\alpha}_q * \text{Median}(y_q)$ is the estimated median level of savings in quantile $q$. $\overline{MPS}_{q,q+1}$ is, by construction, the slope of the estimated relationship between the level of permanent income and the level of savings.
A.2 One-Step Methodology Using PSID Data

Estimating the MPS on SCF data requires the use of an estimated cross section of saving rates. The necessary panel structure, with at least two dates required to compute saving rates, is only available for the years 1983 and 1989.

An alternative approach to estimating the MPS relates the change in saving across time for a given household to the change in its income. Because this requires individual saving rates, for the same household, in two separate periods, this methodology cannot use the 1983-1989 panel data of the SCF (out of which only a single cross-section of saving rates can be produced). DSZ therefore implement this approach using data from the PSID. They define the change in saving as the difference between 1984–89 saving and 1989–94 saving, and estimate the median regression for the change in the saving rate as

$$\Delta Saving \ Rate_{i,q} = \gamma_q 1_{i \in q} + \delta_q 1_{i \in q} \Delta y_i + \zeta_1 \ast 1_{age,i \in (30,39)} + \zeta_2 \ast 1_{age,i \in (50,59)} + \epsilon_{i,q},$$  \hspace{1cm} (15)$$

where $q$ again denotes the income quantile, and $\gamma_q$ is a quantile-specific trend. The advantage of using specification (15) is that the MPS is directly estimated though the coefficient $\delta_q$, rather than implied by the slope of the cross-sectional saving function. But a disadvantage, due to the limitation of PSID data, is that the MPS can only be estimated for the top 20% of the income distribution, and thus very likely underestimates the MPS of the top 5% that is of interest for our paper.

B. Data

The primary source of data is the 1983–89 panel of households from the Survey of Consumer Finances (SCF). The 1983–89 SCF panel contains information on 1,479 households that were surveyed in 1983 and again in 1989. The sample has two parts, households from an area-probability sample and households from a special high-income sample selected on the basis of tax data from the Internal Revenue Service. This second part, which is unique to the SCF, is key to estimating saving rates and marginal propensities to save for top income shares. The SCF contains very high quality information about assets and liabilities, as well as limited data on demographic characteristics and income, for the calendar year prior to the survey. The DSZ sample of working age households excludes households for which the head
is younger than 30 or over 60 in 1986. In addition, it excludes households in which the head
or spouse changed between the two interview dates, and households whose income was less
than $1000 in 1982 or 1988. The resulting sample contains information on 728 households.

The saving rate variable used for the SCF calculations equals the change in real net
worth between 1983 and 1989 divided by six times the average of 1982 and 1988 total real
household income, as shown in equation (11). Because it spans several years, this variable
is likely to be a less noisy measure of average saving than a one-year measure. Net worth
is constructed, for each household, as the total value of assets net of total debts.\footnote{The appendix of DSZ contains a complete list of items entering into the construction of the net worth variable.} All
wealth and income variables are deflated using the NIPA implicit price deflator for personal
consumption expenditures, with a base year of 1994.

An important issue is whether the measures of saving and income used to compute the
saving rate should include \textit{capital gains}. In DSZ, the saving rates computed using SCF data
include capital gains, but they also suggest that the true intentions of households may be
better captured by saving measures that exclude capital gains. This concern is especially
relevant for saving rates of top earners, which are our main focus. We therefore also compute
a measure of top earners’ saving that excludes capital gains.

The second source of data is the PSID, which is used to directly estimate the top 20%
MPS according to (15). DSZ use the PSID income data and the wealth supplement files,
which give detailed information on wealth levels in 1984, 1989, and 1994. DSZ estimation of
MPS using PSID data is based on a sample of 2907 households.

\section*{C. Results}

\subsection*{C.1 Saving Rate Estimates Using SCF Data}

We consider three sets of estimates of saving rates based on the SCF. The first two come
directly from DSZ. They correspond to their Table 3, column 2, which presents median
regression estimates without instrumenting income, and to their Table 5, column 2, which
presents median regression IV estimates using lagged income as an instrument, as in equation
In addition, we produce one more set of estimates, by rerunning the IV estimation of DSZ with saving and income measures corrected for capital gains (KG). The results of the three estimations are presented in Table 1 below.

Table 1 clearly shows one key result of DSZ, namely that saving rates are a steeply increasing function of income. For example, column 2, based on DSZ’s IV results, shows that the saving rate increases from 0.022 in the first quintile to 0.246 in the fifth quantile, and then to 0.385 and 0.455 for the top 5% and top 1%. Adjusting for capital gains leads to a downward correction. Figure 1 illustrates these estimation results graphically, by plotting the estimated levels of savings that correspond to the median income in each quantile.

<table>
<thead>
<tr>
<th></th>
<th>Non-IV Median Reg</th>
<th>IV Median Reg (incl. KG)</th>
<th>IV Median Reg (excl. KG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KG</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quintile 1</td>
<td>0.014 (0.019)</td>
<td>0.022 (0.025)</td>
<td>0.022 (0.024)</td>
</tr>
<tr>
<td>Quintile 2</td>
<td>0.090 (0.029)</td>
<td>0.094 (0.027)</td>
<td>0.091 (0.024)</td>
</tr>
<tr>
<td>Quintile 3</td>
<td>0.111 (0.032)</td>
<td>0.106 (0.036)</td>
<td>0.106 (0.032)</td>
</tr>
<tr>
<td>Quintile 4</td>
<td>0.173 (0.027)</td>
<td>0.167 (0.028)</td>
<td>0.163 (0.027)</td>
</tr>
<tr>
<td>Quintile 5</td>
<td>0.236 (0.040)</td>
<td>0.246 (0.035)</td>
<td>0.214 (0.040)</td>
</tr>
<tr>
<td>Top 5%</td>
<td>0.372 (0.098)</td>
<td>0.397 (0.115)</td>
<td>0.299 (0.121)</td>
</tr>
<tr>
<td>Top 1%</td>
<td>0.512 (0.111)</td>
<td>0.455 (0.088)</td>
<td>0.356 (0.096)</td>
</tr>
<tr>
<td>Age 30-39</td>
<td>-0.041 (0.028)</td>
<td>-0.057 (0.026)</td>
<td>-0.029 (0.027)</td>
</tr>
<tr>
<td>Age 50-59</td>
<td>-0.012 (0.027)</td>
<td>-0.016 (0.027)</td>
<td>0.008 (0.028)</td>
</tr>
</tbody>
</table>

Source

Table 1: Estimated Coefficients of Median Saving Rate Regressions (bootstrapped standard errors in parentheses)

C.2 MPS Estimates

Using the results in Table 1, we can compute the MPS of the top 5% income share by using equation (14), or equivalently by computing the slope of the income-savings relationship shown in Figure 1. In that figure, each vertical set of three dots (for the three regressions of

11Saving rate coefficients for top income groups are shown in the footnote of Table 5 in DSZ.
Table 1) corresponds to the levels of savings at the median incomes of the respective quantiles. Most importantly for the purpose of our paper, the last two sets of dots correspond to the savings of an individual whose income is at the median of the top 5% income bracket and at the median of the top 1% income bracket. However, for consistency with the model, we need to compute the MPS of a household whose income equals not the median but the mean income of the top 5% income group. The mean income of a top 5% household is, as shown in Figure 1, located to the right of the median income of the top 5% bracket, specifically at the 98.5 percentile of the income distribution. This percentile is almost exactly equidistant to that of the median income of the top 5% (97.5 percentile) and the median income of the top 1% (99.5 percentile). The required MPS therefore equals the slope of the saving function in Figure 1 between the median income of the top 5% and the median income of the top 1%,

$$\overline{MPS}_{\text{mean}(top5)} = \frac{\hat{\alpha}_{top1} \times \text{Median}(y_{top1}) - \hat{\alpha}_{top5} \times \text{Median}(y_{top5})}{\text{Median}(y_{top1}) - \text{Median}(y_{top5})},$$

where $\hat{\alpha}_{top1}$ and $\hat{\alpha}_{top5}$ are the estimated saving rates presented in Table 1. Note that since the left and right slopes at the top 5% median income in Figure 1 are close to each other, the MPS of the median household of the top 5% income bracket, which would be computed by averaging the slopes to the right and to the left of the median of the top 5% income bracket, would be similar to the MPS of the mean of the top 5% income bracket.

Table 2 below summarizes our estimates. The first three columns present estimates of the top 5% MPS, $\overline{MPS}_{\text{mean}(top5)}$, based on the three SCF-based regressions reported in Table 1, and computed according to equation (16). The fourth column presents an estimate of the top 20% MPS, based on the PSID-based regression of equation (15). In all cases, the bootstrapped standard errors are shown below the respective coefficients.

In the calibration of the model we treat the DSZ SCF-based IV estimate (0.505) as our upper-bound, the IV estimate excluding capital gains (0.397) as our baseline, and the DSZ PSID-based estimate (0.248) as a very conservative lower bound. We do not use the non-IV estimate (0.61), because this estimate can be contaminated by transitory income shocks.

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12The fact that the mean income of the top 5% is higher than the median income simply reflects the positive skewness of the income distribution within the top 5% income share.
IV. Equating Model-Based and Data-Based MPS

The objective of this appendix is to match the model-based formula for the MPS of top earners with the empirical MPS estimates presented in Table 2, in order to calibrate the
parameters of the preference for wealth specification. The model-based MPS is a function of \( \varphi \) and of the ratio \( \eta / \sigma \). The parameter \( \sigma \), a common parameter of the preferences of top and bottom earners, is set at \( \sigma = 0.5 \), for two reasons. First, it is a value that is used in much of the macroeconomics literature. Second, what determines the MPS is not the levels of \( \sigma \) or \( \eta \), but rather the ratio \( \eta / \sigma \). Our strategy is therefore to fix \( \sigma \) at a commonly used value, and to then explore the sensitivity of our results to different values for the two remaining parameters \( \eta \) and \( \varphi \). Specifically, holding all other properties of the original steady state constant, we initially fix \( \varphi = 0.05 \), and then search for \( \eta \) such that

\[
MPS_{\text{top} 5\%} = \overline{MPS}_{\text{mean(top5)}}.
\]

Because there is a continuum of combinations of \( \varphi \) and \( \eta \) that makes this equality hold, this exercise can be repeated for different \( \varphi \), and we have done so. An important property of the model however is that differences in the combinations of \( \varphi \) and \( \eta \), for a given MPS, have only a small effect on the model’s predictions for bottom earners’ debt-to-income ratio. We demonstrate this in Figure 12 (right subplot) of the paper, by comparing simulations in which the same MPS estimate is matched using alternative combinations of \( \varphi \) and \( \eta \). The main issue is therefore the correct identification of top earners’ MPS, for which we have been able to show a plausible range.

Table 3 presents, for the empirical MPS estimates used in our paper, the values of \( \eta \) that satisfy (17) for the three alternative values of the parameter \( \varphi \) used in the right subplot of Figure 12. All other parameters are held at the baseline calibration values that are discussed in Section IV.A of the paper.

<table>
<thead>
<tr>
<th>MPS</th>
<th>( \varphi = 0.031 )</th>
<th>( \varphi = 0.050 )</th>
<th>( \varphi = 0.080 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPS = 0.248</td>
<td>0.877</td>
<td>0.782</td>
<td>0.706</td>
</tr>
<tr>
<td>MPS = 0.397</td>
<td>1.233</td>
<td>1.082</td>
<td>0.965</td>
</tr>
<tr>
<td>MPS = 0.505</td>
<td>1.561</td>
<td>1.356</td>
<td>1.200</td>
</tr>
</tbody>
</table>

Table 3: Calibration of the Preference Parameter \( \eta \) for Different MPS and \( \varphi \)
Online Appendix for “Inequality, Leverage and Crises”

B. Computational Appendix

Abstract

The solution method that is used in this paper adapts the time-iterative policy function algorithm described by Coleman (1991) to allow for endogenous default decisions. In this appendix we outline the main steps of the algorithm. More detailed explanations accompany the source code, which is available at http://www.mosphere.fr/files/krw2014.

I. Model

The model has four continuous states \((l_t, y_t, z_t, u_t)\) that evolve according to the transition equations

\[
\begin{align*}
  y_t &= 1 + \rho_y (y_{t-1} - 1) + \epsilon^y_t, \\
  z_t &= \bar{z} + \rho_z (z_{t-1} - \bar{z}) + \epsilon^z_t, \\
  l_t &= b_{t-1} (1 - h\delta_t), \\
  u_t &= (1 - \rho_u) u_{t-1} + \gamma_u \delta_t.
\end{align*}
\]

Transitions are driven by two normally distributed exogenous shocks \(\epsilon^y_t\) and \(\epsilon^z_t\) and by the decision to default \(\delta_t\). There are three unknown controls \((p_t, b_t, V_t)\) that are functions of the states. They are defined by the conditions

\[
\begin{align*}
  p_t &= \beta_b E_t \left[ \left( \frac{c^b_{t+1}}{c^b_t} \right)^{-\frac{1}{\sigma}} (1 - h\delta_{t+1}) \right], \\
  p_t &= \beta_T E_t \left[ \left( \frac{c^\tau_{t+1}}{c^\tau_t} \right)^{-\frac{1}{\sigma}} (1 - h\delta_{t+1}) \right] + \varphi \frac{(c^T_t)^{\frac{1}{\sigma}}}{\left(1 + b_t \frac{(1 - \chi)}{\chi}\right)^{\frac{1}{\sigma}}}, \\
  V_t &= \frac{(c^\tau_t)^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + \beta E_t [V_{t+1}],
\end{align*}
\]
where consumption is given by
\[ c_t^r = y_t z_t \frac{1}{\chi} + (l_t - p_t b_t) \frac{(1 - \chi)}{\chi}, \]
\[ c_t^b = y_t (1 - z_t) (1 - u_t) \frac{1}{1 - \chi} - (l_t - p_t b_t). \] (3)

The decision to default is implicitly parameterized by the value function \( V_t. \)\(^1\) We use the definition of the utility cost to rewrite the Euler equations for bottom and top earners. For bottom earners we have
\[ p_t = \beta \beta E_{\epsilon_{t+1}^{y}, \epsilon_{t+1}^{z}} \left[ (1 - \pi_{t+1}) \left\{ \left( \frac{c_{t+1}^b}{c_t^b} \right)^{-\frac{1}{\gamma}} \right\}_{\delta_{t+1}=0} + \pi_{t+1} \left\{ \left( \frac{c_{t+1}^b}{c_t^b} \right)^{-\frac{1}{\gamma}} (1 - h) \right\}_{\delta_{t+1}=1} \right], \] (4)
where we use the c.d.f. of the random utility cost to get
\[ \pi_{t+1} = \text{prob} (\delta_{t+1} = 1) = \mathbb{E} \left( V_{t+1}|_{\delta_{t+1}=1} - V_{t+1}|_{\delta_{t+1}=0} \right). \] (5)

The same approach is used to decompose the Euler equation of top earners.

II. Solution Algorithm

A. Overview

We approximate the optimal decisions as a function of the continuous states \( (l_t, y_t, z_t, u_t) \) and discretize the state-space as a \( 50 \times 10 \times 10 \times 5 \) regular Cartesian grid with boundaries \( 0 \leq l_t \leq 4, 0.9 \leq y_t \leq 1.1, 0.1 \leq z_t \leq 0.4, 0 \leq u_t \leq 0.2. \) This grid is denoted by \( S = (s_n)_{n \in [1,N]} \) and is a \( N \times 4 \) matrix where each row contains the coordinate of one point of the \( N = 25000 \) points of the discretized state-space. The optimal values of \( (b_t, p_t, V_t) \) are exactly solved on these grid points. The numerical solution of the problem is therefore a \( N \times 3 \) matrix \( X = (x_n)_{n \in [1,N]}, \) where each row is the value of the control \( x_n \) for the corresponding grid point \( s_n. \)

We approximate expectations in the Euler equations by using Gauss-Hermite quadrature for \( (\epsilon_{t+1}^{y}, \epsilon_{t+1}^{z}) \) with 5 nodes for each shock. When computing the residuals of the Euler

\(^1\)Note that this approach can also be used in a conventional rational default setting, without the random utility cost.
equations at each iteration step, we interpolate future controls using natural cubic splines, and extrapolate linearly when needed. We look for the controls that satisfy the optimality equations exactly. Default probabilities are defined implicitly as a function of $V_t$ and integrated exactly when computing Euler residuals.

**B. Algorithm**

The time-iterative algorithm that we use to find the solution can be summarized as follows:

1. Start iteration $i$ with an initial guess $X^i = (x^i_n)_{n \in [1,N]}$ for the values of all controls $(b_t, p_t, V_t)$ on each point of the grid. For iteration $i = 1$, use the result from a first order perturbation or an initial guess coming from another calibration of the model.

2. For each point $s_n = (l_t, y_t, z_t, u_t)$ on the grid, find the controls $x_{n}^{i+1} = (b_{t}, p_{t}, V_{t})$ which solve the optimality equations exactly. This involves another, nested, optimization problem. Define a function $f_{n}^{i+1} : x_{n}^{i+1} \in \mathbb{R}^3 \mapsto r_{n}^{i+1} \in \mathbb{R}^3$ that returns the Euler residuals at point $s_n$, at step $i + 1$. Then, use a simple Newton algorithm using $x_{n}^{i}$ as an initial guess, to find $x_{n}^{i+1}$ such that $f_{n}^{i+1} (x_{n}^{i+1}) < 10^{-8}$. For any $x_{n}^{i+1}$, the actual computation of $f_{n}^{i+1} (x_{n}^{i+1})$ is made using the following subprocedure.

   - Take a value $x_{n}^{i+1}$.
   - For each node $\epsilon^{y}_{t+1}, \epsilon^{z}_{t+1}$ given by the Gauss-Hermite quadrature:
     - Assume default (resp. no default) and use the transition equation to compute the future values, generally not on the grid, of the states $(l_{t+1}, y_{t+1}, z_{t+1}, u_{t+1})|_{\delta_{t+1}=1}$ (resp. $(l_{t+1}, y_{t+1}, z_{t+1}, u_{t+1})|_{\delta_{t+1}=0}$).
     - Interpolate the guess for the controls at these future values of the states, i.e. between grid points, in order to get the future value of the controls $(b_{t+1}, p_{t+1}, V_{t+1})|_{\delta_{t+1}=1}$ (resp $(b_{t+1}, p_{t+1}, V_{t+1})|_{\delta_{t+1}=0}$).
     - Use these values and the definition of the auxiliary variable $\pi_{t+1}$ to compute the term inside the square bracket in equation (4), and its counterpart equation for top earners.

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\footnote{In the numerical implementation, we simultaneously solve for all controls at all points on the grid, as a large vectorized system. The algorithmic steps remain exactly equivalent to what is outlined here.}
• Cumulate the result using Gauss-Hermite weights and compute the residuals $r_{n}^{i+1}$ of the Euler equations at point $s^n$.

• Return the residuals $r_{n}^{i+1}$.

3. Denote the solutions of the previous step by $X_i^{i+1} = (x_n^{i+1})_{n \in [1, N]}$. If $\|X_i - X_i^{i+1}\| < 10^{-6}$ then the controls $X_i$ solves the model. Else, start iteration $i + 1$ at step 1, using $X_i^{i+1}$ as an initial guess.
Online Appendix for “Inequality, Leverage and Crises”

C. Literature Review (Long Version)

The central argument of the paper links two strands of the literature that have largely been evolving separately, the literature on income and wealth distribution, and the literature on financial fragility and financial crises. In addition, our modeling approach takes elements from the literature on preferences for wealth, to explain the rise of household debt leverage when the increase in income inequality is permanent, and from the literature on rational default, to endogenize financial crises.

The literature on income and wealth distribution is mostly focused on accurately describing long run changes in the distribution of income and wealth (Piketty and Saez (2003), Piketty (2010), Piketty and Zucman (2013)). It has found that the most significant change in the U.S. income distribution has been the evolution of top income shares. This is reflected in our model, which contains two groups representing the top layer and the remainder of the U.S. income distribution. Our paper focuses only on the macroeconomic implications of increased income inequality, rather than taking a stand on the fundamental reasons for changes in the income distribution.\textsuperscript{1}

The stylized facts section of our paper documents a strong comovement between increases in income inequality and increases in household debt-to-GDP ratios in both the period prior to the Great Depression and the period prior to the Great Recession.\textsuperscript{2} In our model an increase in debt among bottom earners, which empirically has been the main driver of the overall increase in household debt in the period prior to the Great Recession, leads to an increase in crisis risk. This is consistent with the results of Schularick and Taylor (2012) who,

\textsuperscript{1}A large literature focuses on these factors. For a partial review, see Kumhof and Ranciere (2010).

\textsuperscript{2}Note that we focus only on the two largest financial crises in post-WWI U.S. history, which were both preceded by historically unique, decades-long build-ups in household debt-to-income ratios. Bordo and Meisner (2012) look instead at a sample that includes many much smaller crisis episodes.
using a sample of 14 developed countries over the period 1870 - 2008, find that increases in debt-to-GDP ratios have been a powerful predictor of financial crises. Furthermore, in our model the crisis itself is associated with a partial default of bottom earners, accompanied by output costs of default. This mechanism is consistent with the results of Midrigan and Philippon (2011), who show that U.S. regions (states, counties) that had experienced the largest increases in household debt leverage suffered the largest output declines during the Great Recession, and with similar findings by Gärtner (2013), who shows that U.S. states with higher household debt-to-income levels in 1929 experienced a much slower recovery in income, wages, and unemployment between 1933 and 1939.

This paper quantitatively evaluates the role played by income inequality in the 1983-2007 increase in U.S. bottom earners’ debt-to-income ratios and the corresponding increase in crisis risk. Our proposed explanation is that debt increased due to an increase in top earners’ credit supply, in response to a permanent increase in their income share. We find strong empirical support for this mechanism.

However, the literature on financial fragility and financial crises has proposed other explanations for the origins of the Great Recession that focus on domestic and global asset market imbalances. We study the importance of such factors in Section II.F of the paper, and find that they played a role in decade prior to the Great Recession. The global saving glut can account for approximately the fraction of household debt accumulation left unexplained by our model over this period, while complex financial products that spread over the years immediately preceding the crisis can be seen as an additional source of financial fragility.

Rajan (2010) and Reich (2010) share with this paper an emphasis on income inequality as a long-run determinant of leverage and financial fragility. They suggest that sizeable increases in borrowing have been a way for the poor and the middle class to maintain or increase their level of consumption at times when their real earnings were stalling. But these authors do not make a formal case, in the form of a general equilibrium model, to support their argument. The advantage of using a model, as in our paper, is that it permits a quantification of the importance of this channel, and an identification of the respective roles of credit demand and credit supply in the rise of household leverage and crisis risk.

3See Keys et al. (2010), Taylor (2009), and Obstfeld and Rogoff (2009).
4Relatedly, Bertrand and Morse (2013) show empirically that U.S. income inequality is negatively correlated with the saving rate of the middle-class, and positively correlated with personal bankruptcy filings.
Recently, Coibion et al. (2014), using local measures of inequality and credit, have found empirical evidence consistent with supply-side interpretations of debt accumulation patterns during the 2000s.

Another recent literature has related increases in income inequality to increases in household debt (Krueger and Perri (2006), Iacoviello (2008)). In these authors’ approach an increase in the variance of idiosyncratic income shocks across all households generates a higher demand for insurance through credit markets, thereby increasing household debt. This approach emphasizes an increase in income inequality experienced within household groups with similar characteristics, while our paper focuses on the rise in income inequality between two household groups. There is a lively academic debate concerning the relative roles of within- and between-group factors in shaping inequality. But our paper only focuses on changes in one specific type of between-group inequality that can be clearly documented in the data, namely inequality between high-income households and everyone else. Furthermore, the observed increases in household debt-to-income ratios have also been strongly heterogenous between these two income groups.

In theory, if increasing income inequality was accompanied by an increase in intra-generational income mobility, the dispersion in lifetime earnings might be much smaller than the dispersion in annual earnings, as agents move up and down the income ladder throughout their lives. However, a recent study by Kopczuk, Saez and Song (2010)\textsuperscript{5}, using a novel dataset from the Social Security Administration, shows that short-term and long-term income mobility in the United States has been either stable or slightly falling since the 1950s. These authors find that the surge in top earnings has not been accompanied by increased mobility between the top income group and other groups. In addition, they directly measure the persistence of earnings inequality, with inequality defined as the variance of annual log earnings. They show that virtually all of the increase in that variance over recent decades has been due to an increase in the variance of permanent earnings rather than transitory earnings. DeBacker et al. (2013) obtain similar results by using an alternative source, a 1-in-5000 random sample of the population of U.S. tax payers. They find that the increase in cross-sectional earnings inequality over 1987-2006 was permanent for male earnings and predominantly permanent for household income. These studies, while not reflecting a complete consensus in the literature\textsuperscript{6}, suggest that the evolution of contemporaneous income

\textsuperscript{5}See also Bradbury and Katz (2002).

\textsuperscript{6}The findings of Kopczuk et al. (2010) and DeBacker et al. (2013) differ from previous results, based
inequality is close to the evolution of lifetime income inequality. This provides support for one of our simplifying modeling choices, the assumption of two income groups with fixed memberships. It also provides additional support for our calibration, whereby shocks to the income distribution are permanent.

In the baseline version of our model, top earners exhibit preferences for wealth. Wealth in the utility function has been used by a number of authors including Carroll (2000), who refers to it as the “capitalist spirit” specification, Reiter (2004), Piketty (2010), Zou (1994, 1995) and Bakshi and Chen (1996). Wealth in the utility function can represent a number of different saving motives. Our preferred interpretation, following Cole, Mailath and Postelweite (1992), Bakshi and Chen (1996), and Carroll (2000), is that agents derive direct utility from the social status and power conferred by wealth. The reason for introducing this feature is that models with standard preferences have difficulties accounting for the saving behavior of the richest households. For instance, Carroll (2000) shows, using data from the U.S. Survey of Consumer Finances, that the life cycle/permanent income hypothesis model augmented with uncertainty proposed by Hubbard, Skinner and Zeldes (1994) can match the aggregate saving behavior only by over-predicting the saving behavior of median households and by underpredicting the saving behavior of the richest households. By contrast, models featuring wealth in the utility function can match both the aggregate data and the wealth accumulation patterns of the wealthiest households. Francis (2009) shows that introducing preferences for wealth into an otherwise standard life cycle model generates the skewness of the wealth distribution observed in the U.S. data.

Preferences for wealth introduce two additional parameters into the utility function of top earners, the weight of wealth in overall utility and the curvature of utility with respect to wealth. There is very little literature that can be used for guidance on how to calibrate these parameters directly. But there is a small literature on the marginal propensity to save on PSID data, of Gottschalk and Moffitt (1994) and Blundell, Pistaferri and Preston (2008), who attribute a much larger role to increases in the variance of transitory earnings. However, they confirm the results of Primiceri and Van Rens (2009) who, utilizing repeated cross-sections of income from the Consumer Expenditure Survey, find that all of the increase in household income inequality in the 1980s and 1990s reflects an increase in the permanent variance.

An alternative model of saving behavior is the dynastic model (Barro (1974)), where dynasties maximize the discounted sum of utilities of current and future generations. Carroll (2000) surveys evidence suggesting that this model also does not do well in explaining the saving decisions of the richest households. Piketty (2010) shows similar results to Carroll (2000) for France.
(MPS) of different income groups. The key paper is Dynan, Skinner and Zeldes (2004), who find that MPS are steeply increasing in the level of permanent income, and reach values of 0.5 or even higher for the highest income groups. We show that these results can be mapped directly into a calibration of the preference for wealth parameters, thereby giving a solid empirical foundation to the use of these preferences in applied work. This calibration methodology is an important contribution of our paper.

*Endogenous, rational default* of borrowing households is a key feature of our model, with higher leverage leading to a higher crisis probability. Our paper is therefore naturally related to models of consumer bankruptcy as developed in Athreya (2002), Chatterjee et al. (2007) and Livshits et al. (2007). However, these papers are based on economies with a continuum of heterogeneous agents, as in Aiyagari (1994) and Huggett (1993), while our model focuses on aggregate relationships between two groups of agents. It is therefore much closer to the sovereign default literature of Eaton and Gersovitz (1981), Bulow and Rogoff (1989), Aguiar and Gopinath (2006) and Arellano (2008). But our model also exhibits significant differences to that literature. First, lenders in our model are risk-averse, rather than risk neutral as the rest-of-the-world investors of the partial equilibrium sovereign debt literature. Risk-averse investors are also assumed in Borri and Verdelhan (2012), Lizarazo (2013) and Pouzo and Presno (2012). Second, since only a fraction of households will default in any real-world crisis event, we assume that default in our two-agent economy occurs only on a fixed fraction of outstanding debt. More elaborate default setups with partial recovery are studied in Benjamin and Wright (2009), Yue (2010) and Adam and Grill (2013). Third, in much of the sovereign debt literature the state space exhibits a sharp boundary between regions of certain non-default and certain default. But according to Schularick and Taylor (2012), the probability of a major crisis in the United States, while increasing in the level of debt, has nevertheless always remained well below 10% in any given year. Our model replicates this feature by adding a stochastic utility cost of default, similar to Pouzo and Presno (2012). Fourth, the assumptions of partial default and of low default probabilities imply that the level of debt that can be sustained in equilibrium is higher than what is commonly obtained in the sovereign debt literature, even though our output penalties of default are lower than the output losses observed during the largest crises.
References


