Appendix: “Treatment Effects and Informative Missingness with an Application to Bank Recapitalization Programs”

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This appendix contains detailed instructions for the Markov chain Monte Carlo (MCMC) algorithm employed in the paper. The appendix first describes the steps for the algorithm followed by definitions of the vectors and matrices involved. Secondly, the appendix offers details on the location of the data.

MCMC Estimation Algorithm for Censored Outcomes

1. Sample $\beta$ from the distribution $\beta | y, y^*, \theta \setminus \beta$.  
2. Sample $\Omega$ from the distribution $\Omega | y, y^*, \theta \setminus \Omega$ in a one block, multi-step procedure.
3. For $i \in N_1$, sample $y^*_1$ from the distribution $y^*_1 | y, \theta, y^* \setminus y^*_1$.
4. For $i \in N_2$, sample $y^*_2$ from the distribution $y^*_2 | y, \theta, y^* \setminus y^*_2$.
5. For $i \in N_2o$, sample $y^*_3$ from the distribution $y^*_3 | y, \theta, y^* \setminus y^*_3$.
6. For $i \in N_3o$, sample $y^*_4$ from the distribution $y^*_4 | y, \theta, y^* \setminus y^*_4$.
7. For $i \in N_1o$, sample $y^*_5$ from the distribution $y^*_5 | y, \theta, y^* \setminus y^*_5$.

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1The notation “\" represents “except”, e.g., $y^* \setminus y^*_1$ says all elements in $y^*$ except $y^*_1$. 
Step 1: Sampling $\beta$
Sample $\beta \mid y^*, \theta \setminus \beta \sim \mathcal{N}(b, B)$, where
\[
b = B(b_0^{-1}b_0 + \sum_{i \in N_1} J_i'C_iC^{-1}C_i'y_i^* + \sum_{i \in N_2} J_i'D_iD^{-1}D_i'y_i^* + \sum_{j \in N_3} J_j'A_jA^{-1}A_j'y_j^*),
\]
\[
B = (B_0^{-1} + \sum_{i \in N_1} J_i'C_iC^{-1}J_i + \sum_{i \in N_2} J_i'D_iD^{-1}J_i + \sum_{j \in N_3} J_j'A_jA^{-1}J_j)^{-1}.
\]

Step 2: Sampling $\Omega$
Sample $\Omega \mid y, y^*, \theta \setminus \Omega$ in a one block, nine-step procedure by first drawing $\Omega_{11} = \Omega_{tt} - \Omega_{tt}\Omega_{tt}^{-1}\Omega_{tt}$, and then reconstructing $\Omega$ from these quantities
2. (a) $\Omega_{11} = \mathcal{I}(\nu + n, \sum_{i=1}^{N_1} \eta_{i1} \eta_{i1}')$
   \[
   \Omega_{11} = y_{i1} - x_i'J_1\beta,
   \]
   where $J_1 = [1 \ 0 \ 0 \ 0 \ 0]_{1 \times K}$
(b) $\Omega_{22} = \mathcal{I}(\nu + n_2 + n_3, R_{22})$
(c) $B_{12} = \mathcal{MN}(R_{11}^{-1}R_{21}, \Omega_{22} \otimes R_{11}^{-1})$
(d) Define $\Omega_u = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}$
(e) $\Omega_{55} = \mathcal{I}(\nu + n_1, R_{55})$
(f) $B_{15} = \mathcal{MN}(R_{11}^{-1}R_{51}, \Omega_{55} \otimes R_{11}^{-1})$
(g) $\Omega_{33} = \mathcal{I}(\nu + n_2, R_{33})$
(h) $B_{33} = \mathcal{MN}(R_{u}^{-1}R_{3u}, \Omega_{33} \otimes R_{u}^{-1})$
(i) $\Omega_{44} = \mathcal{I}(\nu + n_3, R_{44})$
(j) $B_{44} = \mathcal{MN}(R_{u}^{-1}R_{4u}, \Omega_{44} \otimes R_{u}^{-1})$

where $R = Q + \sum \eta_i \eta_i'$, and the following subsections are obtained by partitioning $R$ to conform to $Q$, and $R_{tt} = R_{tt} - R_{tt}R_{tt}^{-1}R_{tt}$. From these sampling densities, $\Omega$ can be recovered.

Steps 3-7: Sampling $y^*$
\[
y_i^* \mid y, \theta, y^* \setminus y_i^* \sim \mathcal{T}\mathcal{N}_{(-\infty, \theta)}(x_i'1\beta_1 + E(\epsilon_{1i}|\epsilon_{i1}), var(\epsilon_{1i}|\epsilon_{i1})), \quad i \in N_1,
\]
\[
y_i^* \mid y, \theta, y^* \setminus y_i^* \sim \mathcal{T}\mathcal{N}_{(-\infty, \theta)}(x_i'2\beta_2 + E(\epsilon_{1i}|\epsilon_{12}), var(\epsilon_{1i}|\epsilon_{12})), \quad i \in N_2,
\]
\[
y_i^* \mid y, \theta, y^* \setminus y_i^* \sim \mathcal{T}\mathcal{N}_{(-\infty, \theta)}(x_i'3\epsilon_{i1}|\epsilon_{i3}), var(\epsilon_{i3}|\epsilon_{i3})), \quad i \in N_3,
\]
\[
y_i^* \mid y, \theta, y^* \setminus y_i^* \sim \mathcal{T}\mathcal{N}_{(-\infty, \theta)}(x_i'4\epsilon_{i1}|\epsilon_{i4}), var(\epsilon_{i4}|\epsilon_{i4})), \quad i \in N_4,
\]
\[
y_i^* \mid y, \theta, y^* \setminus y_i^* \sim \mathcal{T}\mathcal{N}_{(-\infty, \theta)}(x_i'5\epsilon_{i5}|\epsilon_{i5}), var(\epsilon_{i5}|\epsilon_{i5})), \quad i \in N_5.
\]

2
Definitions

Priors: It is assumed that \( \beta \) has a joint normal distribution with mean \( \beta_0 \) and variance \( B_0 \), and (independently) that the covariance matrix \( \Omega \) has an inverted Wishart distribution with parameters \( v \) and \( Q \),

\[
\pi(\beta, \Omega) = \mathcal{N}(\beta | \beta_0, B_0) \mathcal{IW}(\Omega | v, Q).
\]

Data: For the \( i \)-th observation, define the following vectors and matrices,

\[
y_{iC}^* = (y_{i1}^*, y_{i5}^*)', \quad y_{iD}^* = (y_{i1}^*, y_{i2}^*, y_{i3}^*)', \quad y_{iA}^* = (y_{i1}^*, y_{i2}^*, y_{i4}^*)',
\]

\[
X_{iC} = \begin{pmatrix} x_{i1}' & 0 \\ 0 & x_{i5}' \end{pmatrix}, \quad X_{iD} = \begin{pmatrix} x_{i1}' & 0 & 0 \\ 0 & x_{i2}' & 0 \\ 0 & 0 & (x_{i3}' y_{i1}) \end{pmatrix}, \quad X_{iA} = \begin{pmatrix} x_{i1}' & 0 & 0 \\ 0 & x_{i2}' & 0 \\ 0 & 0 & (x_{i4}' y_{i1} y_{i2}) \end{pmatrix}.
\]

Let \( N_1 = \{ i : y_{i1} = 0 \} \) be the \( n_1 \) observations in the non-selected sample and \( N_2 = \{ i : y_{i1} > 0 \text{ and } y_{i2} = 0 \} \) be the \( n_2 \) observations in the selected untreated sample. Set \( N_3 = \{ i : y_{i1} > 0 \text{ and } y_{i2} > 0 \} \) to be the \( n_3 \) observations in the selected treated sample.

In order to isolate the vectors and matrices that correspond to the 3 different subsets of the sample, define

\[
J_C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}_{2 \times K}, \quad J_D = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}_{3 \times K}, \quad J_A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}_{3 \times K}
\]

where \( K = k_1 + k_2 + k_3 + k_4 + k_5 \), which represents the number of covariates in each equation.

For \( i \in N_1 \) (non-selected sample),

\[
\eta_{iC}^* = y_{iC}^* - X_{iC}J_C \beta,
\]

for \( i \in N_2 \) (selected untreated sample),

\[
\eta_{iD}^* = y_{iD}^* - X_{iD}J_D \beta,
\]

and for \( i \in N_3 \) (selected treated sample),

\[
\eta_{iA}^* = y_{iA}^* - X_{iA}J_A \beta,
\]

Finally, \( N_{2o} \) is defined as the truncated portion of the \( N_2 \) sample in \( y_3 \), \( N_{3o} \) is defined as the truncated region of the \( N_3 \) sample in \( y_4 \), and \( N_{1o} \) is defined as the discrete part of the \( N_1 \) sample in \( y_5 \) since all the equations are Tobit equations with censored dependent variables.
Data

- RFC Card Index to Loans Made to Banks and Railroads, 1934-57
  - National Archives, College Park, MD
  - Record Group 234 / Reconstruction Finance Corp.
- Declined and Cancelled Loans, 1933-1941
  - National Archives, College Park, MD
  - Record Group 234 / Reconstruction Finance Corp.
  - Location: 570
- Paid Loans, 1933-1941
  - National Archives, College Park, MD
  - Record Group 234 / Reconstruction Finance Corp.
  - Location: 570
- Rand McNally Banker’s Directory
  - Years 1932 - 1935

Programs

The MCMC algorithm employed in this paper follows from the steps listed above. This algorithm extends the sampling techniques developed in the below reference:


The GAUSS programs are available in the supplementary materials associated with Chib, Greenberg, and Jeliazkov (2009) found at the below link.