Appendix: Renegotiation Policies in Sovereign Defaults *

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Proposition 1 Country $i$ defaults in the planning problem iff $b_i \geq B_i^p$ where

$$B_i^p = \min\left\{ \frac{\lambda_B}{\lambda_B - \lambda_L} (y_i - y_i^d), \quad y_i - y_i^{nr} \right\} \text{ for } i = 1, 2$$

Proof. We prove this proposition by contradiction. Suppose that $b_1 < B_1^p$ but country 1 defaults in the planning problem. This can only be a solution if it is feasible for the planner to induce country 1 to default and such strategy increases the value for the planner.

To induce default, the planner needs to set the recovery low enough, $\tilde{\phi}_1 < \phi_1^*$ where $y_1^d - \phi_1^* = y_1 - b_1$. If $\phi_1^* \geq 0$, it is feasible for the planner to induce default independent of the default outcome for country 2 because $\tilde{\phi}_1 = 0$ is always feasible. However, inducing country 1 to default is not optimal because $b_1 < \frac{\lambda_B}{\lambda_B - \lambda_L} (y_1 - y_1^d)$ which implies that the additional value from country $i$ defaulting is negative $\lambda_B (y_1^d - y_1) + (\lambda_B - \lambda_L) b_1 < 0$.

Now, consider the more interesting case of $\phi_1^* < 0$. The planner can only induce a default in country 1 by setting $\tilde{\phi}_1 = \phi_1^* < 0$ which requires that country 2 also defaults and pays a recovery $\tilde{\phi}_2 > 0$ that is high enough. By Lemma 3, it is optimal to set $\tilde{\phi}_2 = -\tilde{\phi}_1 = -\phi_1^* > 0$.

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Suppose that $\tilde{\phi}_2 = -\phi_1^*$ does not induce country 2 to default $y_2^d - \tilde{\phi}_2 < y_2 - b_2$. For such $\tilde{\phi}_2$, the planner cannot induce country 1 to default because the lower bound on the recovery for country 1 $\bar{\phi}_1 = 0$ is too high.

Suppose now that we make $\tilde{\phi}_2 = y_2^d - y_2 + b_2$ to induce country 2 to default and $\bar{\phi}_1 = -\tilde{\phi}_2$. Here we have two cases. If $b_2 < B_2^P$ then the gains for the planner for additional two defaults are: $[\lambda_B(y_1^d - y_1) + (\lambda_B - \lambda_L)(b_1 + \tilde{\phi}_2) + \lambda_B(y_2^d - y_2) + (\lambda_B - \lambda_L)(b_2 - \tilde{\phi}_2)] = [\lambda_B(y_1^d - y_1) + (\lambda_B - \lambda_L)b_1 + \lambda_B(y_2^d - y_2) + (\lambda_B - \lambda_L)b_2] < 0$ because $b_1 < B_1^p$ and $b_2 < B_2^p$.

If $b_2 > B_2^P$, then $\tilde{\phi}_2 > 0$ could continue to induce 2 default. The gains for the planner from the additional country 1 default are: $[\lambda_B(y_1^d - y_1) + (\lambda_B - \lambda_L)(b_1 + \tilde{\phi}_2) + \lambda_B(y_2^d - y_2) + (\lambda_B - \lambda_L)(b_2 - \tilde{\phi}_2)] - [\lambda_B(y_2^d - y_2) + (\lambda_B - \lambda_L)b_2] = \lambda_B(y_1^d - y_1) + (\lambda_B - \lambda_L)b_1 < 0$ for $b_1 < B_1^P$ where we have used Lemma 3 that when only one country defaults it is optimal to have $\phi_2^p = 0$. Hence having a default for country 1 when $b_1 < B_1^P$ does not increase the value for the planner. ■