TOGETHER AT LAST: TRADE COSTS, DEMAND STRUCTURE, AND WELFARE*

Monika Mrázová[†]
University of Geneva,
University of Surrey,
and CEP, LSE

J. Peter Neary[‡]
University of Oxford
and CEPR

January 18, 2014
Revised May 6, 2014

Abstract

We show that relaxing the assumption of CES preferences in monopolistic competition has surprising implications when trade is restricted. Integrated and segmented markets behave differently, the latter typically exhibiting reciprocal dumping. Globalization and lower trade costs have different effects: the former reduces spending on all existing varieties, the latter switches spending from home to imported varieties; when demands are less convex than CES, globalization raises whereas lower trade costs reduce firm output. Finally, calibrating gains from trade is harder. Many more parameters are needed, while import demand elasticities typically overestimate the true elasticities, and so underestimate the gains from trade.

Keywords: Additively Separable Preferences; CES Preferences; Iceberg Trade Costs; Quantifying Gains from Trade; Super- and Subconvexity of Demand; Super- and Subconcavity of Utility.

JEL Classification: F12, F15, F17

*An earlier version was presented at the American Economic Association Annual Meetings, Philadelphia, January 2014. This version, except for the Appendix, will appear in the *American Economic Review, Papers and Proceedings*. May 2014. We are grateful to Igor Bykadorov and Sergey Kokovin for invaluable discussions of their related work, and to Swati Dhingra, Jan Haaland, Tony Venables, and Adrian Wood for helpful comments. Mrázová thanks the Fondation de Famille Sandoz for funding under the "Sandoz Family Foundation - Monique de Meuron" Programme for Academic Promotion. Neary thanks the European Research Council for funding under the European Union's Seventh Framework Programme (FP7/2007-2013), ERC grant agreement no. 295669.

†Geneva School of Economics and Management (GSEM), University of Geneva, Bd. du Pont d'Arve 40, 1211 Geneva 4, Switzerland; e-mail: monika.mrazova@unige.ch.

‡Department of Economics, University of Oxford, Manor Road, Oxford OX1 3UQ, UK; e-mail: peter.neary@economics.ox.ac.uk.

International trade, like other branches of applied theory, has made enormous progress in recent decades by building on a central insight of Dixit and Stiglitz (1977): the easiest way to model a taste for variety, an essential foundation for a theory of monopolistic competition, is using a conventional utility function defined over the quantities of all potential commodities. To operationalize this, they considered two alternative specifications of the utility function: additively separable preferences and the CES special case. The former approach was used in Krugman (1979), one of the first applications of monopolistic competition to trade. However, he assumed that trade was unrestricted, and modeled trade liberalization only as an expansion of the global economy. When he and others turned to examine restrictions to trade, it became the norm to consider only the CES case: in the words of Krugman (1980), "it seems worth sacrificing some realism to gain tractability." The result is paradoxical. We now have a clear understanding of many issues in trade under monopolistic competition, and, thanks to Arkolakis, Costinot and Rodríguez-Clare (2012) [ACRC] and others, a clear basis for quantifying the gains from trade, but only under CES assumptions, with their unsatisfactory implication that firms' price-cost mark-ups are invariant to shocks.

A number of authors have considered particular alternatives to the CES. By contrast, the combination of trade costs and general demands has received little attention.[1] In this paper we show that trade costs and additively separable preferences can be combined in a simple model, which is tractable without sacrificing too much realism. Section 1 sketches the model and introduces two key concepts: superconvex demand and superconcave utility. Section 2 compares the implications of integrated and segmented markets for prices and mark-ups. Section 3 shows how the pattern of sales across markets responds to globalization and trade-cost shocks. Section 4 derives the implications for the gains from trade, while Section 5 discusses the problems of calibrating them. Technical details and additional references which

[1] Only two other papers explore this issue. Bertoletti and Epifani (2012) take a similar approach to us but do not consider welfare. Arkolakis et al. (2012) adopt a general specification of demand, but their approach does not nest ours: they assume that demand functions have a "choke price" and are less convex than the CES. These papers are more general than ours in allowing for heterogeneous firms, but, as we hope to show, many interesting issues arise even when we abstract from this.

for reasons of space have had to be omitted are given in Mrázová and Neary (2013).

1 Preliminaries

Except for allowing trade costs, the setting is the same as in Krugman (1979). In each of $\kappa + 1$ identical countries, there is a single monopolistically competitive industry, with a measure n of identical firms, each producing a single symmetrically differentiated variety. International trade incurs symmetric iceberg trade costs τ, but no fixed costs. It follows that trade is all-or-nothing: except when trade costs are prohibitive, every consumer in the world consumes each of the $N \equiv (\kappa + 1)n$ varieties produced in the world. Why do they bother? Because they have a taste for variety, modeled by expressing utility U as a monotonically increasing function of an integral of identical sub-utility functions. With symmetry, we need only distinguish between the consumption of a typical home and imported variety, x and x^* respectively:

$$U = F\left[n\{u(x) + \kappa u(x^*)\}\right] \quad F', u' > 0, u'' < 0 \tag{1}$$

Maximizing this facing given income and prices leads to inverse Frisch demands: $p = \lambda^{-1}u'(x)$ and $p^* = \lambda^{-1}u'(x^*)$; λ is the marginal utility of income, which firms take as given in choosing their optimal sales.

With so much symmetry assumed, the general-equilibrium structure of the model is straightforward. Goods-market clearing requires that each firm's output, denoted by y, meet global demand for its product, with the proviso that τx^* units must be shipped abroad to ensure that x^* arrive:

$$y = L(x + \kappa\tau x^*) \tag{2}$$

Labor is the only factor of production, and the supply of identical worker-consumers in each country is fixed at L. Technology follows the Dixit-Stiglitz specification, perhaps the simplest possible way of allowing for increasing returns. Each firm requires f workers to operate, and c workers to produce a unit of output. Labor-market clearing in every country therefore implies:

$$L = n\left(f + cy\right) \tag{3}$$

We follow the Marx-Keynes-Krugman-Melitz convention of measuring nominal variables in labor units, so the wage is set equal to one by choice of numéraire.

The elasticity of substitution is a sufficient statistic for comparative statics with CES preferences. With general additive preferences, we need to know two statistics to understand the positive effects of exogenous shocks: the elasticity $\varepsilon(x) \equiv -\frac{p(x)}{xp'(x)}$ and convexity $\rho(x) \equiv -\frac{xp''(x)}{p'(x)}$ of demand. In addition, to understand normative implications, we need to know the elasticity of the sub-utility function $\xi(x) \equiv \frac{xu'(x)}{u(x)}$: this is an inverse measure of consumers' taste for diversity, and must lie between zero and one. We write $\varepsilon^* = \varepsilon(x^*)$ and so on for parameters pertaining to imports.[2]

Many implications of these parameters can be summarized using two key properties. The first we call *superconvexity of demand*: a demand function is superconvex at a point if it is more convex than a CES demand function with the same elasticity, $\rho \geq \frac{\varepsilon+1}{\varepsilon}$, otherwise it is subconvex. With superconvexity, the elasticity of demand rises as *per capita* consumption increases, so it is crucial for the difference between a firm's mark-ups on its home and foreign sales. The second property we call *superconcavity of utility*: a sub-utility function is superconcave at a point if it is more concave than a CES sub-utility function with the same elasticity, $\xi \geq \frac{\varepsilon-1}{\varepsilon}$, otherwise it is subconcave.[3] With superconcavity, the elasticity of utility falls, i.e., taste for diversity rises, as *per capita* consumption increases, so it is crucial in determining how consumers trade off changes at the intensive and extensive margins of consumption.

[2] In the CES case, only the elasticity of substitution σ matters: $\{\xi^*, \varepsilon^*, \rho^*\} = \{\xi, \varepsilon, \rho\} = \left\{\frac{\sigma-1}{\sigma}, \sigma, \frac{\sigma+1}{\sigma}\right\}$.

[3] The concavity of an arbitrary sub-utility function is $-\frac{xu''}{u'} = \frac{1}{\varepsilon}$; while, from footnote 2, the concavity of a CES sub-utility function with elasticity ξ is $1-\xi$.

2 Integrated or Segmented Markets?

The first issue we must address is whether home and foreign markets are integrated or segmented. Integrated markets imply that *prices* are equalized, $p^* = \tau p$, segmented markets that *marginal revenues* are equalized, $r_x^* = \tau r_x$.[4] With trade costs, these coincide if and only if demands are CES, since otherwise the ratio of price to marginal revenue differs across markets:

$$p = \frac{\varepsilon}{\varepsilon - 1} r_x, \qquad p^* = \frac{\varepsilon^*}{\varepsilon^* - 1} r_x^* \tag{4}$$

What does this imply for the pattern of price-cost mark-ups across markets? When markets are integrated, mark-ups are the same at home and abroad: $\frac{p}{c} = \frac{p^*}{\tau c}$. However, when markets are segmented, mark-ups differ in a way that depends on the convexity of demand. With subconvexity, the elasticity is higher in export markets and so from (4) the mark-up is lower. Moreover, the price charged abroad is lower than the trade-cost-inclusive home price: $p^* < \tau p$. Segmented markets thus exhibit *reciprocal dumping* but without oligopoly as in Brander and Krugman (1983).[5] All these statements are reversed if demands are superconvex: margins are higher abroad; prices there exceed the trade-cost-inclusive home price; and markets exhibit *reciprocal anti-dumping.* Which is the more likely case? Though there is no clear consensus, the balance of empirical and other evidence suggests that subconvex demands are more realistic than superconvex, implying that reciprocal dumping is the norm.

3 Globalization or Colder Icebergs?

To save space, we focus on the more realistic case of segmented markets. We consider two kinds of trade liberalization: increases in the number of countries κ ("globalization"), and reductions in trade costs τ ("colder icebergs"). These have very different effects on the conditions for firm and industry equilibrium.

[4] $r(x) \equiv xp(x)$ and $r(x^*) \equiv x^* p(x^*)$ denote sales revenue at home and abroad, respectively.
[5] We owe this insight to Sergey Kokovin.

At the firm level, profit maximization equalizes τ-inclusive marginal revenues across markets, as we have seen. Totally differentiating, using "hats" to denote proportional changes ($\hat{x} \equiv d \log x$, $x \neq 0$):

$$\hat{r}_x + \hat{\tau} = \hat{r}_x^* \quad \Rightarrow \quad \eta \hat{x} = \eta^* \hat{x}^* + \hat{\tau} \tag{5}$$

where $\eta \equiv -\frac{x r_{xx}}{r_x} = \frac{2-\rho}{\varepsilon-1}$ is the elasticity of marginal revenue at home.[6] This implies a *positive* relationship between home and foreign sales, as illustrated by the curves labeled "*MR=MC*" in Figure 1. These are shifted upwards by reductions in trade costs, but are unaffected by changes in the number of countries.

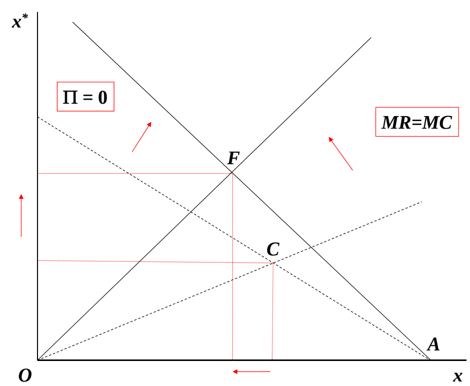

Figure 1: Effects of Lower Trade Costs

At the industry level, free entry requires that operating profits, $\pi + \kappa\pi^*$, equal fixed costs f. From the first-order condition, operating profits in an export market are: $\pi^* = (p^* - \tau c)Lx^* = \frac{\tau c L x^*}{\varepsilon^* - 1}$, and analogously at home. Totally differentiating the free-entry condition:

$$\omega_\pi \varepsilon \eta \hat{x} + (1 - \omega_\pi) \varepsilon^* \eta^* \hat{x}^* = -(1 - \omega_\pi)(\hat{\kappa} + \hat{\tau}) \tag{6}$$

where $\omega_\pi \equiv \frac{\pi}{\pi + \kappa\pi^*}$ is the home-market share in operating profits. This implies a *negative* relationship between home and foreign sales, as illustrated by the curves labeled "$\Pi = 0$" in Figure 1. These are affected in the same way by increases in τ and κ: both pivot the

[6] The firms' first- and second-order conditions require $\varepsilon > 1$ and $\rho < 2$ respectively, so η must be positive.

curve anti-clockwise around the autarky point A (the zero-profit point conditional on not exporting).

Combining these results we can deduce the effects on home and export sales:

$$\bar{\varepsilon}_{\pi}\eta\hat{x} = (1-\omega_{\pi})\left[-\hat{\kappa} + (\varepsilon^{*}-1)\,\hat{\tau}\right] \tag{7}$$

$$\bar{\varepsilon}_{\pi}\eta^{*}\hat{x}^{*} = -(1-\omega_{\pi})\,\hat{\kappa} - \left[1+\omega_{\pi}(\varepsilon-1)\right]\hat{\tau} \tag{8}$$

where $\bar{\varepsilon}_{\pi}$ is an aggregate elasticity weighted by profit shares: $\bar{\varepsilon}_{\pi} \equiv \omega_{\pi}\varepsilon + (1-\omega_{\pi})\,\varepsilon^{*}$. Borrowing terminology from the analysis of devaluation, globalization leads to *expenditure-reduction* (more varieties in the world reduces spending on each individual variety) whereas a fall in trade costs leads to *expenditure-switching* (consumption of imported varieties rises at the expense of domestic ones, as shown in Figure 1).

The two shocks clearly have very different effects on home and foreign sales. Moreover, their effects cannot be aggregated, because the elasticities of marginal revenue differ. There are two exceptions to this rule. One is free trade: the elasticities of marginal revenue are now the same on both home and export sales, and the *MR*=*MC* and $\Pi=0$ loci are straight lines with slopes of 1 and $-\frac{1}{\kappa}$ respectively. The other is CES demands: both loci are now straight lines, and sales adjust to changes in trade costs along a smooth locus *ACF*, mirroring their smooth adjustment to changes in the number of countries along a straight-line *MR*=*MC* locus.[7] This explains why, as noted by ACRC, the effects of both shocks on aggregate welfare in the CES case are isomorphic: both can be summarized in terms of their effects on the home-market share in total output. This isomorphism breaks down when demands are not CES.

Given changes in sales, it is easy to deduce the changes in prices, output, and firm

[7] The *MR*=*MC* locus reduces to $x^{*} = \tau^{-\sigma}x$, and the $\Pi=0$ locus to $x + \kappa\tau x^{*} = y/L$. Eliminating τ gives the *ACF* locus: $x^{*} = \left[(y/L - x)/\kappa\right]^{\frac{\sigma}{\sigma-1}} x^{-\frac{1}{\sigma-1}}$.

numbers. Prices are directly linked to sales by the firms' first-order conditions:

$$\hat{p} = \frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon(\varepsilon - 1)}\hat{x}, \;\; \hat{p}^* = \frac{\varepsilon^* + 1 - \varepsilon^*\rho^*}{\varepsilon^*(\varepsilon^* - 1)}\hat{x^*} + \hat{\tau} \tag{9}$$

Both are increasing with sales if and only if demands are subconvex: as consumption rises, the elasticity of demand falls, and so mark-ups increase. Hence both globalization and lower trade costs reduce all prices.[8]

As for adjustment at the intensive and extensive margins, these follow directly from the market-clearing conditions (2) and (3). From (2), the change in firm output is a weighted average of the changes in sales:

$$\hat{y} = \omega_x\hat{x} + (1 - \omega_x)\left(\hat{\kappa} + \hat{\tau} + \hat{x}^*\right) \tag{10}$$

where $\omega_x \equiv \frac{x}{x+\kappa\tau x^*}$ is the home-market share in total output. This in turn inversely determines the number of active firms and so of produced varieties in each country from the full-employment condition (3):

$$\hat{n} = -\psi\hat{y}, \quad \psi = \frac{\overline{\varepsilon}_h - 1}{\overline{\varepsilon}_h} \tag{11}$$

Here, $\psi \equiv \frac{cy}{f+cy}$ is the share of variable costs in total costs, which is an inverse measure of returns to scale. It is increasing in the aggregate elasticity $\overline{\varepsilon}_h$, which is an output-weighted harmonic mean of the home and foreign demand elasticities: $\overline{\varepsilon}_h \equiv [\omega_x\varepsilon^{-1} + (1 - \omega_x)(\varepsilon^*)^{-1}]^{-1}$.

Just as the changes in mark-ups in (9) hinge on subconvexity of demand, so too do those in output and firm numbers in (10) and (11). In the CES case, output is fixed, and exogenous shocks merely reallocate sales: globalization encourages firms to sell to more markets, but less in each; higher trade costs induce a reduction in production for exports which exactly offsets the increase in home sales. If instead demand is subconvex, mark-ups fall as *per capita*

[8] Lower trade costs always reduce import prices: though the mark-up on foreign sales may rise or fall, the direct effect of the change in trade costs always dominates: $\hat{p}^*/\hat{\tau} = 1 - \frac{\varepsilon^*+1-\varepsilon^*\rho^*}{\varepsilon^*(2-\rho^*)}\frac{1+\omega_\pi(\varepsilon-1)}{\bar{\varepsilon}_\pi} > 0$.

sales fall. Hence, with globalization, the negative effect on profits of lower sales per market exceeds the positive effect of a rise in the number of markets; so, to keep overall profits equal to zero, total output must rise. As for higher trade costs, to keep profits constant requires sales to fall by less in declining markets than they rise in expanding markets; so, here too total output must rise, at least in the neighborhood of free trade, where (7), (8) and (10) imply:[9]

$$\hat{y}\Big|_{\tau=1} = (1-\omega)\left(1-\frac{1}{\varepsilon\eta}\right)(\hat{\kappa}+\hat{\tau}) \tag{12}$$

The key expression on the right-hand side is positive if and only if the elasticity of marginal revenue η exceeds the elasticity of *inverse* demand $\frac{1}{\varepsilon}$, which is equivalent to demand being subconvex.[10] This implies that trade liberalization has opposite effects on total output, and so, from (11), on the number of firms per country, depending on whether it involves an increase in κ or a reduction in τ. Paradoxically, lower trade costs in the neighborhood of free trade *reduce* firm output and *increase* the number of domestic firms if demand is subconvex. Total sales always increase as τ falls, but when demand is subconvex they increase by less than the fall in trade costs.[11]

4 Gains from Trade

We measure welfare changes by the change in equivalent income, Y, needed to keep consumers at their initial utility level:

$$\hat{Y} = \left(\frac{\bar{\varepsilon}_z}{\bar{\varepsilon}_u}\frac{1}{\bar{\xi}_u} - 1\right)\hat{N}_Y - \omega_Y\hat{p} - (1-\omega_Y)\hat{p}^* \tag{13}$$

where $\hat{N}_Y$ is a composite change in the number of varieties.[12] Qualitatively, the change in real income is identical to that in the free-trade case in Mrázová and Neary (2013). Consumers

[9]Here, both weights reduce to: $\omega_x = \omega_\pi = \omega = \frac{1}{\kappa+1}$.

[10]Recalling the definition of η: $\eta - \frac{1}{\varepsilon} = \frac{\varepsilon+1-\varepsilon\rho}{\varepsilon(\varepsilon-1)}$.

[11]At $\tau=1$: $\omega_x\hat{x} + (1-\omega_x)\hat{x}^* = -(1-\omega)\frac{1}{\varepsilon\eta}(\hat{\kappa}+\hat{\tau})$.

[12]$\hat{N}_Y \equiv \left(\frac{\bar{\varepsilon}_z}{\bar{\varepsilon}_u\bar{\xi}_u} - 1\right)[\hat{n} + (1-\omega_u)\hat{\kappa}] + (\omega_z-\omega_u)\hat{\kappa}$.

gain at the extensive margin when the number of varieties increases, and by more the lower is the elasticity of utility ξ, that is, the more they care about variety. They also gain at the intensive margin from any falls in prices. Quantitatively, matters are more complicated. The elasticity of utility in (13), $\bar{\xi}_u$, is a weighted average of those for home and imported varieties, where the weights are the shares of each group in utility, themselves weighted by the elasticities of demand. This elasticity is also adjusted in (13) to take account of any difference between the expenditure- and utility-weighted average demand elasticities $\bar{\varepsilon}_z$ and $\bar{\varepsilon}_u$.[13] Finally, the welfare effects of price changes depend on the shares of each good in expenditure, also adjusted to take account of differences between expenditure- and utility-weighted average demand elasticities.[14]

Using (7) to (11) and (13) we can calculate the gains from trade, but the full expression is not insightful. Differences between weights generate income effects which make even qualitative statements problematic. To provide intuition, consider changes in the neighborhood of free trade:

$$\hat{Y}\Big|_{\tau=1} = (1-\omega)\left(\frac{1-\xi}{\xi}\hat{\kappa} - \hat{\tau}\right) + \frac{\psi - \xi}{\psi\xi}\hat{n} \tag{14}$$

This breaks the change in real income into a direct effect, depending on changes in the exogenous variables, and an indirect one, depending on changes in the number of varieties produced at home. Two sufficient conditions for gains from trade liberalization follow immediately. First is when ψ equals ξ, so the initial equilibrium is *efficient*: for given values of the exogenous variables, no change in n can raise welfare. This obtains either if preferences are CES (a familiar result from Dixit and Stiglitz (1977)), or if a global anti-trust policy continually adjusts firm numbers to ensure efficiency. A second sufficient condition for gains is that $\psi - \xi$ and $\hat{n}$ have the same sign. For example, both are positive when utility is subconcave ($\psi > \xi$, so consumers desire more variety) and demand is subconvex (so, from Section 3, trade liberalization increases the number of varieties).

[13] $\bar{\xi}_u \equiv \omega'_u \xi + (1-\omega'_u)\,\xi^*$ and $\bar{\varepsilon}_z \equiv \omega_z \varepsilon + (1-\omega_z)\,\varepsilon^*$, where $\omega'_u \equiv \frac{\omega_u \varepsilon}{\bar{\varepsilon}_u}$, $\omega_u \equiv \frac{u}{u+\kappa u^*}$, and $\omega_z \equiv \frac{px}{px+\kappa p^* x^*}$.
[14] $\omega_Y \equiv \omega_z + \left(\omega_u \frac{\bar{\varepsilon}_z \xi}{\bar{\varepsilon}_u \bar{\xi}_u} - \omega_z\right)\varepsilon$.

5 Calibrating the Gains

Qualitative results such as those in the previous section are valuable for giving intuition, but the complexity of the general expressions when trade costs are initially positive means that we have to resort to calibration. Space constraints preclude our presenting detailed results, so instead we note some general considerations relating to calibrating the gains from trade.

Our results depend on relatively few parameters: various home-market shares, plus the elasticity and convexity of utility and demand for home and imported varieties. While this is not bad news for calibrationists, "relatively few" is more than two, the number that ACRC showed is needed to calibrate the gains from trade in CES-based models. Here, the same parameters arise as in their case – the home-market share in output and the elasticity of demand – but many variants of each are required.

Consider first the home-market shares. These are all equal in two cases. In free trade, they equal the share of each country in world GNP, $\frac{1}{1+\kappa}$; with CES preferences, they equal $\frac{1}{1+\kappa\tau^{1-\sigma}}$. More generally, they differ from each other, as shown in Table 1. For example, $\omega_\pi > \omega_z > \omega_x$ if and only if demands are subconvex: home sales have higher markups, so they contribute more to profits than to sales value, and more to sales value than to production.[15]

Table 1: Ranking of Home-Market Shares

Demand	Utility	
	Subconcave	Superconcave
Subconvex	$\omega_\pi > \omega_z > \omega_x$	$\omega_\pi > \omega_z > \omega_x$
Superconvex	$\omega_x > \omega_z > \omega_\pi$	$\omega_x > \omega_z > \omega_\pi$
All	$\omega_z > \omega_u$	$\omega_u > \omega_z$

Consider next the average elasticities. These too can be ranked, both relative to each other and relative to the elasticities of demand for home and imported varieties. Some

[15] $\omega_z - \omega_x = \omega_z (1 - \omega_x) \frac{\varepsilon^* - \varepsilon}{\varepsilon(\varepsilon^* - 1)}$, which is positive if and only if demand is subconvex. Similar calculations apply to comparisons of other shares.

rankings are independent of subconvexity: it is always true that $\bar{\varepsilon}_x > \bar{\varepsilon}_z > \bar{\varepsilon}_\pi$.[16] By contrast, ranking the average demand elasticities relative to the elasticities for both kinds of varieties hinges on subconvexity: $\varepsilon < \bar{\varepsilon}_i < \varepsilon^*$ for all i if and only if demand is subconvex. In that case, calibration exercises that use elasticities estimated from import data will *overestimate* the true weighted elasticities. Higher elasticities typically reduce the gains from trade, so using import demand elasticities in calibration exercises will typically *underestimate* the gains from trade.

6 Conclusion

In this paper we have used the approach of Mrázová and Neary (2013) to explore the implications of combining two real-world features typically studied in isolation in general-equilibrium trade models: variable demand elasticities, and barriers to international trade. Even in our simple setting, relaxing the assumption of CES preferences in monopolistic competition has surprising implications when trade is restricted. Integrated and segmented markets behave differently, the latter typically exhibiting reciprocal dumping. Globalization and lower trade costs have very different effects: the former reduces spending on all existing varieties, the latter switches spending from home to imported varieties; when demands are subconvex, globalization raises firm output but lower trade costs reduce it. Finally, calibrating gains from trade is harder. Many more parameters are needed, while import demand elasticities are likely to overestimate the true elasticities, and so underestimate the gains from trade.

[16] Sub- or superconvexity affects the differences in both elasticities and shares in the same direction; e.g., $\bar{\varepsilon}_x - \bar{\varepsilon}_z = (\omega_z - \omega_x)(\varepsilon^* - \varepsilon)$, which is positive from footnote 15. By contrast, comparisons between average elasticities weighted by demand and utility parameters require a concordance between subconvexity of demand and subconcavity of utility; e.g., $\bar{\varepsilon}_z - \bar{\varepsilon}_u = (\omega_z - \omega_u)(\varepsilon - \varepsilon^*)$, but $\omega_z - \omega_u = \omega_z (1 - \omega_u)(\xi - \xi^*)/\xi$.

Appendices

A Notes on the Literature

Because of pressures on space, many relevant references have had to be omitted from the text. Further details can be found in Mrázová and Neary (2013).

Introduction: Quantifying the Gains from Trade with CES Preferences: The results of Arkolakis, Costinot and Rodríguez-Clare (2012) have been further considered by Simonovska and Waugh (2011), and Melitz and Redding (2013). Ossa (2012) explores the implications of elasticities that differ exogenously across industries in a CES framework, whereas we focus on how they differ endogenously between home and foreign markets with non-CES demands.

Introduction: Alternatives to the CES: In discussing papers that have gone beyond the CES, we mention in the text only those that look at broad classes of preferences or demands, such as additive separability, and that explore comparative statics in the presence of trade costs. Many important papers have explored the implications of particular alternatives to the CES, such as quadratic (Melitz and Ottaviano (2008)), translog (Novy (2013) and Feenstra and Weinstein (2010)), or Stone-Geary (Simonovska (2010)). The case of general additive preferences first considered by Dixit and Stiglitz (1977) and Krugman (1979) has been reexamined by Neary (2009), Zhelobodko et al. (2012), Dhingra and Morrow (2011), and Mrázová and Neary (2013), but without trade costs. Related results have been independently presented in Russian by Evgeny Zhelobodko and Sergey Kokovin with Maxim Goryunov and Alexey Gorn. Dhingra and Morrow (2011) also explore how different assumptions about demand affect efficiency.

Section I: The terms "superconvexity" and "superconcavity" were first used in this context in Mrázová and Neary (2011) and Mrázová and Neary (2013) respectively. They will not come as a surprise to the careful reader of Dixit and Stiglitz (1977): see for example their equation (45). Our contribution, apart from the labels, is to present a framework which throws light on the implications of a wide range of assumptions about preferences and demand for comparative statics and calibration of general-equilibrium models with monopolistic competition.

Section II: We follow Jones (1965) in using "hats" (circumflexes) to denote proportional changes.

Section III: Corden (1960) discusses the "expenditure-reduction" and "expenditure-switching" effects of devaluation.

Section V: We present some preliminary calibration exercises in Mrázová and Neary (2013).

B The Change in Compensating Income

B.1 The Direct Utility Function

We wish to express the change in utility in expenditure units. The first step is to totally differentiate the utility function (1), ignoring the transformation function F. This yields:

$$\hat{U} = \hat{N}_u + \omega_u \xi \hat{x} + (1 - \omega_u)\, \xi^* \hat{x}^* \tag{15}$$

where $\hat{N}_u \equiv \hat{n} + (1 - \omega_u)\, \hat{\kappa}$ is the extensive margin change in utility.

B.2 Frisch Demands and the Frisch Indirect Utility Function

The consumer's optimization problem yields the Frisch demand functions: $u'(x) = \lambda p$, $u'(x^*) = \lambda p^*$. Totally differentiating these:

$$\hat{x} = -\varepsilon \left(\hat{\lambda} + \hat{p}\right),\ \hat{x}^* = -\varepsilon^* \left(\hat{\lambda} + \hat{p}^*\right) \tag{16}$$

Substituting the Frisch demands into the direct utility function yields what we can call the "Frisch indirect utility function":

$$V^F(N, p, p^*, \lambda) \equiv U[N, x(p, \lambda), x^*(p^*, \lambda)] \tag{17}$$

In differential form, the change in utility becomes:

$$\hat{U} = \hat{N}_u - \Omega\left[\hat{\lambda} + \omega_\Omega \hat{p} + (1 - \omega_\Omega)\hat{p}^*\right] \tag{18}$$

The coefficient of $\hat{\lambda}$ is a utility-share-weighted average of the home and foreign price-elasticities of V^F:

$$\Omega \equiv \omega_u \xi \varepsilon + (1 - \omega_u)\xi^* \varepsilon^* = \bar{\varepsilon}_u \bar{\xi}_u \tag{19}$$

where: $\bar{\varepsilon}_u \equiv \omega_u \varepsilon + (1 - \omega_u)\varepsilon^*$, $\bar{\xi}_u \equiv \omega_u' \xi + (1 - \omega_u')\xi^*$, and $\omega_u' \equiv \frac{\omega_u \varepsilon}{\bar{\varepsilon}_u}$. The coefficients of price changes are shares in this: $\omega_\Omega \equiv \frac{\omega_u \xi \varepsilon}{\Omega}$. Ω itself is the elasticity of the Frisch indirect utility function with respect to λ; i.e., it tells us how much the consumer would gain from a unit reduction in the marginal utility of income. In the CES case it reduces to: $\Omega = \xi\varepsilon = \frac{\sigma-1}{\sigma}\sigma = \sigma - 1$.

B.3 Solve for the Marginal Utility of Income

Totally differentiating the budget constraint, $I = n[px(p, \lambda) + \kappa p^* x^*(p^*, \lambda)]$, yields:

$$\hat{I} = \hat{N}_I + \omega_z(\hat{p} + \hat{x}) + (1 - \omega_z)(\hat{p}^* + \hat{x}^*) \tag{20}$$

where $\hat{N}_I \equiv \hat{n} + (1 - \omega_z)\hat{\kappa}$ is the extensive margin change in the budget constraint. Substitute from the Frisch demands to solve for $\hat{\lambda}$:

$$\bar{\varepsilon}_z \hat{\lambda} = \hat{N}_I - \omega_z(\varepsilon - 1)\hat{p} - (1 - \omega_z)(\varepsilon^* - 1)\hat{p}^* - \hat{I} \qquad \bar{\varepsilon}_z \equiv \omega_z \varepsilon + (1 - \omega_z)\varepsilon^* \tag{21}$$

This is $\lambda\left(N,p,p^{*},I\right)$ in changes.

B.4 Solve for the Change in Real Income

Eliminating λ from the Frisch indirect utility function gives the familiar Marshallian indirect utility function. In levels this is:

$$\begin{aligned}
V\left(n,\kappa,p,p^{*},I/Y\right) &= V^{F}\left[N,p,p^{*},\lambda\left(N,p,p^{*},I/Y\right)\right] && (22)\\
&= U\left[n,\kappa,x\left\{p,\lambda\left(N,p,p^{*},I/Y\right)\right\},x^{*}\left\{p^{*},\lambda\left(N,p,p^{*},I/Y\right)\right\}\right] && (23)
\end{aligned}$$

In terms of changes:

$$\begin{aligned}
\hat{Y} &= \bar{\varepsilon}_{z}\hat{\lambda}-\hat{N}_{I}+\omega_{z}\left(\varepsilon-1\right)\hat{p}+\left(1-\omega_{z}\right)\left(\varepsilon^{*}-1\right)\hat{p}^{*} && (24)\\
&= \frac{\bar{\varepsilon}_{z}}{\bar{\varepsilon}_{u}\bar{\xi}_{u}}\left[\hat{N}_{u}-\omega_{u}\xi\varepsilon\hat{p}-\left(1-\omega_{u}\right)\xi^{*}\varepsilon^{*}\hat{p}^{*}\right]-\hat{N}_{I}+\omega_{z}\left(\varepsilon-1\right)\hat{p}+\left(1-\omega_{z}\right)\left(\varepsilon^{*}-1\right)\hat{p}^{*} && (25)
\end{aligned}$$

which can be written more compactly as follows:

$$\hat{Y}=\hat{N}_{Y}-\omega_{Y}\hat{p}-\omega_{Y}^{*}\hat{p}^{*} \tag{26}$$

where:

$$\hat{N}_{Y}\equiv\frac{\bar{\varepsilon}_{z}}{\bar{\varepsilon}_{u}\bar{\xi}_{u}}\hat{N}_{u}-\hat{N}_{I}=\left(\frac{\bar{\varepsilon}_{z}}{\bar{\varepsilon}_{u}\bar{\xi}_{u}}-1\right)\hat{n}+\left[\frac{\bar{\varepsilon}_{z}}{\bar{\varepsilon}_{u}\bar{\xi}_{u}}\left(\,1-\omega_{u}\right)-\left(1-\omega_{z}\right)\right]\hat{\kappa} \tag{27}$$

$$\omega_{Y}\equiv\frac{\bar{\varepsilon}_{z}}{\bar{\varepsilon}_{u}\bar{\xi}_{u}}\omega_{u}\xi\varepsilon-\omega_{z}\left(\varepsilon-1\right)\quad\omega_{Y}^{*}\equiv\frac{\bar{\varepsilon}_{z}}{\bar{\varepsilon}_{u}\bar{\xi}_{u}}\left(1-\omega_{u}\right)\xi^{*}\varepsilon^{*}-\left(1-\omega_{z}\right)\left(\varepsilon^{*}-1\right) \tag{28}$$

Alternatively, we can write the weights as follows:

$$\omega_{Y}=\omega_{z}+\left(\omega_{u}\frac{\bar{\varepsilon}_{z}\xi}{\bar{\varepsilon}_{u}\bar{\xi}_{u}}-\omega_{z}\right)\varepsilon\quad\omega_{Y}^{*}=\left(1-\omega_{z}\right)+\left[\left(1-\omega_{u}\right)\frac{\bar{\varepsilon}_{z}\xi^{*}}{\bar{\varepsilon}_{u}\bar{\xi}_{u}}-\left(1-\omega_{z}\right)\right]\varepsilon^{*} \tag{29}$$

Note finally that the weights sum to unity, as they must since the consumer is rational:

$$
\begin{aligned}
\omega_Y + \omega_Y^* &= 1 + \omega_u \frac{\bar{\varepsilon}_z \xi}{\bar{\varepsilon}_u \bar{\xi}_u} \varepsilon + (1 - \omega_u) \frac{\bar{\varepsilon}_z \xi^*}{\bar{\varepsilon}_u \bar{\xi}_u} \varepsilon^* - \omega_z \varepsilon - (1 - \omega_z) \varepsilon^* && (30) \\
&= 1 + \left[\omega_u \frac{\xi}{\bar{\varepsilon}_u \bar{\xi}_u} \varepsilon + (1 - \omega_u) \frac{\xi^*}{\bar{\varepsilon}_u \bar{\xi}_u} \varepsilon^* - 1 \right] \bar{\varepsilon}_z && (31) \\
&= 1 + \left[\frac{\omega_u \varepsilon \xi + (1 - \omega_u) \varepsilon^* \xi^*}{\bar{\varepsilon}_u \bar{\xi}_u} - 1 \right] \bar{\varepsilon}_z = 1 && (32)
\end{aligned}
$$

Hence (26) is equation (13) in the text.

C Solving for Welfare Change With Trade Costs

We want to evaluate the change in real income given by (26), where the change in varieties is given by (27). To do this, we need to use equations (9), (10), and (11) in the text for the changes in prices, firm output, and firm numbers respectively. Evaluating the latter at initial free trade gives:

$$
\hat{p} = \frac{\varepsilon + 1 - \varepsilon \rho}{\varepsilon (\varepsilon - 1)} \hat{x} \qquad \hat{p}^* = \frac{\varepsilon^* + 1 - \varepsilon^* \rho^*}{\varepsilon^* (\varepsilon^* - 1)} \hat{x}^* + \hat{\tau} \tag{33}
$$

$$
\hat{y} = [\omega \hat{x} + (1 - \omega) \hat{x}^*] + (1 - \omega) (\hat{\kappa} + \hat{\tau}) \qquad \hat{n} = -\psi \hat{y} \tag{34}
$$

From (33) we can calculate the average change in prices:

$$
\omega \hat{p} + (1 - \omega) \hat{p}^* = \frac{\varepsilon + 1 - \varepsilon \rho}{\varepsilon (\varepsilon - 1)} [\omega \hat{x} + (1 - \omega) \hat{x}^*] + (1 - \omega) \hat{\tau} \tag{35}
$$

From the free-entry condition, equation (6) in the text, we can calculate the change in total sales:

$$
\varepsilon \eta [\omega \hat{x} + (1 - \omega) \hat{x}^*] = - (1 - \omega) (\hat{\kappa} + \hat{\tau}) \tag{36}
$$

Add this to $\hat{y}$ to express the change in total sales as a function of the change in output only:

$$\omega\hat{x} + (1-\omega)\,\hat{x}^* = -\frac{1}{\varepsilon\eta - 1}\hat{y} \tag{37}$$

Hence the average change in prices becomes:

$$\omega\hat{p} + (1-\omega)\,\hat{p}^* = -\frac{1}{\varepsilon}\hat{y} + (1-\omega)\,\hat{\tau} = \frac{1}{\varepsilon\psi}\hat{n} + (1-\omega)\,\hat{\tau} \tag{38}$$

where we make use of the fact that: $\varepsilon\eta - 1 = \varepsilon\frac{2-\rho}{\varepsilon-1} - 1 = \frac{\varepsilon+1-\varepsilon\rho}{\varepsilon-1}$.

We now have all we need to calculate the change in real income. At initial free trade, equation (26) simplifies to:

$$\hat{Y}\Big|_{\tau=1} = \left(\frac{1}{\xi} - 1\right)\hat{n} + \left(\frac{1}{\xi} - 1\right)(1-\omega)\,\hat{\kappa} - [\omega\hat{p} + (1-\omega)\,\hat{p}^*] \tag{39}$$

Substituting for the changes in prices from (38) gives:

$$\hat{Y}\Big|_{\tau=1} = \frac{\psi - \xi}{\xi\psi}\hat{n} + (1-\omega)\left(\frac{1-\xi}{\xi}\hat{\kappa} - \hat{\tau}\right) \tag{40}$$

where we use the definition of ψ to simplify the coefficient of $\hat{n}$: since $\psi = \frac{\varepsilon-1}{\varepsilon}$, and so $\varepsilon = \frac{1}{1-\psi}$, it follows that $\frac{1-\xi}{\xi} - \frac{1}{\varepsilon\psi} = \frac{1-\xi}{\xi} - \frac{1-\psi}{\psi} = \frac{\psi-\xi}{\xi\psi}$. This is equation (14) in the text. If desired, we can substitute from equation (11) for the change in firm numbers to calculate $\hat{Y}$ explicitly:

$$\hat{n} = -\psi\hat{y} = -\psi\,(1-\omega)\left(1 - \frac{1}{\varepsilon\eta}\right)(\hat{\kappa} + \hat{\tau}) \tag{41}$$

This gives:

$$\hat{Y}\Big|_{\tau=1} = (1-\omega)\left[\frac{1}{\xi}\left\{1 - \psi + \frac{\psi-\xi}{\varepsilon\eta}\right\}\hat{\kappa} - \left\{1 + \frac{\psi-\xi}{\xi}\left(1 - \frac{1}{\varepsilon\eta}\right)\right\}\hat{\tau}\right] \tag{42}$$

This is harder to interpret, though it shows clearly that efficiency (the sign of $\psi - \xi$) matters for the welfare effects of both shocks, while super- versus subconvexity (the sign of $\varepsilon\eta - 1$) matters for the welfare effects of trade liberalization. In the CES case this reduces to:

$$\hat{Y}\Big|_{\tau=1,CES} = (1-\omega)\left(\frac{1}{\sigma-1}\hat{\kappa} - \hat{\tau}\right) \tag{43}$$

as in Arkolakis et al. (2012).

It can be checked that the coefficient of $\hat{\kappa}$ in (42) is identical to the expression given in Mrázová and Neary (2013) for the effect of globalization on welfare. (To see this, note that if $k = \kappa + 1$ is the total number of countries, then $(1-\omega)\hat{\kappa} = \hat{k}$; also recall that $\psi = \frac{\varepsilon-1}{\varepsilon}$ and $\eta = \frac{2-\rho}{\varepsilon-1}$.) That paper discusses the implications of the expression in detail, and presents a quantitative analysis of the welfare impact of globalization, as a function of ε and ρ, for two widely-used families of demand functions, due to Bulow and Pfleiderer (1983) and Pollak (1971) respectively. It is straightforward to repeat these exercises for the coefficient of $\hat{\tau}$ in (42) which gives the welfare effects of changes in trade costs in the neighborhood of free trade.

References

Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare. 2012. "New Trade Models, Same Old Gains?" *American Economic Review*, 102(1): 94–130.

Arkolakis, Costas, Arnaud Costinot, David Donaldson, and Andrés Rodríguez-Clare. 2012. "The Elusive Pro-Competitive Effects of Trade." Yale University.

Bertoletti, P., and P. Epifani. 2012. "Monopolistic Competition: CES Redux." University of Bocconi.

Brander, James, and Paul Krugman. 1983. "A 'Reciprocal Dumping' Model of International Trade." *Journal of International Economics*, 15(3-4): 313–321.

Bulow, Jeremy I., and Paul Pfleiderer. 1983. "A Note on the Effect of Cost Changes on Prices." *Journal of Political Economy*, 91(1): pp. 182–185.

Corden, W Max. 1960. "The Geometric Representation of Policies to Attain Internal and External Balance." *Review of Economic Studies*, 28(1): 1–22.

Dhingra, Swati, and John Morrow. 2011. "The Impact of Integration on Productivity and Welfare Distortions Under Monopolistic Competition." Princeton University.

Dixit, Avinash K., and Joseph E. Stiglitz. 1977. "Monopolistic Competition and Optimum Product Diversity." *American Economic Review*, 67(3): 297–308.

Feenstra, Robert C., and David E. Weinstein. 2010. "Globalization, Markups, and the U.S. Price Level." NBER Working Paper No. 15749.

Jones, Ronald W. 1965. "The Structure of Simple General Equilibrium Models." *Journal of Political Economy*, 73(6): pp. 557–572.

Krugman, Paul. 1980. "Scale Economies, Product Differentiation, and the Pattern of Trade." *American Economic Review*, 70(5): 950–959.

Krugman, Paul R. 1979. “Increasing Returns, Monopolistic Competition, and International Trade.” *Journal of International Economics*, 9(4): 469–479.

Melitz, Marc J, and Stephen J Redding. 2013. “Firm Heterogeneity and Aggregate Welfare.” NBER Working Paper No. 18919.

Mrázová, Monika, and J. Peter Neary. 2011. “Selection Effects with Heterogeneous Firms.” Department of Economics, University of Oxford Discussion Paper No. 588.

Mrázová, Monika, and J. Peter Neary. 2013. “Not so Demanding: Preference Structure, Firm Behavior, and Welfare.” Department of Economics, University of Oxford Discussion Paper No. 691.

Neary, J. Peter. 2009. “Putting the ‘New’ into New Trade Theory: Paul Krugman’s Nobel Memorial Prize in Economics.” *Scandinavian Journal of Economics*, 111(2): 217–250.

Novy, Dennis. 2013. “International Trade without CES: Estimating Translog Gravity.” *Journal of International Economics*, 89(2): 271–282.

Ossa, Ralph. 2012. “Why Trade Matters After All.” NBER Working Paper No. 18113.

Pollak, Robert A. 1971. “Additive Utility Functions and Linear Engel Curves.” *Review of Economic Studies*, 38(4): 401–414.

Simonovska, Ina. 2010. “Income Differences and Prices of Tradables.” NBER Working Paper No. 16233.

Simonovska, Ina, and Michael E Waugh. 2011. “The Elasticity of Trade: Estimates and Evidence.” NBER Working Paper No. 16796.

Zhelobodko, Evgeny, Sergey Kokovin, Mathieu Parenti, and Jacques-François Thisse. 2012. “Monopolistic Competition: Beyond the Constant Elasticity of Substitution.” *Econometrica*, 80(6): 2765–2784.