1 Estimation Appendix

The stylized facts documented in Tables 2 and 3 are also confirmed in semi-parametric regressions of campaign contributions on the vote shares. The use of semi-parametric estimation is appropriate given the highly non-linear relationships predicted by the model. We start with the model:

\[ \text{Contributions}_{i,p,t} = f(\text{votes}_{i,p,t}) + \sum_p \sum_t \delta_{p,t} D_{p,t} + \epsilon_{i,p,t} \quad (1) \]

where \( \text{Contributions}_{i,p,t} \) are the contributions made to candidate \( i \) from party \( p \) in election-cycle \( t \), \( \text{votes}_{i,j,t} \) is the two-party vote share (henceforth vote share) of that candidate, \( f() \) is a non-parametric function, \( D_{p,t} \) are party * election-cycle dummies, and \( \epsilon_{i,p,t} \) is the error term. We estimate the parametric terms \( \hat{\delta}_{p,t} \) using the differencing method described in Yatchew (1999). We initially order the candidates in increasing order of their two-party vote share. Under the assumption that \( f(\text{votes}_{i,p,t}) - f(\text{votes}_{i-1,p(i-1),t(i-1)}) \approx 0 \), we can difference (1) in order to eliminate the non-parametric term and estimate:

\[ \text{Contributions}_{i,p,t} - \text{Contributions}_{i-1,p(i-1),t(i-1)} = \sum_p \sum_t \hat{\delta}_{p,t} (D_{p(i),t(i)} - D_{p(i-1),t(i-1)}) + v_{i,p,t} \]

where \( v_{i,p,t} \) is the error term. Once \( \hat{\delta}_{p,t} \) has been estimated, we obtain the non-parametric term:

\[ f(\text{votes}_{i,p,t}) = \text{Contributions}_{i,p,t} - \sum_p \sum_t \hat{\delta}_{p,t} D_{p,t} \]

We estimate \( f() \) using Fan’s (1992) locally weighted regression, with quartic kernel weights. Our estimates at a point with vote-share \( v_1 \) are based on a linear regression that weights an observation with vote-share \( v_2 \) by:

\[ w_{v_1}(v_2) = \begin{cases} \frac{15}{16} \left(1 - \left(\frac{v_1 - v_2}{\lambda}\right)^2\right)^2 & \text{if } |v_1 - v_2| < \lambda \\ 0 & \text{otherwise} \end{cases} \]
where $\lambda$ is the bandwidth parameter. In our estimates, we use a bandwidth of 0.05.\footnote{The results are similar when different bandwidths are used. A smaller bandwidth makes the results noisier in regions where there are fewer observations. A larger bandwidth makes the results smoother, but that smoothness can dampen the rapid changes that occur around a vote share of 50%, which is the main region of interest.}

\section{Proof Appendix}

**Proposition 1** A solution to the contract theory problem gives the equilibrium levels $[\tau_A, \tau_B, M_A, M_B]$ to a solution of the game theory problem, and the levels to a solution of the game theory problem give a solution to the contract theory problem.

**Proof.** Assume a multilateral contracting problem in game theory form. We will show that any solution of the game theory problem is representable as a solution to the contract theory problem and vice versa. Let us define the game theory problem as:

\[
\begin{align*}
\max_{M_A(\tau_A, \tau_B), M_B(\tau_A, \tau_B)} & \quad U_{SIG}[\tau^*_A (M^*_A, M^*_B), \tau^*_B (\tau_A, \tau_B), M_A (\tau^*_A, \tau^*_B), M_B (\tau^*_A, \tau^*_B)] \\
\text{s.t.} \quad & \tau^*_A (M^*_A, M^*_B) = \max_{\tau_A(M^*_A, M^*_B)} U_A[\tau_A, M^*_A (\tau_A, \tau_B), M^*_B (\tau_A, \tau_B)] \\
\text{s.t.} \quad & \tau^*_B (M^*_A, M^*_B) = \max_{\tau_B(M^*_A, M^*_B)} U_B[\tau_B, M^*_A (\tau_A, \tau_B), M^*_B (\tau_A, \tau_B)]
\end{align*}
\]  

(2)

The above problem is a very complicated game theory problem with a solution using optimal control theory. We will show that the compensation levels and levels of support of any solution can be obtained by solving a simpler contract theory problem where the principal (the SIG) chooses the compensation and support levels subject to the constraint that each agent (candidates) gets an outside option which would obtain if the agent didn’t support the SIG at all, received no compensation and her opponent received the maximum contribution $M_{SIG}$. That is, the solution can be obtained from:

\[
\begin{align*}
\max_{\tau_A, \tau_B, M_A, M_B} & \quad U_{SIG}[\tau_A, \tau_B, M_A, M_B] \\
\text{s.t.} \quad & U_A[\tau_A, \tau_B, M_A, M_B] \geq U_A[0, \tau_B, 0, M_{SIG}] \\
\text{s.t.} \quad & U_B[\tau_A, \tau_B, M_A, M_B] \geq U_B[\tau_A, 0, M_{SIG}, 0]
\end{align*}
\]  

(3)
at \([\tau_A^*, 0, M_{SIG}, 0]\) and \([\tau_B^*, M_A^*, M_B^*]\) for candidate B. For instance, it could use a differentiable payment function \(M_k(\tau_k, \tau_{-k}) = W(\tau_k) - R(\tau_k)\), where \(R(\tau_k)\) is a differentiable function over the positive real numbers with the following properties: (1.) \(R(0) = 0\), (2.) \(R(\tau_k^*) = 0\) and (3.) \(W(\tau_k) > R(\tau_k) > 0\) \(\forall \tau_k \neq 0, \tau_k^*\). Thus, \([\tau_A^*, \tau_B^*, M_A^*, M_B^*]\) is in the constraint set for the equilibrium values of the game theory problem: \(G^I \supset C\).

Now we show that the constraint set of the contract theory problem contains the equilibrium values for the constraint set of the game theory problem: \(C \supset G^I\). Suppose that the vector \([\tau_A^*, \tau_B^*, M_A^*, M_B^*]\) contains the equilibrium values of an element of the constraint set to the game theory problem. In any subgame where the interest group chooses a policy \(M_k(\tau_k, \tau_{-k}), M_k \geq 0 \Rightarrow U_k[\tau_k, \tau_{-k}, M_k, M_{-k}] \geq (since the politician can reject the offer and the interest group can condition the payment to the other politician on rejection with a maximum of contributing \(M_{SIG}\)) \(U_k[0, \tau_{-k}, 0, M_{SIG}] \Rightarrow U_k[\tau_k^*, \tau_{-k}, M_k^*(\tau_k^*, \tau_{-k}), M_{-k}] \geq U_k[0, \tau_{-k}, 0, M_{SIG}] \Rightarrow \) the vector of equilibrium-path values \([\tau_A^*, \tau_B^*, M_A^*, M_B^*]\) is feasible in (3): \(C \supset G^I\). Thus \(C = G'\).

Since the constraint sets for the two problems are the same and the objective functions are the same, the set of solutions must be the same. Thus, \([\tau_A^*, \tau_B^*, M_A^*, M_B^*]\) is a solution of (3) if and only if

\[
[\tau_A^* (M_A^*(\tau_A^*, \tau_B^*)), M_B^*(\tau_A^*, \tau_B^*))], \tau_B^* (M_A^*(\tau_A^*, \tau_B^*), M_B^*(\tau_A^*, \tau_B^*))], M_A^*(\tau_A^*, \tau_B^*), M_B^*(\tau_A^*, \tau_B^*)
\]

is a solution of (2).