Definitions of the Marginal Cost of Funds for Capital and Labor

The MCPF for the capital and labor tax are given by

\begin{equation}
\eta_K \equiv \frac{rK}{rK + \tau_K \frac{\partial(rK)}{\partial \tau_K} + \tau_L \frac{\partial(wL)}{\partial \tau_K}}
\end{equation}

and

\begin{equation}
\eta_L \equiv \frac{wL}{wL + \tau_K \frac{\partial(rK)}{\partial \tau_L} + \tau_L \frac{\partial(wL)}{\partial \tau_L}}
\end{equation}

respectively. In each of these expressions, the numerator is the cost to the representative agent of a marginal increase in the tax, while the denominator is the marginal revenue from that tax increase. Thus the ratio is the MCPF.\(^1\)

Derivation of Equation (11)

Taking a total derivative of utility (1) with respect to \(\tau_Z\), substituting in the consumer first-order conditions (3), subtracting total derivatives of the production functions (4) and (5) with respect to \(\tau_Z\), substituting in the firm first-order conditions (6) and (7), substituting in total derivatives of the factor-market clearing conditions (8) and (9) and the government budget

\(^1\) Each expression is a “nonenvironmental” MCPF, one that ignores any welfare effects stemming directly from policy-induced changes in emissions or environmental quality.
constraint (10) with respect to \( \tau_z \), and rearranging yield the following equation for the marginal change in welfare for a marginal change in either the CES or C&T:

\[
\frac{1}{\lambda} dU = \tau_z \frac{dZ}{d\tau_z} + \tau_Y \frac{dY}{d\tau_Z} + \tau_K \frac{d(rk)}{d\tau_Z} + \tau_L \frac{d(wL)}{d\tau_Z}
\]

Expanding the total derivatives allows this to be rewritten as

\[
\frac{1}{\lambda} dU = \frac{dZ}{d\tau_z} + \tau_K \frac{\partial(rk)}{\partial \tau_z} + \tau_L \frac{\partial(wL)}{\partial \tau_z} \left[ \tau_K \frac{\partial(rK)}{\partial \tau_k} + \tau_L \frac{\partial(wL)}{\partial \tau_L} \right]
\]

The next step is to derive expressions for \( d\tau_k/d\tau_z \) and \( d\tau_L/d\tau_z \).

Taking a total derivative of the government budget constraint (10) and substituting in \( dG = 0 \) and \( \tau_Y = 0 \) yields

\[
\left( Z + \tau_z \frac{dZ}{d\tau_z} + \tau_L \frac{\partial(wL)}{\partial \tau_z} + \tau_K \frac{\partial(rK)}{\partial \tau_z} \right) d\tau_z + \left( rK + \tau_L \frac{\partial(wL)}{\partial \tau_K} + \tau_K \frac{\partial(rK)}{\partial \tau_K} \right) d\tau_K
\]

\[
+ \left( wL + \tau_L \frac{\partial(wL)}{\partial \tau_L} + \tau_K \frac{\partial(rK)}{\partial \tau_L} \right) d\tau_L = 0
\]

The share \( \alpha_K \) of marginal revenue from the carbon tax (the first term in (B.5) is used to reduce the capital tax and the share \( \alpha_L \) is used to reduce the labor tax. Together with (B.5), these imply

\[
\frac{d\tau_k}{d\tau_z} = -\alpha_K \frac{Z + \tau_z \frac{dZ}{d\tau_z} + \tau_L \frac{\partial(wL)}{\partial \tau_z} + \tau_K \frac{\partial(rK)}{\partial \tau_z}}{rK + \tau_L \frac{\partial(wL)}{\partial \tau_K} + \tau_K \frac{\partial(rK)}{\partial \tau_K}}
\]

and

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\[
\frac{d\tau_L}{d\tau_Z} = -\alpha_L \frac{Z + \tau_Z \frac{dZ}{d\tau_Z} + \tau_L \frac{\partial(wL)}{\partial\tau_Z} + \tau_K \frac{\partial(rK)}{\partial\tau_L}}{wL + \tau_L \frac{\partial(wL)}{\partial\tau_L} + \tau_K \frac{\partial(rK)}{\partial\tau_L}}
\]

Substituting (B.6) and (B.7) into (B.5) and rearranging, using (12), (B.1), and (B.2), yields

\[
\frac{1}{\lambda} \frac{dU}{d\tau_Z} = \tau_Z \frac{dZ}{d\tau_Z} + \eta_R \left[ \tau_K \frac{\partial(rk)}{\partial\tau_Z} + \tau_L \frac{\partial(wL)}{\partial\tau_Z} \right] + (\eta_R - 1) \left( Z + \tau_Z \frac{dZ}{d\tau_Z} \right)
\]

The last remaining step is to expand the tax-interaction term (the second term on the right-hand side of (B.8)). To do this, recognize that the marginal burden of \( \tau_Z \) is \( Z \), and the marginal burden of \( \tau_K \) is \( rK \). So having \( \gamma_{ZK} \) share of the burden fall on capital implies that portion of the tax-interaction effect is equivalent to an increase in \( \tau_K \) of \( \gamma_{ZK} Z rK \). Following the analogous step for \( \tau_L \) lets us express the tax-interaction term as

\[
\eta_R \left[ \tau_K \frac{\partial(rk)}{\partial\tau_Z} + \tau_L \frac{\partial(wL)}{\partial\tau_Z} \right] = \eta_R Z \left[ \frac{\gamma_{ZK}}{rK} \left( \tau_K \frac{\partial(rk)}{\partial\tau_K} + \tau_L \frac{\partial(wL)}{\partial\tau_K} \right) + \frac{\gamma_{ZL}}{wL} \left( \tau_K \frac{\partial(rk)}{\partial\tau_L} + \tau_L \frac{\partial(wL)}{\partial\tau_L} \right) \right]
\]

Rearranging (B.9), using (B.1) and (B.2), yields

\[
\eta_R \left[ \tau_K \frac{\partial(rk)}{\partial\tau_Z} + \tau_L \frac{\partial(wL)}{\partial\tau_Z} \right] = \eta_R Z \left[ \frac{\gamma_{ZK}}{\eta_K} + \frac{\gamma_{ZL}}{\eta_L} \right]
\]

Substituting (B.10) and (13) into (B.8) and dividing through by \( dZ/d\tau_Z \) yields (11).

**Derivation of Equation (14)**

Expanding the total derivatives from (B.3) for the case of the CES
yields

\[
\frac{1}{\lambda} \frac{dU}{d\tau_Z} = \tau_Z \frac{dZ}{d\tau_Z} + \tau_Y \frac{dY}{d\tau_Z} + \tau_K \frac{\partial (rK)}{\partial \tau_Z} + \tau_L \frac{\partial (wL)}{\partial \tau_Z} \\
+ \frac{d\tau_Y}{d\tau_Z} \left[ \tau_K \frac{\partial (rK)}{\partial \tau_Y} + \tau_L \frac{\partial (wL)}{\partial \tau_Y} \right] + \frac{d\tau_K}{d\tau_Z} \left[ \tau_K \frac{\partial (rK)}{\partial \tau_K} + \tau_L \frac{\partial (wL)}{\partial \tau_K} \right] \\
+ \frac{d\tau_L}{d\tau_Z} \left[ \tau_K \frac{\partial (rK)}{\partial \tau_L} + \tau_L \frac{\partial (wL)}{\partial \tau_L} \right]
\]

(B.11)

The CES is equivalent to a revenue-neutral combination of \( \tau_Y \) and \( \tau_Z \). This implies that

\[
\frac{d\tau_Y}{d\tau_Z} = -\frac{Z + \tau_Z \frac{dZ}{d\tau_Z}}{Y + \tau_Y \frac{dY}{d\tau_Z}} 
\]

(B.12)

Note however, that because the CES still affects the revenue from other taxes, it still implies a change in \( \tau_K \) and \( \tau_L \). Following a similar set of steps to those that gave (B.6) and (B.7) yields

\[
\frac{d\tau_K}{d\tau_Z} = -\alpha_K \left[ \tau_L \frac{\partial (wL)}{\partial \tau_Z} + \tau_K \frac{\partial (rK)}{\partial \tau_Z} + \frac{d\tau_Y}{d\tau_Z} \left[ \tau_L \frac{\partial (wL)}{\partial \tau_Y} + \tau_K \frac{\partial (rK)}{\partial \tau_Y} \right] \right]
\]

(B.13)

and

\[
\frac{d\tau_L}{d\tau_Z} = -\alpha_L \left[ \tau_L \frac{\partial (wL)}{\partial \tau_Z} + \tau_K \frac{\partial (rK)}{\partial \tau_Z} + \frac{d\tau_Y}{d\tau_Z} \left[ \tau_L \frac{\partial (wL)}{\partial \tau_Y} + \tau_K \frac{\partial (rK)}{\partial \tau_Y} \right] \right]
\]

(B.14)

Substituting (B.13) and (B.14) into (B.11) and rearranging, using (12), (B.1), and (B.2), yields
\[
\frac{1}{\lambda} \frac{dU}{d\tau_Z} = \tau_Z \frac{dZ}{d\tau_Z} + \tau_Y \frac{dY}{d\tau_Z}
\]

\[= \tau_Z \frac{dZ}{d\tau_Z} + \tau_Y \frac{dY}{d\tau_Z} + \eta_R \left\{ \tau_k \frac{\partial (rk)}{\partial \tau_Z} + \tau_L \frac{\partial (WL)}{\partial \tau_Z} + \frac{d\tau_Y}{d\tau_Z} \left[ \tau_k \frac{\partial (rk)}{\partial \tau_Y} + \tau_L \frac{\partial (WL)}{\partial \tau_Y} \right] \right\}
\]

which is analogous to (B.8). Then to get from (B.15) to (14), follow the same steps that led from (B.8) to (11).