

A Theory of Outsourcing and Wage Decline
 By Thomas J. Holmes and Julia Thornton Snider
 Web Appendix
 Results for the Cobb-Douglas Case

1 Preliminaries

Assume now that the linespeed functions are Cobb-Douglas, i.e. for type $i \in \{L, K\}$ the linespeed is,

$$y_i = A_i l^{\alpha_i} k^{1-\alpha_i},$$

with $\alpha_L > \alpha_K$. Without loss of generality, we can set

$$A_i = \frac{1}{\alpha_i^{\alpha_i} (1 - \alpha_i)^{1-\alpha_i}},$$

in which case the cost to produce one unit of type i linespeed is $c_i(w, r) = w^{\alpha_i} r^{1-\alpha_i}$. Normalizing $r = 1$, the cost function reduces to

$$c_i = w^{\alpha_i}. \tag{1}$$

Moreover, the cost-minimizing labor demand to attain linespeed y_i is

$$l_i(w_i, y_i) = \alpha_i w_i^{-(1-\alpha_i)} y_i.$$

In the limiting cases of $\alpha_L = 1$ and $\alpha_K = 0$ considered in the text, $c_L = w$, $l_L = y_L$ and $c_K = 1$, $l_K = 0$.

Given a unit linespeed cost of c_i , the problem of a specialist firm of type i facing price p_i is same as the problem considered in the text. Therefore the optimal line speed equals

$$y_i^* = \gamma^{\frac{1}{1-\gamma}} p_i^{\frac{1}{1-\gamma}} c_i^{-\frac{1}{1-\gamma}}. \tag{2}$$

As discussed in the main text, for our result we condition on intermediate goods prices p_L and p_K being such that the linespeed is the same for both type L and type K firms,

$y_L^* = y_K^*$. From (2), this implies

$$\frac{p_L}{c_L} = \frac{p_K}{c_K},$$

or using $p_F = \lambda_L p_L + p_K p_K$ (i.e. final good price equals the sum of the component prices), we have

$$p_i = \frac{c_i}{\lambda_L c_L + \lambda_K c_K} p_F. \quad (3)$$

Note that equality of linespeed across types, $y_L^* = y_K^*$, implies that if the share of type i production line capital $spec_share_i$ allocated to specialist firms in equilibrium is the same for both types, $spec_share_L = spec_share_K$, then when the intermediates are aggregated to make final good among just the specialist firms, the proportions of type L and type K intermediates are exactly right so there are no leftover parts. In particular, if $spec_share_L = spec_share_K = 1$, there is full specialization, and market clearing in the intermediate goods market implies (3) must necessarily hold.

2 The Condition

In this section we derive a condition under which a firm that initially has vertical structure (m_L, m_K) strictly prefers selling off all production lines, $s_L = m_L$, $s_K = m_K$, to going into stage 2 to with vertical structure (m_L, m_K) . As discussed in the text, the market price of production lines is bounded from below by the profit from operating them as specialist firms. So it is sufficient to show that the combined profit of m_L and m_K specialist production lines of type L and type K exceeds the profit of in integrated firm with with vertical structure (m_L, m_K) .

Recall from the paper that maximized profit at price p and unit linespeed cost c equals

$$\pi^* = \omega p^{\frac{1}{1-\gamma}} c^{-\frac{\gamma}{1-\gamma}}, \quad (4)$$

and that this formula works for both integrated and specialist firms with the appropriate choice of p and c .

Let Π_s be the combined profit of running the m_L and m_K lines as speciality firms after dividing through through by ω to get rid of the multiplicative constant. Using (4), this equals

$$\Pi_s = m_L p_L^{\frac{1}{1-\gamma}} c_{L,s}^{-\frac{\gamma}{1-\gamma}} + m_K p_K^{\frac{1}{1-\gamma}} c_{K,s}^{-\frac{\gamma}{1-\gamma}}$$

where $c_{L,s}$ and $c_{K,s}$ are the equilibrium unit linespeed costs determined below. Substituting in for p_i from (3), we can write this as

$$\Pi_s = \frac{p_F^{\frac{1}{1-\gamma}}}{(\lambda_L c_{L,s} + \lambda_K c_{K,s})^{\frac{1}{1-\gamma}}} (m_L c_{L,s} + m_K c_{K,s}).$$

The analogous profit of an integrated firm with m_L and m_K lines equals

$$\begin{aligned} \Pi_v &= p_v^{\frac{1}{1-\gamma}} c_v^{-\frac{\gamma}{1-\gamma}} = (m_L p_L + m_K p_K)^{\frac{1}{1-\gamma}} c_v^{-\frac{\gamma}{1-\gamma}} \\ &\quad \left(m_L p_F \frac{c_{L,s}}{\lambda_L c_{L,s} + \lambda_K c_{K,s}} + m_K p_F \frac{c_{K,s}}{\lambda_L c_{L,s} + \lambda_K c_{K,s}} \right)^{\frac{1}{1-\gamma}} c_v^{-\frac{\gamma}{1-\gamma}} \\ &= \frac{p_F^{\frac{1}{1-\gamma}}}{(\lambda_L c_{L,s} + \lambda_K c_{K,s})^{\frac{1}{1-\gamma}}} (m_L c_{L,s} + m_K c_{K,s})^{\frac{1}{1-\gamma}} c_v^{-\frac{\gamma}{1-\gamma}} \end{aligned}$$

It is immediate that $\Pi_s > \Pi_v$ if and only if

$$m_L c_{L,s} + m_K c_{K,s} < c_v,$$

or substituting in for (1),

$$m_L w_{L,s}^{\alpha_L} + m_K w_{K,s}^{\alpha_K} < m_L w_v^{\alpha_L} + m_K w_v^{\alpha_K}. \quad (5)$$

It is this condition that we evaluate numerically below, after we obtain the formulas for $w_{L,s}$, $w_{K,s}$ and w_v .

3 Equilibrium Wages

A union facing a specialist of type i sets the wage to maximize

$$\begin{aligned} (w_i - w^\circ) l_i &= (w_i - w^\circ) \alpha_i w_i^{-(1-\alpha_i)} y_i^* \\ &= (w_i - w^\circ) \alpha_i w_i^{-(1-\alpha_i)} \gamma^{\frac{1}{1-\gamma}} p_i^{\frac{1}{1-\gamma}} w_i^{-\alpha_i \frac{1}{1-\gamma}} \\ &= \alpha_i \gamma^{\frac{1}{1-\gamma}} p_i^{\frac{1}{1-\gamma}} (w_i - w^\circ) w_i^{-\frac{1-(1-\alpha_i)\gamma}{1-\gamma}} \end{aligned}$$

It is straightforward to derive the solution to this problem,

$$w_{i,s} = \frac{1 - (1 - \alpha_i) \gamma}{\alpha_i \gamma} w^\circ. \quad (6)$$

Next consider the problem of a union of an integrated firm with m_L and m_K production lines. Given a wage of w , the unit linespeed cost of the vertically integrated firm is

$$c_v = m_L w^{\alpha_L} + m_K w^{\alpha_K}, \quad (7)$$

Labor demand for the vertically integrated firm is

$$l_v = m_L \alpha_L w^{-(1-\alpha_L)} y_v + m_K \alpha_K w^{-(1-\alpha_K)} y_v$$

The problem of the union is to pick w to maximize

$$(w - w^\circ) l_v = (w - w^\circ) (m_L \alpha_L w^{-(1-\alpha_L)} + m_K \alpha_K w^{-(1-\alpha_K)}) y_v,$$

where y_v solves (2) with c_v given by (7) and $p_i = p_F$. We can rewrite this problem as

$$\max_w (w - w^\circ) (caw^{-1+a} + dbw^{-1+b}) (cw^a + dw^b)^{-\frac{1}{1-\gamma}} \quad (8)$$

where to ease the notational burden we substitute $a = \alpha_L$, $b = \alpha_K$, $c = m_L$ and $d = m_K$ and drop the multiplicative constant $\gamma^{\frac{1}{1-\gamma}} p_v^{\frac{1}{1-\gamma}}$. The FONC of problem (8), after dividing through by $(cw^a + dw^b)^{-\frac{1}{1-\gamma}-1}$ equals

$$\begin{aligned} & (caw^{-1+a} + dbw^{-1+b}) (cw^a + dw^b) \\ & - (w - w^\circ) (ca(1-a)w^{-2+a} + db(1-b)w^{-2+b}) (cw^a + dw^b) \\ & - (w - w^\circ) \frac{1}{1-\gamma} (caw^{-1+a} + dbw^{-1+b}) (caw^{a-1} + dbw^{b-1}) \\ & = 0 \end{aligned}$$

Multiplying through by w^2 yields

$$\begin{aligned}
& w (caw^a + dbw^b) (cw^a + dw^b) \\
& - (w - w^\circ) (ca(1-a)w^a + db(1-b)w^b) (cw^a + dw^b) \\
& - (w - w^\circ) \frac{1}{1-\gamma} (caw^a + dbw^b)^2 = 0
\end{aligned}$$

We are unable to solve this for an explicit expression for the optimal wage w . However, we can go the other direction and for each w , determine an equation for the w° that results in w for the optimal wage. This equation is

$$w^\circ = w \frac{B(w)}{A(w)}, \quad (9)$$

where

$$\begin{aligned}
A(w) & \equiv (ca(1-a)w^a + db(1-b)w^b) (cw^a + dw^b) \\
& \quad + \frac{1}{1-\gamma} (caw^a + dbw^b)^2. \\
B(w) & \equiv A(w) - (caw^a + dbw^b) (cw^a + dw^b).
\end{aligned}$$

It is straightforward to see that the ratio $B(w)/A(w)$ depends upon only the ratio of c to d , so without loss of generality we can assume $c + d = 1$ (or $m_L + m_K = 1$).

We note that for some parameter values $B(w) \leq 0$. We have examined examples where this happens and in these examples, at $w^\circ = 0$, the optimal wage in the vertically integrated case is strictly bounded above zero. In contrast, with specialization, the optimal wage goes to zero as w° goes to zero, for both types. (See (6)). In such cases, for low w° , the optimal wage w_v under integration is strictly higher than both $w_{L,s}$ and $w_{K,s}$, so condition (5) holds immediately.

4 Numerical Analysis

Restating (5), we need to show

$$m_L w_{L,s}^{\alpha_L} + m_K w_{K,s}^{\alpha_K} < m_L w_v^{\alpha_L} + m_K w_v^{\alpha_K} \quad (10)$$

for

- $1 \geq \alpha_L > \alpha_K \geq 0$
- $m_L > 0$ and $m_K > 0$, for $m_L + m_K = 1$.
- $0 < \gamma < 1$
- $w_{L,s}$, $w_{K,s}$ and w_v as defined above.

We show this numerically as follows. We take $(\alpha_L, \alpha_K, m_L, \gamma)$ and a value of w_v . We first check if $B(w) \leq 0$ and if so we are done, following the discussion above. If $B(w) > 0$, we use formula (9) to back out the implied value of w° . We then use (6) to calculate $w_{L,s}$ and $w_{K,s}$ and then verify that (10) holds. We did this on a grid of $(\alpha_L, \alpha_K, m_L, \gamma)$ on increments of .01. We set $w_v = 2^j$, for integers j from -20 to 20 (a range of w_v from .000001 to a million). We evaluated approximately 20 million different cases. The condition was satisfied everywhere at a tolerance of ten decimal places. That is, either $B(w) \leq 0$, or $B(w) > 0$ and (10). The event $B(w) \leq 0$ tends to happen when γ is small, making labor demand inelastic. The following table shows how the likelihood that $B(w) \leq 0$ varies with γ :

Table 1

Fraction of Cases Where w_v is Bounded Above Zero at $w^\circ = 0$.
 (That is, where $B(w) \leq 0$)

Range of γ	Fraction of cases
(0, .1]	.328
(.1, .2]	.157
(.2, .3]	.100
(.3, .4]	.068
(.4, .5]	.047
(.5, .6]	.031
(.6, .7]	.020
(.7, .8]	.011
(.8, .9]	.005
(.9, 1.0)	.002