Online Appendix for
Intermediate Goods and Weak Links: A Theory of Economic Development

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This appendix contains outlines of the proofs of the propositions reported in the paper.

Proof of Proposition 1. The Symmetric Allocation, Given Capital

Follows directly from the fact that $Q_i = A_i m$, where $m = (K^\alpha H^{1-\alpha})^{1-\sigma} X^\sigma$ is constant across activities. QED.

Proof of Proposition 2. The Competitive Equilibrium, Given Capital

1. The first order conditions from the Variety $i$ Problem are

$$(1 - \tau_i)p_i \alpha (1 - \sigma) \frac{Q_i}{K_i} = r + \delta$$

$$(1 - \tau_i)p_i (1 - \alpha)(1 - \sigma) \frac{Q_i}{H_i} = w$$

$$(1 - \tau_i)p_i \sigma \frac{Q_i}{X_i} = q.$$

Substituting these conditions back into the production function yields an equation that characterizes the price of good $i$:

$$p_i = \frac{mc}{A_i(1 - \tau_i)\epsilon}, \quad (1)$$

where $mc \equiv ((r + \delta)^\alpha w^{1-\alpha})^{1-\sigma} q^\sigma$ is a key piece of the marginal cost and $\epsilon \equiv (\alpha^\alpha (1 - \alpha)^{1-\alpha})^{1-\sigma} (1 - \sigma)^{1-\sigma} \sigma^\sigma$. 

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2. Integrating the Variety $i$ first order conditions above gives

\[(r + \delta)K = \alpha(1 - \sigma) \int (1 - \tau_i) p_i Q_i \, di\]  \hspace{1cm} (2)

\[wH = (1 - \alpha)(1 - \sigma) \int (1 - \tau_i) p_i Q_i \, di\]  \hspace{1cm} (3)

\[qX = \sigma \int (1 - \tau_i) p_i Q_i \, di\]  \hspace{1cm} (4)

where the limits of the integration are understood to be 0 to 1. Note that

\[\int p_i c_i \, di = Y, \quad \int p_i z_i \, di = qX, \quad \int p_i Q_i \, di = Y + qX.\]

Define $\tau \equiv \frac{E}{Y + qX}$ to be distortion revenues as a share of gross output. Then

\[\int (1 - \tau_i) p_i Q_i \, di = (1 - \tau)(Y + qX).\]

Substituting this expression into (2), (3), and (4) gives

\[(r + \delta)K = \alpha(1 - \sigma) \frac{1 - \tau}{1 - \sigma} \frac{1 - \tau}{1 - \sigma(1 - \tau)} Y\]  \hspace{1cm} (5)

\[wH = (1 - \alpha)(1 - \sigma) \frac{1 - \tau}{1 - \sigma} \frac{1 - \tau}{1 - \sigma(1 - \tau)} Y\]  \hspace{1cm} (6)

\[qX = \frac{\sigma(1 - \tau)}{1 - \sigma(1 - \tau)} Y.\]  \hspace{1cm} (7)

These expressions allow us to solve for $mc$ (see the definition under (1)) as

\[mc = \frac{1 - \tau}{1 - \sigma(1 - \tau)} \cdot \epsilon \cdot \frac{Y}{(K^\alpha H^{1-\alpha})^{1-\sigma} X^\sigma}.\]  \hspace{1cm} (8)

3. Next, consider the first-order conditions from the Final Good and Intermediate Good Problems. For each of these problems, take the first order condition and then integrate it back into the firm’s production function. For the final good, this gives

\[\left(\int p_i \frac{\sigma}{\rho} \, di\right)^{-\frac{1+\rho}{\rho}} = 1\]  \hspace{1cm} (9)
and for the intermediate good

\[
\left( \int p_i^{\frac{\theta}{1-\rho}} \, di \right)^{-\frac{\theta}{\rho}} = q
\]

(10)

Now substitute (1) into (9) to get

\[
mc = \epsilon B_\theta
\]

(11)

where

\[
B_\theta \equiv \left( \int_0^1 (A_i(1 - \tau_i))^{\frac{\theta}{\tau - \theta}} di \right)^{\frac{1-\theta}{\tau-\theta}}.
\]

(12)

Combining (1) with this expression, we can solve (10) to find

\[
q = \frac{B_\theta}{B_\rho}
\]

(13)

where \(B_\rho\) is defined analogously to \(B_\theta\). Combining (8), (11), (7), and (13) yields the main result in the proposition.

4. Finally, we need to solve for \(\tau\). From the first-order conditions for the Final Goods Problem and the Intermediate Goods Problem we get

\[
p_i Q_i = p_i c_i + p_i z_i = p_i^{\frac{\theta}{1-\theta}} Y + \left(p_i/q\right)^{\frac{\theta}{1-\rho}} (qX).
\]

Multiplying this expression by \(\tau_i\), integrating, and then using (1), (11), and (13) leads to the solution for \(\tau\):

\[
\tau = (1 - \sigma(1 - \tau))T_\theta + \sigma(1 - \tau)T_\rho
\]

(14)

where \(T_\rho \equiv \int_0^1 \tau_i \left( \frac{A_i(1 - \tau_i)}{B_\rho} \right)^{\frac{\theta}{\tau-\theta}} \, di\). That is, \(T_\rho\) is a weighted average of the sector-specific distortions, where the weights depend on \(\rho\); \(T_\theta\) is defined analogously. QED.
Proof of Proposition 3. The Competitive Equilibrium in Steady State

Straightforward using (5) and the Euler equation from the Household Problem. QED.

Proof of Proposition 4. Symmetric Wedges

Straightforward evaluation given earlier results. QED.

Proof of Proposition 5. Random Productivity and Wedges

1. Define \( B(\eta) \equiv \left( \int (A_i(1 - \tau_i))^{\eta} d\eta \right)^{1/\eta} \). Define \( m_i \equiv \eta(a_i + \omega_i) \). Then \( m_i \sim N(\eta(\mu_m + \mu_a), \eta^2 \nu^2) \), where \( \nu^2 \equiv \nu_a^2 + \nu_w^2 + 2\nu_a \omega \). Therefore,

\[
B(\eta) = (E(e^{m_i}))^{1/\eta} = e^{\mu_a + \mu_w + \frac{1}{2} \nu^2}.
\]

Let \( \bar{B} \equiv [B(\frac{\theta}{1-\theta})]^{1-\sigma} [B(\frac{\rho}{1-\rho})]^\sigma \). Then

\[
\log \bar{B} = \mu_a + \mu_w + \frac{1}{2} \cdot \left( (1 - \sigma) \frac{\theta}{1-\theta} + \sigma \frac{\rho}{1-\rho} \right) \nu^2.
\]

Making the substitution \( 1 - \bar{\tau} = e^{\mu_w + \nu_w^2/2} \) then yields term \( \bar{\theta} \).

2. To get term \( \bar{\theta} \), we need to solve for \( \tau \). From equation (14), one can obtain

\[
1 - \tau = \frac{1 - T_\theta}{1 - \sigma(T_\theta - T_\rho)}.
\]

Evaluating the integrals in \( T_\theta \) and \( T_\rho \) as above gives

\[
T_\theta = 1 - \exp\{\mu_w + \frac{1}{2} \cdot \frac{1 + \theta}{1 - \theta} \nu_w^2 + \frac{\theta}{1 - \theta} \nu_a \omega \}
\]

and \( T_\rho \) is the analogous expression. Straightforward algebra then delivers term \( \bar{\eta} \). QED.