Confucianism and the East Asian Miracle

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Appendix

This appendix contains proofs and derivations of Propositions 1-3 in the main text.

The aggregate dynamics of the economy can be conveniently described by a system of deterministic differential equations involving the variables $C_1$, $C_2$, $N_{11}$, and $N_{12}$. Maximization of utility by the two representative households results in the familiar Euler equations (11) and (21). As in the text, we denote the two constant growth rates of $C_1$ and $C_2$ as $\gamma_{c1}$ and $\gamma_{c2}$ where $\gamma_{c2} > \gamma_{c1}$ because $\rho_2 < \rho_1$. The striking aspect of equations (11) and (21) is that consumption growths do not depend on the number of intermediates, $N_s$.

To study the dynamic behavior of $N_{11}$ and $N_{12}$, we must solve the system of simultaneous differential equations (14) and (20). The equality between $w_1$ and the marginal product of labor implies

\[ w_1 = (1 - \alpha) \cdot (Y_1 / L_1). \]

After some manipulation, the interest rate, given by equations (7) and (8) can be written as

\[ r_1 = (1 / \eta) \cdot (1 - \alpha) \cdot \alpha \cdot (Y_1 / N_1). \]

Hence, aggregate income, $w_1 L_1 + r_1 \eta_1 N_1$, equals $Y_1 - \alpha^2 Y_1$. It follows that the Western household’s budget constraint in equation (14) becomes

\[ \eta \dot{N}_{11} = Y_1 - \alpha^2 Y_1 - C_1 - r_1 \eta N_{12} \]
\[ = (1 - \alpha^2) Y_1 - C_1 - r_1 \eta N_{12} \]
\[ = (1 + \alpha) (1 - \alpha) Y_1 - C_1 - r_1 \eta N_{12}. \]

If we substitute for $Y_1$ and $r_1$ from equations (5) and (8) into (A3) and also use equation (7), we get a formula for the Western household’s budget constraint.
By a similar process of substitution into equation (20), using equations (5), (6), (7), (8) and (18), we can also get a formula for the Eastern household’s budget constraint

\[ \eta N_{11} = \left[ \pi_1 \cdot \left( 1 + \alpha \right) / \alpha \right] \cdot N_{11} \cdot \left( \pi_1 / \alpha \right) \cdot N_{12} - C_1. \]

We now must solve the system of simultaneous differential equations (A4) and (A5). We can substitute \( (\lambda \pi_2 / \pi_1) \cdot (A4) \) into equation (A5) to get

\[ \frac{\dot{N}_{12}}{2} = r_1 N_{12} + a \cdot C_1 - b \cdot C_2, \]

where \( a \) and \( b \) are defined as

\[ a = \nu \lambda / \left[ (\nu \lambda + \eta) \cdot \eta \right], \]
\[ b = 1 / (\nu \lambda + \eta). \]

(A5) is a first-order, linear differential equation in \( N_{12} \). The general solution of this equation is

\[ N_{12}(t) = (\text{constant}) \cdot e^{r_1 t} - \left[ a / (r_1 - \gamma_{C_1}) \right] \cdot C_1(t) + \left[ b / (r_1 - \gamma_{C_2}) \right] \cdot C_2(t). \]

We assume that the production function is sufficiently productive to ensure growth in consumption, but not so productive as to yield unbounded utility

\[ r_1 > \rho > r_1 \cdot (1 - \theta). \]

The first part of this condition guarantees that \( \gamma_{C_2} > 0 \). The second part ensures that the attainable utility is bounded and implies that \( r_1 - \gamma_{C_2} > 0 \). The transversality condition for the dynamic optimization by the Eastern household implies

\[ \lim_{t \to \infty} \left\{ N_{12}(t) \cdot e^{-r_1 t} \right\} = 0. \]

If we substitute for \( N_{12}(t) \) from equation (A7) into the transversality condition in equation (A9), we get
\[
\lim_{t \to \infty} \{ \text{constant} - \left[ a \left( r_1 - \gamma_{c1} \right) \right] \cdot C_1(T_0) \cdot e^{-\left( r_1 - \gamma_{c1} \right) t} \\
+ \left[ b \left( r_1 - \gamma_{c2} \right) \right] \cdot C_2(T_0) \cdot e^{-\left( r_1 - \gamma_{c2} \right) t} \} = 0.
\]

Since \( C_1(T_0) \) and \( C_2(T_0) \) are finite and \( r_1 - \gamma_{c2} > 0 \), \( r_1 - \gamma_{c1} > 0 \), the second and the third terms in the braces converge toward zero. Hence, the transversality condition requires the constant to be zero. The solution of \( N_{12} \) becomes

\[
N_{12}(t) = - \left[ a \left( r_1 - \gamma_{c1} \right) \right] \cdot C_1(t) + \left[ b \left( r_1 - \gamma_{c2} \right) \right] \cdot C_2(t).
\]

If we substitute \( N_{12}(t) \) from equation (A11) into equation (A4), we get

\[
\dot{N}_{11} = \left\{ \frac{\gamma_{c1}}{\alpha} \cdot (1 + \alpha) / \alpha - \gamma_{c1} \right\} \cdot N_{11} - f \cdot C_1 + g \cdot C_2,
\]

where \( f \) and \( g \) are defined as

\[
f = \frac{a \cdot r_1}{\left\{ \alpha \left( r_1 - \gamma_{c1} \right) \right\} + 1 / \eta},
\]

\[
g = \frac{b \cdot r_1}{\left\{ \alpha \left( r_1 - \gamma_{c2} \right) \right\}}.
\]

(A12) is a first-order, linear differential equation in \( N_{11} \). By repeating the same procedure as above, we can solve for \( N_{11} \) as

\[
N_{11}(t) = \left\{ \frac{f}{\left[ r_1 \cdot (1 + \alpha) / \alpha - \gamma_{c1} \right]} \right\} \cdot C_1(t)
\]

\[
- \left\{ \frac{g}{\left[ r_1 (1 + \alpha) / \alpha - \gamma_{c2} \right]} \right\} \cdot C_2(t).
\]

Finally, if we add (A11) and (A13) together, we get

\[
N_1(t) = N_{11}(t) + N_{12}(t)
\]

\[
= m \cdot C_1(t) + n \cdot C_2(t)
\]

where \( m \) and \( n \) are defined as

\[
m = \frac{b}{\left[ r_1 \cdot (1 + \alpha) / \alpha - \gamma_{c1} \right]},
\]

\[
n = \frac{b}{\left[ r_1 \cdot (1 + \alpha) / \alpha - \gamma_{c2} \right]}.
\]

The transversality conditions imply that \( r_1 - \gamma_{c1} > 0 \) and \( r_1 - \gamma_{c2} > 0 \), and hence \( [r_1 \cdot (1 + \alpha) / \alpha] - \gamma_{c1} > 0 \), \( [r_1 \cdot (1 + \alpha) / \alpha] - \gamma_{c2} > 0 \). Since \( a, b, f, g, m, n > 0 \), the dynamic analysis of \( N_{12}, \) \( N_{11} \) and \( N_1 \) as described in the text can be derived from the
following three theorems of real numbers:

Let \( A, B, C \) be real numbers and functions of time, \( t \). \( A(T_0), B(T_0), C(T_0) > 0 \) at \( t = T_0 \).

**Theorem 1:** If \( A = B - C \), and both \( B \) and \( C \) grow at positive constant rates with \( \dot{B}/B > \dot{C}/C \), then \( \dot{A}/A > \dot{B}/B \) initially and \( \dot{A}/A \) declines monotonically toward \( \dot{B}/B \) as \( t \to \infty \).

**Theorem 2:** If \( A = B - C \), and both \( B \) and \( C \) grow at positive constant rates with \( \dot{B}/B < \dot{C}/C \), then \( \dot{A}/A < \dot{B}/B \) initially and \( \dot{A}/A \) declines monotonically. Let \( \dot{A}/A > 0 \) initially, then \( \dot{A}/A \) declines until at the some point, \( t = T_1 > T_0 \), \( \dot{A}/A = 0 \) and \( A \) begins to decline from then on for \( t > T_1 \); \( A \) will eventually decline until at \( t = T_2 > T_1 \), when \( B = C \), \( A = 0 \). Immediately after \( t > T_2 \), \( \dot{A}/A > \dot{C}/C \) and then declines monotonically toward \( \dot{C}/C \) as \( t \to \infty \).

**Theorem 3:** If \( A = B + C \), and both \( B \) and \( C \) grow at positive constant rates with \( \dot{B}/B > \dot{C}/C \), then \( \dot{B}/B > \dot{A}/A > \dot{C}/C \) initially and \( \dot{A}/A \) rise monotonically toward \( \dot{B}/B \) as \( t \to \infty \).