Online Appendix to “Understanding Markups in the Open Economy”

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Deriving the distribution of markups

Malik and Trudel (1982, equation (5.17)) use Mellin transforms to derive the following distribution for the ratio \( u = \frac{c_1}{c_2} \), the ratio of the first order statistic to the second, given that the sample is from the Weibull distribution \( G(c) \) in the main text. This distribution is also reported in Rinne (2009, p.244, equation (5.42c)):

\[
\hat{h}(u) = \frac{r(r-1)\theta u^{\theta-1}}{[(r-1) + u^\theta]^2},
\]

Since we specify the (unrestricted) markup as \( m = \frac{c_2}{c_1} \), we note that \( m \) is a function of \( u \), \( m = \frac{1}{u} \), and apply a straightforward Jacobian transformation:

\[
h(m) = \frac{\left(r(r-1)\theta \left(\frac{1}{m}\right)^{\theta-1}\right)}{\left[(r-1) + \left(\frac{1}{m}\right)^\theta\right]^2} \left(\frac{1}{m^2}\right) = \frac{r(r-1)\theta m^{-(\theta+1)}}{\left[(r-1) + m^{-\theta}\right]^2}.
\]

Similarly, Malik and Trudel (1982) use a Mellin transform to derive the following distribution for \( u = \frac{c_1}{c_2} \) given a Pareto distribution of efficiency draws (a power law distribution of cost draws)

\[
\hat{h}(u) = \theta u^{\theta-1}.
\]

Again, we specify the (unrestricted) markup as \( m = \frac{c_2}{c_1} \), implying that \( m = \frac{1}{u} \), and apply the Jacobian transformation:

\[
h(m) = \frac{\theta m^{1-\theta}}{m^2} = \frac{\theta m^{-(\theta+1)}}{1}
\]
Since the distribution of markups is the same for all goods $j$, we drop the goods index below for simplicity. Taking (natural) logs, the expression decomposes into

\[
\ln \left(1 + \frac{\beta f}{L}\right) + \ln E[M^{-\sigma}] = \ln E[M^{1-\sigma}].
\]

Since the natural log is a concave function, Jensen’s inequality implies $E[\ln M^{1-\sigma}] \leq \ln E[M^{1-\sigma}]$ and $E[\ln M^{-\sigma}] \leq \ln E[M^{-\sigma}]$. The function $M^{-\sigma}$ has a greater degree of convexity than $M^{1-\sigma}$, so $\ln E[M^{-\sigma}] - E[\ln M^{-\sigma}] \geq \ln E[M^{1-\sigma}] - E[\ln M^{1-\sigma}]$.

This last inequality implies that

\[
E[\ln M^{1-\sigma}] \geq \ln \left(1 + \frac{\beta f}{L}\right) + E[\ln M^{-\sigma}],
\]

as taking the log inside the expectation reduces the right-hand side more than the left-hand side. We note that for any constant $k$, $E[\ln M^k] = kE[\ln M]$, yielding

\[
E[\ln M] \geq \ln \left(1 + \frac{\beta f}{L}\right).
\]

**B1. Uniqueness**

Standard properties of expectations tell us that $E[M^{(j)^{1-\sigma}}] > E[M^{(j)^{-\sigma}}]$ for $\infty > \sigma > 1$ and $M^{(j)} \geq 1$. In Proposition 1, we showed that $E[M^{(j)}]$ is decreasing in the number of rivals. Thus, $E[M^{(j)^{1-\sigma}}]$ is increasing in $r$ and $E[M^{(j)^{-\sigma}}]$ is increasing even faster. Thus, $E[M^{(j)^{1-\sigma}}]/E[M^{(j)^{-\sigma}}]$ is greater than 1 and decreasing in $r$ toward 1, meaning that there can only be one $r$ for which the ratio equals the constant $\left(1 + \frac{\beta f}{L}\right)$.

**B2. Upper- and lower-bounds for the number of rivals.**

The distribution of the markup does not yield a closed-form solution for the expected markup $E[M]$ or for the expected log markup, $E[\ln M]$. However, we know from Proposition 1 that the mean markup $E[M]$ is decreasing in $r$. Therefore, we determine an upper- and lower-bound for $r$. Specifically, we can express the minimum number of rivals as a function of the expected log markup and derive a clean closed-form solution for the maximum number of rivals. Let $V = \ln M$. Then the probability density for $V$ is a
simple transformation of $h(m)$,

$$h_V(v) = e^v h(e^v) I_{\mathbb{R}^+}(v) = e^v r(r - 1) \theta (e^v)^{-(\theta + 1)} \left\{ (r - 1) + (e^v)^{-\theta} \right\}^2.$$

The probability that $V \geq \bar{m}$ (or any other positive constant) is then

$$\int_{\ln(\bar{m})}^{\infty} e^v r(r - 1) \theta (e^v)^{-(\theta + 1)} \left\{ (r - 1) + (e^v)^{-\theta} \right\}^2 dv = \frac{r}{1 + (r - 1)e^{\theta \bar{m}}}.$$

Using a generalized version of Chebyshev’s inequality, we can characterize a lower-bound for the number of rivals:

$$\bar{m} \Pr [\ln M \geq \bar{m}] \leq E[\ln M]$$

$$\frac{r \bar{m}}{1 + (r - 1)e^{\theta \bar{m}}} \leq E[\ln M]$$

$$r \geq \frac{E[\ln M](e^{\theta \bar{m}} - 1)}{E[\ln M]e^{\theta \bar{m}} - \bar{m}}.$$

As noted previously, the expected markup and the number of rivals is inversely related, a relationship seen here in the lowerbound for $r$. When $E[M]$ falls, the lowerbound increases, reflecting the fact that more rivals will enter when the expected markup is high (and vice versa). We know from equation B.2 that the expected log gross markup $E[\ln M]$ must be at least as large as the gross log per-period cost of production, $\ln(1 + \beta f L)$, producing an upperbound for $r$. Thus, we know that $r$ lies within the following bounds:

$$\frac{\ln \left( 1 + \frac{\beta f}{L} \right) (e^{\theta \bar{m}} - 1)}{\ln \left( 1 + \frac{\beta f}{L} \right) e^{\theta \bar{m}} - \bar{m}} \geq r \geq \frac{E[\ln M](e^{\theta \bar{m}} - 1)}{E[\ln M]e^{\theta \bar{m}} - \bar{m}}.$$

THE DISTRIBUTION OF MARKUPS UNDER TRADE

First, we consider the case that the best two rivals for a destination market originate in the same country. Let $\psi_{ni}$ be the probability that the two best rivals to supply country $n$ both originate in country $i$. Then, it must be that the two best rivals in a particular industry in country $i$ are more efficient (have lower marginal costs) than any other potential suppliers of the good to country $n$. Let $c_{2i}$ be the second-best cost draw for an industry

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1 See Theorem 5 in Mood, Graybill, and Boes (1974, p.71): For a random variable $X$, a nonnegative function $g(\cdot)$, and a scalar $k > 0$, then $kP[g(X) \geq k] \leq E[g(X)]$. 

in country $i$. Then the probability that it is lower than the best draw for the same industry in any country $\hat{i} \neq i$ is

\[
\psi_{ni} = \int \int_{c_{1i}} g_{2ni}(c_{2i}) \prod_{\hat{i} \neq i} [1 - G_{1ni}(c_{2i})] \, dc_{2i} dc_{1i}
\]

\[
= \pi_{ni} \psi'_{ni},
\]

where we define $\psi'_{ni} = (r_{i-1})T_i(w_{di})^{-\theta} \Phi_n - T_i(w_{di})^{-\theta}$, the probability that the second best producer in $i$ will be the second best supplier to country $n$ in the world market as a whole, given that the best producer of a good in $i$ is also the best supplier to $n$ worldwide.\(^2\) The distribution of markups in this case is a simple application of our autarkic distribution, renaming $r$ in equation (3) as $r_i$.

The second step is to compute the probability that the best supplier to $n$ is from country $i$ and the best rival supplier to supply country $n$ is in country $\hat{i} \neq i$, denoted $\psi_{nii}$. The unconditional probability that this occurs is the probability that the best supplier native to country $\hat{i}$ has some marginal cost $c_{1\hat{i}}$, which lies between the first- and second-best draws in country $i$, $c_{1i}$ and $c_{2i} > c_{1i}$, while the best rivals from all third countries ($v \neq \hat{i}, i$) have a marginal cost that is larger than $c_{1\hat{i}}$. This probability is given by

\[
\psi_{nii} = \int \int_{c_{1i}} \left( \int g_{1ni}(c_{1i}) \prod_{v \neq \hat{i}, i} [1 - G_{1nv}(c_{1v})] \, dc_{1v} \right) g_{1ni,2ni}(c_{1i}, c_{2i}) \, dc_{2i} dc_{1i}
\]

\[
(C.1) \quad = \psi'_{nii} \pi_{ni}(1 - \psi'_{ni}),
\]

where $\psi'_{nii} = \frac{r_i T_i(w_{di})^{-\theta}}{\Phi_n - T_i(w_{di})^{-\theta}}$, the probability that the second best supplier to $n$ is in country $\hat{i}$ conditional on the best supplier being from $i$ while the second best is not. Note that $\sum_{\hat{i} \neq i} \psi'_{nii} = 1$.

Finally, we compute the distribution of markups charged in country $n$ given that the best rival to supply a good is in $i$ and the second-best is in country $\hat{i}$, which we call $h_{nii}(m)$. We use the formula for the distribution of the ratio of two independent random variables, $C_{1ni}(j)$ and $C_{1ni}(j)$, described by Mood, Graybill and Boes (1974, pp.187-88), given that $C_{1ni}(j)$ is greater than $C_{1ni}(j)$ but less than $C_{2ni}(j)$ and the best supplier

\(^2\)Under symmetry, the probability $\psi_{ni}$ collapses to the very intuitive expression $\psi = \frac{1}{N} \times \frac{r-1}{N_r-1}$. 
to $n$ from any third country:\footnote{We show in Appendix C that because the distribution of efficiency levels in each country is independent, terms for third countries cancel out of this conditional distribution. We can integrate the density over the domain $[1, \bar{m}]$, noting the mass point at $\bar{m}$ and see that it forms a well behave cumulative distribution function that integrates to one.}

$$
\hat{h}_{ni}(m) = \int_{0}^{\infty} \int_{mc_{i1}}^{\infty} c_{1i} g_{1ni,2ni}(c_{1i}, c_{2i}) g_{1nu}(mc_{1i}) dc_{2i} dc_{1i} \\
= \frac{\theta T_1(w_i d_{ni})^{-\theta} r_i T_1(w_i d_{ni})^{-\theta} m^{\theta-1}}{[T_1(w_i d_{ni})^{-\theta} + r_i T_1(w_i d_{ni})^{-\theta} (m^{\theta} - 1)]^2}
$$

Then, the full distribution of markups in country $n$ under trade, $\hat{h}_n(m)$, is given by

$$
\hat{h}_n(m) = \sum_{i=1}^{N} \psi_{ni} h_i(m) + \sum_{i=1}^{N} \sum_{i \neq i} \psi_{ni} h_{ni i}.
$$

The difference between $h_{ni}(m)$ and $\hat{h}_{ni}(m)$ arises due to the fact that outcomes $c_{1ni}$ and $c_{1ni}$ come from the best of $r_i$ draws from country $i$’s distribution, which is independent of the realizations of the $r_i$ draws in country $i$ from which the best firm in country $i$ emerges. In contrast, the distribution of two ordered draws in country $i$ is not independent and thus the difference between them can not be constructed from two independent distributions, rather from one joint distribution.

**Gains from trade**

To close the model under autarky or trade, we use a market clearing condition. Let $\lambda_D$ be the share of variable costs in expenditures for each country, given the vector of trade costs $D$ that it faces when exporting. We can use the free entry condition to show that under autarky, $\lambda_D$ equals $\left(1 + \frac{\beta f}{L} \right)^{-1}$. Similarly, under free trade with symmetric countries, $\lambda$ equals $\left(1 + \frac{\beta f}{L(1 + \frac{N}{2})} \right)^{-1}$. Given our unit cost specification, the share of labor in these variable costs is $\beta$. Then, the labor market clearing condition stipulates that payments to labor equal labor’s share in production costs:

$$
\omega_n L_n = \beta \lambda P_n Y_n.
$$

We use the wage as our numeraire, $\omega \equiv 1$. Then, we can compare output under autarky with output under free trade in a world with $N$ symmetric countries:

$$
\frac{Y^t}{Y^a} = \left(1 + \frac{\beta f}{\frac{\beta f}{L(1 + \frac{N}{2})}} \right) \left( \frac{P^a}{P^t} \right).
$$
The first term on the right-hand side is greater than one and reflects the fact that aggregate revenues and average firm profits fall under trade versus autarky because opening to foreign competition squeezes markups. We already know from Propositions 2 and 3a that the autarkic price level is greater than the price level under free trade. To find out how much greater, we must substitute in our formulas for the aggregate price level under autarky and free trade,

\[
\frac{P_a}{P^t} = \left( \frac{1 + (R - 1)\bar{m}^\theta}{1 + (r - 1)\bar{m}^\theta} \right) \left( \frac{\bar{m}^\theta R^{\frac{\sigma - 1}{\sigma}} + (R - 1)(\bar{m}^\theta - 1)\left[ R(r - 1)^{\frac{\sigma - 1}{\sigma}} - (R - 1)r^{\frac{\sigma - 1}{\sigma}} \right]}{(1 + (R - 1)\bar{m}^\theta)\left[ R(r - 1)^{\frac{\sigma - 1}{\sigma}} - (R - 1)r^{\frac{\sigma - 1}{\sigma}} \right]} \right)^{\frac{1}{\sigma}}
\]

Even under symmetry, the level of gains from trade clearly depends upon the number of domestic rivals before liberalization. In Figure 3, we show that they are lower for countries with a high level of contestability \((r^a)\) to begin with, as these countries already have lower average markups than their trading partners.

Trade increases contestability for any given market, which reduces markups, generating a gain from trade that is new to the BEJK framework, though not to models with alternative environments with imperfect competition, such as Bergin and Feenstra (2009), Devereux and Lee (2001), Melitz and Ottaviano (2008), and Rodriguez (2011). However, in our model, an increase in entry due to market scale effects can shift the distribution of efficiency levels among active firms to the right at the same time it changes the shape of the distribution of markups, a combination not captured by any of these papers. Thus, the increase in entry acts both as a technological advance and an increase in the intensity of competition. Geography, in the form of trade frictions, interferes with welfare gains from both Ricardian efficiency effects and contestability.

**D1. Gains from free trade vs. autarky**

To the degree that free trade results in an increased number of rivals for any particular market, it shifts the entire distribution of marginal costs to the left, similar to an innovation in available technology \(T\). A particularly clean case occurs when countries are identical and that trade is costless, so that \(T_i = T, \omega_i = \omega \equiv 1, \) and \(d_{ni} = 1\) for all \(i\). Then we see that the distribution for the lowest unit cost among all potential suppliers to any country \(n\) in equation (11) reduces to the Weibull distribution

\[
G_{1n}(c_1) = 1 - e^{-rNTc_1^\theta},
\]

which is observationally equivalent to a world with \(R = rN\) rivals who all draw from an underlying distribution that takes the same form as the distribution of cost parameters for any individual country, \(G(c) = 1 - e^{-Tc^\theta}\). The distribution of markups in this special

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4If we do not normalize the wage \(\omega\) to equal 1, this expression is the ratio of the real wage under trade, relative to the real wage under autarky.

5The distribution of first order statistics for samples drawn from a Weibull distribution is also Weibull.
case takes the form
\[ h(m) = \frac{R(R - 1)\theta m^{-(\theta + 1)}}{[R - 1 + m^{-\theta}]^2}. \]

The implication is clear: trade has the same effect on the distribution of markups as increasing contestability and therefore reduces the number of firms charging the unconstrained markup and, all else equal, the aggregate price level, which takes the same form as under autarky, but with the total number of rivals for each market, \( R \). Defining \( r^a \) as the number of rivals under autarky, we will show below that \( R > r^a \). Proposition 3 follows directly from this increase in contestability under trade.

**Proposition 3:** In a world with symmetric countries, free trade (a) increases the total number of rivals competing to supply a destination market, (b) reduces the aggregate price level, and (c) reduces the expected markup, as well as the probability that firms will charge the unconstrained markup.

**PROOF:**

Part a) To show that the total number of rivals under trade equals a number \( R > r^a \), we use the open economy version of the free entry condition and a labor market clearing condition that takes the same form as the closed economy version in equation (7) for each country. If all countries are identical and trade is costless, we have

\[ \text{(D.1)} \quad E_t \left[ (1 + \frac{N - 1}{N}) \left( P_{t+s}(j)Y_{t+s}(j) - C_1(j)Y_{t+s}(j) \right) | R \right] \geq f \\
E_t \left[ (1 + \frac{N - 1}{N}) \left( P_{t+s}(j)Y_{t+s}(j) - C_1(j)Y_{t+s}(j) \right) | R + 1 \right] < f \]

The condition simplifies to

\[ \text{(D.2)} \quad \frac{E[M^{1-\sigma} | r]}{E[M^{-\sigma} | r]} \geq 1 + \frac{\beta f}{(1 + \frac{(N-1)}{N})L} \\
\frac{E[M^{1-\sigma} | r]}{E[M^{-\sigma} | r] + 1} < 1 + \frac{\beta f}{(1 + \frac{(N-1)}{N})L}. \]

Since the left-hand side is decreasing in \( r \) and \( \frac{\beta f}{1 + \frac{(N-1)}{N}} < 1 + \frac{\beta f}{L} \), it is clear that the possibility of exporting strictly increases entry. Thus, leaping from autarky to free trade increases the number of rivals competing to produce any good \( (R = Nr > r^a) \), in addition to reducing prices by reallocating production to more efficient producers.\(^6\)

Part b) \( E[C_2(j)^{1-\sigma}] \) under free trade and symmetry takes the exact form of its counterpart under autarky, only substituting \( R > r^a \) for the number of rivals, making \( E[C_2(j)^{1-\sigma}] \) greater than its counterpart under autarky. From the discussion in Lemma 1, we know that \( E[C_{1n}(j)^{1-\sigma}] \) must also be greater than its counterpart under autarky. Therefore,\(^6\)

\(^6\)We assume the fixed cost of exporting is zero for simplicity, but one can also derive a reasonable restriction on the size of a fixed cost of exporting that preserves this result.
$(P)^{1-\sigma}$ must be greater than its counterpart under autarky $(P^a)^{1-\sigma}$, revealing that the aggregate price level falls under trade: $P < P^a$.

Part c) It follows directly from the derivative in Proposition 1 and the fact that $R > r^a$ that the average markup falls under trade. Similarly, the likelihood of charging the unconstrained markup falls when opening to trade.

The results from Proposition 3 echo those of Bergin and Feenstra (2009) and Melitz and Ottaviano (2008), but now within the homothetic preference structure of Bernard, Eaton, Jensen, and Kortum (2003). Atkeson and Burstein (2007 and 2008) show the results in Parts (b) and (c) numerically, while de Blas and Russ (2011) demonstrate that having a large number of rivals under autarky reduces the impact of trade liberalization on markups. Note also that increasing the number of trading partners has a similar effect to increasing the number of rivals in any trading partner, seen in numerical solutions calculated by Garetto (2012). As in classic studies of trade and endogenous market structure, geographic frictions here increase the market power of domestic producers, dampening the effect of foreign industrial structure on domestic markups and prices.

D2. Proof of Lemma 1: The aggregate price level falls under trade

**PROOF:**

A country will never import a good with a higher price than it pays under autarky and the second-best competitor will never be less efficient than the second-best competitor under autarky. To quantify the impact on the aggregate price level, we can compute

$$P_n^{1-\sigma} = E[P_n(j)^{1-\sigma}] = \Pr [M_n(j) > \bar{m}] \bar{m}^{1-\sigma}E[C_{1n}(j)^{1-\sigma}] + \Pr [M_n(j) \leq \bar{m}] E[C_{2n}(j)^{1-\sigma}]$$

and note that the relevant moment of the supplier’s expected marginal cost is given by

$$E[C_{1n}(j)^{1-\sigma}] = \int_0^\infty c_1^{1-\sigma} g_{1n}(c_1) dc_1 = (\Phi_n)^{\frac{\sigma}{\sigma-1}} \Gamma \left( \frac{\theta - (\sigma - 1)}{\theta} \right).$$

Since $\sigma > 1$, this moment is strictly greater than its counterpart under autarky. We also can compute the same $1 - \sigma^{th}$ moment for the marginal cost of the second-best rival by using the probability that it is in the same source country $i$ as the actual supplier, $\psi_{ni}$:

$$E[(C_{2n})^{1-\sigma}] = \sum_{i=1}^N \psi_{ni} E[(C_{2ni})^{1-\sigma}] + \sum_{i=1}^N \sum_{i \neq i} N \psi_{ni} E[(C_{1ni})^{1-\sigma}],$$

which we know is at least as great as its counterpart under autarky because the second-best rival producer of a good $j$ in the entire world (including the home country) by definition could not have a marginal cost any higher than the second-best rival under autarky.
**Competition and Price Volatility**

Suppose that there is a shock to marginal cost $\varepsilon$ such that a shock $\varepsilon > 1$ reduces efficiency and increases the marginal cost of an industry’s low-cost supplier in country $n$. Using equation (15), the probability that pass-through occurs under trade is now

\[
\Pr[M_{nii}(j) = \frac{C_{2n}(j)}{\varepsilon C_{1n}(j)} \geq \bar{m}] = \frac{T_i(w_i d_{ni})^{-\theta}}{T_i(w_i d_{ni})^{-\theta} + r_i T_i(w_i d_{ni})^{-\theta} \left[ (\varepsilon \bar{m})^\theta - 1 \right]}. \tag{E.1}
\]

By Proposition 3, it follows that pass-through is increasing in the technological advantage of the exporting country, $\frac{r_i}{r}$, and decreasing in $d_{ni}$, $\frac{w_i}{w_i}$, and $r_i$—the same Ricardian factors that govern the average markup for exporters shipping to destination country $n$.\(^7\)

**Price Adjustment in the Closed Economy**

In the simple, closed-economy framework, a lower number of rivals leads to more frequent price changes in response to idiosyncratic shocks to marginal costs. The reason is clear from Figure 1. When $r$ is low, more firms charge the unconstrained CES markup—their prices are not tightly bounded by the marginal costs of their next-best rival, so they are better able to pass on idiosyncratic increases in marginal cost to their customers. The fraction of firms that set their price equal to the marginal cost of the next-best rival are unable to do this. Since firms will not change prices in response to an idiosyncratic shock unless they charge the unconstrained markup, Figure 1 suggests that at least half of firms will never be able to adjust their prices upward ever, unless they experience a shock common to all rivals and which affects all rivals at exactly the same time. We apply a lognormally distributed idiosyncratic shock with the log of the shock being distributed $N(0,10)$, so that the standard deviation of the shock is 10%.\(^8\) After 1000 simulations, using the same parameters as in Figure 1, we compute that, all else equal, 73.3% of firms adjust their price in response to a shock when $r = 2$, while the figure falls to 64.8% when $r = 20$. This is consistent with results from Nakamura and Steinsson (2010), who find that no price changes are observed for 40% of products over the period 1982-2007, as well as Gopinath and Rigobon (2008) and Gopinath, Itskhoki, and Rigobon (2010), who report static prices for approximately 30% of their sample. The following corollary to Proposition 1 formalizes this result.

**COROLLARY 1:** The probability that shocks to marginal cost are reflected in a firm’s price is falling in the number of rivals $r$, as is price volatility.

\(^7\)Although several studies have shown that pass-through depends on the choice of currency invoicing, Goldberg and Tille (2009) demonstrate that this currency invoicing choice also depends on the degree of competition in the destination market, so we view our market structure approach as quite relevant.

\(^8\)This is in line with calibration by Feenstra, Obstfeld, and Russ (2011) for micro-level shocks, drawing on empirical estimates by Basu, Fernald, and Kimball (2006) and Foster, Haltiwanger, and Syverson (2008).
PROOF:

For some random i.i.d. shock $\varepsilon$ to firm-specific marginal cost with probability density $\eta(\varepsilon)$ over the domain $(0, \bar{\varepsilon})$, we can compute the fraction of firms that will raise prices in response to an idiosyncratic increase in marginal costs. Suppose a shock occurs such that $\varepsilon > 1$, increasing the marginal cost for a particular active firm but not its rivals in the industry.

First, we note that only firms charging the unconstrained CES markup would be able to increase their prices, since firms setting prices bounded by the marginal cost of their next-best rival can not. Then, the probability that a firm will pass an idiosyncratic increase in marginal cost fully to buyers by raising its price is equal to the probability that the current price ($\bar{m}$ times marginal cost) times the shock does not exceed the marginal cost of the next best rival,

$$\Pr[\bar{m}\varepsilon C_1(j) \leq C_2(j)] = \Pr \left[ \frac{C_2(j)}{C_1(j)} \geq \bar{m}\varepsilon \right] = \Pr[M(j) \geq \bar{m}\varepsilon].$$

Since the distribution of markups is independent of $\varepsilon$, we can compute this probability as

$$\Pr[M(j) \geq \bar{m}\varepsilon] = \int_{-\infty}^{\infty} \int_{-\infty}^{\bar{m}} h(\varepsilon m)\eta(\varepsilon)d\varepsilon dm.$$

(F.1)

It follows from Corollary 1 that regardless of the probability distribution for $\varepsilon$, as long as the marginal cost shock is independent of the markup, the probability of full pass-through under autarky is decreasing in the number of rivals.\(^9\) Multiplying $\varepsilon$ above by some positive constant less than one, we see that the result is general to any degree of pass-through, not just full pass-through.\(^10\) The intuition also applies for a downward cost shock, which is omitted here for the sake of brevity. In this case, all firms charging the unconstrained markup would have to lower their prices, otherwise their markup would rise above $\bar{m}$, implying marginal revenues less than marginal costs. Further, some portion of firms charging a price equal to $C_2(j)$ would also lower prices, namely those for whom leaving the price at $C_2(j)$ resulted in a markup greater than $\bar{m}$. Thus, downward adjustment is most likely when firms are more likely to have relatively inefficient rivals, which is the case when $r$ is low. Note that having less complete and less frequent price

\(^9\)That is, given the calculus used to prove Proposition 1, equation (F.1) implies that the probability of the markup being high enough to permit adjustment to positive price shocks is decreasing in the number of rivals $r$.

\(^10\)Our assumption that firms pay a fixed cost when they become active prevents the lowest-cost producer from having to adjust prices in response to temporary idiosyncratic shocks hitting its next-best rivals. The rivals will not find it profitable to try to undercut an existing producer unless they experience a transitory shock large enough to cover the entire fixed cost. We assume that the variance of costs is small enough that the likelihood of such a large shock is negligible.
adjustment in response to idiosyncratic shocks implies lower price volatility.

Limited pass-through of the marginal cost shock does not depend on the elasticity of substitution being infinite. In Figure 3, we vary the elasticity of substitution between varieties within an industry \( j \) from 4 (just greater than our calibrated value for \( \sigma \)) to 10,000 and show that pass-through by the low-cost producer is limited even at this low elasticity and decreases continuously as the elasticity of substitution increases, in response to a large shock (\( \varepsilon = 1 \)) to marginal cost, given two rivals in each industry. The degree of pass-through for idiosyncratic shocks in the closed economy varies the size of the shock and the number of rivals, but always displays this continuous decreasing behavior.\(^{11}\)

\(^{11}\)We are very grateful to Ariel Burstein for alerting us to the fact that this continuity can be shown numerically and Konstantin Kucheryavyy for sharing insights and code for a numerical illustration of markup behavior within his proof in Kucheryavyy (2012).
Figure F1. Passthrough declines continuously in the elasticity of substitution between rival’s goods within an industry $j$. 