A.1 Definition of Equilibrium

We consider a stationary competitive equilibrium for a small open economy. For the purposes of defining an equilibrium in a compact way, with a slight abuse of notation, we suppress the individual states into vectors \( \{j, x^i\} \), \( i \in \{W, R, P, G\} \), such that \( x^W \equiv (a, \bar{e}, d, z) \) for workers, \( x^R \equiv (a, \bar{e}, h) \) for retirees not in a nursing home, \( x^P \equiv (a, \bar{e}) \) for private nursing home residents and \( x^G \equiv (\bar{e}) \) for public nursing home residents. Accordingly, we redefine decision rules, value functions, income taxes and means-tested transfers to be functions of the individual states \( (j, x^i) \). Define the individual state spaces:

\[
X^W \subset [0, \infty) \times (0, \infty) \times [0, \infty) \times (-\infty, \infty), \\
X^R \subset [0, \infty) \times (0, \infty) \times (-\infty, \infty), \\
X^P \subset [0, \infty) \times (0, \infty), \\
X^G \subset [0, \infty),
\]

and denote by \( \Xi(X) \) the Borel \( \sigma \)-algebra on \( X \in \{X^W, X^R, X^P, X^G\} \). Let \( \Psi^i_j(X) \) be a probability measure of individuals with state \( x^i \in X \) in cohort \( j \). Note that these agents constitute \( \eta_j \Psi^i_j(X) \) fraction of the total population.

DEFINITION. Given a fiscal policy \( \{\tau_e(e), S(\bar{e}), G, p^G, \xi^w, \xi^m, \xi^n\} \) and a real interest rate \( r \), a steady-state competitive equilibrium consists of policies and associated value functions \( \{c^i(j, x), a^h(j, x), V^i(j, x)\}_{i \in \{W, R, P, G\}} \), nursing home policy \( g(j, x) \), bequest transfers \( \chi(\bar{e}) \), taxes and transfers \( \{\tau^i(x), T^i(x)\}_{i \in \{W, R, P\}} \), wage rate \( w \), cost of private nursing home care, \( M^n \), capital stocks \( \{\bar{K}, K\} \), and an invariant distribution \( \{\Psi^i_j\}_{j \in \{1, \ldots, J\}} \) such that

1. Given prices, tax and transfer schedules, the policy functions \( c^i(j, x) \), \( a^h(j, x) \) and \( g(j, x) \) solve the dynamic programming problems of the individuals.

2. Given prices, firms choose their inputs so as to maximize profits: \( w = F_L(K, L) \) and \( r = F_K(K, L) - \delta \).

3. Aggregate private wealth is given by \( \sum_{j,i} \eta_j \int_{X^i} a^h(j, x) d\Psi^i_j = (1 + n)\bar{K} \).

4. Markets clear:
(a) Goods: Equation (15) in the paper holds where 
\[
C = \sum_{j,i} \eta_j \int_{X^i} c^i(j, x) d\Psi^i_j \quad \text{and} \quad \tilde{M} = \sum_{j=R}^J \eta_j \left\{ \int_{X^R} M(j, h) d\Psi^R_j + \sum_{i \in \{P, G\}} \int_{X^i} \Psi^i_j \right\}.
\]

(b) Labor: \( \sum_j \eta_j \int_{X^W} \Omega(j, d, z) d\Psi^W_j = L. \)

5. Distributions of agents are consistent with individual behavior for each age \( j \) and agent type \( k \in \{W, R, P\} \) and type \( G \):

\[
\Psi^k_{j+1}(X_0) = \int_{X_0} \left\{ \int_{X^i} Q^k_{j}(x, x') d\Psi^i_j \right\} dx',
\]

\[
\Psi^G_{j+1} = s^n_j \left( \int_{X^G} d\Psi^G_j + \int_{X^P} g(j, x) d\Psi^P_j \right),
\]

for all \( X_0 \in X^k \), where \( Q^k_{j}(x, x') \) is the probability of an age-\( j \) agent going from current state \( x \) to future state \( x' \).

6. The government’s budget is balanced: \( IT + ST = MT + PP + G \). Income taxes:

\[
IT = \sum_{j,i} \eta_j \int_{X^i} \tau^y_i(y^i(j, x)) d\Psi^i_j + \frac{1}{1 + n} \sum_{j=R}^{J-1} \eta_j \left\{ \int_{X^R} (1 - s_j) \tau^R_y(y^R(j, x)) d\Psi^R_j \right\} + \int_{X^i} (1 - s^n_j) \tau^R_y(y^i(j, x)) d\Psi^i_j \right\},
\]

where \( y^i(j, x) \) is taxable income. Social security taxes: \( ST = \sum_{j=1}^R \eta_j \int_{X^W} \tau_e(e) d\Psi^W_j \).

Total means-tested transfer payments:

\[
MT = \sum_{j,i} \eta_j \int_{X^i} T^i(j, x) d\Psi^i_j + \sum_{j=R+1}^J \eta_j \int_{X^G} \left[ p^G(M^n + \xi^n) - S(\bar{e}) + \tau^R_g(S(\bar{e})) \right] d\Psi^G_j
\]

\[
- \sum_{j=R+1}^J \eta_j \int_{X^P} g(j, x) \left[ a(1 + r) + S(\bar{e}) - \tau^R_j(y(j, x)) \right] d\Psi^P_j,
\]

and pension payments: \( PP = \sum_{j,i \in \{R, P, G\}} \eta_j \int_{X^i} S(\bar{e}) d\Psi^i_j \).

7. Total transfers to new retirees, \( B = \eta_R \int_{X^R} \chi(\bar{e}) d\Psi^R_{R+1} \), are equal to total accidental
bequests or,

$$B = \frac{1}{1+n} \sum_{j=R+1}^{J} \eta_{j-1} \left\{ \int_{X_R} (1-s_j) \left[ (1+r) a'(j-1, x) - \tau^R_y(y(j-1, x)) \right] d\Psi_{j-1}^{R} + \int_{X_P} (1-s_j^n) \left[ (1+r) a'(j-1, x) - \tau^R_y(y(j-1, x)) \right] d\Psi_{j-1}^{P} \right\}. $$

8. The nursing home sets $M^n$ to balance its budget:

$$(1-p^G) (M^n + \xi^n) \sum_{j=R+1}^{J} \eta_j \int_{X_G} d\Psi_{j}^{G} = (M^n - M^n) \sum_{j=R+1}^{J} \eta_j \int_{X_P} d\Psi_{j}^{P}.$$ 

A.2 Model Version With Long-term Care Insurance

In this version of the model agents can purchase a long-term care insurance (LTCI) policy. Under the policy an agent pays a fixed premium each period he is not in a nursing home and receives a fixed benefit each period he is in one.\(^1\) Let $b$ denote the benefit amount received each period and $p(b)$ the insurance premium. We assume that $b \in \{0, \hat{b}\}$, where $b = 0$ means an individual is not a policy-holder ($p(0) = 0$) and $\hat{b}$ is fixed exogenously. For simplicity, we assume that individuals make LTCI decisions at age 65 before retiring and observing their initial health shock and that they cannot modify their insurance coverage later in life.

The problem of an age $j < R$ year-old worker is the same as in the baseline model. The problem of an age $R$ individual is

$$V^W(R, a, \bar{e}, d, z) = \max_{c, a \geq 0, b \in \{0, \hat{b}\}} \left\{ U(c) + \beta s_R(1 - \bar{\theta} R) E[V^R(R + 1, a', \bar{e}, h', b)] + \beta s_R \bar{\theta} R \max [V^P(R + 1, a', \bar{e}, b), V^G(R + 1, b)] \right\} $$

subject to

$$c + a' + p(b) = a + y - \tau^W(\bar{y}) + T^W [y - \tau^W(\bar{y}), a],$$

$$y = ra + e - \tau_e(e),$$

$$\bar{y} = \max \{0, y - \max [0, p(b) - \kappa y]\},$$

$$e = w \Omega(j, d, z),$$

$$\bar{e}' = (e + j \hat{e})/(j + 1),$$

$$T^W [y - \tau^W(\bar{y}), a] = \max \{0, \omega - [y - \tau^W(\bar{y}) + a]\},$$

\(^1\)This is the same LTCI contract used by Brown and Finkelstein (2008).
and the initial distribution of $h'$.

Retired individuals solve similar problems to those in the baseline model. Those not in a nursing home solve (12) in the paper with the extra state variable $b$ and subject to

$$
c + a' + M(j, h) + p(b) = a + \tilde{y} + T^R[\tilde{y}, a, M(j, h)],
$$

$$
\tilde{y} = ra + S(\bar{e}) - \tau^R [ra, S(\bar{e}), M(j, h) + p(b)] + 1\{j = R + 1\} \chi(\bar{e}),
$$

$$
T^R[\tilde{y}, a, M(j, h)] = \max \{0, \xi^m + M(j, h) - (\tilde{y} + a)\}.
$$

and the initial distribution and law of motion for $h$. Note that while LTCI premiums are tax-deductible expenses, they are not covered by the means-tested insurance program. Private nursing home residents solve (13) with the extra state variable $b$ and budget constraint

$$
c^n + a' + M^n = (1 + r)a + S(\bar{e}) + b - \tau^R [ra, S(\bar{e}), \xi^n + M^n - b] + 1\{j = R + 1\} \chi(\bar{e}),
$$

A.2.1 LTCI Firms

LTCI companies behave competitively. However, to account for fact that LTCI policy prices are marked up substantially above actuarially fair levels we assume that a portion of LTCI premiums is used to pay administrative costs. Thus both the benefit amount $\hat{b}$ and the premium $p(\hat{b})$ are set exogenously. We set $\hat{b}$ such that the daily benefit amount received is $102. This is the average daily benefit amount for nursing home care over the period 1990 to 2005 according to a study by Life Plans, Inc.\footnote{Source: Life Plans, Inc. (2007). “Who buys long-term care insurance?” Prepared for: America’s Health Insurance Plans.} The premium is set such that 7.6 percent of agents purchase the contract. This is the percentage of individuals 65 and over in the HRS who own a LTCI policy for at least half of the waves in which they are observed. The resulting value for $p(\hat{b})$ corresponds to an annual premium amount of $3,854 which is higher than the annual premium Brown and Finkelstein (2008) use of $1,816. It may seem puzzling that a higher premium than what is observed in the data is required to obtain the low LTCI take-up rates. However, we view this higher premium as capturing a number of problems with the LTCI market such as the high declination rates of LTCI applications and the risk of premium hikes for existing policyholders. See footnote 11 of the paper for more information.

The administrative cost $\phi$ is then set such that the following budget constraint holds:

$$
\sum_j \eta_j \int_X I[b(j, x) = \hat{b}] (I[l(j, x) = 0]p(b) - I[l(j, x) > 0]b)d\Psi_j = \phi.
$$
Table A.1: Percentage shares of wealth held due to the presence of health expenses and their risks in the economy with LTCI.

<table>
<thead>
<tr>
<th></th>
<th>Nursing</th>
<th>Nursing</th>
<th>Medical</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Savings for OOP Expenses</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aggregate</td>
<td>6.25</td>
<td>13.02</td>
<td></td>
</tr>
<tr>
<td><strong>Savings for Cross-Sectional OOP Expense Risk</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aggregate</td>
<td>2.95</td>
<td>3.63</td>
<td></td>
</tr>
</tbody>
</table>

The numbers are the percentage decline in the benchmark wealth levels when a particular expense or risk is removed. The first column shows the decline in wealth when nursing home expenses or risk are removed. The second column shows the decline when both nursing home and medical expenses or risks are removed.

A.2.2 Results

We now assess the effects of adding LTCI to the benchmark economy as calibrated in Section 4. LTCI is purchased almost exclusively by the top lifetime earnings quintile, where 36.1 percent choose to buy it, compared to 0.1 percent of the fourth earnings quintile. The only moment of interest that is substantially affected is Medicaid’s share of nursing home expenses: it decreases from 45 to 41 percent. This is because LTCI insurance reduces the need for Medicaid transfers to nursing home residents who are in the top lifetime earnings quintile.

Table A.1 shows the results we get when we repeat the four main experiments involving modifying nursing home expenses in our analysis in the LTCI economy: removing nursing home expenses, removing cross-sectional OOP nursing home expense risk, removal of all health expenses, and removal of cross-sectional OOP health expense risk. Effects of nursing home risk and expenses are slightly lower with LTCI. However, the impact of OOP nursing home expenses and OOP health expenses decreases by less than 10 percent. For example, while OOP nursing home expenses account for 6.84 percent of savings in the baseline model they account for 6.25 percent in the version with LTCI. Since the economy with LTCI behaves very similarly to the benchmark economy, all of our conclusions with respect to the relative importance of nursing home risk and expenses hold in the economy with LTCI.