1 Appendix: Empirical Models for Fiscal Policy Simulation

The analysis of the effects of austerity requires the choice of an empirical model to measure them, that is one needs to choose an empirical model that maps fiscal plans into macroeconomic outcomes. In this appendix we will first describe the general structure of models used for fiscal policy simulation. Then we shall illustrate the specific model used in AFG 2019 to simulate the effect of fiscal plans on output growth and the debt to GDP ratio.

We start from the specification, estimation and simulation of a model that describes the behavior of a set of macro variables, \( Y_t \), as a function of their past values, \( Y_{t-1} \), the past values of a few policy variables \( P_{t-1} \) (in our case fiscal policy variables) and macroeconomic shocks. Similarly, the dynamics of policy variables can be decomposed into a "rule" – which describes the response of current policy to past policy and past macroeconomic conditions – and deviations from the rule, that include our fiscal plans. Plans are constructed considering a sequence of announced and implemented fiscal adjustments that replicate the in-sample correlation between announcements and unexpected measures. The estimated parameters of the equations used to describe plans allow to simulate the average fiscal plan in the data: they do so because, when the effects of an unanticipated shift in some fiscal variable is simulated, announcements should move consistently with what has been observed in the sample. For the same reason, when we simulate an EB or a TB plan we do not move taxes (spending) keeping spending (taxes) constant because this has almost never happened in the plans we reconstructed. Instead, we move taxes and spending according to what we have observed, on average, in the EB or TB consolidations present in our sample. Finally, by simulating plans that explicitly include announcements, the approach based on narratively identified fiscal plans addresses the "fiscal foresight" problem (see Leeper (2010)).

The more general model one can use to map fiscal plans into macroeconomic variables can be written as:

\[
Y_t = f_1(Y_{t-1}, P_{t-1}, \Theta_1) + f_2(\text{plan}_t, \Theta_2) + u_{1t} \tag{1}
\]

\[
P_t = f_3(Y_{t-1}, P_{t-1}, \Theta_3) + f_4(\text{plan}_t, \Theta_4) + u_{2t} \tag{2}
\]

\[
\text{plan}_t = g(e_{i,t}, e_{i,t-1}, e_{i,t-1}^a, \Phi) + u_{3t} \tag{3}
\]

Once the variables to be included in \( Y_t \) (the macro variables) and \( P_t \) (the policy variables) are chosen (a choice that is limited by the scarcity of data), in order to use the model to run a simulation a functional form for \( f_1, f_2, f_3, f_4 \) must be also chosen.
and the parameters $\Theta_1, \Theta_2, \Theta_3, \Theta_4$ must be estimated. Once the model is estimated, simulations allow to construct an impulse response ($IR$) that describes the difference between the forecast of the macro variables conditional on the scenario in which a fiscal plan is implemented and the forecast for the same variables absent fiscal plans:

$$IR(t, s) = E(\text{Y}_{i,t+s} \mid \text{plans}_t; I_t) - E(\text{Y}_{i,t+s} \mid \text{no plans}_t; I_t) \quad s = 0, 1, 2, ...$$

The impact of fiscal plan is then usually reported in the form of multipliers.

There are several approaches to experimenting with empirical models. Independently of the preferred model, a number of conditions need to be satisfied to obtain a valid simulation. First, empirical reduced forms must be simulated keeping all parameters constant: this is the reason why the literature typically chooses deviations from a policy rule as the relevant policy experiment. In addition, if deviations from the policy rule occur via plans, i.e. through correlated, unexpected and announced fiscal adjustment measures, simulation with constant parameters is only possible if the relation between the different fiscal measures in simulation is coherent with that observed in sample. Counterfactual experiments are very risky. It is tempting to try and answer the question on what is the response of the economy to some fiscal plan constructed differently from the estimated pattern within sample (for example exclusively via announced measures or via unanticipated measures, while in the sample a mix of them has been typically adopted). Running such experiments would require simulating the model choosing for some of the parameters values that are different from the estimated ones. This is risky because changing some parameters while leaving others unchanged might lead the model astray. An obvious case is the one in which the variation of the parameters perturbed implies a change in the parameters that are arbitrarily kept constant (see Lucas (1976)). Deviations from the policy rule (our plans) must satisfy three further conditions (see Ramey (2016)): (1) they must be exogenous for the estimation of the model parameters; (2) they must be uncorrelated with other structural macroeconomic shocks; (3) they should not mix anticipated with unanticipated shifts in policy variables. Condition (1) allows to identify the relevant information from the observed correlation in the data: if we can identify fiscal actions that are exogenous with respect to current fluctuations in output, then we can measure the output effect of fiscal policy analyzing the response of output to such policy actions. Condition (2) allows simulation of the effect of a shift in fiscal policy muting other potential sources of macroeconomic fluctuations (i.e. shifts in technology, or in monetary policy, or in consumers’ preferences), so that their effect can be assessed by keeping all the other shocks constant. Condition (3) allows to identify the response of economic agents to changes in the information set from their response to the implementation of fiscal measures.
The empirical model used in AFG 2019 to compute fiscal multipliers is a linear dynamic model (a VAR) or a non-linear one (a Smooth-Transition VAR) – used when the dynamic path of the economy depends on the probability of being in different regimes, e.g. in an economic expansion or recession or with high or low debt over GDP ratios. The use of a dynamic model has several advantages. First, including in the VAR changes in revenues and spending (as a fraction of GDP) allows to track the impact of the narratively identified shifts in fiscal variables on total revenues and total spending thus checking the strength of narratively identified instruments – for instance it allows to verify if, following a positive shift in taxes, revenues indeed increase. Second, in a dynamic model the estimated coefficients on the narratively identified shifts in fiscal variables measure the effect on output growth of the component of such adjustments that is orthogonal to lagged included variables: thus the estimated multipliers are not affected by the possible predictability of plans on the basis of the lagged information included in the model. Third, a dynamic model allows to compute multipliers in two different ways: with respect to an initial fiscal impulse and with respect to the cumulated change in fiscal variables. The limited set of variables in the specification of a dynamic model in this case does not affect the identification of the exogenous fiscal measures because these are not derived from VAR innovations but are directly observed. Estimates of the output response to a fiscal plan, however, might also depend on the effect that plans have on variables not included in the VAR: this omission will not affect the measurement of the final effect but it prevents the identification of different transmission channels. Finally, dynamic models allow naturally to reconstruct the response of the debt over GDP ratio to fiscal adjustment. This is achieved by appending to the model the dynamic identity that describes the evolution of the debt/GDP given the average cost of financing the debt, real growth and the ratio of primary surplus to GDP.

The empirical literature based on narratively identified adjustments has traditionally adopted a simplified version of the full dynamic models by using a single equation approach to the computation of impulse responses. Romer and Romer (2010) have inaugurated this tradition by using the moving average representation of output in terms of the narratively identified fiscal adjustment to derive the impulse response function that describes the tax multiplier. The validity of such an approach requires the orthogonality of the included adjustments to structural shocks in the economy and the appropriate choice of the truncation of the length of the lag of policy instruments. Jordà (2005) has refined this approach to propose a Local Projection method to compute impulse responses via estimation of a series of single equations that captures the effect of exogenous adjustments on a given variable at each period after implementation of the policy. If the underlying model is lin-
ear and the structural shocks are correctly identified, then LP recovers exactly the impulse response computed from the VAR. If instead the underlying model is non linear, Local Projections can be interpreted as a linear approximation of the true model (see e.g. the applications in Auerbach and Gorodnichenko (2016); Ramey and Zubairy (2015, 2017)). The validity of the Local Projections method requires that the exogenous adjustment variable is not correlated over time, which makes the application of this method practically impossible when plans are the relevant adjustment. Moreover, in the presence of non linearities — arising, for instance, because the dynamic response to a fiscal plan depends on the regime the economy is in when the plan is introduced — Local Projections do not offer a good approximation of the non-linearity described by a Smooth Transition VAR (STVAR). This is because in a STVAR impulses responses depend on the state of the economy in each period from the initial one, in which the impulse occurs, and the final one in which the response of the relevant variable is observed. Such a non-linearity cannot be replicated when the Local Projection Method is adopted. 1

Further refinements to the econometric approach are proposed by Jordà and Taylor (2016), who question the validity of the narrative fiscal instrument used by Guajardo et al. (2014). Building on evidence initially provided by De Cos and Moral-Benito (2016) they transform the narrative fiscal instrument into a binary ”treatment” variable to show that it is predictable. Consolidation is more likely when public debt to GDP is high, when the economy is growing below potential, when growth slows down (in contrast with the common-sense timing of countercyclical policies) and when some consolidation has been introduced in the past. Predictability, however, does not per se imply the failure of exogeneity: it would do so if the (excluded) controls, that are good predictors, were correlated with output growth. Jordà and Taylor (2016) propose a statistical design based on taking ”triple insurance” against the potential endogeneity. First, they take all episodes of consolidation from the IMF narrative instruments as a subset of all consolidation episodes that are a candidate for random allocation; second they add all the statistically significant predictors as covariates in the regression that measures the effect of the instruments on the macroeconomic outcome of interest; third they use inverse probability score weighting to re-randomise allocation of the IMF consolidation events (the more predictable is an event the less is the weight attributed to it in order to measure the macroeconomic outcome of fiscal consolidation). Finally, they propose to measure the macroeconomic consequences of consolidations by using an IPWRA (Inverse Probability Weighted Regression Adjusted) estimator.

1Batini et al. (2012) clearly illustrate the importance of allowing the regime to evolve as function of the fiscal impulse.
The methodology adopted by Jordà and Taylor (2016) suffers, in our view, from a potentially serious problem of loss of information that occurs when the narrative adjustments are transformed into a binary treatment variable. There are two sources of identification of narrative adjustments: the timing of a fiscal correction and its size. Transforming fiscal adjustments into a 0/1 dummy completely neglects the importance of size as a source of identification. This is a crucial shortcoming for an analysis of the effects of fiscal policy. Alesina et al. (2018) show that an indicator variable that takes the value of 1 when an adjustment is implemented and 0 otherwise, explains a very low share of the variance of the narrative instrument, supporting the conjecture that the main source of identification is the size of the adjustment, not its timing. In addition, the evidence that the timing of narrative adjustments can be predicted does not imply that the fiscal correction itself is predictable because, as we have seen, its size cannot be predicted. In other words, fiscal policy is different from a medical treatment in which a group of patients are given the same dose of a medicine and a control group no medicine. How much medicine is given matters a lot and the dose is different across different "patients". The evidence that the timing of narrative adjustments can be predicted does not imply that the fiscal correction itself is predictable if its size cannot be predicted.

1.1 An example

To illustrate the practical implementation of model specification we give a detailed representation of the model used to derive the dynamic response of the debt/GDP ratio to fiscal adjustment plans. The dynamics of the debt ratio, $d$, for country $i$ is

$$d_{it} = \frac{1 + i_{it}}{1 + x_{it}} d_{it-1} + g_{i,t} - \tau_{i,t} + u_{6,i,t}$$

$$x_{it} \equiv \Delta p_{it} + \Delta y_{it} + \Delta p_{it} \Delta y_{it}$$

where $i_{it}$ is the nominal average net cost of financing the debt, $x_{it}$ nominal output growth, $\Delta p_{it}$ is GDP inflation, $\tau_{i,t}$ is tax revenue as a fraction of GDP, and $g_{i,t}$ is primary government spending, also as a fraction of GDP. $u_{6,i,t}$ is a stock-flow adjustment, namely a term that tracks the difference between the actual change in the debt ratio and the change associated with the three variables in the foregoing equation. The need for stock-flow adjustment arises, for example, in the presence of revenues from sales or purchases of financial and nonfinancial assets, revaluations (in the case the debt is valued at market prices), debt write-offs, and so forth, all items that do not enter the definition of the primary surplus ($g_{i,t} - \tau_{i,t}$). To track the effect on the debt ratio of austerity plans the model must be specified so that
\( Y_t = (\Delta y_{i,t}, \Delta p_{i,t}, i_{it}, d_{it}) \), \( P_t = (\Delta g_{i,t}, \Delta \tau_{i,t}) \). We therefore adopt the following specification:

\[
\begin{bmatrix}
\Delta y_{i,t} \\
\Delta p_{i,t} \\
\Delta g_{i,t} \\
\Delta \tau_{i,t}
\end{bmatrix},
\begin{bmatrix}
a_{1,i} \\
a_{2,i} \\
a_{3,i}
\end{bmatrix}
\]
similarly for \( b_i \)

\[
\Delta y_{i,t} = A_1(L) z_{i,t-1} + \begin{bmatrix} a_1' & b_1' \end{bmatrix} \begin{bmatrix} g_{i,t} \\ g_{i,t-1,t} \\ \tau_{i,t}^{u} \\ \tau_{i,t}^{a} \\ \tau_{i,t-1,t} \\ \tau_{i,t-1,t} \\ \tau_{i,t,t+1} \\ \tau_{i,t,t+1} \end{bmatrix} + \lambda_{1,i} + \chi_{1,t} + u_{1,i,t}
\]

\[
\Delta p_{i,t} = A_2(L) z_{i,t-1} + \begin{bmatrix} a_2' & b_2' \end{bmatrix} \begin{bmatrix} g_{i,t} \\ g_{i,t-1,t} \\ \tau_{i,t}^{u} \\ \tau_{i,t}^{a} \\ \tau_{i,t-1,t} \\ \tau_{i,t-1,t} \\ \tau_{i,t,t+1} \\ \tau_{i,t,t+1} \end{bmatrix} + \lambda_{2,i} + \chi_{2,t} + u_{2,i,t}
\]

\[
\Delta g_{i,t} = A_3(L) z_{i,t-1} + \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \end{bmatrix} \begin{bmatrix} g_{i,t} \\ g_{i,t-1,t} \\ \tau_{i,t}^{u} \\ \tau_{i,t}^{a} \\ \tau_{i,t-1,t} \\ \tau_{i,t-1,t} \\ \tau_{i,t,t+1} \\ \tau_{i,t,t+1} \end{bmatrix} + \lambda_{3,i} + \chi_{3,t} + u_{3,i,t}
\]

\[
\Delta \tau_{i,t} = A_4(L) z_{i,t-1} + \begin{bmatrix} \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \end{bmatrix} \begin{bmatrix} g_{i,t} \\ g_{i,t-1,t} \\ \tau_{i,t}^{u} \\ \tau_{i,t}^{a} \\ \tau_{i,t-1,t} \\ \tau_{i,t-1,t} \\ \tau_{i,t,t+1} \\ \tau_{i,t,t+1} \end{bmatrix} + \lambda_{4,i} + \chi_{4,t} + u_{4,i,t}
\]

\[
i_{it} = A_5(L) z_{i,t-1} + \lambda_{5,i} + \chi_{5,t} + u_{5,i,t}
\]

\[
d_{it} = \frac{1 + i_{it}}{(1 + x_{it})} d_{i,t-1} + g_{i,t} - \tau_{i,t} + u_{6,i,t}
\]

\[
x_{it} \equiv \Delta p_{i,t} + \Delta y_{i,t} + \Delta p_{i,t} \Delta y_{i,t}
\]

To recover the effect of adjustment plans on the fiscal and macroeconomic variables, the empirical model for \( Y_t \) and \( P_t \) must be accompanied by a set of equations describing the response of announcements to contemporaneous corrections and the relative
weights of tax and spending measures within a plan. We allow both correlations to be different according to the type of plan, \textit{TB} versus \textit{EB}. In other words, we allow for plans to have a different intertemporal and intratemporal structure according to their type. The following equations complete the model:

\[
\begin{align*}
\tau_{i,t}^u &= \delta_{TB}^0 e_{i,t}^{u} \ast TB_{i,t} + \delta_{EB}^0 e_{i,t}^{u} \ast EB_{i,t} + \epsilon_{0,i,t} \\
g_{i,t}^u &= \vartheta_{TB}^0 e_{i,t}^{u} \ast TB_{i,t} + \vartheta_{EB}^0 e_{i,t}^{u} \ast EB_{i,t} + \upsilon_{0,i,t} \\
\tau_{i,t,t+j}^a &= \delta_{j}^{TB} e_{i,t}^{u} \ast TB_{i,t} + \delta_{j}^{EB} e_{i,t}^{u} \ast EB_{i,t} + \epsilon_{j,i,t} \quad j = 1, 2 \\
g_{i,t,t+j}^a &= \vartheta_{j}^{TB} e_{i,t}^{u} \ast TB_{i,t} + \vartheta_{j}^{EB} e_{i,t}^{u} \ast EB_{i,t} + \upsilon_{j,i,t} \quad j = 1, 2
\end{align*}
\]

where the first two equations describe the average tax (\(\delta\)) and spending (\(\vartheta\)) share of \textit{EB} and \textit{TB} plans. The next two equations describe the relation between unexpected shifts and those announced for years \(t+1\) and \(t+2\), differentiating between \textit{EB} and \textit{TB} plans. (These auxiliary regressions allow us to construct the \(e_{i,t,t+j}^{a} = \tau_{i,t,t+j}^a + g_{i,t,t+j}^a\) needed to compute impulse responses). The coefficients in the equations describing the dynamic evolution of the plans are allowed to vary across the type of plan. This is to capture the fact that \textit{TB} plans tend to be front-loaded relative to \textit{EB} plans because cutting expenditures takes longer than raising taxes. The model is non linear and therefore impulse response will depend on initial conditions, this is the reason why in the text we report different impulse response according to different levels of the initial level of debt and of the initial cost of debt servicing.

\section*{Bibliography}


