Online Appendix:

Strategic Performance of Deferred Acceptance in Dynamic Matching Problems

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Appendix D

In this Appendix, we study in depth some issues that we discussed briefly in the paper.

A. Strategy-Proofness as the Market Size Grows

We now show that if $\bar{\eta}_t$ (the DA-IP matching in a continuum economy) is not a unique statically stable matching in a static economy $\bar{F}_t(\bar{\eta}_{t-1})$ for some $t \geq 1$ then the fraction of agents who can manipulate the DA-IP mechanism does not necessarily converge to 0. In this example, we relax some of the assumptions we used in the paper to simplify the presentation. Specifically, we will not assume that the agents born in each period are distributed identically in the continuum economy $\bar{F} = (\bar{\nu}, \bar{\mathbf{r}})$ to which the sequence of finite economies converges. In addition, the measure of the agents born in each period is not necessarily 1. Finally, of course in one of the periods the DA-IP matching of that period will not be a unique statically stable matching.

EXAMPLE 6: *There are 4 schools, s_1, s_2, s_3 and s_4. Consider the following continuum economy $\bar{F} = (\bar{\nu}, \bar{r})$ such that $\bar{r}^{s_1} = \bar{r}^{s_2} = \bar{r}^{s_3} = \bar{r}^{s_4} = 0.25$, $\bar{\nu}(\bar{I}_0) = 0$, $\bar{\nu}(\bar{I}_1) = 0.5$, $\bar{\nu}(\bar{I}_2) = 0.75$, $\bar{\nu}(\bar{I}_3) = 0.25$ and $\nu(\bar{I}_t) = 0$, for all $t > 3$. We assume that each agent's preference satisfies strong rankability. In addition, $\bar{I}_1$ is partitioned into $\bar{I}_1^1$ and $\bar{I}_1^2$ where $\nu(\bar{I}_1^1) = \nu(\bar{I}_2^2) = 0.25$. Furthermore, $\bar{I}_2$ is partitioned into $\bar{I}_2^1$, $\bar{I}_2^2$ and $\bar{I}_2^3$ where $\nu(\bar{I}_2^1) = \nu(\bar{I}_2^2) = \nu(\bar{I}_2^3) = 0.25$. The preference rankings of the agents satisfies the following conditions:*

$$
\begin{aligned}
i \in \bar{I}_1^1 :& (s_1, s_1) \\
i \in \bar{I}_1^2 :& (s_2, s_2) \\
i \in \bar{I}_2^1 :& (s_1, s_1) \succ_i (h, h) \succ_i (s_4, s_4) \\
i \in \bar{I}_2^2 :& (s_4, s_4) \succ_i (s_2, s_2) \succ_i (s_3, s_3) \\
i \in \bar{I}_2^3 :& (s_3, s_3) \succ_i (s_2, s_2) \succ_i (s_4, s_4) \\
i \in \bar{I}_3 :& (s_4, s_4) \succ_i (s_1, s_1).
\end{aligned}
$$

The priority scores are distributed continuously as follows:

$$i \in \bar{I}_2^1 : x_i^{s_4} \in [0.75, 0.85) \ \& \ x_i^{s_1} \in [0.5, 0.75)$$
$$i \in \bar{I}_2^2 : x_i^{s_3} \in [0.75, 1) \ \& \ x_i^{s_4} \in [0.5, 0.75)$$
$$i \in \bar{I}_2^3 : x_i^{s_4} \in [0.85, 1) \ \& \ x_i^{s_3} \in [0.5, 0.75)$$
$$i \in \bar{I}_3 : x_i^{s_4} \in [0.85, 1) \ \& \ x_i^{s_1} \in [0.75, 1)$$

For this economy, the DA-IP matching, $\bar{\eta}$, is given in the following table.

	Period 1	Period 2	Period 3
s_1	$\bar{I}_1^1$	$\bar{I}_1^1$	$\bar{I}_3$
s_2	$\bar{I}_1^2$	$\bar{I}_1^2$	$\emptyset$
s_3	$\emptyset$	$\bar{I}_2^3$	$\bar{I}_2^3$
s_4	$\emptyset$	$\bar{I}_2^2$	$\bar{I}_2^2$
h	$\emptyset$	$\bar{I}_2^1$	$\bar{I}_2^1$

Consider now the following matching $\bar{\mu}$:

	Period 1	Period 2	Period 3
s_1	$\bar{I}_1^1$	$\bar{I}_1^1$	$\bar{I}_2^1$
s_2	$\bar{I}_1^2$	$\bar{I}_1^2$	$\bar{I}_2^2$
s_3	$\emptyset$	$\bar{I}_2^2$	$\bar{I}_2^3$
s_4	$\emptyset$	$\bar{I}_2^3$	$\bar{I}_3$
h	$\emptyset$	$\bar{I}_2^1$	$\emptyset$

Clearly, each agent in $\bar{I}_2^1$ prefers $\bar{\mu}$ to the DA-IP matching η. We will show below that there exists a sequence of finite economies that converges to the continuum economy $\bar{F}$ and such that there is a suitable profitable manipulation for any agent in $\bar{I}_1^1$. Note that the assumption of market thickness is violated in the example and also that $\bar{\mu}_t$ is statically stable matching in economy $\bar{F}(\bar{\mu}_{t-1})$ for all $t = 1, 2, 3$ and, thus, there are multiple statically stable matching in economy $\bar{F}(\bar{\mu}_1) = \bar{F}(\bar{\eta}_1)$.

Now we construct a sequence of economies $\{E^k\}$ that converges to $\bar{F}$. For each given $k = 1, \cdots, \infty$, economy E^k is as follows: $r^{ks_1} = r^{ks_2} = r^{ks_3} = r^{ks_4} = k$. In addition, $|I_0^k| = 0$, $|I_1^k| = 2k$, $|I_2^k| = 3k$, $|I_3^k| = k$ and $|I_t^k| = 0$, for all $t > 3$. We assume that each agent's preference satisfies strong rankability. In addition, I_1^k is partitioned into I_1^{k1} and I_1^{k2} where $|I_1^{k1}| = |I_1^{k2}| = k$. Furthermore, I_2^k is partitioned into $\bar{I}_2^{k1}$, I_2^{k2} and I_2^{k3} where $|I_2^{k1}| = |I_2^{k2}| = |I_2^{k3}| = k$. The preference rankings of the agents mirror those in the continuum economy $\bar{F}$ specified above. We set $\tilde{r}^{ks_1} = \tilde{r}^{ks_2} = \tilde{r}^{ks_3} = \tilde{r}^{ks_4} = 0.25 = k/4k$ and $\tilde{v}(\{i\}) = 1/4k$. Furthermore, we set the priority scores so that (1) no two agents have the same priority score (2)

	Period 1	Period 2	Period 3
s_1	I_1^{k1}	I_1^{k1}	I_2^{k1}
s_2	I_1^{k2}	I_1^{k2}	I_2^{k2}
s_3	$\emptyset$	I_2^2	I_2^3
s_4	$\emptyset$	I_2^3	I_3
h	$\emptyset$	I_2^1	$\emptyset$

the range of the agents' priority scores are the same as in continuum economy $\bar{F}$, and (3) $\tilde{\nu}^k$ converges to $\bar{\nu}$ in weak topology. With these assumptions, E^k converges to $\bar{F}$.*

Observe here that the DA-IP matching in economy E^k mirrors the one in the continuum economy $\bar{F}$. However, suppose that an agent in $i^* \in I_2^{k1}$ reports a strongly rankable preference that satisfies

$$(s_1, s_1) \succ_{i^*} (s_4, s_4) \succ_{i^*} (h, h).$$

In this case, the DA-IP matching is as follows: As a result of this manipulation, i^* definitely improves. Therefore, no matter how high k is, all the agents in I_2^{k1} can manipulate the DA-IP mechanism. Clearly, the percentage of the children who have a manipulation of the DA-IP mechanism stays constant for each k.

Example 6 has a very specific structure in the sense that in period 2 only one agent's false preference report leads to a matching that is significantly different from the DA-IP matching under truth telling. In other words, if the market is very big this false report leads to a very long chain of rejections and proposals in the DA algorithm. Indeed, in any randomly generated economy, we suspect that one agent's report having such a big impact is very unlikely. In this sense Examples similar to 6 is unlikely to occur in reality.

B. *Algorithm to Find Attainable Matchings*

We present the full version of our algorithm that find all the attainable schools. Recall that we are working with a fixed static economy (J, r) and are investigating which matchings i induce. All the agents other than i report their (isolated) preferences truthfully (as given in economy (J, r)).

We now prove Lemma 2 studied in the main text of the paper.

PROOF OF LEMMA 2:

To prove this lemma it suffices to show that the DA mechanism allocates agent i to the attainable school listed highest in the submitted preference report of agent i. Let P_i^* be the submitted preference report of agent i. Let s^* be the highest ranked attainable school in i's report. Contrary to the claim, suppose that i is not allocated to s^*. Clearly, by the definition of the non-attainable schools, i cannot be allocated to any non-attainable school. Thus, i must be allocated to an attainable school that is listed after s^*. However, because s^* is an attainable school, the DA must allocate i to s^* for some report of i^*. As a result, if i's true preferences were P_i^*, she would have had a successful manipulation of the DA mechanism. This contradicts the strategy-proofness of the DA mechanism in static settings.

We now diverge somewhat from the material in Section 6 where for simplicity of presentation, we introduced a simpler version of our algorithm.

Fix an attainable school s and we now look for ways to find all the s-attainable matchings. Fix a set $S^s \subseteq S^{NA}$. For (s, S^s), we split the non-attainable schools into two groups: redundant and non-redundant. If we set $S^s = \emptyset$, then the current definition of (s, S^s)-redundant school is equivalent to s-redundant school defined in Section 6.

DEFINITION 11: *Fix a pair (s, S^s) where $s \in S^A$ and $S^s \subset S^{NA}$. A school s' is (s, S^s)-redundant if (i) $s' \in S^{NA}$ and (ii) the DA mechanism produces the same matching when i submits any two preference reports, P_i^s and $\tilde{P}_i^s$, such that*

1) *both rank s as the highest attainable school*
2) *the sets of schools ranked higher than s under P_i^s and $\tilde{P}_i^s$ both contain S^s, and they differ only in that the one under P_i^s does not contain s' while the one under $\tilde{P}_i^s$ does.*

A school s' is (s, S')-non-redundant if s' is non-attainable and in addition, it is not (s, S')-redundant. We use the notations $S^R(s, S')$ and $S^{NR}(s, S')$ to denote the (s, S')-redundant and non-redundant schools, respectively.

Fix any $S' \subset S^{NA}$ and $S'' \subset S^{NA}$ such that $S' \subseteq S''$. From the definition above it is clear that

(i) If $s' \in S^R(s, S')$, then $s' \in S^R(s, S'')$.

(ii) If $s' \in S^{NR}(s, S'')$, then $s' \in S^R(s, S')$.

For a pair (s, S^s) with $s \in S^{NA}$ and $S^s \subset S^{NA}$, we write $\hat{P}_i^s$ to denote a preference report of i in which the set of schools ranked higher than s is S^s. We also write $\hat{\sigma}^s$ to denote the DA matching when i reports $\hat{P}_i^s$.

LEMMA 9: *Fix a pair (s, S^s) with $s \in S^{NA}$ and $S^s \subset S^{NA}$. A school s' is (s, S^s)-redundant if and only if*

$$x_i^{s'} < \min_{j \in \hat{\sigma}^s(s')} \{x_j^{s'}\}$$

PROOF:

(a) We first prove the if part.

Fix any preference reports, P_i^s or $\tilde{P}_i^s$, such that

1) they rank s as their highest attainable school
2) the sets of non-attainable schools ranked higher than s under P_i^s and $\tilde{P}_i^s$ both contain S^s, and these sets differ only in that one under P_i^s does not contain s' while the one under $\tilde{P}_i^s$ does.

We need to prove that the DA mechanism produces the same matching if i submits either P_i^s or $\tilde{P}_i^s$. Let σ^s and $\tilde{\sigma}^s$ be the corresponding DA matchings when i submits P_i^s and $\tilde{P}_i^s$. We will show that $\sigma^s = \tilde{\sigma}^s$.

By Lemma 2, $\hat{\sigma}^s(i) = \sigma^s(i) = \tilde{\sigma}^s(i)$. Thus, agent i is indifferent between the three matchings. Let SE^s, $\tilde{SE}^s$ and $\hat{SE}^s$ be the economies which differ from SE only in that agent i's preferences are P_i^s, $\tilde{P}_i^s$ and $\hat{P}_i^s$ respectively. It is well-known that σ^s, $\tilde{\sigma}^s$ and $\hat{\sigma}^s$ are the agent-optimal or school-worst stable matchings in the corresponding economies SE^s, $\tilde{SE}^s$ and $\hat{SE}^s$.

By Lemma 6, σ^s is stable in economy $\hat{SE}^s$. Because $\hat{\sigma}^s$ is the school-worst stable matching in economy $\hat{SE}^s$, we obtain that

$$\min_{j \in \hat{\sigma}^s(s')} \{x_j^{s'}\} \leq \min_{j \in \sigma^s(s')} \{x_j^{s'}.\}$$

Combining the condition above with the condition given in the lemma, we get

$$x_i^{s'} < \min_{j \in \sigma_t^s(s')} \{x_j^{s'}\}. \tag{1}$$

By Lemma 6, $\tilde{\sigma}^s$ is stable in economy SE^s. Because σ^s is the agent-optimal stable matching in SE^s, every agent j weakly prefers σ^s to $\tilde{\sigma}^s$. If we show the opposite, i.e., every agent j weakly prefers $\tilde{\sigma}^s$ to σ^s, then we are done. It suffices to show that σ^s is stable in economy $\tilde{SE}^s$ because in this economy, matching $\tilde{\sigma}^s$ is the agent-optimal stable matching. Suppose that σ^s is not stable in $\tilde{SE}^s$. Given that the schools' priorities and each agent $j \neq i$'s preferences are the same in $\tilde{SE}^s$ and SE^s, no agent $j \neq i$ and any school $\tilde{s}$ can block σ^s in economy $\tilde{SE}^s$ because σ^s is stable in economy SE^s. Therefore, there must exist $\tilde{s}$ such that i and $\tilde{s}$ block σ^s in economy $\tilde{SE}^s$. Now recall that i is matched to s under both σ^s and $\tilde{\sigma}^s$. Thus, to be a part of a blocking pair, $\tilde{s}$ must be ranked higher than s in $\tilde{P}_i^s$. By the conditions given in the lemma, if $\tilde{s} \neq s'$ then $\tilde{s}$ is again ranked higher than s in P_i^s. Then, i and s' should have blocked σ^s in SE^s, which contradicts that σ^s is stable in SE^s. Thus, $\tilde{s} = s'$. However, due to (1), i and s' cannot block σ^s in economy $\tilde{SE}^s$. Consequently, σ^s is stable in economy $\tilde{SE}^s$.

(b) We now prove the only if part. Suppose that

$$x_i^{s'} > \min_{j \in \hat{\sigma}_t^s(s')} \{x_j^{s'}\}.$$

We show s' is non-redundant for (s, S^s). On the contrary suppose that s' is redundant for (s, S^s). Consider i's isolated-preference report, $\tilde{P}_i^s$, in which the set of schools listed ahead of s is $S^s \cup s'$. We would reach a contradiction if we show that the DA mechanism does not produce $\hat{\sigma}^s$ when i reports $\tilde{P}_i^s$. Let $\tilde{SE}^s$ be the economy which differs from SE only in that i's preferences are $\tilde{P}_i^s$. Given that the DA produces a stable matching in SE^s, all we need to show is that $\hat{\sigma}^s$ is not stable in $\tilde{SE}^s$. This is clear because now i and s' will block $\hat{\sigma}^s$ as i ranks s' ahead s and $x_i^{s'} > \min_{j \in \hat{\sigma}^s(s')}\{x_j^{s'}\}$.

We now prove the two Lemmas that are used in the paper.

PROOF OF LEMMA 4:

As usual, for all the preference profiles we consider here, every agent except i reports truthfully. Let P and $\tilde{P}$ be the preference profiles in which s and h are the most preferred schools for i respectively. Denote the corresponding DA matchings under P and $\tilde{P}$ by σ and $\tilde{\sigma}$. It is well-known that when an agent drops out of the market, the schools are worse off, i.e, for each $\tilde{s} \neq h$,

$$\min_{j \in \tilde{\sigma}(\tilde{s})} x_j^{\tilde{s}} \leq \min_{j \in \sigma(\tilde{s})} x_j^{\tilde{s}}.$$

Because $s' \in S^{NR}(s)$, we have that

$$\min_{j \in \sigma(s')} x_j^{s'} < x_i^{s'}.$$

By combining the above two relations with Lemma 3 we prove that s' is a non-redundant school for h.

PROOF OF LEMMA 5:

Fix $S' \subseteq S^{NR}(s)$. In all preference profiles we consider in this proof, every player's preferences are the true ones except i's. Let P^1 and $\tilde{P}^1$ be the preference profiles in which i's report ranks only the members of S and $\tilde{S}$ ahead of s. Similarly, let P^2 and $\tilde{P}^2$ be the preference profiles in which i's report rank only the members of $S' \cup S$ and $S' \cup \tilde{S}$ ahead of s. Denote the matching that results from P^1 and $\tilde{P}^1$ by σ. Similarly, let σ^2 and $\tilde{\sigma}^2$ be the DA matchings which result from P^2 and $\tilde{P}^2$ respectively. We will now show that $\sigma^2 = \tilde{\sigma}^2$. Suppose otherwise.

We first show that σ^2 (or $\tilde{\sigma}^2$) is staticly stable under $\tilde{P}^1$ (P^1 respectively). Suppose otherwise. From Lemma 6 (in the appendix), we know that σ^2 is a stable matching under P^1. Because P^1 and $\tilde{P}^1$ differ only in agent i's preferences, if σ^2 is not stable under $\tilde{P}_1$, then i must be a part of any blocking pair. In addition, the schools which are not ranked ahead of s under $\tilde{P}_i^1$ but not under P_i^1 are those in $\tilde{S} \setminus S$. Thus, some school s' in this set and i must block σ^2 under $\tilde{P}_1$. In other words,

$$s' \tilde{P}_i^1 \sigma^2(i) = s = \sigma(i)$$
$$x_i^{s'} > x_{\sigma^2(s')}^{s'}$$

Furthermore, under P_1, σ is the agent optimal stable matching while σ^2 is a stable matching. Thus, we find

$$x_i^{s'} > x_{\sigma^2(s')}^{s'} > x_{\sigma(s')}^{s'}.$$

These two relations imply that i and s' must block σ under $\tilde{P}^1$. This contradicts that σ is stable under $\tilde{P}^1$.

We now show that σ^2 ($\tilde{\sigma}^2$) is staticly stable under $\tilde{P}^2$ (P^2, respectively). We know that σ^2 is stable under P^2. Because P^2 and $\tilde{P}^2$ only differ in i's preferences and the schools which are not ranked ahead of s under $\tilde{P}_i^2$ but not under P_i^2 are those in $\tilde{S} \setminus S$. Hence, some s' in this set and i must block σ^2 under $\tilde{P}^2$. At the same time, we know that σ^2 is stable under $\tilde{P}^1$. The schools which are not ranked ahead of s under $\tilde{P}_i^2$ but not under $\tilde{P}_i^1$ are those in $S' \setminus \tilde{S}$. Thus, s' is in $S' \setminus \tilde{S}$. But there is no common element in $\tilde{S} \setminus S$ and $S' \setminus \tilde{S}$. Thus, σ^2 ($\tilde{\sigma}^2$) is staticly stable under $\tilde{P}^2$ (P^2, respectively).

Finally, because σ^2 is stable and $\tilde{\sigma}^2$ is the agent optimal stable matching under $\tilde{P}^2$, each agent prefers $\tilde{\sigma}^2$ to σ^2. At the same time, under P^2, this relation is reversed. Consequently, $\sigma^2 = \tilde{\sigma}^2$.

Now we are ready to present our algorithm to find all the attainable matchings under which i is allocated to the same attainable school.

The Algorithm to Find the Attainable Matchings

Fix an attainable school s.

Round 0. Fix a preference report of i in which s is ranked first. Find the DA matching when i submits this preference report and $\mathcal{M}(s)$ be the set that consists of this matching. Find all the redundant and non-redundant schools for $(s, \emptyset)$. Let $\mathcal{S}(s)$ be the set of all subsets of $S^{NR}(s, \emptyset)$. Call $\mathcal{S}_1(s) = \{S' \in \mathcal{S}(s) : |S'| = 1\}$.

Round 1. Sequentially consider sets in $\mathcal{S}_1(s)$. For each $S' \in \mathcal{S}_1(s)$, fix a preference report of i in which s is the highest ranked attainable school and in which the set of schools that are ranked higher than s is S'. If the DA matching corresponding to this report is already found, then eliminate all the sets containing S' from $S(s)$. If not, add the DA matching to $\mathcal{M}(s)$. If school $s' \in S^{NA}$ is redundant for (s, S') where $S' \in \mathcal{S}_1(s)$, then we eliminate each $S'' \supseteq \{S' \cup s'\}$ from $\mathcal{S}^0(s)$. Once all the members of $\mathcal{S}_1(s)$ are considered, set $\mathcal{S}_2(s) = \{S' \in \mathcal{S}(s) : |S'| = 2\}$.

Round k. Sequentially consider sets in $\mathcal{S}_k(s)$. For each $S' \in \mathcal{S}_k(s)$, fix a preference report of i in which s is the highest ranked attainable school and in which the set of schools that are ranked higher than s is S'. If the DA matching corresponding to this report is already found, then eliminate all the sets containing S' from $S(s)$. If not, add the DA matching to $\mathcal{M}(s)$. If school $s' \in S^{NA}$ is redundant for (s, S') where $S' \in \mathcal{S}_1(s)$, then we eliminate each $S'' \supseteq \{S' \cup s'\}$ from $\mathcal{S}^0(s)$. Once all the members of $\mathcal{S}_k(s)$ are considered, set $\mathcal{S}_{k+1}(s) = \{S' \in \mathcal{S}(s) : |S'| = k+1\}$.

The algorithm stops at step $\bar{k}$ where $\mathcal{S}_{\bar{k}+1}(s) = \emptyset$ and the set of the DA matchings $\mathcal{M}(s)$ is the one found in the last round.

We now show that $\mathcal{M}(s)$ resulting from the above algorithm is the set of attainable matchings under which i is matched to s.

PROPOSITION 3: *Fix $s \in S^A$. The algorithm above yields all the s-attainable matchings.*

PROOF:

This proposition is a direct consequence of Lemmas 9 and 5.

The algorithm above is more elaborate than the one in the main text of the paper but is potentially much faster. Secondly, by running the above algorithm for each attainable school, we find all the matchings that the DA mechanism delivers for various reports of i. As we noted in the paper, if there is only one h-attainable match, then all the attainable matchings can be found without running the above algorithm.

C. Simulation Result

In this subsection, we report our simulation results for two cases: (i) there are twice as many agents as the total number of seats at schools and (ii) homecare is the worst option for every agent.

First, to increase competition we consider markets in which the number of agents is twice as large as the total number of seats. Otherwise, the current simulation exercise is identical to the one considered in Table 2.

Under priorities satisfying IPA, the manipulation percentage drops significantly. There are two reasons for this: (i) competition decreases one's chance of getting into a better school significantly in the next period and (ii) half of the agents from period 1 is forced to stay home in period 0. Both of these forces reduce one's chances of manipulating. Under Danish priorities however there are two opposing forces: competition means that agents are being allocated to lower ranked schools on average, which suggests that by staying home improves one's priorities in the following period. Hence, manipulation is more likely to be successful. On the other hand, competition in unbalanced markets also forces many students out of school under truthtelling. These students will not be able to manipulate under Danish priorities. In addition, those who manipulate by staying home do not

	Priorities Satisfying IPA			Danish Priorities		
Schools' Capacity	10 schools	50 schools	100 schools	10 schools	50 schools	100 schools
1	0.19%	0.04%	0.04%	9.14%	14.63%	16.39%
5	0.08%	0.00%	0.00%	13.45%	16.67%	17.78%
10	0.04%	0.00%	0.00%	13.82%	17.30%	18.02%
15	0.02%	0.00%	0.00%	14.15%	16.93%	18.30%
20	0.0%	0.00%	0.00%	14.45%	16.83%	18.07%

Table 3—The percentage of markets in which an agent can manipulate the DA-IP Mechanism and the number of agents is twice as large as the total number of seats at schools.

get better priority than those who are forced to stay home. This again reduces the possibility of manipulation under Danish prioirities. The opposing two forces explain why the manipulation percentages under Danish priorities in Table 3 are almost half of those in Table 2.

In the simulations considered in Table 2, any agent's payoff from staying home is drawn according to a uniform distribution on the $[0, 1]$ interval. Consequently, many agents rank home ahead of some schools, which could be somewhat restrictive and affect the manipulation percentage. Thus, we repeat our simulation exercise by assuming that each agent's payoff from staying home is 0. For each combination of the number of schools and capacity, we randomly generate 10,000 markets and report the results in Table 4.

	Priorities Satisfying IPA			Danish Priorities		
Schools' Capacity	10 schools	50 schools	100 schools	10 schools	50 schools	100 schools
1	1.41%	2.93%	3.59%	3.58%	2.74%	3.45%
5	0.55%	1.13%	1.08%	1.02%	1.10%	1.22%
10	0.36%	0.56%	0.58%	0.75%	0.63%	0.84%
15	0.29%	0.41%	0.48%	0.5%	0.42%	0.45%
20	0.20%	0.26%	0.31%	0.38%	0.25%	0.31%

Table 4—(Homecare as Worst Option) The percentage of markets in which an agent can manipulate the DA-IP Mechanism where the home is the worst option.

The percentage of markets in which an agent can manipulate indeed increased significantly under priorities satisfying IPA. The main reason behind this result is that when each agent ranks home as the worst option, the number of possible matchings an agent can induce by misreporting her preferences increases significantly. When a child ranks a school ahead of the school she obtains under truth telling, it generally leads to a sequence of rejections and new applications in the DA algorithm. When home is not the worst option, the chance that this sequence ends with some agent choosing home increases greatly. Therefore, an increase of the manipulable markets is expected under the current simulations. However, the manipulation percentage still converges to 0 as the capacities of schools increases. In fact, already when the schools' capacities reach 20, the manipulation percentage is negligible.

Under Danish priorities, the manipulation percentage is higher when there are 10 schools compared to those under priorities satisfying IPA. On the other hand, these numbers are comparable when there are 50 or 100 schools. The main reason is the following: As we discussed earlier whether the gain in terms of the period 2 payoffs exceeds the loss in terms of the period 1 payoff determines if the agent has a successful manipulation or not. Here the cost is sizable because staying home brings one the payoff of 0. From Pittel (1988), we know that when there are n schools and n agents, the average rank of schools agents obtain under the DA-IP matching is $\ln n$. Consequently, as n increases, the loss in terms of the period 1 payoff being low decreases significantly. Hence, staying home is a successful manipulation more frequently when there are 10 schools. In our simulations with 50 or 100 schools, the payoff from truth telling exceeded 1 for agent 1 without exception which means that staying home was never profitable for agent 1. As a result, the manipulation percentage stays comparable under both priority structures when the number of schools 50 or 100.

Based on Tables 2 and 4, we can conclude that (i) the Danish priorities leads to more manipulations in general, (ii) This increase is more pronounced when the cost of staying home is low and (iii) The Danish priorities do not lead to additional manipulations when the number of schools is relatively big and the cost of staying home is severe.