Majority Choice of an Income Targeted Educational Voucher[1]

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Online Appendix

A. Representative Democracy Model with a Continuum of Types. This part of the appendix is largely an adaptation of Proposition 2 and Corollary 1 in Besley and Coate (1997) to the case of a continuum of types. Their central assumption is that the policy preference of every population member is known and, if a population member is elected, then that policy will be implemented. Their Corollary 1 states roughly that existence of a Condorcet winner among preferred policy vectors in the population implies single candidate equilibrium exists for sufficiently low cost of becoming a candidate, with the individual having the Condorcet winning policy vector the candidate (and winner). Besley and Coate (BC) assume an integer number of voters, any of whom can be a candidate. Our model assumes a continuum of voters and thus potential candidates, indexed by endowed income y, with continuous distribution F(y) and density f(y), the latter positive on the support of y. We make analogous assumptions about preferences and equilibrium as do BC. We next summarize those assumptions and introduce a bit more notation, and then report the results of interest. We provide an additional result about uniqueness of equilibrium, but under a strong modification to BC's model.

A population member has indirect utility function V = V(p,y), where p is a policy vector. In the voucher-model application, a voucher, income tax rate, eligibility threshold, and per student level of public expenditure arise in equilibrium, but the policy vector in the indirect utility function V is tri-variate as one variable is eliminated by the government balanced-budget requirement. Let $p^*(y)$ denote the preferred policy choice of voter y, which we assume is unique. Let P^* denote the set of p^* values; i.e., $P^* \equiv \{p^*(y) \mid f(y) > 0\}$. The results regard the case where there is a Condorcet winner p^w:

p^w is a Condorcet winner if $p^w \in P^*$ and $V(p^w,y) \geq V(p,y)$ for at least one-half the measure of voters for all $p \in P^*$.

Let y^w satisfy $p^w = p^*(y^w)$. Income y^w may or may not be unique though it is unique in the voucher application. However, we have multiple voters with income y^w, consistent with the notion that f(y) is positive. Let Y^w denote the set of y^w values.

Equilibrium assumes voters first decide whether to become candidates, followed by voting. Any voter can become a candidate at 0 cost, and voters choose to be candidates or not simultaneously. Given the slate of candidates, assumed non-empty at the moment, voters simultaneously and costlessly vote by voting for one candidate, though any voter can abstain. If a voter is indifferent between candidates and votes, then the voter randomizes with equal probabilities among them. The candidate receiving the highest measure of votes is the winner and that candidate's preferred policy is implemented. If there is a tie among the highest vote getters, then a winner is selected among them with equal probabilities. If no candidate enters the race or if a positive measure of votes fails to materialize, then a relatively lousy policy p^0 is

[1] Part A of this appendix tracks closely part of the on-line appendix for Epple and Romano (2014).

implemented, which is worse for everyone than a positive measure of policies $p \in P^*$.[2] It is also assumed that voters never choose a weakly dominated strategy when voting.

Two preliminary results are:

<u>Result 1</u>: If two candidates enter from the set whose policies are preferred to p^0, then a candidate that is majority preferred will win.

<u>Result 2</u>: If one candidate enters from the set whose policies are preferred to p^0, then that candidate is elected.

Result 1 is implied by the assumption that voters never choose a weakly dominated strategy. Given that it is costless to vote, a voter is never worse off and sometimes better off voting for their preferred candidate if there are just two candidates from this set.[3] As well, sincere voting is implied with two candidates. Result 1 and the sincerity implication are results in BC.

Result 2 follows since everyone prefers the election of any candidate from this set to the lousy default outcome. It is not an equilibrium for a zero measure of voters to vote.

The main result is as follows:

<u>Result 3</u>: Assuming a Condorcet winner among preferred policies: (i) a single candidate equilibrium having a candidate y^w exists, with that candidate elected; and (ii) a single candidate equilibrium must have a y^w elected.

<u>Proof of Result 3</u>: (i) If only a y^w becomes a candidate, then that candidate will be elected by Result 2.[4] A y^w becoming a singleton candidate is an equilibrium, since, by Result 1, any $y \notin Y^w$ would not be elected and then gains nothing by also entering; nor would another y^w entering gain since his preferred policy arises anyway. (ii) It is not an equilibrium for any $y \notin Y^w$ to be the only entrant, since, by Result 1, a y^w would enter and win the election. ■

We emphasize that Result 3 is a simple adaptation of BC's Corollary 1.

We can modify the BC model to generate uniqueness of equilibrium with equilibrium policy p^w. Assume two parties that simultaneously offer their party's candidacy to any voter. Only party candidates run. Once the slate is set, voters simultaneously vote as above. Preferences are as above, in particular party affiliation of a candidate does not affect preferences. Such a political process might arise if running for election is prohibitively costly for a non-

[2] The notion is that a lack of leadership if no one is elected results in a worse policy than that advocated preferred by some potentially elected households (though this assumption is becoming increasingly difficult to defend). Note, too, that BC assumed, if only one candidate enters, that candidate is automatically the winner (as that candidate could vote for himself if no one else votes). Since we require a positive measure of votes to win given the continuum, we must modify the assumption a bit.

[3] If the two candidates are equally preferred by everyone, then everyone still votes in equilibrium to avoid the possibility that no one votes and the lousy default policy arises.

[4] The y^w candidate must have a policy in the set majority preferred to p^0, since p^w is majority preferred in P^*.

affiliated candidate, while the party bears all running costs from exogenous funds for their affiliated candidate. A party wants to win the election. Under these assumptions and assuming a Condorcet winner:

Result 4:[5] *Equilibrium has each party offer their candidacy to a y^w, at least one accepts the offer, and the resulting policy is p^w.*

Obviously, the parties offer a candidacy to a y^w. At least one accepts the candidacy offer to avoid the default policy if neither runs. Whether one or both potential candidates run, voting equilibrium obviously implies that p^w is implemented.

B. Propositions with Strongly Religious Households. We refer to the strongly religious as just religious, and the other households as non-religious. We assume the same income distributions for both types, and that the proportion of religious types equals γ. Both types have the same utility function (as in the text) if public school is attended. If private school is attended, school quality for religious types equals κq, with $\kappa > 1$, where q is per student expenditure in the private school attended.

As in the text, the initial analysis here concerns equilibrium effects with religious households assuming the superintendent chooses public school. Note that such a superintendent might be religious; the preferred policy choice of a religious and non-religious household is the same if public school is attended.

We first establish the analogues of Proposition1-c. Let $y_T^R(t,z,g)$ denote the minimum income of a religious households that would choose private school given (t,z,g), with z either equal to 0 or v, assuming eligibility for the voucher in the latter case. It is obvious that $y_T^R(t,z,g) < y_T(t,z,g)$ for all relevant policies. The comparative statics in (1) in the text hold for $y_T^R(t,z,g)$ just like for $y_T(t,z,g)$. Just to avoid tedium, we assume policies and an income distribution such that religious types will attend both public and private schools, i.e., $y_{min} < y_T^R(t,v,g) < y_T^R(t,0,g) < y_{max}$, where the second inequality is implied by the comparative statics.

The elected superintendent will continue to choose (v,y_m) to minimize the tax rate given his preferred choice of g (i.e., the analogue of Proposition 1b obviously holds). We can state the problem as minimizing t given $tY = B(y_m,v,g,t)$, for given g and where B is equilibrium public expenditure. Thus, B must be minimized over (y_m,v). We show the equivalent of Proposition 1-cii using this streamlined approach. (As with no religious types, it is convenient to prove part cii before part ci. The modified proposition is:

Proposition R1-cii. *Targeting: Assume equilibrium has a superintendent that chooses public school and let g^* denote the superintendent's choice of g. If $v > 0$ is strictly optimal (i.e.,*

[5] Jackson, Mathevet, and Mattes (2007) provide a similar result. See their Propositions 1 and 2.

someone uses the voucher), then y_m^* *equals either* $y_T(t,0,g^*)$ *or* $y_T^R(t,0,g^*)$*, which value depending on parameters.*

<u>*Proof of Proposition R1-cii*</u>*. To economize on notation, we just write g for* g^* *in this proof. First we argue that* $v < g$ *is optimal. If* $v = g$*, then the superintendent saves nothing by providing any private school students the voucher, implying* $v = 0$ *is also optimal (a contradiction). To show the remaining results, we must consider two cases, delineated by whether* $y_T^R(t,0,g) \le$ *or* $> y_T(t,v,g)$*.*

<u>*Case 1*</u>*:* $y_T^R(t,0,g) > y_T(t,v,g)$*. In this case, it must be that:*
$y_{min} < y_T^R(t,v,g) < y_T(t,v,g) < y_T^R(t,0,g) < y_T(t,v,g) < y_{max}$*. The latter values delineate 5 ranges within which* y_m *can fall, and over which* $B(y_m,v,g,t)$ *differs. We show that B is minimized over* y_m *at either* $y_T(t,0,g)$ *or* $y_T^R(t,0,g)$*. To show this, we write out B for each range and compute its derivate with respect to* y_m*. We have:*

<u>*Range 1*</u>*:* $y_m \in [y_{min}, y_T^R(t,v,g)]$*. Here no one gets a voucher and*
$B = g \cdot \left[\int_{y_{min}}^{y_T^R(t,v,g)} \gamma f(y)dy + \int_{y_{min}}^{y_T(t,v,g)} (1-\gamma)f(y)dy \right]$*. Thus,* $\partial B / \partial y_m = 0.$

<u>*Range 2*</u>*:* $y_m \in (y_T^R(t,v,g), y_T(t,v,g)]$*. Here*
$B = v \cdot \int_{y_T^R(t,v,g)}^{y_m} \gamma f(y)dy + g \cdot \left[\int_{y_{min}}^{y_T^R(t,v,g)} \gamma f(y)dy + \int_{y_m}^{y_T^R(t,0,g)} \gamma f(y)dy + \int_{y_{min}}^{y_T(t,0,g)} (1-\gamma)f(y)dy \right]$ *Thus,*
$\partial B / \partial y_m = (v-g)\gamma f(y_m) < 0,$ *the inequality since we know* $v < g$*.*

<u>*Range 3*</u>*:* $y_m \in (y_T(t,v,g), y_T^R(t,0,g)]$*. Here*

$$B = v \cdot \left[\int_{y_T^R(t,v,g)}^{y_m} \gamma f(y)dy + \int_{y_T(t,v,g)}^{y_m} (1-\gamma)f(y)dy \right] + g \cdot \left[\int_{y_{min}}^{y_T^R(t,v,g)} \gamma f(y)dy + \int_{y_m}^{y_T^R(t,0,g)} \gamma f(y)dy + \int_{y_{min}}^{y_T(t,v,g)} (1-\gamma)f(y)dy + \int_{y_m}^{y_T(t,0,g)} (1-\gamma)f(y)dy \right].$$

Thus,
$\partial B / \partial y_m = (v-g)f(y_m) < 0.$

<u>*Range 4*</u>*:* $y_m \in (y_T^R(t,0,g), y_T(t,0,g)]$*. Here*

$$B = v \cdot \left[\int_{y_T^R(t,v,g)}^{y_m} \gamma f(y)dy + \int_{y_T(t,v,g)}^{y_m} (1-\gamma)f(y)dy \right] + g \cdot \left[\int_{y_{min}}^{y_T^R(t,v,g)} \gamma f(y)dy + \int_{y_m}^{y_T(t,0,g)} (1-\gamma)f(y)dy + \int_{y_{min}}^{y_T(t,v,g)} (1-\gamma)f(y)dy + \int_{y_m}^{y_T(t,0,g)} (1-\gamma)f(y)dy \right].$$

Thus,
$\partial B / \partial y_m = [v-(1-\gamma)g]f(y_m),$ *the sign of which depends on the relative value of v and* $(1-\gamma)g$

<u>*Range 5*</u>*:* $y_m \in (y_T(t,0,g), y_{max}]$*.*
$B = v \cdot \left[\int_{y_T^R(t,v,g)}^{y_m} \gamma f(y)dy + \int_{y_T(t,v,g)}^{y_m} (1-\gamma)f(y)dy \right] + g \cdot \left[\int_{y_{min}}^{y_T^R(t,v,g)} \gamma f(y)dy + \int_{y_{min}}^{y_T(t,v,g)} (1-\gamma)f(y)dy \right]$*. Here,*

$\partial B/\partial y_m = vf(y_m) > 0.$ *Noting that B is continuous, using the derivatives of B in the 5 ranges it is implied that if* $v > g(1-\gamma)$, *then the minimum is at* $y_m = y_T^R(t,0,g)$; *and if* $v < g(1-\gamma)$, *then the minimum is at* $y_m = y_T(t,0,g)$. *(If* $v = g(1-\gamma)$, *then any* y_m *between and including the two threshold is optimal, but this will not arise generically.)*

<u>Case 2</u>: $y_T^R(t,0,g) \le y_T(t,v,g)$. *In this case, it must be that:* $y_{min} < y_T^R(t,v,g) < y_T^R(t,0,g) \le y_T(t,v,g) < y_T(t,0,g) < y_{max}$. *Again, there are five ranges into which* y_m *might fall, and we proceed as in Case 1.*

<u>Range 1</u>: $y_m \in [y_{min}, y_T^R(t,v,g)]$. *Here no one gets a voucher and*

$$B = g \cdot \left[\int_{y_{min}}^{y_T^R(t,v,g)} \gamma f(y)dy + \int_{y_{min}}^{y_T(t,v,g)} (1-\gamma) f(y)dy \right].$$ *Thus,* $\partial B/\partial y_m = 0.$

<u>Range 2</u>: $y_m \in (y_T^R(t,v,g), y_T^R(t,0,g)]$. *Here*

$$B = v \cdot \int_{y_T^R(t,v,g)}^{y_m} \gamma f(y)dy + g \cdot \left[\int_{y_{min}}^{y_T^R(t,v,g)} \gamma f(y)dy + \int_{y_m}^{y_T^R(t,0,g)} \gamma f(y)dy + \int_{y_{min}}^{y_T(t,0,g)} (1-\gamma) f(y)dy \right]$$ *Thus,*

$\partial B/\partial y_m = (v-g)\gamma f(y_m) < 0$, *the inequality since we know* $v < g$.

<u>Range 3</u>: $y_m \in (y_T^R(t,0,g), y_T(t,v,g)]$. *Here*

$$B = v \cdot \left[\int_{y_T^R(t,v,g)}^{y_m} \gamma f(y)dy \right] + g \cdot \left[\int_{y_{min}}^{y_T^R(t,v,g)} \gamma f(y)dy + \int_{y_{min}}^{y_T(t,0,g)} (1-\gamma) f(y)dy \right].$$ *Thus,* $\partial B/\partial y_m = v\gamma f(y_m) > 0.$

<u>Range 4</u>: $y_m \in (y_T(t,v,g), y_T(t,0,g)]$. *Here*

$$B = v \cdot \left[\int_{y_T^R(t,v,g)}^{y_m} \gamma f(y)dy + \int_{y_T(t,v,g)}^{y_m} (1-\gamma) f(y)dy \right]$$
$$+ g \cdot \left[\int_{y_{min}}^{y_T^R(t,v,g)} \gamma f(y)dy + \int_{y_m}^{y_T(t,0,g)} (1-\gamma) f(y)dy + \int_{y_{min}}^{y_T(t,v,g)} (1-\gamma) f(y)dy \right].$$ *Thus,*

$\partial B/\partial y_m = [v-(1-\gamma)g] f(y_m)$, *the sign of which depends on the relative value of v and* $(1-\gamma)g$

<u>Range 5</u>: $y_m \in (y_T(t,0,g), y_{max}]$.

$$B = v \cdot \left[\int_{y_T^R(t,v,g)}^{y_m} \gamma f(y)dy + \int_{y_T(t,v,g)}^{y_m} (1-\gamma) f(y)dy \right] + g \cdot \left[\int_{y_{min}}^{y_T^R(t,v,g)} \gamma f(y)dy + \int_{y_{min}}^{y_T(t,v,g)} (1-\gamma) f(y)dy \right].$$ *Here,*

$\partial B/\partial y_m = vf(y_m) > 0.$ *Noting that B is continuous, using the derivatives of B in the 5 ranges it is implied that if* $v > g(1-\gamma)$, *then the minimum is at* $y_m = y_T^R(t,0,g)$; *and if* $v < g(1-\gamma)$, *then there are two local minima at* $y_m = y_T^R(t,0,g)$ *and* $y_m = y_T(t,0,g)$. *Either of the latter might be optimal depending on parameters.*

Thus, we have shown that one of $y_m = y_T^R(t,0,g)$ *or* $y_m = y_T(t,0,g)$ *is optimal in any case, completing the proof.*

We now show the analogue of Proposition 1-ci, beginning with a statement of it (though its statement is exactly as for the case with no religious types).

Proposition R1-ci. Assuming equilibrium has a superintendent that chooses public school, a voucher $v \in (0, g^*)$ *is optimal.*

Proof of Proposition R1-ci Again, we drop the '' from g. We have already shown* $v < g$ *is optimal assuming* $v > 0$*. Thus, we show* $v = 0$ *is not optimal, specifically by showing B can be decreased with a positive v. In the vicinity of* $v = 0$ *(with non-negative v), only Case 2 in the Proof of Proposition 1-ci can arise; i.e., it must be that* $y_T^R(t,0,g) \le y_T(t,v,g)$*, this since* $y_T^R(t,0,g) < y_T(t,0,g)$*. Suppose that the superintendent sets* $y_m = y_T^R(t,v,g)$*. We show that B is decreasing in v in the vicinity of* $v = 0$ *with the latter choice, implying a positive v is optimal, since the alternative potential optimizing choice of* y_m *(from Proposition R1-cii) would imply lower yet B if such a* y_m *is optimal. Then:*

$$B = v \cdot \int_{y_T^R(t,v,g)}^{y_T^R(t,0,g)} \gamma f(y)dy + g \cdot \left[\int_{y_{min}}^{y_T^R(t,v,g)} \gamma f(y)dy + \int_{y_{min}}^{y_T(t,0,g)} (1-\gamma) f(y)dy \right].$$

Differentiating with respect to v:

$$\partial B / \partial v = \int_{y_T^R(t,v,g)}^{y_T^R(t,0,g)} \gamma f(y)dy + v \cdot \partial [\int_{y_T^R(t,v,g)}^{y_T^R(t,0,g)} \gamma f(y)dy] / \partial v + g \cdot [\partial y_T^R(t,v,g) / \partial v] \gamma f(y_T^R(t,v,g)).$$

The first two terms in the latter approach 0 as v approaches 0, while the last term is negative since $\partial y_T^R(t,v,g)/\partial v < 0$*.*

Next we develop the analogue of Proposition 1-ciii regarding voting coalitions in equilibrium with the elected superintendent choosing public school. First note that those choosing public school in equilibrium with the same income have the same local policy and thus voting preferences whether religious or not. However, because religious types have stronger preference to attend private school, the sets of income types that choose public school generally differs. Let $f^{pub}(y)$ denote the population density of nonreligious types that attend public school and $f^{pubR}(y)$ the same for religious types. $f^{pub}(y)$ equals $(1-\gamma)f(y)$ if public school is chosen by non-religious household with income y and 0 otherwise. $f^{pubR}(y)$ equals $\gamma f(y)$ if public school is chosen by religious household with income y and 0 otherwise. The ranges of income where public school is chosen vary with parameters and are implicit in the Proof of Proposition R1-ci. Those that choose private school and receive no voucher prefer lower t (and care not about g and v) whether religious or not. These income sets can differ, too, but we need not provide notation for this. The sign of (4) in the text again determines the local voting preferences of those that choose private school and receive a voucher. The equilibrium income sets for which this is positive or negative will in general differ between religious and non-religious types, both because U_q will differ and because those that choose private school (and get a voucher) can differ. Let Y^+ denote the nonreligious income set that obtain a voucher and for which $U^{v\prime} > 0$ and let Y^{+R} the analogous set of religious households.

Proposition R1-ciii. Assuming equilibrium has a superintendent that chooses public school, y^w *must satisfy: (i)* $\int_{y \ge y^w} [f^{pub}(y) + f^{pubR}(y)]dy + \int_{Y^+} (1-\gamma) f(y)dy + \int_{Y^{+R}} \gamma f(y)dy = .5;$ *and* *(ii) Households in (i) other than y^w prefer a candidate with marginally higher income and the remaining households prefer a candidate with marginally lower income.*

The proof is exactly as for Proposition 1-ciii and is omitted. The specifics of the voting coalitions are of interest, but vary with policy characteristics as noted above. An example is that in Figure 3 in the text.

Proposition 2 in the text describes the set of potential optima for all candidates, which is necessary to confirm existence of equilibrium. To provide the analogue of Proposition 2 when there are religious types, the following lemma is useful.

Lemma R1. An elected superintendent with income y_s that would choose private school and set $v = g > 0$, would choose $y_m = Max\ [\ y_s, y_T^R(t,0,g)]$.

Proof of Lemma R1. Given $v = g$, the superintendent's objective is to minimize the tax rate while making sure to be eligible himself for the voucher. Minimizing the tax rate corresponds to maximizing the measure of households that attend private school with no voucher since the public cost of all those who take a voucher or attend private school with a voucher is the same. Only religious types with $y \geq y_T^R(t,0,g)$ and non-religious types with $y \geq y_T(t,0,g)$ will attend private school with no voucher. Given the own-eligibility constraint and that $y_T^R < y_T$, the choice of y_m then maximizes the measure of households that attend private school with no voucher. If $y_s < y_T^R$, then the superintendent maintains his own eligibility and maximizes the measure of households that attend private school with no voucher with any $y_m \in [\ y_s, y_T^R\ ]$, but then chooses $y_m = y_T^R$ due to the benevolence assumption (A8) because this allows more households to get a voucher and subsidize it rather than attend public school.

With Lemma R1 in hand, the analogue of Proposition 2 is very similar.

Proposition R2: A household that chooses the policy vector and itself chooses public school follows the policy described in Propositions R1-ci and R1-cii A household y that chooses the policy vector and itself chooses private school either: (i) chooses t=g=v=0 with optimal private consumption; or (ii) sets $v = g > 0$, with $y_m = Max\,[\ y, y_T^R(t,0,g\ ]$, t satisfying:

$$(B.1)\quad \int_{y_{min}}^{Max[\ y, y_T^R\ ]} \gamma f(y)dy + \int_{y_{min}}^{Max[\ y, y_T\ ]} (1-\gamma) f(y)dy = tY\,,$$

and with v solving: $\begin{array}{l} Max_{v,t,y_m,g}\ U^R(v) \\ s.t.\ y_m = Max[\ y, y_T^R(t,0,g)];(B.1);\ and\ v = g. \end{array}$ *Assuming a household choosing private school faces a strictly quasi-concave optimization problem, policy type (i) [(ii)] is optimal if y > [<] Y/F(y).*

The proof parallels that of Proposition 2 and is omitted, though we make a few comments. In addition to the difference in the eligibility threshold as compared to the case with no religious types (Lemma 1), the budget constraint for a superintendent that chooses policy (i) is modified to account for the alternative schooling choices of religious and non-religious types. A difference that is not apparent from comparing the two versions of the proposition is that the threshold income where an elected superintendent is indifferent to choosing public school and private school, each with the superintendent setting policy, is lower for religious than non-religious households. This is very intuitive and seen in the example developed in the text.[6] What is perhaps surprising is that the income level where an elected superintendent choosing private school transitions to the 0-tax policy does not differ between religious and non-religious types, i.e., satisfies $y = Y/F(y)$. The intuition is that the decision as to whether to employ taxes to fund private consumption is purely a fiscal one, dependent on income but not on the demand for private consumption.

C. Computational Program Summary.

Overview: We first summarize the computational strategy for the case when there only non-religious households. We then explain how the strategy is extended to incorporate preferences of religious households.

The program calculates a feasible set of policy alternatives, S, that includes the most-preferred policy of each citizen. The set of most preferred policies, s, is then selected from S. This is the set of citizen-candidate proposals. The Condorcet winner, if any, is then obtained by finding the policy from s that defeats all other policies in s in pairwise voting.

As detailed in Section 2, a citizen candidate's policy proposal is a tuple comprised of the tax rate, voucher, expenditure per student in public school, and highest income eligible for the voucher: t,v,g,y_m. The approach of the program is to consider a policy triple (t,v,g) while exploiting Proposition 1 of Section 3 to determine y_m. From Proposition 1, a citizen candidate who proposes a positive voucher will propose a targeted voucher with the income-eligibility limit set equal to the income of the individual indifferent between public and private when not receiving a voucher: $y_m=y_T(t,0,g)$. It is also useful in the describing the computations to refer to the income, denoted y_a in the program, of the lowest-income individual indifferent who will take up the voucher. This is the individual eligible for the voucher who is indifferent between public school with no voucher and private school with the voucher: $y_a = y_T(t,v,g)$.

As demonstrated in Section 3, there are three possible regime types:

1. (t,v,g) with $g>v\geq 0$
2. (t,v,v) with $v>0$
3. (0,0,0)

An individual who would attend public school under his/her most-preferred policy will propose a Regime 1 policy. An individual who would attend private school with a voucher under his/her

[6] The argument is this. For given income, the optimal policy choice and utility level for religious and non-religious types are the same conditional on choosing public school. However, for given income and optimal policy choices utility is higher for religious types than non-religious types if private school is chosen. The latter is implied by the fact that religious types value private schooling more and would therefore have higher utility even if choosing the same policy values as is optimal for a non-religious type in private school. Thus, the range of income where the private alternative is preferred by an elected superintendent must be wider.

most-preferred policy will propose a Regime 2 policy. In Regimes 1 and 2, the income eligibility limit, y_m, is determined by Proposition 1 as described above. An individual who would attend private school without a voucher under his/her most-preferred policy will propose the Regime 3 policy, with $y_m=y_{min}$.

Citizen Candidates: Discretize the income distribution by selecting an equally spaced grid of values of incomes spanning the support of the income distribution $[y_{min},y_{max}]$. Denote these incomes y_j for j=1,…,J,. This is the set of citizen-candidates in the model. In the program, the space between points on this grid is $1,000.

Feasible Policies: We next describe the strategy for calculating the set S that will contain s as a proper subset. Let a citizen-candidate with income y_j be named y_j.

Regime 1: Calculate the (t,v,g) allocation most preferred by y_j assuming y_j attends public school (even if y_j does not prefer public school under his most-preferred policy). Do this calculation for each y_j for j=1,…,J,. Let (t_j,v_j,g_j) be j's policy from this calculation. This calculation also provides the incomes, $y_{a,j} = y_T(t_j,v_j,g_j)$ and $y_{m,j} = y_T(t_j,0,g_j)$. These income boundaries are used in the calculation of votes. Hence, for each y_j, the program saves the row vector $(t_j,v_j,g_j, y_{a,j}, y_{m,j})$. These row vectors are "stacked" vertically to obtain a matrix of dimension JX5.
Calculation of (t_j,v_j,g_j) for y_j for Regime 1 proceeds as follows. The citizen-candidate choosing a most-preferred Regime 1 policy faces the following constraints:

1) Government budget constraint.
2) Boundary-indifference condition for $y_{a,j}$
3) First-order condition for private school expenditure, $e_{a,j}$ if $y_{a,j}$ attends private school
4) Boundary-indifference condition for $y_{m,j}$
5) First-order condition for $e_{m,j}$ if $y_{m,j}$ attends private school

The computational approach entails solving a system of 17 nonlinear simultaneous equations. The five constraints implicitly express five variables as functions of g and v: t(g, v), e_a(g, v), y_a(g, v), e_m(g, v), y_m(g, v). Differentiate each of the five constraints with respect to g to obtain the derivatives of the preceding five functions with respect to g. Differentiate each of the five constraints with respect to v to obtain the derivatives of the preceding five functions with respect to v. Together, this yields 15 equations for the constraints and their derivatives. The two additional equations are the first-order conditions for the citizen-candidate's most-preferred g and v. These are obtained by differentiating the citizen-candidate's utility function with respect to g and with respect to v. (Note that the derivative of the citizen-candidate's utility function yields dt(g,v))/dv=0.) For each citizen candidate, y_j, this system of 17 nonlinear simultaneous equations is solved.

Regime 2: All feasible policies satisfying v=g are calculated regardless of whether they are preferred policy of any candidate. The computations proceed similarly to those for Regime 1, by solving a system of nonlinear simultaneous equations while invoking the g=v constraint. This calculation also provides the income, $y_{a,j}$, of the individual indifferent between attending public school with spending $g_j = v_j$ and attending private school with a voucher v_j. For each y_j, the program saves the row vector $(t_j,v_j,v_j,y_{a,j},y_{m,j})$. These are stacked vertically to obtain a matrix of dimension Jx5.

Regime 3: For regime three, there is a single policy (0,0,0). Hence this policy yields the vector $(0,0,0,y_{min},y_{min})$ where y_{min} is the lower bound of the support of the income distribution.

The set S: The union of the above three sets of policies yields set S. In the program, the union of these policies is obtained by vertically stacking the above to obtain a matrix of dimension (2J+1)x5. Let K=2J +1. Hence, S has dimension Kx5.

Proposals: Calculate the utility of candidate y_j for each every policy in S. The policy that maximizes the utility of y_j is then the proposal of candidate y_j. This calculation for each candidate y_j then yields the set of policy proposals s.

Voting: A random sample (e.g., 100,000) of incomes is drawn from the income distribution. These are voters. A guess is made of the income of the winning candidate, and this income is the contender, y_c, against which others are paired. The proposal of y_c is voted against the proposal of the lowest-income individual, y_1, and the fraction of voters favoring the proposal of y_c is calculated. If the fraction exceeds .5, the proposal of y_c is voted against the proposal of the next-highest income, y_2. If the proposal of y_c is not defeated by any alternative, y_c this step of the computations is completed. If the proposal of y_c is defeated by the proposal of some candidate, say, y_i, then y_i becomes the new contender, i.e. $y_c=y_i$. The proposal of the new y_c is voted successively against the proposal of y_{i+1}, y_{i+2}, etc until either y_c defeats all remaining alternatives or y_c is replaced by a new contender. This process continues until the reigning y_c is paired against all proposals through that of y_J. Winning this series of votes is a necessary condition to be a Condorcet winner. Let y^w denote the income of this winner. The proposal of y^w is then paired against every other proposal. If y^w defeats all alternatives, then y^w is the Condorcet winner. If the proposal of some other candidate defeats y^w, there is no Condorcet winner, and no citizen-candidate equilibrium exists.

Religious Candidates: When there are both religious and non-religious types, there are again the same three regime types. Now, however, the system of nonlinear equations for Regime 1 must take account of the fact that there are different income thresholds for take-up of the voucher, i.e., $y_T(t,v,g)$ differs between religious and non-religious types. Similarly, the income thresholds for attending of private school when not eligible for a voucher also differ. In addition, the most-preferred Regime 1 policy alternative for a given y_j differs between religious and non-religious types. Hence, the number of equations that need to be solved to obtain an element of S for Regime 1 is roughly double that for the case with only non-religious types. In addition, this system of equations needs to be solved twice as many times to obtain the most preferred policy for each y_j for each religious type. Similar generalization is required to solve for elements of S for Regime 2. In total, S then contains (4*J+1) elements. The elements of s then are then chosen, each element being a most-preferred alternative of either a religious or a non-religious individual on the grid of incomes. Voting to select the winner, if any, proceeds as before with utility for the elements of s differing by income and by religious type.

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