The Rise of Intangible Capital and the Macroeconomic Implications

Andrea Chiavari and Sampreet Singh Goraya

Online Appendix

Contents

I Empirical Appendix **1**

I.I Data . . . 1

I.I.I Main Sample, Variables, and Summary Statistics . . . 1

I.I.II User Cost of Tangible and Intangible Capital . . . 1

I.I.III Intangible Capital Measurement and Accounting Standards . . . 2

I.I.IV Additional Validations for Firm-Level Intangible Capital . . . 5

I.II Production Function Estimation . . . 7

I.II.I Ackerberg-Caves-Frazer . . . 7

I.II.II Cost Shares . . . 9

I.III Robustness Production Function Estimation . . . 9

I.III.I Unconstrained Returns to Scale . . . 10

I.III.II Decreasing Returns to Scale . . . 11

I.III.III Sector-Level Production Technology . . . 11

I.III.IV Translog Production Function . . . 11

I.III.V Alternative Variable Input. . . . 12

I.III.VI Excluding Internally Generated Intangible Capital ($k_{R\&D}$). . . . 12

I.III.VII Adding Goodwill to Balance Sheet Intangible Capital. . . . 12

I.III.VIII Using a Different Deflators. . . . 12

I.III.IX Controlling for Output and Input Price Variation . . . 13

I.III.X Controlling for Measurement Error. . . . 14

I.IV Robustness Investment Rates . . . 14

I.IV.I Investment Rates across Sectors . . . 14

I.IV.II Investment Rates across Time . . . 15

I.IV.III Investment Rates across Firms of Different Age, Size, Leverage, and Liquidity Groups . . . 16

I.IV.IV Investment Rates across Different Types of Intangible Capital . . . 17

I.IV.V Investment Rates Calculated with Alternative Specification . . . 17

I.IV.VI Investment Rates in the Presence of Measurement Error . . . 18

I.IV.VII Investment Rates with Goodwill as Part of Balance Sheet Intangible Capital . . . 19

I.IV.VIII Investment Rates with an Alternative Deflator 19
I.V Robustness for Marginal Revenue Product of Both Types of Capital 20
I.V.I Responsiveness 21
I.V.II Dispersion 22

II Quantitative Appendix **22**
II.I Alternative Modeling Assumptions 22
II.I.I Griliches (1979)' Knowledge Capital Model 22
II.I.II Intangible Capital as a Demand Shifter 22
II.I.III Intangible Capital, Returns to Scale, and Market Power 23
II.II Additional Comparisons between Model and Data over Time 24
II.III Additional Robustness Quantitative Implications of IBTC 25

I Empirical Appendix

I.I Data

I.I.I Main Sample, Variables, and Summary Statistics

We use the Compustat dataset from 1980 to 2015. We linearly interpolate SALE, COGS, XSGA, EMP, PPEGT, PPENT, XRD, INTAN, GDWL, and AM. We exclude utilities (SIC codes between 4900 and 4999) because their prices are heavily regulated. We also exclude financial firms (SIC codes between 6000 and 6999) because their balance sheets are different from those of other firms. For data quality, we interpret as mistakes if SALE, PPEGT, PPENT, COGS, EMP, or XSGA are zero, negative, or missing, and we drop those observations. Moreover, if XSGA is missing or negative, we drop it as well. Finally, if XRD, INTAN, AM, or GDWL are negative or missing, we treat them as zeros. To obtain a real measure of the main variables, we deflate them with the GDP deflator, and we deflate investment in tangible and intangible capital by the appropriate deflators.[1] Table I presents summary statistics for our variables.

Table I: Summary Statistics (1980-2015)

	Sales	Cost of Goods Sold	Employment	Tangible Capital Stock	Intangible Capital Stock
Mean	2,312,994	1,574,325	7,973	1,573,609	284,841
25^{th} Percentile	27,516	14,899	131	8,009	2
Median	153,043	89,267	687	51,104	3,103
75^{th} Percentile	810,332	510,746	3,632	349,972	34,096
No. Obs.	187,919	187,919	187,919	187,919	187,919

Note. Summary statistics of cleaned Compustat dataset between 1980 and 2015. All variables are in thousands of US$. Sales and Cost of Goods Sold are deflated with the GDP deflator with base year 2012, and both types of capital stock are deflated using the appropriate investment deflator with base year 2012.

I.I.II User Cost of Tangible and Intangible Capital

Using the cost shares approach requires a measure of the user cost of capital. To this end, we define the user cost of capital as $r_{j,t} = i_t - \mathbb{E}_t \pi_{t+1} + \delta_j$, $j \in \{T, I\}$, where i_t equals the nominal interest rate, $\mathbb{E}_t \pi_{t+1}$ is expected inflation at time t, and δ_j is the capital-specific depreciation rate. We take the annual Moody's Seasoned Aaa Corporate Bond Yield as an empirical proxy of the nominal interest rate and use the annual growth rate of the Investment Nonresidential

[1] Deflators are taken from the NIPA tables.

Price Deflator to calculate expected inflation.[2,3] The depreciation rate of tangible capital is calibrated to $\delta = 0.07$, and the firm-level depreciation rate of intangible capital is computed as a weighted average of the depreciation rates used to construct the intangible capital stock.

I.I.III Intangible Capital Measurement and Accounting Standards

Measuring intangible capital is a difficult task as the accounting standards (US GAAP) are insufficient to satisfactorily book the intangible assets on the balance sheets.[4] In this section, we explain which assumptions are needed to compute intangible capital using the balance sheet for stocks and the income statements for flows.

Intangible capital differs fundamentally from tangible capital as a portion is internally generated by firms. Unlike tangible assets, which are recorded on the balance sheet at the purchase price and depreciated over their useful life, internally generated intangible assets, like knowledge and organizational capital, follow different accounting standards. Investments in internal intangible capital, such as R&D, advertising, or employee training, are fully expensed in the period they are incurred, creating a distinction in accounting treatment.[5]

Figure I: Advertising Expenses of Coca Cola

Selling, General and Administrative Expenses

The following table sets forth the significant components of selling, general and administrative expenses (in millions):

Year Ended December 31,	2016	2015	2014
Stock-based compensation expense	$ **258**	$ 236	$ 209
Advertising expenses	**4,004**	3,976	3,499
Selling and distribution expenses	**5,177**	6,025	6,412
Other operating expenses	**5,823**	6,190	7,098
Selling, general and administrative expenses	$ **15,262**	$ 16,427	$ 17,218

The Coca-Cola Company annually invests billions in promoting its products, like Coca-Cola and Dasani, anticipating future benefits in increased sales and margins. Despite this, accounting rules prevent recognizing these assets on its balance sheet. For instance, in 2016, Coca-Cola spent approximately $4 billion on advertising (Figure I). Google Inc. similarly allocated substantial funds, around $16 billion for research and development and $13 billion for sales and marketing in 2017 (Figures IIa and IIb). Because of the significance of these costs and their long-lasting nature we build a measure of knowledge capital.

[2]Moody's Seasoned Aaa Corporate Bond Yield: https://fred.stlouisfed.org/series/AAA. The Investment Price Deflator: https://fred.stlouisfed.org/series/A008RD3Q086SBEA.

[3]We estimate an AR(1) process on the annual growth rate of the Investment Nonresidential Price deflator and define the contemporaneous expected inflation as $\mathbb{E}_t \pi_{t+1} = \mu + \rho \pi_t$.

[4]Lev and Gu (2016).

[5]Exceptions exist, e.g., in US GAAP, under ASC 985, mandates the capitalization and amortization of computer software development costs after reaching technological feasibility.

Figure II: Intangible Investments by Google

Research and Development

The following table presents our R&D expenses (in millions):

	Year Ended December 31,		
	2015	2016	2017
Research and development expenses	$ 12,282	$ 13,948	$ 16,625
Research and development expenses as a percentage of revenues	16.4%	15.5%	15.0%

R&D expenses consist primarily of:

- Compensation expenses, including SBC, and facilities-related costs for employees responsible for R&D of our existing and new products and services; and
- Depreciation and equipment-related expenses.

(a) Research and Development Expenses

Sales and Marketing

The following table presents our sales and marketing expenses (in millions):

	Year Ended December 31,		
	2015	2016	2017
Sales and marketing expenses	$ 9,047	$ 10,485	$ 12,893
Sales and marketing expenses as a percentage of revenues	12.1%	11.6%	11.6%

Sales and marketing expenses consist primarily of:

- Advertising and promotional expenditures related to our products and services; and
- Compensation expenses, including SBC, and facilities-related costs for employees engaged in sales and marketing, sales support, and certain customer service functions.

(b) Marketing Expenses

On the contrary, externally acquired intangible capital is capitalized on balance sheets at fair value under US GAAP, following ASC 350 guidelines (formerly FAS 142), reflected in INTAN in Compustat. ASC 820 (formerly FAS 157) guides its fair value determination during acquisition, offering options like replacement cost estimation, market comparison, or Discounted Cash Flow model use. Identifiable intangible assets, meeting separability or contractual-legal criteria, i.e., the control of the future economic benefits is warranted by contractual or legal rights, are individually capitalized, e.g., brand names and patents. Intangibles not meeting these criteria, like corporate culture or advertising effectiveness, are recorded as goodwill (GDWL). Goodwill and intangible assets with indefinite lives undergo periodic impairment tests. Figure III shows Coca-Cola's externally purchased intangibles.

Internally generated intangible capital: potential issues. The inability to capitalize internally produced intangible capital on firms' balance sheets raises concerns about potential double-counting. For instance, if firm 1 internally produces intangible capital at a cost of x, and later sells it to firm 2, this transaction will not appear as a negative cost on firm 1's income statement. However, firm 2 will record the acquired intangible on its balance sheet at fair value y. While the total amount of intangible capital remains unchanged, the transaction might inaccurately suggest an increase from x to $x + y$ in the overall stock of intangible capital.

Figure III: Coca-Cola's Externally Purchased Intangibles

Coca-Cola Co.

Balance sheet: goodwill and intangible assets

US$ in millions

	Dec 31, 2019	Dec 31, 2018	Dec 31, 2017	Dec 31, 2016	Dec 31, 2015
Trademarks	9,266	6,682	6,729	6,097	5,989
Bottlers' franchise rights	109	51	138	3,676	6,000
Goodwill	16,764	10,263	9,401	10,629	11,289
Other	110	106	106	128	164
Indefinite-lived intangible assets	**26,249**	**17,102**	**16,374**	**20,530**	**23,442**
Customer relationships	344	185	205	392	493
Bottlers' franchise rights	341	30	213	487	604
Trademarks	177	186	182	228	211
Other	55	88	94	179	97
Definite-lived intangible assets, gross carrying amount	**917**	**489**	**694**	**1,286**	**1,405**
Accumulated amortization	(400)	(321)	(432)	(688)	(715)
Definite-lived intangible assets, net	**517**	**168**	**262**	**598**	**690**
Intangible assets	**26,766**	**17,270**	**16,636**	**21,128**	**24,132**

Based on:10-K (filing date: 2020-02-24),10-K (filing date: 2019-02-21),10-K (filing date: 2018-02-23),10-K (filing date: 2017-02-24),10-K (filing date: 2016-02-25).

While theoretically concerning, we are confident this double-counting issue is rare and of minimal quantitative relevance. Many intangible assets are acquired through whole-firm acquisitions, causing the target firm, along with its intangible assets, to exit the sample (Peters and Taylor, 2017; Ewens et al., 2025). Additionally, a significant portion of intangible capital is purchased as final goods from other firms, eliminating double-counting concerns. Furthermore, our empirical measure indicates a declining trend in internally produced relative to total intangible capital, reducing the relevance of this issue over time. Thus, we conclude that this concern is not quantitatively appealing.

Externally acquired intangible capital: potential issues. When a firm acquires another, the capitalization of acquired assets involves three steps. Tangible assets are capitalized at fair value p^T, identifiable intangible assets at fair value p^I, and the residual value, reflecting unidentifiable intangibles like synergies or organizational culture, is placed in goodwill. In the data, this translates to $\text{GDWL} = p^y - p^T - p^I$.

To address concerns that unidentifiable intangible assets might represent discounted future market power coming from the strategic acquisition of competitors, we follow two approaches. First, we use the IPP deflator to deflate intangible capital, addressing aggregate trends in its input price but not the firm-level heterogeneity. Unfortunately, more granular investment deflators are unavailable, a common limitation for all inputs in firm-level data. Second, recognizing that goodwill captures the potential rise in prices related to unidentifiable assets, we exclude goodwill from the total intangible capital on the balance sheet.

Accounting standards for software: a special case. The accounting standards for internal software development or external purchases differ from those for other intangible assets.

FASB ASC subtopic 350-40 guides the accounting for computer software developed or obtained for internal use, allowing capitalization of costs during the development stage, which ceases post-implementation. Subtopic 985-20 guides costs incurred for software meant for sale, leased, or marketed, permitting capitalization post-technological feasibility until the software's general release. Figure IV illustrates the accounting treatment of software with the example of Athena Health Inc.'s software investments. The company capitalized $113.9 million in 2017 for software development and reported $53.8 million for external software acquisitions.

Figure IV: Software Capitalization of Athena Health

6. CAPITALIZED SOFTWARE COSTS

Capitalized software consisted of the following:

	As of December 31,	
	2017	2016
Capitalized internal-use software development costs	$ 113.9	$ 122.7
Acquired third-party software licenses for internal use	53.8	47.5
Total gross capitalized software for internal-use	167.7	170.2
Accumulated amortization	(74.8)	(82.9)
Capitalized internal-use software in process	46.8	38.5
Total capitalized software costs	$ 139.7	$ 125.8

Capitalized software amortization expense totaled $71.3 million, $73.5 million, and $53.4 million for the years ended December 31, 2017, 2016, and 2015, respectively. Future amortization expense for all capitalized software placed in service as of December 31, 2017 is estimated to be:

Years ending December 31,	Amount
2018	$ 50.5
2019	25.4
2020	10.7
2021	5.5
2022	0.8

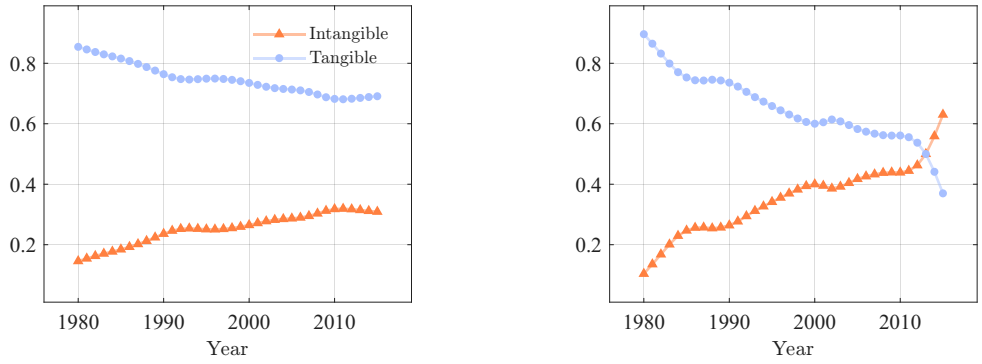
Software used in research and development follows subtopic 730-10. Generally, purchased software for research and development with alternative future uses is capitalized and amortized as an intangible asset. However, if software purchased for a specific research and development project lacks alternative uses and separate economic value, it is considered a research and development cost and is expensed. Thus, our measure captures most software-related intangible capital through balance sheet intangible capital or capitalized knowledge capital.

I.I.IV Additional Validations for Firm-Level Intangible Capital

Here we compare trends in intangible capital investment between BEA aggregate data and our Compustat measure. Figure V shows the share of tangible and intangible capital investment in total investment from 1980 to 2015. Both sources depict a similar story: in 1980, a substantial part of investment was in tangible capital, reducing to roughly 70% in BEA and 40% in Compustat by 2015. Despite the overall coherence, differences exist. The decline in tangible

capital's share in Compustat is more pronounced, potentially due to undercapitalization of true IPP capital in BEA or the selection of intangible-intensive firms in Compustat.

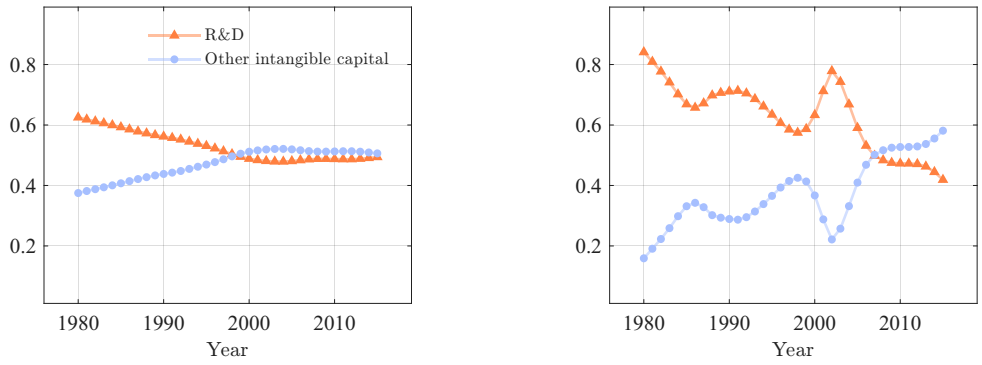
Figure V: Investment Components Share



(a) BEA **(b) Compustat**

Note. The figures show the evolution of the share of tangible capital investment and of intangible capital investment over total investment in both BEA data and Compustat data for the period 1980-2015. The data are detrended with an HP filter with $\lambda = 6.25$.

Figure VI: Intangible Capital Components Share

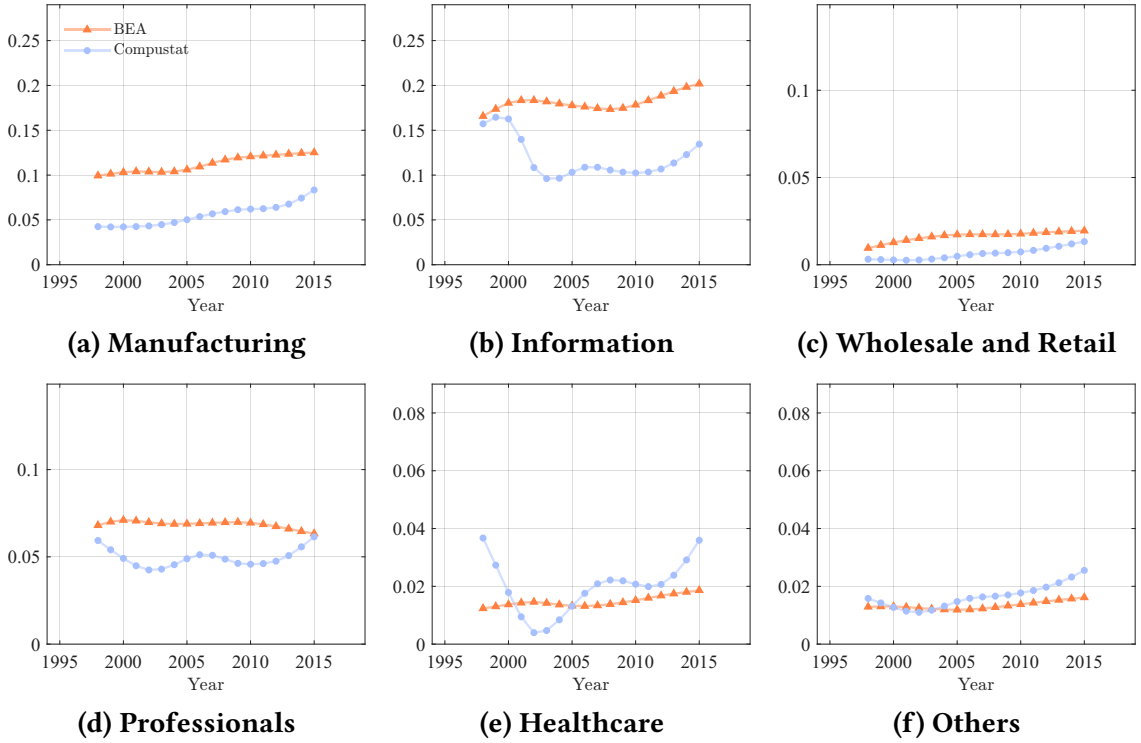


(a) BEA **(b) Compustat**

Note. The figures show the evolution of the share of knowledge capital investment (R&D) and other intangible capital investment (intangible capital investment different from R&D) over total intangible capital investment in both BEA data and Compustat data for the period 1980-2015. The data are detrended with an HP filter with $\lambda = 6.25$.

Figure VI depicts the evolution of the different components of intangible capital investment in both BEA and Compustat from 1980 to 2015. Both sources show a consistent trend: in 1980, research and development dominated intangible capital investment, decreasing to less than 50% by 2015. In Figure VII, we compare sector-level intangible capital investment shares between BEA and Compustat for 1998-2015. While trends align, there are level differences in some sectors. Identifying the exact sources of these discrepancies is challenging, but overall, our firm-level intangible capital measure reasonably captures tendencies present in aggregate data.

Figure VII: Intangible Capital Components Share by Sector



Note. The figures show the evolution of the intangible capital investment share across different sectors of the US economy for both BEA-KLEMS data and Compustat data between 1998 and 2015. The data are detrended with an HP filter with $\lambda = 6.25$.

I.II Production Function Estimation

I.II.I Ackerberg-Caves-Frazer

To overcome some of the criticisms in Gandhi et al. (2020), we work with a structural value-added specification, as in Ackerberg et al. (2015) and De Loecker and Scott (2016), given by

$$\mathcal{Q}_{ft} = \min\left\{K_{T,ft}^{\alpha} K_{I,ft}^{\nu} L_{ft}^{1-\alpha-\nu} \exp(\omega_{ft} + \varepsilon_{ft}),\ \beta M_{ft}\right\}, \tag{1}$$

where $\mathcal{Q}_{ft}$ is output, $K_{T,ft}$ is tangible capital, $K_{I,ft}$ is intangible capital, L_{ft} is labor, ω_{ft} is log productivity, ε_{ft} is the error term, and M_{ft} is material. This structural value-added production function yields the following first-order condition:

$$\mathcal{Q}_{ft} = K_{T,ft}^{\alpha} K_{I,ft}^{\nu} L_{ft}^{1-\alpha-\nu} \exp(\omega_{ft} + \varepsilon_{ft}), \tag{2}$$

justifying the regression of $\mathcal{Q}_{ft}$ on tangible capital, intangible capital, and labor while ignoring materials. Using equation (1), the estimation of the firm-level production function reduces to

$$q_{ft} = \alpha k_{T,ft} + \nu k_{I,ft} + (1 - \alpha - \nu)\ell_{ft} + \omega_{ft} + \varepsilon_{ft}, \tag{3}$$

where $q_{ft} = \log(\mathcal{Q}_{ft})$, $k_{T,ft} = \log(K_{T,ft})$, $k_{I,ft} = \log(K_{I,ft})$, and $\ell_{ft} = \log(L_{ft})$. The main identification challenge to the production function estimation is the simultaneity bias induced by the unobserved time-varying firm-level productivity, ω_{ft}. We follow the control function literature (Ackerberg et al., 2015) to estimate the production function in (3) using a two-step approach.

The identification relies on the observation that a firm's tangible capital investment demand is given by a policy function of the form $x_{T,ft} = \chi_T(k_{T,ft}, k_{I,ft}, \omega_{ft})$. Then, provided that the policy function is invertible, the productivity process can be proxied by a control function given by $\omega_{ft} = \omega(k_{T,ft}, k_{I,ft}, x_{T,ft})$ where $\omega(\cdot) = \chi_t^{-1}(\cdot)$.

Therefore, in the first stage of this estimation procedure, we can clean the firm-level output value from measurement errors and unanticipated productivity shocks, regressing output on a polynomial of tangible capital, intangible capital, and labor, given by

$$q_{ft} = \mathcal{P}_t(k_{T,ft}, k_{I,ft}, \ell_{ft}, x_{T,ft}) + \varepsilon_{ft}. \tag{4}$$

Then, in the second stage, using the estimate $\widehat{\mathcal{P}}_t$ from the previous stage, we can construct a measure of productivity that does not depend on the measurement error ε_{ft}, given by

$$\omega_{ft}(\alpha, \nu) = \widehat{\mathcal{P}}_t(k_{T,ft}, k_{I,ft}, \ell_{ft}, x_{T,ft}) - \alpha k_{T,ft} - \nu k_{I,ft} - (1 - \alpha - \nu)\ell_{ft}. \tag{5}$$

Finally, taking advantage of the assumption that productivity follows an AR(1) process, it is possible to construct a measure of productivity innovations, given by

$$\xi(\alpha, \nu, \rho) = \omega_{ft}(\alpha, \nu) - \rho\omega_{ft-1}(\alpha, \nu). \tag{6}$$

Therefore, using the productivity innovations, we can construct a set of moment condi-

tions to estimate the parameters of the production function, given by

$$\mathbb{E}(\xi(\alpha, \nu, \rho) \times \mathbf{z}_{ft}) = \mathbf{0}_{Z\times 1}, \tag{7}$$

where Z ≥ 3 and, under the assumption that firms react to unanticipated productivity shocks contemporaneously and that capital is predetermined, the set of admissible instruments is $\mathbf{z}_{ft} \in \{\ell_{ft}, k_{T,ft}, k_{I,ft}, \ell_{it-1}, k_{T,ft-1}, k_{I,ft-1}, \dots\}$.

I.II.II Cost Shares

The cost shares approach has been prominently adopted in Foster et al. (2008) and exploits the first-order conditions of the firm. To make fruitful use of the first-order conditions of the firm, two assumptions are needed, namely: (i) constant returns to scale in production and (ii) all inputs are variable. Under these assumptions, the output elasticities can be calculated as

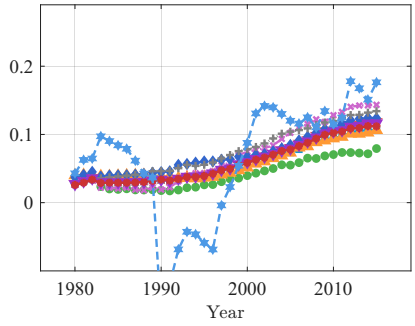
$$\alpha = \text{med}\left\{\frac{r_t^T k_{T,ft}}{w_{ft}\ell_{ft} + r_t^T k_{T,ft} + r_t^I k_{I,ft}}\right\} \quad \text{and} \quad \nu = \text{med}\left\{\frac{r_t^I k_{I,ft}}{w_{ft}\ell_{ft} + r_t^T k_{T,ft} + r_t^I k_{I,ft}}\right\}, \tag{8}$$

where $w_{ft}\ell_{ft}$ is the wage bill, $r_t^T k_{T,ft}$ is the rental cost of tangible capital, and $r_t^I k_{I,ft}$ is the rental cost of intangible capital.

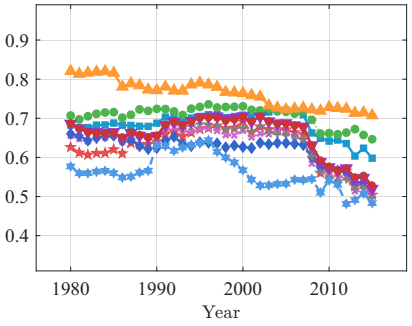
I.III Robustness Production Function Estimation

Here, we test the robustness of our results against the following alternative specifications: (i) unconstrained returns to scale; (ii) imposing decreasing returns to scale; (iii) technology at the two-digit sector level (NAICS 2); (iv) a translog production function; (v) using the cost of goods sold as a variable input; (vi) excluding internally generated intangible capital ($k_{R\&D}$); (vii) including goodwill in the measure of balance sheet intangible capital; (viii) using an alternative deflator for intangible capital; (ix) accounting for output and input price variation in the ACF estimation; and (x) controlling for measurement error in intangible capital. Figure VIII illustrates results from these alternatives, while below we present all exercises in detail.

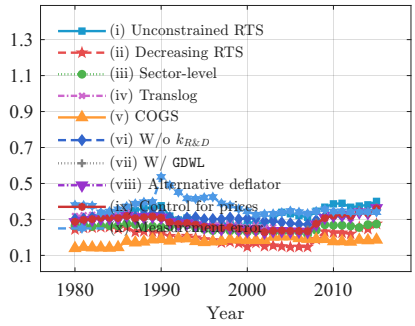
Figure VIII: Trends in Input Shares: Robustness



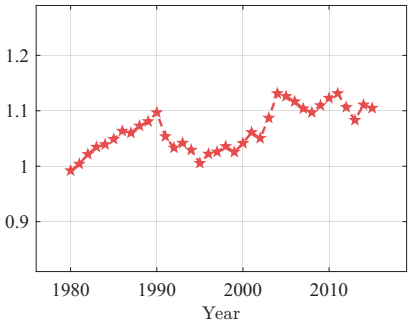
(a) Intangible Capital Share



(b) Labor Share



(c) Tangible Capital Input Share



(d) Returns to Scale

Note. The figures present the output elasticities estimated using the ACF approach with unconstrained returns to scale (dashed light blue lines with squares), with the ACF approach and decreasing returns to scale equal to 0.9 (dashed red lines with pentagrams), with the sector-level ACF approach (dotted green lines with circles), with the translog ACF approach (dash-dotted purple lines with crosses), with the ACF approach using COGS as a variable input (solid orange lines with triangles), with the ACF approach using only externally purchased intangible capital (dashed dark blue lines with diamonds), with the ACF approach goodwill as part of balance sheet intangible capital (dotted gray lines with plus signs), with the ACF approach controlling for alternative deflators (dash-dotted violet lines with downward-pointing triangles), with the ACF approach controlling for unobservable input and output price variation (dash-dotted orange lines with stars), and with the ACF approach controlling for measurement error (dashed teal lines with hexagrams). The elasticities are estimated using 10-year rolling windows over time.

I.III.I Unconstrained Returns to Scale

To test the robustness of our results to a specification with unconstrained returns to scale, we estimate with the ACF approach the following production function:

$$q_{ft} = \alpha k_{T,ft} + \nu k_{I,ft} + \beta \ell_{ft} + \omega_{ft} + \varepsilon_{ft}. \tag{9}$$

With this alternative specification, the set of moment conditions becomes

$$\mathbb{E}(\xi(\alpha, \nu, \beta, \rho) \times \mathbf{z}_{ft}) = \mathbf{0}_{Z \times 1}, \tag{10}$$

where $Z \geq 4$.

I.III.II Decreasing Returns to Scale

To test the robustness of our results to a specification with decreasing returns to scale equal to 0.9, common in firm dynamics literature, we estimate with the ACF approach the following production function:

$$q_{ft} = \alpha k_{T,ft} + \nu k_{I,ft} + (0.9 - \alpha - \nu)\ell_{ft} + \omega_{ft} + \varepsilon_{ft}. \tag{11}$$

With this alternative specification, the set of moment conditions becomes

$$\mathbb{E}(\xi(\alpha, \nu, \rho) \times \mathbf{z}_{ft}) = \mathbf{0}_{Z\times 1}, \tag{12}$$

where Z ≥ 4.

I.III.III Sector-Level Production Technology

We relax the assumption of common technology across sectors by allowing for a sector-specific production technology, given by

$$q_{ft} = \alpha_s k_{T,ft} + \nu_s k_{I,ft} + (1 - \alpha_s - \nu_s)\ell_{ft} + \omega_{ft} + \varepsilon_{ft}, \tag{13}$$

With this specification, the average output elasticities will be computed using a sales-weighted average.

I.III.IV Translog Production Function

We also test the robustness of our results to a more flexible translog production function (a second-order approximation of a CES production technology), given by

$$\begin{aligned} q_{ft} = {} & \alpha k_{T,ft} + \nu k_{I,ft} + (1 - \alpha - \nu)\ell_{ft} \\ & - \beta k_{T,ft}k_{I,ft} - \beta k_{T,ft}\ell_{ft} - \beta k_{I,ft}\ell_{ft} + \beta k_{T,ft}^2 + \beta k_{I,ft}^2 + \beta \ell_{ft}^2 + \omega_{ft} + \varepsilon_{ft}. \end{aligned} \tag{14}$$

Therefore, with this alternative specification, the set of moment conditions becomes

$$\mathbb{E}(\xi(\alpha, \nu, \beta, \rho) \times \mathbf{z}_{ft}) = \mathbf{0}_{Z\times 1}, \tag{15}$$

where $Z \geq 4$. Finally, the endogenous output elasticities will be given by

$$\theta^T = \text{med}\big(\alpha - \beta k_{I,ft} - \beta \ell_{ft} + 2\beta k_{T,ft}\big), \tag{16}$$

$$\theta^I = \text{med}\big(\nu - \beta k_{T,ft} - \beta \ell_{ft} + 2\beta k_{I,ft}\big), \tag{17}$$

$$\theta^\ell = \text{med}\big(1 - \alpha - \nu - \beta k_{T,ft} - \beta k_{T,ft} + 2\beta \ell_{ft}\big). \tag{18}$$

I.III.V Alternative Variable Input.

We use a different variable input: cost of goods sold. This input, unlike employment, does not keep track of scientists or designers employed by firms to produce intangible capital but instead tracks only the variable expenditures used in production. This specification shows patterns similar to our benchmark.

I.III.VI Excluding Internally Generated Intangible Capital ($k_{R\&D}$).

We use only externally acquired intangible capital (k_{BS}) as the intangible capital measure. This robustness shows patterns similar to our benchmark, suggesting that any overlap between capitalized R&D and labor is unlikely to drive our main findings.

I.III.VII Adding Goodwill to Balance Sheet Intangible Capital.

We incorporate 38% of Goodwill into balance sheet intangible capital, as reported by Ewens et al. (2025), resulting in the calculation $k_{BS} = \texttt{INTAN} + \text{AM} - (1 - 0.38)\text{GDWL}$. Our analysis indicates that this adjustment does not affect the results.

I.III.VIII Using a Different Deflators.

We employ an alternative deflator, replacing the IPP deflator used in the baseline analysis with the R&D deflator. This approach assesses whether our results are influenced by the significant decline in the relative price of software. Overall, we find that the choice of deflator has minimal quantitative impact on our estimates.

I.III.IX Controlling for Output and Input Price Variation

In the absence of firm-level deflators the structural value-added production function takes the following form:

$$q_{ft} + p_{ft} = \alpha(k_{T,ft} + p_t^T) + \nu(k_{I,ft} + p_t^I) + (1 - \alpha - \nu)(\ell_{ft} + p_{ft}^\ell) + \omega_{ft} + \varepsilon_{ft}, \tag{19}$$

where p_{ft} is the output price, p_t^T is the common user cost of tangible capital, p_t^I is the common user cost of intangible capital, and p_{ft}^ℓ is the price of labor. This empirical specification produces the following structural error term:

$$\omega_{ft} + p_{ft} - \alpha p_t^T - \nu p_t^I - (1 - \alpha - \nu)p_{ft}^\ell. \tag{20}$$

We follow De Loecker et al. (2016) and let the wedge between the output and input price (scaled by the output elasticity) be a function of the demand shifters and the productivity difference. The inclusion in the control function of demand shifters $\mathbf{d}_{ft}$, constructed using measures of market shares as in De Loecker et al. (2020), should therefore capture the relevant output and input market forces that generate differences in the output and input price.[6]

The first-stage estimation procedure to clean from measurement error becomes

$$q_{ft} = \mathcal{P}_t(k_{T,ft}, k_{I,ft}, \ell_{ft}, x_{T,ft}) + \boldsymbol{\vartheta}'\mathbf{d}_{ft} + \varepsilon_{ft}, \tag{21}$$

where $\mathcal{P}(\cdot)$ is a polynomial taking as inputs the firm's state variables and the control function, and $\mathbf{d}_{ft}$ is a vector of firm-level sales shares controlling for the pass-through of input price to output price variation. Under this alternative specification, in the second stage, using the estimates for $\widehat{\mathcal{P}}$ and $\widehat{\boldsymbol{\vartheta}}'$, we can construct a measure of productivity that does not depend on measurement error and unobservable input prices, given by

$$\omega_{ft} = \widehat{q}_{ft} - \alpha k_{T,ft} - \nu k_{I,ft} - (1 - \alpha - \nu)\ell_{ft} - \widehat{\boldsymbol{\vartheta}}'\mathbf{d}_{ft}, \tag{22}$$

where $\widehat{q}_{ft} = \widehat{\mathcal{P}}(k_{T,ft}, k_{I,ft}, \ell_{ft}, x_{T,ft}) + \widehat{\boldsymbol{\vartheta}}'\mathbf{d}_{ft}$. Notice that equation (22) is identical to the main specification up to the estimate of $\widehat{\boldsymbol{\vartheta}}'\mathbf{d}_{ft}$.

[6] As discussed in De Loecker et al. (2016), this is an exact control when output prices, conditional on productivity, reflect input price variation and when demand is of the (nested) logit form.

I.III.X Controlling for Measurement Error.

We address measurement error in intangible capital by employing a methodology proposed by Collard-Wexler and De Loecker (2021). The presence of measurement error typically biases downward estimated input shares. Thus, without it, we would expect an even higher estimate of the input share. The methodology relies on using intangible capital investment as an instrument, which is challenging due to limited data availability, as noted in Levinsohn and Petrin (2003). Despite the trade-off between addressing measurement error and maintaining statistical power, we proceed and find a larger, though less precisely estimated, increase in the intangible capital share over the sample period.[7]

I.IV Robustness Investment Rates

Here, we demonstrate the robustness of our findings related to the distribution of investment rate. We explore alternative investment rate distributions (i) across sectors; (ii) over time; (iii) among different types of firms; (iv) for various types of intangible capital; (v) with an alternative specification; (vi) adding part of Goodwill to balance sheet intangible capital; and (vii) using an alternative deflator.

I.IV.I Investment Rates across Sectors

Table II details the investment rate distribution across the SIC1 sector—mining, construction, manufacturing, transportation, and public utilities, wholesale, retail, and services. Across all sectors, albeit with some variation, intangible capital investment consistently exhibits higher positive spike rates and serial correlation compared to tangible capital. Even in the retail sector, with the lowest positive spike rate and serial correlation, rates remain notably higher at 42% and 0.14 respectively.

[7]Due to limited intangible capital investment data, we employ the inverse hyperbolic sine transformation and use cost of goods sold instead of employee numbers as the variable input in our production function estimator. This approach aims to maximize available observations. However, for a few data points, the estimator yields a negative intangible capital input share, attributed to data loss from measurement error correction. This is a limitation linked to convergence issues in the GMM method when dealing with a small number of observations, as noted in Gao and Kehrig (2017).

Table II: Investment Rates Moments by Sector

Investment rates	MIN	CON	MAN	TCU	WHO	RET	SRV
Average	0.23	0.20	0.37	0.34	0.19	0.18	0.31
Positive fraction, $i > 1$	0.62	0.88	0.91	0.83	0.76	0.75	0.90
Negative fraction, $i < -1$	0.05	0.09	0.02	0.03	0.09	0.07	0.04
Inaction rate	0.33	0.03	0.07	0.14	0.15	0.18	0.07
Spike rate, $\|i\| > 20$	0.52	0.67	0.82	0.61	0.52	0.45	0.71
Positive spikes, $i > 20$	0.52	0.58	0.81	0.60	0.49	0.42	0.69
Negative spikes, $i < -20$	0.01	0.09	0.01	0.01	0.03	0.03	0.01
Standard Deviation	0.27	0.25	0.23	0.34	0.25	0.27	0.25
Serial correlation, Corr(i_t, i_{t-1})	0.39	0.45	0.32	0.33	0.21	0.14	0.35

Note. This table shows the moments of the investment rate distribution of intangible capital across different sectors. The statistics are computed for a balanced panel between 1980 and 1990. MIN is the mining sector. CON is the construction sector. MAN is the manufacturing sector. TCU is the transportation and public utilities sector. WHO is the wholesale sector. RET is the retail sector. SRV is the services sector.

I.IV.II Investment Rates across Time

Table III reports the investment rate distribution across different time periods. We find that the evolution of the investment distribution of intangible capital is remarkably stable over different time periods. This is particularly true for the positive spike rate (75% in the second decade and 70% in the last part of the sample) and for the serial correlation (0.25 in the second decade and 0.23 in the last part of the sample), which remain substantially higher than the ones associated with tangible capital investment for the entire period of our analysis.

Table III: Investment Rates Moments by Period

Investment rates	1991-2000	2001-2015
Average	0.34	0.31
Positive fraction, $i > 1$	0.89	0.90
Negative fraction, $i < -1$	0.03	0.06
Inaction rate	0.08	0.04
Spike rate, $\|i\| > 20$	0.76	0.73
Positive spikes, $i > 20$	0.75	0.70
Negative spikes, $i < -20$	0.01	0.03
Standard deviation	0.26	0.30
Serial correlation, Corr(i_t, i_{t-1})	0.25	0.23

Note. This table shows the moments of the investment rate distribution of intangible capital over time. The statistics are computed for a balanced panel of firms between 1991 and 2000 and between 2001 and 2015.

I.IV.III Investment Rates across Firms of Different Age, Size, Leverage, and Liquidity Groups

Table IV examines the investment rate distribution across various firm groups—differentiating between young and old firms (below and above median age), small and large firms (below and above median sales), high and low leverage firms (below and above median leverage), and low and high liquidity firms (below and above median liquidity). Calculating leverage and liquidity as in Jeenas (2019), we observe consistent stability in the moments of the investment rate distribution for intangible capital across these different groups. The positive spike rate, always higher than that associated with tangible capital, ranges from 69% to 83%, while serial correlation, higher than tangible capital, varies from 0.27 to 0.38. These results suggest that distinctive features in the investment rate distribution of intangible and tangible capital are unlikely to be attributed to the behavior of specific firm groups, such as young, small, high-leverage, or low-liquidity firms.

Table IV: Investment Rates Moments by Age, Size, Leverage, and Liquidity

Investment rates	Age		Size		Leverage		Liquidity	
	Young	Old	Small	Large	High	Low	Low	High
Average	0.37	0.33	0.35	0.35	0.31	0.37	0.33	0.37
Positive fraction, $i > 1$	0.86	0.90	0.84	0.91	0.85	0.92	0.88	0.91
Negative fraction, $i < -1$	0.02	0.03	0.02	0.02	0.03	0.01	0.02	0.01
Inaction rate	0.12	0.08	0.14	0.07	0.12	0.07	0.10	0.08
Spike rate, $\|i\| > 20$	0.73	0.79	0.70	0.82	0.70	0.84	0.78	0.79
Positive spikes, $i > 20$	0.72	0.77	0.70	0.81	0.69	0.83	0.77	0.79
Negative spikes, $i < -20$	0.01	0.01	0.01	0.01	0.02	0.00	0.01	0.01
Standard deviation	0.30	0.23	0.29	0.23	0.28	0.21	0.25	0.25
Serial correlation, Corr(i_t, i_{t-1})	0.27	0.32	0.29	0.34	0.37	0.38	0.35	0.31

Note. This table shows the moments of the investment rate distribution of intangible capital across different types of firms. The statistics are computed for a balanced panel of firms between 1980 and 1990. Young firms are firms with age below the median. Old firms are firms with age above the median. Small firms are firms with sales below the median. Large firms are firms with sales above the median. High leverage firms are firms with leverage above the median. Low leverage firms are firms with leverage below the median. Low liquidity firms are firms with liquidity below the median. High liquidity firms are firms with liquidity above the median.

Our results imply also that, despite the recognized role of intangible capital in amplifying financial frictions (Falato et al., 2022; Caggese and Pérez-Orive, 2022), the distinctive features in the investment rate distribution of intangible and tangible capital are not significantly influenced by variations in financial frictions proxies such as age, size, leverage, and liquidity (Cloyne et al., 2023; Gertler and Gilchrist, 1994; Ottonello and Winberry, 2020; Jeenas, 2019).

I.IV.IV Investment Rates across Different Types of Intangible Capital

Table V details the investment rate distribution of balance sheet intangible capital (k_{BS}) and knowledge capital ($k_{R\&D}$). While differences highlight inherent heterogeneity among intangible capital types, some variations stem from construction: knowledge capital, capitalized from non-negative expenditures, lacks negative investments. We also notice that inaction in intangible capital investments is fully explained by knowledge capital, hence our conservative choice to focus only on spike rates and serial correlation. Despite these differences, both intangible capital types consistently exhibit higher positive spike rates (ranging from 48% to 78%) and serial correlation (ranging from 0.19 to 0.47) compared to tangible capital, reinforcing the robustness of our benchmark findings across diverse intangible assets.

Table V: Investment Rates Moments by Type

Investment rates	BS	R&D
Average	0.23	0.35
Positive fraction, $i > 1$	0.88	0.83
Negative fraction, $i < -1$	0.11	0.00
Inaction rate	0.01	0.17
Spike rate, $\|i\| > 20$	0.53	0.78
Positive spikes, $i > 20$	0.48	0.78
Negative spikes, $i < -20$	0.05	0.00
Standard deviation	0.32	0.23
Serial correlation, Corr(i_t, i_{t-1})	0.19	0.47

Note. This table shows the moments of the investment rate distribution across different types of intangible capital. The statistics are computed for a balanced panel of firms between 1980 and 1990. BS is the balance sheet stock of capital. R&D is the stock of knowledge capital.

I.IV.V Investment Rates Calculated with Alternative Specification

Table VI reports the investment rate distribution of intangible and tangible capital when calculated with the following alternative specification:

$$\frac{x_{j,ft}}{k_{j,ft-1}} \equiv \frac{k_{j,ft} - k_{j,ft-1}}{k_{j,ft-1}} + \delta_j, \quad j \in \{T, I\}. \tag{23}$$

We find that intangible capital still has a higher positive spike rate, 73% compared to 21% for tangible capital, and a higher serial correlation, 0.29 compared to 0.09 for tangible capital. These findings corroborate our benchmark results, suggesting that the way investment rates are calculated in the data does not play a role in the different behavior of the investment rates

Table VI: Investment Rates Moments Alternative Calculations

Investment rates	Intangible	Tangible
Average	0.35	0.13
Positive fraction, $i > 1$	0.89	0.86
Negative fraction, $i < -1$	0.04	0.11
Inaction rate	0.08	0.03
Spike rate, $\|i\| > 20$	0.75	0.25
Positive spikes, $i > 20$	0.73	0.21
Negative spikes, $i < -20$	0.02	0.03
Standard deviation	0.30	0.22
Serial correlation, Corr(i_t, i_{t-1})	0.29	0.09

Note. This table shows the moments of the investment rate distribution of intangible and tangible capital. The statistics are computed for a balanced panel of firms between 1980 and 1990.

of these two types of capital.

I.IV.VI Investment Rates in the Presence of Measurement Error

Here we test the implications of measurement error for the investment rate distribution. In particular, we assume that the intangible capital stock $k_{I,ft}$ is incorrectly measured, such that $k_{I,ft} = k^*_{I,ft} \exp(\omega_{ft})$ where $k^*_{I,ft}$ is the true intangible capital stock and $\exp(\omega_{ft})$ is the measurement error. Our specification follows Collard-Wexler and De Loecker (2021) and assumes that $\exp(\omega_{ft})$ is a classical measurement error. Hence, it is uncorrelated with true intangible capital stock but it is serially correlated over time. We assume that ω_{ft} follows a AR1 process given by

$$\omega_{ft} = \rho\omega_{ft-1} + \eta_{ft}, \quad \eta_{ft} \sim \mathcal{N}\left(0, \sigma^2_\eta\right); \tag{24}$$

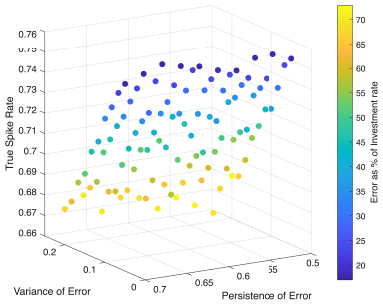
where ρ is the persistence and σ^2_η is the variance of the i.i.d. component.

Under this measurement error specification, we can rewrite the true intangible capital investment rate as

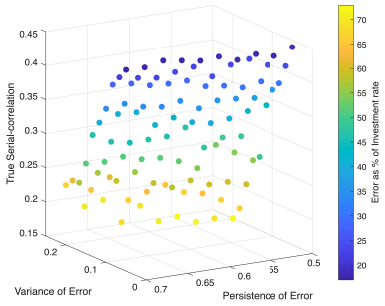
$$\frac{x^*_{I,ft}}{k^*_{I,ft-1}} \approx \frac{x_{I,ft}}{k_{I,ft-1}} - (1-\rho)\omega_{ft-1} - \eta_{ft}. \tag{25}$$

As the parameters governing the measurement are unknown, we compute the spike rate and the correlation of the true investment under different values of persistence and the variance of the shock, simulating the measurement error process over many realizations, and plot them in Figure IX.

Figure IX: True Spike Rate and Serial Correlation of Investment with Measurement Error



(a) True Spike Rate



(b) True Serial Correlation

Note. Figure IXa and IXb show the true spike rate and true serial correlation of investment on the z-axis, respectively. The x-axis shows the value for persistence ρ and the y-axis shows the values for variance σ_η^2 of the measurement error. The legend (yellow to dark blue) reports the level of measurement error as the proportion of the observed investment rate in the data, where dark blue represents low levels of measurement error (approx. 20%) and yellow represents high levels of measurement error (approx. 70%).

Varying levels of measurement error can distort the observed capital stock and, consequently, the recorded investment rate. Assessing this impact on the actual investment rate, we find that changes in measurement error do influence the true spike rate and correlation. However, this impact remains modest, even when measurement error constitutes a substantial portion of the observed investment rate, reaching up to around 70%. Crucially, both the spike rate and serial correlation, critical for determining adjustment costs in the model, consistently maintain significantly higher values compared to their tangible investment rate counterparts.

I.IV.VII Investment Rates with Goodwill as Part of Balance Sheet Intangible Capital

In order to asses the important of the exclusion of Goodwill from our baseline measure of balance sheet intangible capital, we present an alternative measure which capitalizes 38% of it, as suggested in Ewens et al. (2025), i.e., $k_{BS} = \texttt{INTAN} - 0.62 \times \texttt{GDWL} + \texttt{AM}$. Our findings are summarized in Table VII. Overall, we see that changes in the stock of intangible capital produce only minimal changes in the derived investment distribution. Thus, we conclude that the exclusion of Goodwill from our baseline measure is unlikely to be driving our findings.

I.IV.VIII Investment Rates with an Alternative Deflator

Here we show that using the R&D deflator instead of the IPP deflator, which is not influenced by the large decline in the real price of software does not alter our findings. Table VIII il-

Table VII: Investment Rates when Goodwill is Capitalized

Investment rates	Baseline	W/ Goodwill
Average	0.35	0.33
Positive fraction, $i > 1$	0.88	0.88
Negative fraction, $i < -1$	0.02	0.03
Inaction rate	0.10	0.09
Spike rate, $\|i\| > 20$	0.77	0.73
Positive spikes, $i > 20$	0.76	0.72
Negative spikes, $i < -20$	0.01	0.02
Standard deviation	0.26	0.28
Serial correlation, Corr(i_t, i_{t-1})	0.31	0.28

Note. This table shows the moments of the investment rate distribution of intangible capital under different assumptions. The statistics are computed for a balanced panel of firms between 1980 and 1990.

lustrates this points. While some differences are observed, they are quantitatively minimal, indicating that variations due to different deflators do not appear to drive our main empirical results.

Table VIII: Investment Rates when Using a Different Deflator

Investment rates	Baseline	R&D deflator
Average	0.35	0.34
Positive fraction, $i > 1$	0.88	0.88
Negative fraction, $i < -1$	0.02	0.02
Inaction rate	0.10	0.10
Spike rate, $\|i\| > 20$	0.77	0.76
Positive spikes, $i > 20$	0.76	0.75
Negative spikes, $i < -20$	0.01	0.01
Standard deviation	0.26	0.26
Serial correlation, Corr(i_t, i_{t-1})	0.31	0.32

Note. This table shows the moments of the investment rate distribution of intangible capital under different assumptions. The statistics are computed for a balanced panel of firms between 1980 and 1990.

I.V Robustness for Marginal Revenue Product of Both Types of Capital

In this section, we examine the robustness of our results on the higher relative responsiveness and dispersion of the marginal revenue product of intangible capital compared to tangible capital by considering the impact of firm-level heterogeneous markups, a common feature in the data (De Loecker et al., 2020). Heterogeneous markups distort the marginal revenue

product of both types of capital because firms with different levels of market power have different incentives to suppress output and hence input demand (Peters, 2020; Edmond et al., 2023). In the presence of these markups, without adjustment frictions, we derive the following relation between the marginal revenue product of capital, markups (μ), and the marginal cost of capital:

$$r + \delta_j \propto \frac{MRPK_j}{\mu}, \quad j \in \{T, I\}. \tag{26}$$

Equation (26) shows that now the relevant object of interest is not the $MRPK_j$ but the ratio $MRPK_j/\mu$, referred to as the adjusted marginal revenue product of capital.[8]

I.V.I Responsiveness

We examine the responsiveness of our adjusted marginal revenue product. Table IX reports regression coefficients, comparing the baseline and alternative specifications. Positive and significant coefficients ($\gamma_1 > 0$) persist across all specifications for both tangible and intangible capital. Notably, the adjusted marginal revenue product of intangible capital exhibits a stronger reaction to revenue productivity shocks than its tangible counterpart, supporting our main result that intangible capital faces greater adjustment frictions.

Table IX: Heterogeneous Response of $MRPK_T/\mu$ and $MRPK_I/\mu$ to $TFPR$ Shocks

	Baseline Specification		Alternative Specification	
	(1)	(2)	(3)	(4)
Dependent Variable	$MRPK_{T,ft}/\mu_{ft}$	$MRPK_{I,ft}/\mu_{ft}$	$MRPK_{T,ft}/\mu_{ft}$	$MRPK_{I,ft}/\mu_{ft}$
ε_{ft}	0.84***	1.11***	0.81***	1.13***
	(0.01)	(0.01)	(0.01)	(0.01)
Time dummies	✓	✓	✓	✓
Firm dummies	✓	✓	✓	✓
Controls	✓	✓	✓	✓
Observations	88,964	88,964	80,485	80,485

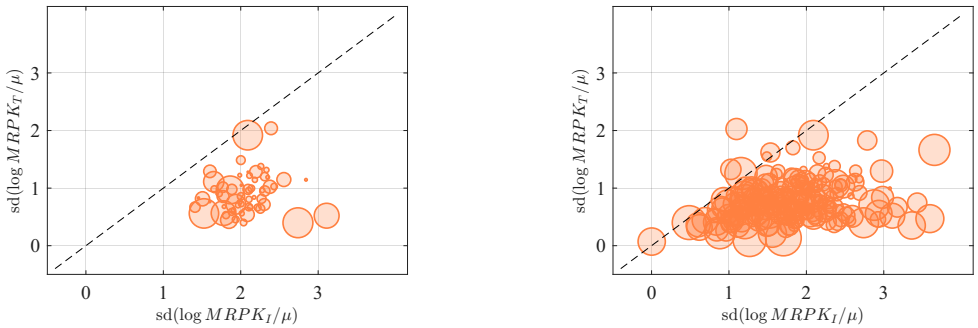
Notes. We report the coefficients from the regressions of marginal revenue product of tangible capital, $MRPK_{T,ft}$, and marginal revenue product of intangible capital, $MRPK_{I,ft}$, on revenue productivity shocks, ε_{ft}. The controls include sales, leverage, and liquidity. The baseline specification, which controls for classical (fixed and iid) measurement error, is shown in the main text. The alternative specification, which controls for serially correlated measurement error, is presented also in the main text. Standard errors are in parentheses. *** p-value<0.01, ** p-value<0.05, * p-value<0.1.

[8] Markups calculations follow De Loecker et al. (2020)

I.V.II Dispersion

In Figure X, considering both SIC2 and SIC3 levels, our robustness exercise reaffirms that the adjusted marginal revenue product of intangible capital is consistently more dispersed than that of tangible capital across the majority of sectors.

Figure X: Sector-Level Dispersion in $MRPK_I$ and $MRPK_T$



(a) SIC2-Level **(b) SIC3-Level**

Note. The figures show the standard deviation of $MRPK_I/\mu$ (x-axis) and the standard deviation of $MRPK_T/\mu$ (y-axis). Standard deviations are calculated within sectors and averaged across the years. Marginal revenue products are constructed as described in the text. The dashed black line shows the 45-degree line. Figure Xa is constructed by calculating standard deviations at the SIC2 level; each circle represents a SIC2 sector, where the size of the circle is proportional to its size (sale weighted) in Compustat. Figure Xb is constructed by calculating standard deviations at the SIC3 level; each circle represents a SIC3 sector, where the size of the circle is proportional to its size (sale weighted) in Compustat.

II Quantitative Appendix

II.I Alternative Modeling Assumptions

II.I.I Griliches (1979)' Knowledge Capital Model

This model interprets intangible capital as an endogenous productivity shifter. The resulting production function aligns with the production function estimated in the main text, where $\nu k_{I,ft} + \omega_{ft}$ represents total productivity, with the first component being endogenous and the second exogenous. While maintaining the benchmark structure, parameter ν signifies intangible capital's importance for firm productivity.

II.I.II Intangible Capital as a Demand Shifter

In this section, we show that introducing intangible capital as a demand shifter in the presence of a standard CES demand leads to a model that is isomorphic to the one used in the paper.

Assuming that the demand function faced by firm f is given by

$$q = p^{-\sigma} k_I^{\nu} C, \tag{27}$$

where k_I is the stock of intangible capital, p is the price charged by the firm, and C is aggregate consumption. Notice that this demand function can easily be microfounded through a standard CES structure, where intangible capital influences the value that the final consumer experiences from a given variety (Sedláček and Sterk, 2017). The production technology, in this case, is simply given by

$$q = e^{z} k_T^{\alpha} \ell^{1-\alpha}, \tag{28}$$

where k_T is tangible capital, ℓ is labor, and z is the idiosyncratic productivity. In this environment, the static profit maximization problem of the firm is

$$\begin{aligned} \pi &= \max_{p,\ell} pq - W\ell, \\ q &= p^{-\sigma} k_I^{\nu} C, \\ q &= e^{z} k_T^{\alpha} \ell^{1-\alpha}; \end{aligned} \tag{29}$$

which can alternatively be restated as

$$\pi = \max_{\ell} e^{\widehat{z}} k_T^{\widehat{\alpha}} k_I^{\widehat{\nu}} \ell^{\widehat{1-\alpha-\nu}} - W\ell, \tag{30}$$

where $e^{\widehat{z}} \equiv e^{z\frac{\sigma-1}{\sigma}} C^{\frac{1}{\sigma}}$, $\widehat{\alpha} \equiv \alpha(\sigma-1)/\sigma$, $\widehat{\nu} \equiv \nu/\sigma$, and $\widehat{1-\alpha-\nu} \equiv (1-\alpha)(\sigma-1)/\sigma$. Hence, the problem stated in equation (30) is isomorphic to the problem stated in the main text of the benchmark model and proposes an additional rationalization of the empirical approach proposed in the main text.

II.I.III Intangible Capital, Returns to Scale, and Market Power

In this section, we show that in the presence of a standard CES demand the presence of empirically plausible increasing returns to scale would lead to a model that is isomorphic to the one used in the quantitative theoretical part of the main text. We assume that the demand

function faced by firm f is given by

$$q = p^{-\sigma} C, \tag{31}$$

where p is the price charged by the firm and C is aggregate consumption. Notice that this demand function can easily be microfounded through a standard CES structure. The production technology, in this case, is simply given by

$$q = e^{z} \left(k_T^{\alpha} k_I^{\nu} \ell^{1-\alpha} \right)^{\omega}, \tag{32}$$

where k_T is tangible capital, k_I is intangible capital, ℓ is labor, and z is the idiosyncratic productivity. In this environment, the static profit maximization problem of the firm is

$$\begin{aligned} \pi &= \max_{p,\ell} pq - W\ell, \\ q &= p^{-\sigma} C, \\ q &= e^{z} \left(k_T^{\alpha} k_I^{\nu} \ell^{1-\alpha} \right)^{\omega}; \end{aligned} \tag{33}$$

which can alternatively be restated as

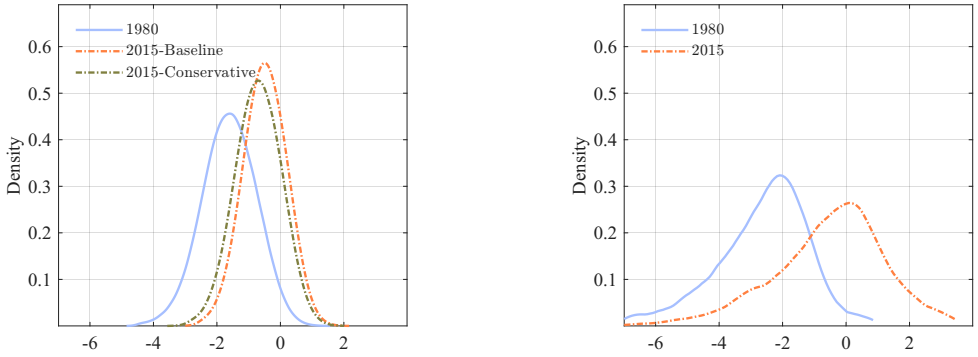
$$\pi = \max_{\ell} e^{\widehat{z}} \left(k_T^{\alpha} k_I^{\nu} \ell^{(1-\alpha-\nu)} \right)^{\widehat{\omega}} - W\ell, \tag{34}$$

where $e^{\widehat{z}} \equiv e^{z \frac{\sigma-1}{\sigma}} C^{\frac{1}{\sigma}}$ and $\widehat{\omega} = \omega(\sigma - 1)/\sigma$ is the curvature of the revenue function. Hence, the problem stated in equation (34) is isomorphic to the problem stated in the main text of the benchmark model. This finding shows that calibrating a competitive economy with decreasing returns to scale or a monopolistically competitive model with CES demand, mild increasing returns to scale, and empirically meaningful market power is observationally equivalent.

II.II Additional Comparisons between Model and Data over Time

Figure XI illustrates the changing distribution of intangible intensity over time, comparing model predictions with data. Despite some qualitative differences, both exhibit a rightward shift, indicating an increasing use of intangible capital relative to labor by firms. In Figure XIIa, the evolution of the $TFPR$ distribution in both model and data for 2015 reveals increased dispersion, signaling a decline in allocative efficiency.

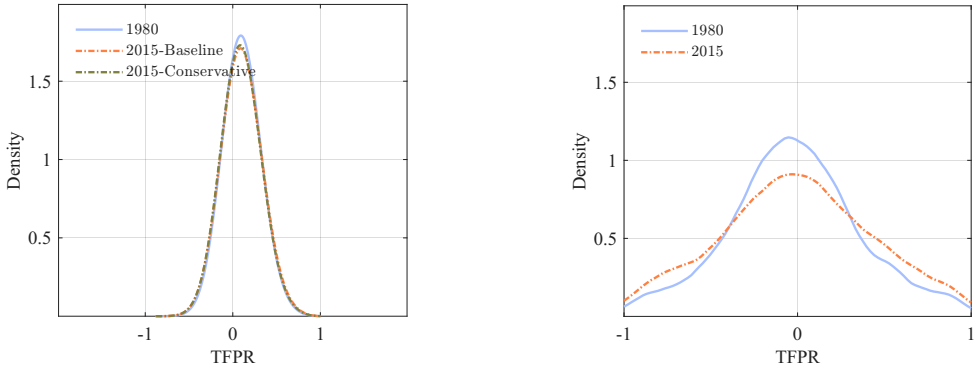
Figure XI: Intangible Intensity



(a) Model **(b) Data**

Note. Figure XIa shows the distribution of log intangible intensity in be 1980 (solid light blue line) and 2015 (dashed orange line) from the model. Figure XIb shows the same distributions from the data.

Figure XII: Total Factor Productivity Revenue



(a) Model **(b) Data**

Note. Figure XIIa shows the distribution of $TFPR$ in 1980 (solid light blue line) and 2015 (dashed orange line) from the model. Figure XIIb shows the same distributions from the data. All distributions are demeaned.

II.III Additional Robustness Quantitative Implications of IBTC

Table X displays the moment fit of the different calibrations, while XI shows the implied parameters. In all calibrations, the model aligns well with the targeted moments. Notably, the results consistently indicate higher adjustment costs for intangible capital compared to tangible capital. Specifically, when using inaction rates, the model infers a more substantial difference in fixed adjustment costs between intangible and tangible capital than in the benchmark calibration. This underscores that targeting spike rates across capital types is a conservative calibration choice. Overall, the conclusion that intangible capital faces higher adjustment costs remains robust across different calibration strategies.

For the CES production function, we assume the following production function:

$$y = e^z \left(\nu k_I^{\frac{\sigma_I - 1}{\sigma_I}} + \left(\alpha k_T^{\frac{\sigma_T - 1}{\sigma_T}} + (1 - \alpha - \nu) \ell^{\frac{\sigma_T - 1}{\sigma_T}} \right)^{\frac{\sigma_T}{\sigma_T - 1} \frac{\sigma_I - 1}{\sigma_I}} \right)^{\frac{\gamma \sigma_I}{\sigma_I - 1}}.$$

where, ν, α, and $1 - \alpha - \nu$ are input weights for intangible capital, tangible capital, and labor, respectively. For the counterfactual experiment where the input weights change, we increase ν by 7 percentage points and reduce the labor weight by the same amount.

Table X: Alternative Calibrations: Moments

	1980				2015	
Target Moments	Convex Costs Only	Matching Inaction Rates	CES	Data	Alternative Adj. Costs	Data
Investment Rate Distributions						
Average investment rate x_T	0.18	0.18	0.19	0.11	0.15	0.11
Average investment rate x_I	0.39	0.38	0.38	0.35	0.38	0.32
corr $(x_{T,ft}, x_{T,ft-1})$	0.09	0.10	0.11	0.09	0.09	0.09
corr $(x_{I,ft}, x_{I,ft-1})$	0.31	0.31	0.27	0.31	0.23	0.22
Positive spike rate x_T	–	–	0.24	0.19	0.23	0.19
Positive spike rate x_I	–	–	0.52	0.76	0.55	0.75
Inaction rate x_T	–	0.03	–	0.03	–	–
Inaction rate x_I	–	0.11	–	0.10	–	–
Firm Dynamics						
Entry rate	0.11	0.10	0.13	0.12	–	–
Average firm size	21.0	20.7	25.8	20.6	–	–
Average entrant size	5.96	6.15	5.50	6.11	–	–
Wage	1.00	1.00	1.00	–	–	–

Table XI: Alternative Calibrations: Parameters

	Convex Costs Only	Matching Inaction Rates	Alternative Adj. Costs	CES	
Fitted Parameters	Value				Description
Investment Adjustment Costs					
γ_T	0.038	0.039	0.045	0.075	Convex adjustment cost k_T
γ_I	0.680	0.700	0.370	0.450	Convex adjustment cost k_I
f_T	0	0.002	0.030	0.044	Fixed adjustment cost k_T
f_I	0	0.022	0.035	0.084	Fixed adjustment cost k_I
Firm Dynamics					
c_e	0.120	0.120	–	0.350	Entry cost
c_f	1.950	1.950	–	1.950	Operating cost
η	2.705	2.710	–	3.845	Scale parameter
m	0.024	0.025	–	0.015	Measure of potential entrants
CES					
σ_T	–	–	–	0.450	Tangible Capital & Labor
σ_I	–	–	–	2.550	Composite bundle and Intangible capital
ν	–	–	–	0.150	Intangible Capital Weight
α	–	–	–	0.290	Tangible capital Weight
$1 - \nu - \alpha$	–	–	–	0.560	Labor Weight

References

Ackerberg, D. A., K. Caves, and G. Frazer (2015). Identification properties of recent production function estimators. *Econometrica 83*(6), 2411–2451.

Caggese, A. and A. Pérez-Orive (2022). How stimulative are low real interest rates for intangible capital? *European Economic Review 142*, 103987.

Cloyne, J., C. Ferreira, M. Froemel, and P. Surico (2023). Monetary policy, corporate finance and investment. *Journal of the European Economic Association*, jvad009.

Collard-Wexler, A. and J. De Loecker (2021). Production function estimation with measurement error in inputs. Technical report, National Bureau of Economic Research.

De Loecker, J., J. Eeckhout, and G. Unger (2020). The rise of market power and the macroeconomic implications. *The Quarterly Journal of Economics 135*(2), 561–644.

De Loecker, J., P. K. Goldberg, A. K. Khandelwal, and N. Pavcnik (2016). Prices, markups, and trade reform. *Econometrica 84*(2), 445–510.

De Loecker, J. and P. T. Scott (2016). Estimating market power evidence from the us brewing industry. Technical report, National Bureau of Economic Research.

Edmond, C., V. Midrigan, and D. Y. Xu (2023). How costly are markups? *Journal of Political Economy 131*(7), 1619–1675.

Ewens, M., R. H. Peters, and S. Wang (2025). Measuring intangible capital with market prices. *Management Science 71*(1), 407–427.

Falato, A., D. Kadyrzhanova, J. Sim, and R. Steri (2022). Rising intangible capital, shrinking debt capacity, and the us corporate savings glut. *The Journal of Finance 77*(5), 2799–2852.

Foster, L., J. Haltiwanger, and C. Syverson (2008). Reallocation, firm turnover, and efficiency: selection on productivity or profitability? *American Economic Review 98*(1), 394–425.

Gandhi, A., S. Navarro, and D. A. Rivers (2020). On the identification of gross output production functions. *Journal of Political Economy 128*(8), 2973–3016.

Gao, W. and M. Kehrig (2017). Returns to scale, productivity and competition: Empirical evidence from us manufacturing and construction establishments. *Productivity and Competition: Empirical Evidence from US Manufacturing and Construction Establishments (May 1, 2017)*.

Gertler, M. and S. Gilchrist (1994). Monetary policy, business cycles, and the behavior of small manufacturing firms. *The Quarterly Journal of Economics 109*(2), 309–340.

Griliches, Z. (1979). Issues in assessing the contribution of research and development to productivity growth. *The bell journal of economics*, 92–116.

Jeenas, P. (2019). Firm balance sheet liquidity, monetary policy shocks, and investment dynamics. *Work 5*.

Lev, B. and F. Gu (2016). *The end of accounting and the path forward for investors and managers*. John Wiley & Sons.

Levinsohn, J. and A. Petrin (2003). Estimating production functions using inputs to control for unobservables. *The review of economic studies 70*(2), 317–341.

Ottonello, P. and T. Winberry (2020). Financial heterogeneity and the investment channel of monetary policy. *Econometrica 88*(6), 2473–2502.

Peters, M. (2020). Heterogeneous markups, growth, and endogenous misallocation. *Econometrica 88*(5), 2037–2073.

Peters, R. H. and L. A. Taylor (2017). Intangible capital and the investment-q relation. *Journal of Financial Economics 123*(2), 251–272.

Sedláček, P. and V. Sterk (2017). The growth potential of startups over the business cycle. *American Economic Review 107*(10), 3182–3210.