Supplemental Appendix to *Fiscal Policy in a Networked Economy* by Becko, Flynn, and Patterson

B. Rationing Equilibrium Microfoundation

In this appendix, we set up a generalization of our baseline model to allow for fully general rationing on either the supply or the demand side of the market when interest rates do not adjust to clear labor markets. We show that this model reduces to our baseline model under the following four economic conditions: 1) agents cannot be forced to work more or employ more workers than they would like, 2) rationing is minimally efficient in the sense that it meets either demand or supply, 3) aggregate labor demand is smaller than aggregate labor supply, and 4) there are no income effects in household labor supply. Finally, we sketch an extension of the model that accommodates multiple types of labor.

B.1. A general model of labor rationing

As in the main text, a finite number of competitive household types $n \in N$ with mass μ_n and firms $i \in I$ respectively supply and demand a homogeneous labor factor in order to produce goods over two periods $t \in \{1, 2\}$. Wages in both periods are sticky and so are normalized to one, i.e., $w^t = 1$. Due to a binding ZLB on nominal interest rates, the real interest rate r is exogenous. We denote by $p^t = \{p_i^t\}_{i \in I}$ the vector of prices in period t.

Given prices, each household n forms a hypothetical first-period labor supply ℓ_n^{*1} consistent with optimization subject to only (a) a borrowing constraint in the form of minimum savings $\underline{s}_n$ and (b) a budget constraint incorporating lump-sum taxes $\tau = (\tau_n^1, \tau_n^2)$.

$$\begin{aligned} \ell_n^{*1} \in \arg\max_{\ell^1} \max_{\ell^2, c^1, c^2} & \sum_{t=1,2} \beta_n^{t-1} u_n^t(c^t, \ell^t) \\ \text{s.t. } & p^1 c^1 + \frac{p^2 c^2}{1+r} + \tau_n^1 + \frac{\tau_n^2}{1+r} = \ell^1 + \frac{\ell^2}{1+r} \\ & \ell^1 - p^1 c^1 - \tau_n^1 \geqslant \underline{s}_n \end{aligned} \tag{A26}$$

Similarly, each firm i forms a hypothetical first-period labor demand L_n^{*1} consistent with profit maximization, given a CRS production function $F_i^{t=1}$ that incorporates the single labor factor as well as a vector of inputs (any goods).

$$L_i^{*1} \in \arg\max_{L^1} \max_{X^1} \; p_i^1 F_i^1(X^1, L^1) - p^1 X^1 - L^1 \tag{A27}$$

Next, a non-price mechanism that we refer to as the *rationing function* assigns to each house-

hold and firm its realized first-period labor supply and demand, respectively, as a function of the vectors $\ell^{*1} = \{\ell_n^{*1}\}_{n\in N}$ and $L^{*1} = \{L_i^{*1}\}_{i\in I}$ of all hypothetical, preferred labor supplies and demands, respectively.

$$\ell_n^1 = R_n^S\left(\ell^{*1}, L^{*1}\right) \qquad\qquad L_i^1 = R_i^D\left(\ell^{*1}, L^{*1}\right) \tag{A28}$$

Although we remain agnostic to the details of labor assignment, we assume the rationing function assigns equal amounts of labor supply and demand, i.e., $\sum_{n\in N} \mu_n R_n^S\left(\ell^{*1}, L^{*1}\right) = \sum_{i\in I} R_i^D\left(\ell^{*1}, L^{*1}\right)$.

Taking its rationed labor supply as given, each household n chooses second-period labor supply ℓ_n^2 and vectors of first- and second-period consumption $c_n^t = \{c_{ni}^t\}_{i\in I}$.

$$\begin{aligned}(\ell_n^1, \ell_n^2, c_n^1, c_n^2) \in \underset{\ell^1,\ell^2,c^1,c^2}{\arg\max} & \sum_{t=1,2} \beta_n^{t-1} u_n^t(c^t, \ell^t) \\ \text{s.t. } & p^1 c^1 + \frac{p^2 c^2}{1+r} + \tau_n^1 + \frac{\tau_n^2}{1+r} = \ell^1 + \frac{\ell^2}{1+r} \\ & \ell^1 - p^1 c^1 - \tau_n^1 \geqslant \underline{s}_n \\ & \ell^1 = \ell_n^1\end{aligned} \tag{A29}$$

Taking its rationed labor demand as given, each firm i chooses a vector of first-period inputs $X_i^1 = \{X_{ij}^1\}_{j\in I}$ to maximize profits; in the second period, i chooses both labor L^2 and inputs $X_i^2 = \{X_{ij}^2\}_{j\in I}$ to maximize profits given a CRS production function $F_i^{t=2}$.

$$\begin{aligned}(X_i^t, L_i^t) \in \underset{X^t, L^t}{\arg\max}\ & p_i^t F_i^t(X^t, L^t) - p^t X^t - L^t \\ \text{s.t. } & L^1 = L_i^1\end{aligned} \tag{A30}$$

In addition to levying lump-sum taxes τ_n^t on households, the government purchases G_i^t units of good $i \in I$ subject to running a balanced budget over the two periods.

$$\sum_{n\in N} \mu_n \left(\tau_n^1 + \frac{1}{1+r}\tau_n^2\right) = p^1 G^1 + \frac{1}{1+r} p^2 G^2 \tag{A31}$$

Finally, goods markets and the second period labor market clear.

$$F_i^t(X_i^t, L_i^t) = \sum_{j\in I} X_{ji}^t + \sum_{n\in N} \mu_n c_{ni}^t + G_i^t, \qquad\qquad \sum_{i\in I} L_i^2 = \sum_{n\in N} \mu_n l_n^2 \tag{A32}$$

A *general* rationing equilibrium is therefore defined as follows:

Definition 2. *Given a profile of government spending and transfers* $\{\tau_n^t, G_i^t\}_{t\in\{0,1\}, n\in N, i\in I}$ *that satisfies (A31), a general rationing equilibrium is a profile of prices and quantities* $\{p_i^t, \ell_n^{*1}, L_i^{*1}, l_n^t, c_n^t, L_i^t, X_{ij}^t\}_{t\in\{0,1\}, n\in N, i,j\in I}$ *that satisfy conditions (A26) – (A32).*

B.2. Microfounding the Reduced-Form Model

In this section we show how the more reduced-form model of rationing in the main text can be obtained as a special case of the more general rationing model above. This occurs under three conditions: That the rationing function satisfies feasibility and allocative efficiency properties, demand is deficient, and that there are no income effects in labor supply.

We begin by imposing two intuitive assumptions on the rationing function.

Assumption 1. *The rationing function (R^S, R^D) satisfies the following conditions:*

1. *Feasibility: For all ℓ^{*1} and L^{*1}, for all $i \in I$ and $n \in N$,*

$$R_n^S\left(\ell^{*1}, L^{*1}\right) \leqslant \ell_n^{*1} \qquad \text{and} \qquad R_i^D\left(\ell^{*1}, L^{*1}\right) \leqslant L_i^{*1}. \tag{A33}$$

2. *Allocative efficiency: For all ℓ^{*1} and L^{*1}, there do not exist $i \in I$ and $n \in N$ such that*

$$R_n^S\left(\ell^{*1}, L^{*1}\right) < \ell_n^{*1} \qquad \text{and} \qquad R_i^D\left(\ell^{*1}, L^{*1}\right) < L_i^{*1}. \tag{A34}$$

Feasibility captures the idea that firms and households may not be forced to hire more workers or work more than they wish, respectively. Allocative efficiency posits that the rationing mechanism ensures there are not firms and workers who would like to be matched, but are not. We view this as a reasonable assumption when wage stickiness—and not for example search frictions—is the main friction.

We now impose that we are in a region where aggregate ideal labor demand is less than aggregate ideal labor supply.

Assumption 2. *There is strictly deficient demand in any equilibrium $\sum_{i \in I} L_i^{*1} < \sum_{n \in N} \mu_n \ell_n^{*1}$.*

Finally, we assume that households have the intratemporal preferences of Greenwood, Hercowitz, and Huffman (1988), so that there are no income effects in labor supply:

Assumption 3. *Household preferences are GHH, i.e., for all $n \in N, t \in \{1, 2\}$, there exist concave functions U_n^t, strictly increasing and homothetic functions ν_n^t, and strictly convex functions Λ_n^t, all real-valued and increasing, such that for all consumption vectors c and labor supplies ℓ,*

$$u_n^t(c, \ell) = U_n^t\left(\nu_n^t(c) - \Lambda_n^t(\ell)\right). \tag{A35}$$

The following result shows that rationing equilibria and general rationing equilibria coincide under these assumptions:

Proposition 3. *Under Assumptions 1, 2, and 3, each household n has a unique preferred level of labor supply $\ell_n^{*1} = \bar{\ell}_n^{*1}$, independent of government spending and transfer policies. Moreover, under*

any government spending and transfer policies, we have that $\mu_n \ell_n^1 = R_n^1(L^{*1}) \equiv R_n^S(\bar{\ell}^{*1}, L^{*1})$ *and* $L_i^1 = L_i^{*1}$*. Thus, the set of rationing equilibria and general rationing equilibria are equal.*

Proof. Since, by Assumption 3, ν_n^t is strictly increasing and homothetic, and since p^t is independent of policy, each household consumes a fixed basket of goods α_n^t with non-negative weights on each good. We can therefore write $\nu_n^t(c^t) = \nu_n^t(\{\alpha_{ni}^1 \tilde{c}_n\}_{i \in \mathcal{I}}) = \omega_n^t \tilde{c}_n^t$ for some constant $\omega_n^t > 0$. Each household's choice of consumption can then be reduced to choosing $\tilde{c}_n^t$. Recall we can express the household's budget constraint as:

$$p^1 \alpha_n^1 \tilde{c}^1 + \frac{p^2 \alpha_n^2 \tilde{c}^2}{1+r} + \tau_n^1 + \frac{\tau_n^2}{1+r} = \ell^1 + \frac{\ell^2}{1+r} \tag{A36}$$

By the intratemporal Euler equation of the household, by strict convexity of Λ_n^t, we can express its optimal labor supply as:

$$\ell_n^{*t} = \bar{\ell}_n^{*t} \equiv \left(\Lambda_n^{t\,\prime}\right)^{-1} \left(\omega_n^t\right) \tag{A37}$$

if $\left(\Lambda_n^{t\,\prime}\right)^{-1} (\omega_n^t)$ lies in the positive reals and zero otherwise. The definition of generalized rationing equilibrium therefore implies $\mu_n \ell_n^1 = R_n^S(\ell^{*1}, L^{*1}) = R_n^S(\bar{\ell}^{*1}, L^{*1}) \equiv R_n^1(L^{*1})$, as desired.

By Assumption 2, we have that demand is strictly deficient $\sum_{i \in I} L_i^{*1} < \sum_{n \in N} \ell_n^{*1}$. Next, we claim Assumption 1 implies that if there is strictly deficient demand, then $L_i^1 = L_i^{*1}$. To see this, first note that the feasibility property and deficient demand together imply there exists some n such that $\ell_n^1 < \ell_n^{*1}$. But then if there is some i such that $L_i^1 < L_i^{*1}$, allocative efficiency cannot hold, a contradiction. □

B.3. Extension to multiple labor types and/or labor markets

The microfoundation above can be extended to accommodate many labor types (firm preferences over workers) and many labor markets (worker preferences over firms) under a two key additional assumptions. We sketch the arguments here, as they are essentially identical to the ones above. First, *all* first period wages are rigid, as are the expectations of all second period wages (or their absolute and relative inflation rates). This prevents any adjustment through prices, as in the one-factor model. Second, there is deficient demand not in just one labor market, but in all of them. This guarantees that firms are rationed the labor they demand. Again, households' GHH preferences ensure that fiscal policies do not affect their preferred levels of labor supply locally, so that we obtain the same formulation as before. Since the rationing function in the reduced-form model in the main text conditions on the identities of all workers and all firms, we may interpret it as matching the appropriate labor types to firms in the appropriate labor markets, under these conditions of extreme rigidity and demand deficiency.

C. Additional Results

In this appendix, we present results on properties (including existence) of rationing equilibrium (C.1), provide comparative statics for the multiplier (C.2), and analyze benchmark cases in which various network adjustments to the Keynesian multiplier are zero (C.3).

C.1. Equilibrium Properties

In this appendix, we ensure our analysis of the multiplier is well-posed and eliminate any nuisance terms that unnecessarily complicate the analysis. To this end, we first provide a non-substitution theorem that ensures prices are technologically determined—and thus independent of fiscal policies—and, second, prove the existence of a rationing equilibrium.

The following technical conditions on production technologies and household preferences are sufficient for the non-substitution theorem. Assumption 4 provides basic technical conditions on production and Assumption 5 imposes a simple positivity condition on demand such that there is demand for all goods.

Assumption 4. *For all $i \in N$ and $t \in \{1,2\}$, the production functions F_i^t are continuous, weakly increasing, strictly quasi-concave, and homogeneous of degree one. Further, labor is essential in production, i.e., $F_i^t(X_i^t, 0) \equiv 0$, and production is strictly increasing in labor. Finally, for $t \in \{1,2\}$, there exists some $\overline{p}^t \in \mathbb{R}^I_{>0}$ and $\{\overline{X}_i^t, \overline{L}_i^t\}_{i\in I}$ s.t. for all i, $F_i^t(\overline{X}_i^t, \overline{L}_i^t, z_i^t) \geqslant 1$ and $\overline{p}^t\overline{X}_i^t + \overline{L}_i^t \leqslant \overline{p}_i^t$.*[42]

Assumption 5. *For any p, ℓ^1, τ, θ: for each good i and time t, some household type n has $c_{ni}^t > 0$.*

Under these two rather weak assumptions, we can show that:

Proposition 4. *Fix a profile of government spending and transfers $\mathcal{P} = \{\tau_n^t, G_i^t\}_{t\in\{0,1\},n\in N,i\in I}$ that satisfies (4) and suppose that Assumptions 4 and 5 hold. There exists a unique $p^t(\mathcal{P}) \in \mathbb{R}^I_{\geqslant 0}$ consistent with rationing equilibrium and, moreover, $p^t(\mathcal{P}) \gg 0$. Furthermore, $p^t(\mathcal{P})$ is invariant to $\mathcal{P}$.*

Proof. We prove the existence of demand-independent prices with lattice-theoretic argument similar to that of Acemoglu and Azar (2020) and their uniqueness with an argument similar to that of Stiglitz (1970).

Preliminaries. Fix a time period $t \in \{1,2\}$ and a firm $i \in I$. We define i's unit cost function (at this time and technology) $\kappa_i : \mathbb{R}^I_{>0} \to \mathbb{R}$ as the function that maps any strictly positive vector of prices to i's least cost of production:

$$\kappa_i^t(p) \equiv \min_{X_i^t, L_i^t \geqslant 0,\ F_i^t(X_i^t, L_i^t)\geqslant 1} pX_i^t + L_i^t. \tag{A38}$$

[42] A sufficient but not necessary condition is that every good can be produced using only labor.

This function is well-defined because, by Assumption 4, i can produce a unit of output with the input bundle $\overline{X}_i^t, \overline{L}_i^t$. Thus, we may restrict the domain over which the firm optimizes in (A38) to the set

$$\left\{ X_i^t \in \mathbb{R}^I_{\geqslant 0}, L_i^t \in \mathbb{R}_{\geqslant 0} \;\middle|\; \forall j \in I, X_{ij}^t \leqslant \frac{\overline{\kappa}_i^t(p)}{p_j},\ L_i^t \leqslant \overline{\kappa}_i^t(p),\ F_i^t(X_i^t, L_i^t) \geqslant 1 \right\} \tag{A39}$$

where $\overline{\kappa}_i^t(p) \equiv p\overline{X}_i^t + \overline{L}_i^t$. This set is compact by continuity of F_i^t (from Assumption 4). Thus, since the objective of (A38) is continuous, a minimum exists by Weierstrass' Theorem.

Now note that by Assumption 5 and market clearing, each firm $i \in I$ has strictly positive output in any equilibrium. Firm optimization and CRS (homogeneity of degree one of F_i^t from Assumption 4) thus imply that each firm's price is equal to its marginal cost, which also equals its average cost. Since labor is essential (from Assumption 4) and has a wage normalized to one, this cost must moreover be strictly positive in any equilibrium. We conclude that any equilibrium price vector at time $t \in \{1, 2\}$ $p^t \in \mathbb{R}^I_{\geqslant 0}$ must be strictly positive and satisfy:

$$\forall i \in I, \quad p_i^t = \kappa_i^t(p^t). \tag{A40}$$

The remainder of the proof shows that such a price vector exists, is unique, and invariant to $\mathcal{P}$.

Existence. Recall the price vector $\overline{p}^t$ from Assumption 4. By that assumption and the definition of κ_i^t, we have

$$\forall i \in I, \quad \kappa_i^t(\overline{p}^t) \leqslant \overline{p}_i^t. \tag{A41}$$

Moreover, we claim that there exists an $\alpha > 0$ such that:

$$\forall i \in I, \quad \kappa_i^t(\alpha\overline{p}^t) \geqslant \alpha\overline{p}_i^t. \tag{A42}$$

To see this, suppose it does not hold. Then there exists a sequence $\{\alpha^k\}$ such that $\alpha^k \to 0$, a firm i, and a sequence of input bundles $\{X_i^k, L_i^k\}$ such that $X_i^k \in \mathbb{R}^I_{\geqslant 0}, L_i^k \in \mathbb{R}_{\geqslant 0}$ and for all $k \in \mathbb{N}$,

$$F_i^t(X_i^k, L_i^k) \geqslant 1 \qquad \text{and} \qquad \alpha\overline{p}^t X_i^k + L_i^k < \alpha\overline{p}_i^t. \tag{A43}$$

Since by assumption all components of $\overline{p}$ are strictly positive, this implies $L_i^k \to 0$ and each $X_i^k \leqslant \frac{\overline{p}_i^t}{\overline{p}^t}$. But then F_i^t's continuity and the Bolzano-Weierstrass Theorem imply that $F_i^t(X_i^\infty, 0) \geqslant 1$ at some limit point X_i^∞ of $(X_i^k)_{k\in\mathbb{N}}$. This contradicts that labor is essential (Assumption 4).

Since—as is immediate from the definition (A38)—each cost function κ_i is monotonically increasing, the observations above imply that the function $\kappa = (\kappa_1, ..., \kappa_{|I|})$ maps the set

$$\mathcal{D} \equiv \prod_{i\in I}[\alpha\overline{p}_i, \overline{p}_i] \tag{A44}$$

to itself for some $\alpha > 0$, namely one sufficiently small that the previous argument carries. Since $\mathcal{D}$ endowed with the standard component-wise partial order is a complete lattice, Tarski's fixed point theorem implies that the set of fixed points of κ on $\mathcal{D}$ is a complete lattice. In particular, it is non-empty. This implies the existence of a strictly positive solution to the fixed-point problem (A40).

Uniqueness. Suppose that p^t and $p^{t'}$ are two strictly positive solutions (A40). We will show $p^t = p^{t'}$. Let $(X_i^t)_{i\in I}$ and $(L_i^t)_{i\in I}$ be any vectors of firm-specific cost-minimizing unit input demands at p^t, and let $\widehat{\boldsymbol{X}}^{\boldsymbol{t}}$ be the $I \times I$ matrix with (i,j) entry X_{ji}^t and $\widehat{\boldsymbol{L}}^{\boldsymbol{t}}$ be the $I \times I$ diagonal matrix with ii^{th} entry L_i^t. The price-cost fixed point equation (A40) at p^t can therefore be written:

$$p^t = (\widehat{\boldsymbol{X}}^{\boldsymbol{t}})^T p^t + \widehat{\boldsymbol{L}}^{\boldsymbol{t}} \mathbb{1} \tag{A45}$$

We moreover claim that $\boldsymbol{I} - (\widehat{\boldsymbol{X}}^{\boldsymbol{t}})^T$ is invertible, so that we have

$$p^t = \left(\boldsymbol{I} - (\widehat{\boldsymbol{X}}^{\boldsymbol{t}})^T\right)^{-1} \widehat{\boldsymbol{L}}^{\boldsymbol{t}} \mathbb{1}. \tag{A46}$$

To see why, recall that since labor is essential for each good, $p_i^t > ((\widehat{\boldsymbol{X}}^{\boldsymbol{t}})^T p^t)_i$ for all $i \in I$. This implies $\boldsymbol{I} - (\widehat{\boldsymbol{X}}^{\boldsymbol{t}})^T$ is invertible, because if for any vector v, $v = (\widehat{\boldsymbol{X}}^{\boldsymbol{t}})^T v$, then for all $\alpha \in \mathbb{R}$ such that $p^t + \alpha v \geqslant 0$, the fact that $(\widehat{\boldsymbol{X}}^{\boldsymbol{t}})^T$ has all positive entries implies then $p^t + \alpha v > (\widehat{\boldsymbol{X}}^{\boldsymbol{t}})^T (p^t + \alpha v) \geqslant 0$. Since p^t is strictly positive, this is only possible when $v = 0$.

By cost-minimization, we have the component-wise inequality

$$p^{t'} \leqslant (\widehat{\boldsymbol{X}}^{\boldsymbol{t}})^T p^{t'} + \widehat{\boldsymbol{L}}^{\boldsymbol{t}} \mathbb{1}, \qquad \text{which implies} \qquad p^{t'} \leqslant \left(\boldsymbol{I} - (\widehat{\boldsymbol{X}}^{\boldsymbol{t}})^T\right)^{-1} \widehat{\boldsymbol{L}}^{\boldsymbol{t}} = p^t. \tag{A47}$$

The same argument implies $p^{t'} \geqslant p^t$, and so $p^t = p^{t'}$.

Invariance. We observe that the previous step constructed the unique equilibrium price vector and this construction was invariant to $\mathcal{P}$, completing the proof. □

The existence of unique, positive prices $p^t \in \mathbb{R}^I_>$ consistent with rationing equilibrium and that are invariant to policy is what allows us to set $p_i^t = 1$ for all $i \in I$ and $t \in \{1, 2\}$ in our analysis. Moreover, combining Proposition 4 with constant returns to scale technology implies a simple form for aggregate input and labor demands. To see this, first define the unit input and labor demands

$$\left(\widehat{X}_i^t, \widehat{L}_i^t\right) \equiv \arg \min_{(X_i^t, L_i^t) \text{ s.t. } F_i^t(X_i^t, L_i^t) \geqslant 1} p^t X_i^t + L_i^t \tag{A48}$$

where by Proposition 4, p^t is the unique price vector consistent with rationing equilibrium. These demands are well-defined because (a) a minimum cost exists by the argument in the first step of

the proof of Proposition 4 and (b) production is strictly quasi-concave by Assumption 4. We now claim:

Corollary 1. *Suppose that Assumptions 4 and 5 hold. In any rationing equilibrium, input demands X^t and labor demands L^t are given by:*

$$X^t = \widehat{\boldsymbol{X}}^{\boldsymbol{t}} Q^t \quad L^t = \widehat{\boldsymbol{L}}^{\boldsymbol{t}} Q^t \tag{A49}$$

where $\widehat{\boldsymbol{X}}^{\boldsymbol{t}}$ is the matrix with i^{th} column $\widehat{X}_i^t$ and $\widehat{\boldsymbol{L}}^{\boldsymbol{t}}$ is the diagonal matrix with (i,i) entry $\widehat{L}_i^t$.

Proof. CRS implies that for a firm producing Q_i^t units in equilibrium,

$$X_i^t = Q_i^t \widehat{X}_i^t \quad L_i^t = Q_i^t \widehat{L}_i^t \tag{A50}$$

Stacking these equations over I yields the result. □

Proposition 4 implies two additional, useful results. First, the Leontief-inverse matrix always exists. Second, one can use the Leontief-inverse to obtain a useful closed-form expression for the policy-independent prices. This is stated formally in the following corollary:

Corollary 2. *Suppose that Assumptions 4 and 5 hold. For any $t \in \{1,2\}$, the Leontief-inverse matrix $(\boldsymbol{I} - \widehat{\boldsymbol{X}}^{\boldsymbol{t}})^{-1}$ exists. Moreover, prices are given uniquely by the following expression:*

$$p^t = \left(\boldsymbol{I} - (\widehat{\boldsymbol{X}}^{\boldsymbol{t}})^T\right)^{-1} \widehat{\boldsymbol{L}}^{\boldsymbol{t}} \mathbb{1} \tag{A51}$$

Proof. This follows from the argument in the "Uniqueness" step of the proof of Proposition 4 that any unit-inputs matrix corresponding to non-substitution prices is invertible. □

We now proceed to establish that the analysis of equilibrium is well posed by providing regularity conditions under which equilibria exist. To this end, we assume basic continuity properties of demand and that household consumption in the first period is bounded away from fully consuming first period income as income grows large.

Because it does not complicate the analysis, we prove the result when consumption and second-period labor supply take the general functional forms $c_n^t = c_n^t(p, \ell_n^1, \tau_n)$ and $\ell_n^2 = \ell_n^2(p, \ell_n^1, \tau_n)$.

Assumption 6. *The primitives satisfy the following properties:*

1. *The consumption and labor functions c_n^t and ℓ_n^2 are continuous in ℓ_n^1.*
2. *For any p, τ: there exists $\overline{y} \in \mathbb{R}_+$ and $\overline{c} < 1$ such that for all $n \in N$ and $\ell_n^1 > \overline{y}$, we have that $p^1 c_n^1(p, \ell_n^1, \tau_n) \leqslant \overline{c}\ell_n^1$.*

With this additional structure, we are now able to prove the existence of rationing equilibria for the economy under consideration.

Proposition 5. *Fix a profile of government spending and transfers $\mathcal{P} = \{\tau_n^t, G_i^t\}_{t\in\{0,1\},n\in N,i\in I}$ that satisfies (4) and suppose that Assumptions 4, 5, and 6 hold. There exists a rationing equilibrium.*

Proof. In each period $t \in \{1, 2\}$, prices p^t are given by Corollary 2. Moreover, by Assumption 6 we have the following fact: for any p^1, p^2, τ: there exists some $\overline{y} \in \mathbb{R}_+$ and some $\overline{c} < 1$ such that $p^1 c_n^1(p, \ell_n^1, \tau_n) \leqslant \overline{c}\ell_n^1$ for all n, $\ell_n^1 > \overline{y}$.

Thus, first period aggregate spending $(p^1)^T C^1 + (p^1)^T G^1$ satisifes:

$$(p^1)^T C^1 + (p^1)^T G^1 \leqslant \overline{c}\,\overline{y} + \overline{c}\mu^T \ell^1 + (p^1)^T G^1 \tag{A52}$$

Since $\overline{c} < 1$, it follows that there exists $\overline{Y}$ such that if $\ell^1 \in \mathcal{Y}^1 \equiv \{\ell^1 \in \mathbb{R}_+^N \mid \mu^T \ell^1 \leqslant \overline{Y}\}$, then aggregate spending is weakly less than $\overline{Y}$; since all spending flows to wages, i.e., $\mu^T \ell^1 = \mathbb{1}^T L^1 = (p^1)^T (C^1 + G^1)$, aggregate income is also then less than $\overline{Y}$. Formally:

$$\forall \ell^1 \in \mathcal{Y}^1 : R^1 \left(\widehat{\boldsymbol{L}}^{\mathbf{1}} (1 - \widehat{\boldsymbol{X}}^{\mathbf{1}})^{-1} \left(C^1(p, \ell^1, \tau) + G^1 \right) \right) \in \mathcal{Y}^1 \tag{A53}$$

This observation allows us to define a function $\Psi : \mathcal{Y}^1 \to \mathcal{Y}^1$ given by:

$$\Psi(\ell^1) = R^1 \left(\widehat{\boldsymbol{L}}^{\mathbf{1}} (1 - \widehat{\boldsymbol{X}}^{\mathbf{1}})^{-1} \left(C^1(p, \ell^1, \tau) + G^1 \right) \right) \tag{A54}$$

where the previous argument establishes that $\Psi(\ell^1)$ is indeed contained in $\mathcal{Y}^1$. Moreover, continuity of $C^1(\cdot)$ establishes that Ψ is a continuous function.

Since Ψ is a continuous function on a compact, convex domain, it has a fixed point ℓ^1 by Brouwer's theorem.

Given this fixed point ℓ^1 of Ψ, we can construct a rationing equilibrium as follows: Let p^t be the non-substitution-theorem prices. Let c_n^t and ℓ_n^2 by the relevant functions taking in prices p^t and incomes ℓ^1. Let production in each period be:

$$Q^t = (\boldsymbol{I} - \widehat{\boldsymbol{X}}^{\boldsymbol{t}})^{-1} (G^t + C^t) \tag{A55}$$

The definitions of the consumption and labor supply functions ensure that household budget constraints hold. The construction of Q^t ensures that each goods market clears. Because ℓ^1 is a fixed point, first period income is consistent with the rationing function and the first period labor market clears. Finally, the second period labor market clears by Walras' law. □

As we have established natural conditions under which an equilibrium exists, our analysis of equilibria is well-posed.

C.2. Comparative Statics for the Multiplier

We use the structure of the multiplier from Proposition 1 to provide comparative statics of the multiplier in the various objects that contribute towards it. To this end, define the matrix:

$$\mathcal{M} = \boldsymbol{C}^{\mathbf{1}}_{\boldsymbol{h}^{\mathbf{1}}} \boldsymbol{l}^{\mathbf{1}}_{\boldsymbol{L}^{\mathbf{1}}} \widehat{\boldsymbol{L}}^{\mathbf{1}} \left(\boldsymbol{I} - \widehat{\boldsymbol{X}}^{\mathbf{1}} \right)^{-\mathbf{1}} \tag{A56}$$

and assume that all entries of this matrix are non-negative.[43] $\mathcal{M}$ serves the role of a generalized MPC in our multiplier expression. We first consider the effect of arbitrary changes in this object on the response of final output to an arbitary shock.

Proposition 6. *Consider a change in the economy such that $\mathcal{M}$ is replaced with $\mathcal{M}' = \mathcal{M} + \varepsilon\mathcal{E}$. The effect on dY^1 of this change is given to first order in ε by:*

$$\frac{d}{d\varepsilon} dY^1|_{\varepsilon=0} = (\boldsymbol{I} - \mathcal{M})^{-1} \mathcal{E} (\boldsymbol{I} - \mathcal{M})^{-1} \partial Y^1 \tag{A57}$$

Proof. We start from the multiplier derived in Proposition 1 and then differentiate with respect to ε:

$$\frac{d}{d\varepsilon} dY^1|_{\varepsilon=0} = \frac{d}{d\varepsilon} \left(\boldsymbol{I} - \mathcal{M} \right)^{-1}|_{\varepsilon=0} \partial Y^1 = \left(\boldsymbol{I} - \mathcal{M} \right)^{-1} \mathcal{E} \left(\boldsymbol{I} - \mathcal{M} \right)^{-1} \partial Y^1 \tag{A58}$$

This uses the standard formula from matrix calculus that $\partial \left(\boldsymbol{A}^{-\mathbf{1}} \right) = -\boldsymbol{A}^{-\mathbf{1}} (\partial \boldsymbol{A}) \boldsymbol{A}^{-\mathbf{1}}$, taking $\boldsymbol{A} = \boldsymbol{I} - \mathcal{M}$, and noting that $\partial \boldsymbol{A} = -\mathcal{E}$. □

Thus, for changes in network structure given by any $\mathcal{E}$, we can compute how the multiplier from any shock changes.

Corollary 3. *Suppose that either (i) all household MPCs increase or (ii) at any firm, the share of income rationed to some zero-MPC household decreases and the share rationed to all other households increases. For any $\partial Y^1 \geqslant 0$, dY^1 increases in all dimensions.*

Proof. See in both cases that $\frac{d}{d\varepsilon} dY^1|_{\varepsilon=0} = \left(\boldsymbol{I} - \mathcal{M} \right)^{-1} \mathcal{E} \left(\boldsymbol{I} - \mathcal{M} \right)^{-1} \partial Y^1$, where $\mathcal{E} \geqslant 0$. The result follows immediately. □

While the general formulae above permit exact computation of the effects on the full vector of final output, given the potentially unrestricted network structures that we allow, it is hard to draw qualitative conclusions. For the remainder of this analysis, we report comparative statics of total final output in the empirically relevant case where there exists some reference incidence h^*

[43] Recall we have also assumed in the main text that the modulus of $\mathcal{M}$ is strictly below 1.

around which the bias and homophily effects are zero for all possible ∂h^1. In this case, the analysis of Appendix A.3 shows that the total effect of a purchases shock on output follows

$$\mathbb{1}^T dY^1 \propto \boldsymbol{m}\boldsymbol{R}^{\mathbf{1}}_{\boldsymbol{L^1}}\widehat{\boldsymbol{L}}^{\mathbf{1}}\left(\boldsymbol{I}-\widehat{\boldsymbol{X}}^{\mathbf{1}}\right)^{-1}\partial G^1 \tag{A59}$$

It is then immediate that a rationing matrix that places higher entries on higher MPC households increases the output effect of any uniformly positive or negative shock. A more interesting result is shown in the following proposition, which establishes that adding IO linkages to an economy without them leads to a contraction in the range of output multipliers across sectors:

Proposition 7. *Consider two economies in which the bias and homophily effects are zero for all possible shocks and that are identical except that one features no input-output linkages* $\widehat{\boldsymbol{X}}^{\mathbf{1}} = 0$ *and the other has some arbitrary input output matrix* $\widehat{\boldsymbol{X}}^{\mathbf{1}}$*. The maximum purchases multiplier in the first economy is larger than the maximum purchases multiplier in the second economy and the minimum purchases multiplier is smaller in the first economy than the minimum purchases multiplier in the second economy.*

Proof. The maximum purchases multiplier in the first economy is given (up to constants irrelevant for this comparison) by the following:

$$\max_{i\in\mathcal{I}} \mathbb{1}^T \boldsymbol{m}\boldsymbol{R}^{\mathbf{1}}_{\boldsymbol{L^1}} e_i = \max_{i\in\mathcal{I}} (\mathbb{1}^T \boldsymbol{m}\boldsymbol{R}^{\mathbf{1}}_{\boldsymbol{L^1}})_i \tag{A60}$$

where e_i is the vector with one in dimension i and zeros elsewhere. In the second economy, see that the maximum purchases multiplier is given by:

$$\max_{i\in\mathcal{I}} \mathbb{1}^T \boldsymbol{m}\boldsymbol{R}^{\mathbf{1}}_{\boldsymbol{L^1}}\widehat{\boldsymbol{L}}^{\mathbf{1}}\left(\boldsymbol{I}-\widehat{\boldsymbol{X}}^{\mathbf{1}}\right)^{-1} e_i = \max_{i\in\mathcal{I}} \left(\mathbb{1}^T \boldsymbol{m}\boldsymbol{R}^{\mathbf{1}}_{\boldsymbol{L^1}}\right)\widehat{\boldsymbol{L}}^{\mathbf{1}}\left(\boldsymbol{I}-\widehat{\boldsymbol{X}}^{\mathbf{1}}\right)^{-1}_{.,i} \tag{A61}$$

As production is CRS, recall by Corollary 2 that the columns of $\widehat{\boldsymbol{L}}^{\mathbf{1}}\left(\boldsymbol{I}-\widehat{\boldsymbol{X}}^{\mathbf{1}}\right)^{-1}$ sum to one. Thus, the maximum multiplier is some weighted average of the elements of $\mathbb{1}^T \boldsymbol{m}\boldsymbol{R}^{\mathbf{1}}_{\boldsymbol{L^1}}$. The value of which is necessarily bounded above by the maximum element of $\mathbb{1}^T \boldsymbol{m}\boldsymbol{R}^{\mathbf{1}}_{\boldsymbol{L^1}}$, which is the maximum purchases multiplier in the first economy. The proof that the minimum purchases multiplier is smaller in the first economy than the minimum purchases multiplier in the second economy follows the same argument, where we instead note that the weighted average is necessarily bounded below by the minimum. □

C.3. Special Cases Where Incidence, Bias, and Homophily Effects Vanish

In the main text, we briefly discussed a simple case where all of the incidence, bias, and homophily in shock propagation vanish. Here, we state and more formally discuss this case.

Proposition 8. *Suppose that all industries have a common rationing-weighted average MPC,* $m = \sum_{n \in N} (\boldsymbol{R}^{\mathbf{1}}_{\boldsymbol{L}^{\mathbf{1}}})_{ni} m_n$. *The incidence, bias, and homophily effects are zero, and the change in GDP corresponding to any change in government purchases is:*

$$dGDP^1 = \frac{1}{1-m} \mathbb{1}^T dG^1 \tag{A62}$$

Proof. To begin, recall from Proposition 1 and the definition $\widehat{\boldsymbol{C}^{\mathbf{1}}}\boldsymbol{m} = \boldsymbol{C}^{\mathbf{1}}_{\boldsymbol{h}^{\mathbf{1}}}$ that for any shock to first-period government purchases we have

$$\begin{aligned} \mathbb{1}^T dY^1 &= \mathbb{1}^T \left(\boldsymbol{I} - \boldsymbol{C}^{\mathbf{1}}_{\boldsymbol{h}^{\mathbf{1}}} \boldsymbol{R}^{\mathbf{1}}_{\boldsymbol{L}^{\mathbf{1}}} \widehat{\boldsymbol{L}^{\mathbf{1}}} (\boldsymbol{I} - \widehat{\boldsymbol{X}^{\mathbf{1}}})^{-1} \right)^{-1} dG^1 = \sum_{k=0}^{\infty} \mathbb{1}^T \left(\boldsymbol{C}^{\mathbf{1}}_{\boldsymbol{h}^{\mathbf{1}}} \boldsymbol{R}^{\mathbf{1}}_{\boldsymbol{L}^{\mathbf{1}}} \widehat{\boldsymbol{L}^{\mathbf{1}}} (\boldsymbol{I} - \widehat{\boldsymbol{X}^{\mathbf{1}}})^{-1} \right)^k dG^1 \\ &= \sum_{k=0}^{\infty} m^k \mathbb{1}^T dG^1 = \frac{1}{1-m} \mathbb{1}^T dG^1 \end{aligned} \tag{A63}$$

where the last line follows from the fact that $\mathbb{1}^T \left(\boldsymbol{C}^{\mathbf{1}}_{\boldsymbol{h}^{\mathbf{1}}} \boldsymbol{R}^{\mathbf{1}}_{\boldsymbol{L}^{\mathbf{1}}} \widehat{\boldsymbol{L}^{\mathbf{1}}} (\boldsymbol{I} - \widehat{\boldsymbol{X}^{\mathbf{1}}})^{-1} \right) = m \mathbb{1}^T$. It remains to show that this fact follows from the conditions provided in the statement of the Proposition. Namely, we must show that

$$\mathbb{1}^T \boldsymbol{C}^{\mathbf{1}}_{\boldsymbol{h}^{\mathbf{1}}} \boldsymbol{R}^{\mathbf{1}}_{\boldsymbol{L}^{\mathbf{1}}} = m \mathbb{1}^T \quad \Longrightarrow \quad \mathbb{1}^T \boldsymbol{C}^{\mathbf{1}}_{\boldsymbol{h}^{\mathbf{1}}} \boldsymbol{R}^{\mathbf{1}}_{\boldsymbol{L}^{\mathbf{1}}} \widehat{\boldsymbol{L}^{\mathbf{1}}} (\boldsymbol{I} - \widehat{\boldsymbol{X}^{\mathbf{1}}})^{-1} = m \mathbb{1}^T \tag{A64}$$

This is immediate from the transpose of the no-profit condition

$$p^1 = (\boldsymbol{I} - (\widehat{\boldsymbol{X}^{\mathbf{1}}})^T)^{-1} \widehat{\boldsymbol{L}^{\mathbf{1}}} \mathbb{1}, \tag{A65}$$

and our normalization $p = \mathbb{1}$. □

The proposition imposes that each firm's marginal employees have the same average MPC as one another. This eliminates the incidence, bias, and homophily effects, leaving only the classical Keynesian multiplier. That is, wherever in the economy a purchases shock is directed, and however it spreads through directed consumption and the IO network, the change in aggregate consumption generated by the reduction in firm revenue is the same. Of course, a particular special case that satisfies these conditions is when there is a single good and a single household (in which case $\boldsymbol{R}^{\mathbf{1}}_{\boldsymbol{L}^{\mathbf{1}}} = 1$). Note that even when the traditional Keynesian multiplier obtains, the aggregate MPC need not equal either the average MPC or the income-weighted average MPC of the population; this is the case only when each firm's marginal employees have that average MPC.

D. Model Extensions and Results

In this appendix, we extend the baseline model to allow for imperfect competition with fixed markups (D.1) and many periods (D.2).

D.1. Imperfect Competition

In this appendix, we show how to incorporate imperfect competition in the form of fixed markups on marginal costs. Now, instead of each sector being populated by a continuum of perfectly competitive firms, we suppose that for all $i \in I$ there is a single monopolist producing each good, charging a fixed markup of $\frac{\widehat{\Pi}_i^t}{1-\widehat{\Pi}_i^t}$ over their marginal cost and making (and distributing) profits $\Pi_i^t = \frac{\widehat{\Pi}_i^t}{1-\widehat{\Pi}_i^t} p_i^t Q_i^t$.[44] Despite this, we argue that a non-substitution theorem still holds and we can obtain analogous multiplier formulae once we augment labor income rationing with profit rationing. To do this, we have to slightly modify Assumption 4:

Assumption 7. *For each $t \in \{1,2\}$ there exists some $\overline{p}^t \in \mathbb{R}_+^I$ and $\{X_i^t, L_i^t\}_{i\in I}$ such that for all $i \in I$: $F_i^t(X_i^t, L_i^t) \geqslant 1$ and $(1 + \frac{\widehat{\Pi}_i^t}{1-\widehat{\Pi}_i^t})(\overline{p}^t X_i^t + L_i^t) \leqslant \overline{p}_i^t$*

Under this modified assumption, we can state and prove the modified non-substitution theorem with markups:

Proposition 9. *Fix a profile of government spending and transfers $\mathcal{P} = \{\tau_n^t, G_i^t\}_{t\in\{0,1\},n\in N,i\in I}$ that satisfies (4) and suppose that Assumptions 4, 5, and 7 hold. There exists a unique $p^t(\mathcal{P}) \in \mathbb{R}^I_{\geqslant 0}$ consistent with rationing equilibrium and, moreover, $p_t(\mathcal{P}) \gg 0$. Furthermore, $p^t(\mathcal{P})$ is invariant to $\mathcal{P}$.*

Proof. We modify the proof of proposition 4. Each firm i in period t now sets a price $p_i^t = (1 + \frac{\widehat{\Pi}_i^t}{1-\widehat{\Pi}_i^t})\kappa_i(p^t) = \kappa_i(p^t)/(1 - \widehat{\Pi}_i^t)$, where κ_i^t is i's unit cost function in period t. That is, i prices goods as though it were a competitive firm with production function $(1-\widehat{\Pi}^t)F_i^t(X_i^t, L_i^t)$. Consider now a modified economy without markups and production functions given by the previously-stated markup-adjusted production functions. Assumption 7 implies that Assumption 4 holds in this modified economy. The result then follows by application of Proposition 4. □

Since, by Proposition 9, prices are invariant to policy, we may normalize them to one without loss of generality. This implies that firms earn profits per unit sold of $\frac{\widehat{\Pi}_i^t}{1-\widehat{\Pi}_i^t} / \left(1 + \frac{\widehat{\Pi}_i^t}{1-\widehat{\Pi}_i^t}\right) = \widehat{\Pi}_i^t$. Each firm i and time t therefore pays $\widehat{\Pi}_i^t Q_i^t$ to its shareholders, in close analogy to its payments $\widehat{L}_i^t Q_i^t$ to employees and its expenditures $\widehat{X}_{ij}^t Q_i^t$ on inputs from each other firm j.

We assume that profits from each firm are distributed to households in each period t according to an exogenous dividend (a.k.a. profit rationing) function $D^t : \mathbb{R}^I \to \mathbb{R}^N$ satisfying $\sum_{i\in\mathcal{I}} \Pi_i^t = \sum_{n\in N} D^t(\Pi^t)_n$ for all $\Pi^t \in \mathbb{R}^{\mathcal{I}}$. With profits, household income is comprised of rationed first-period labor income, chosen second-period labor income, and (not chosen) dividend income in both periods. For simplicity, we continue to assume that (by the additive separability of labor supply

[44] One microfoundation for constant markups is that industries are comprised of a continuum of firms, with each other firm's and household's demands having the same CES aggregator for these firms' varieties.

and consumption preferences) each household n's consumption in each period depends on its labor and profit incomes only through its total income $\{h_n^t\}_{t=1,2}$.

We can now state a profit-inclusive Keynesian cross. The main difference to Proposition 1 comes from the need to account for changes in profits and how these are distributed to households as dividends. Additionally, the presence of profits links the first period to the second—because changes in future profits affect lifetime incomes, which affect consumption today.

Proposition 10. *For any shock inducing a partial equilibrium effect $\partial Y = \boldsymbol{C_\tau} d\tau + dG$, the general equilibrium response in production satisfies:*

$$dQ = \widehat{\boldsymbol{X}} dQ + \boldsymbol{C_{h^1}} \boldsymbol{R^1_{L^1}} \widehat{\boldsymbol{L}}^{\boldsymbol{1}} dQ^1 + \boldsymbol{C_h} \boldsymbol{D_\Pi} \widehat{\boldsymbol{\Pi}} dQ + \partial Y \tag{A66}$$

where $\boldsymbol{C_h}$ is the matrix of household directed MPCs, where $\boldsymbol{D_\Pi}$ is the block diagonal matrix composed of $\boldsymbol{D^1_{\Pi^1}}$ and $\boldsymbol{D^2_{\Pi^2}}$—which are each $N \times I$ matrices with entries $D^t_{\Pi_i^t}(\Pi^t)_n$—and where $\widehat{\boldsymbol{\Pi}}$ is the block diagonal matrix composed of $\widehat{\boldsymbol{\Pi}}^{\boldsymbol{1}}$ and $\widehat{\boldsymbol{\Pi}}^{\boldsymbol{2}}$—themselves each diagonal matrices with entries $\widehat{\Pi}_i^t$. All quantities are evaluated at the initial equilibrium.

Proof. Stacking the vectors that represent periods 1 and 2, we perturb the goods market equilibrium conditions:

$$dQ \;\; = \widehat{\boldsymbol{X}} dQ + \boldsymbol{C_{h^1}} \boldsymbol{\mu} d\ell^1 + dG + \boldsymbol{C_h} \boldsymbol{D_\Pi} \widehat{\boldsymbol{\Pi}} dQ \tag{A67}$$

Plugging in for $\boldsymbol{\mu} d\ell^1 = \boldsymbol{R^1_{L^1}} \widehat{\boldsymbol{L}}^{\boldsymbol{1}} dQ^1$, we obtain the result. □

In the special case where there are no profits in the second period, i.e., $\widehat{\Pi}_i^2 = 0$ for all i, restricting (A66) to the first period and using $dY_1 = (\boldsymbol{I} - \widehat{\boldsymbol{X}}^{\boldsymbol{1}}) dQ_1$ implies

$$dY^1 = \left[\boldsymbol{I} - \boldsymbol{C^1_{h^1}} (\boldsymbol{R^1_{L^1}} \widehat{\boldsymbol{L}}^{\boldsymbol{1}} + \boldsymbol{D^1_\Pi} \widehat{\boldsymbol{\Pi}}^{\boldsymbol{1}}) (\boldsymbol{I} - \widehat{\boldsymbol{X}}^{\boldsymbol{1}})^{-1} \right]^{-1} \partial Y_1. \tag{A68}$$

This is the same expression as our baseline multiplier, except for that both labor income and profit income is rationed to households. This expression microfounds our quantitative calibration with firm profits in Section 4.

D.2. Multiple Time Periods

Consider the benchmark model from Section 2, but with many periods. In particular, suppose that real interest rates are constrained not just for a single period but for all T time periods, so that we may take them as exogenously fixed at rates r^t. Labor is rationed in periods $1, ..., T-1$, whereas in period T, labor is supplied competitively. Household consumption c_n^t and final period

labor supply ℓ_n^T is chosen as a function of preferences, taxes, and total income in each period in a way that satisfies the dynamic budget constraint:

$$\sum_{t\leqslant T}\frac{\ell_n^t}{\prod_{t'\leqslant t}1+r^{t'}}=\sum_{t\leqslant T}\frac{p^tc_n^t+\tau_n^t}{\prod_{t'\leqslant t}1+r^{t'}} \tag{A69}$$

Similarly, lump-sum taxes and spending $\{\{\tau_n^t\}_{n\in N},\{G_i^t\}_{i\in I}\}_{t\leqslant T}$ satisfy a lifetime budget constraint for the government:

$$\sum_{n\in N}\mu_n\Big(\sum_{t\in T}\frac{1}{\prod_{t'\leqslant t}1+r^{t'}}\tau_n^t\Big)=\sum_{t\in T}\frac{1}{\prod_{t'\leqslant t}1+r^{t'}}p^tG^t \tag{A70}$$

At all periods $t\leqslant T-1$—those in which labor is rationed—and for all $n\in N$, we have $\mu_n\ell_n^t=R_n^t(L^t)$ for some period-specific rationing function R^t satisfying $\sum_{n\in N}\mu_nR_n^t(L^t)=\sum_{n\in N}L_n^t$.

Definition 3. *(Dynamic rationing equilibrium) Given a profile of government spending and transfers $\{\tau_n^t,G_i^t\}_{t\in\{0,...,T\},n\in N,i\in I}$ that satisfies (A70), a dynamic rationing equilibrium is a profile $\{p_i^t,\ell_n^t,c_n^t,L_i^t,X_{ij}^t\}_{t\in\{0,...,T\},n\in N,i,j\in I}$ that satisfies firm optimization in every period (as in (1)), consumption and final period labor supply consistent with household optimization, labor rationing in periods $t\leqslant T-1$, and goods and labor market clearing.*

Under this dynamic equilibrium concept, we obtain an analogous fiscal multiplier to that of our two period model. In order to present this result, let us introduce a bit of notation: Since prices are exogenous, normalize the units of all goods so that $p_i^t=1$ for all $t\leqslant T$, $i\in I$. Below, for any T-length vector v, we will use the notation v^{-T} to denote the $(T-1)$-length $(v_1,...,v_{T-1})$. In a similar fashion, we let $\widehat{\boldsymbol{L}}^{-\boldsymbol{T}}$ and $\widehat{\boldsymbol{X}}^{-\boldsymbol{T}}$ denote the block-diagonal $\big((T-1)\times I\big)\times\big((T-1)\times I\big)$ matrices comprised of the within-period-t unit labor demand and IO matrices on the t^{th} block. $\boldsymbol{R}^{-\boldsymbol{T}}_{\boldsymbol{L}^{-\boldsymbol{T}}}$ is the analogous block-diagonal $\big((T-1)\times N\big)\times\big((T-1)\times I\big)$ matrix corresponding to marginal labor rationing stacked across $T-1$ periods. Somewhat more interestingly, let $\boldsymbol{m}^{-\boldsymbol{T}}$ denote the $\big((T-1)\times N\big)\times\big((T-1)\times N\big)$ matrix whose (t,n,t',n') entry is zero if $n\neq n'$ and—if $n=n'$—is equal to n's MPC into period t consumption spending out of period t' income. Finally, let $\widehat{\boldsymbol{C}}^{-\boldsymbol{T}}$ be the $\big((T-1)\times N\big)\times\big((T-1)\times I\big)$ matrix that captures the direction of this marginal spending, i.e., the matrix with (t,i,t',n) entry equal to the fraction of n's period-t'-income-induced spending into period t that is directed toward good i.

Proposition 11. *Given any rationing equilibrium, the local change in equilibrium final output dY^{-T} in periods $1,\dots,T-1$ following a fiscal shock with direct effect ∂Y^{-T} on final output in periods $1,\dots,T-1$ is given by:*

$$dY^{-T}=\left(\boldsymbol{I}\ -\ \widehat{\boldsymbol{C}}^{-\boldsymbol{T}}\ \boldsymbol{m}^{-\boldsymbol{T}}\ \boldsymbol{R}^{-\boldsymbol{T}}_{\boldsymbol{L}^{-\boldsymbol{T}}}\ \widehat{\boldsymbol{L}}^{-\boldsymbol{T}}\ \left(\boldsymbol{I}-\widehat{\boldsymbol{X}}^{-\boldsymbol{T}}\right)^{-1}\right)^{-1}\partial Y^{-T} \tag{A71}$$

Proof. The proof is directly analogous to that of Proposition 1. □

The presence of many periods with rationing introduces one key qualitative difference: Shocks can spill over across periods with labor rationing. As a result, it is no longer sufficient to consider the directed MPC of households; one must work with the directed *intertemporal* MPC of households that represents marginal changes in consumption across goods and time. These intertemporal MPCs are precisely the object of study of Auclert et al. (2018). Indeed, (A71) coincides with their intertemporal Keynesian cross when there is a single good.

E. Additional Tables and Figures

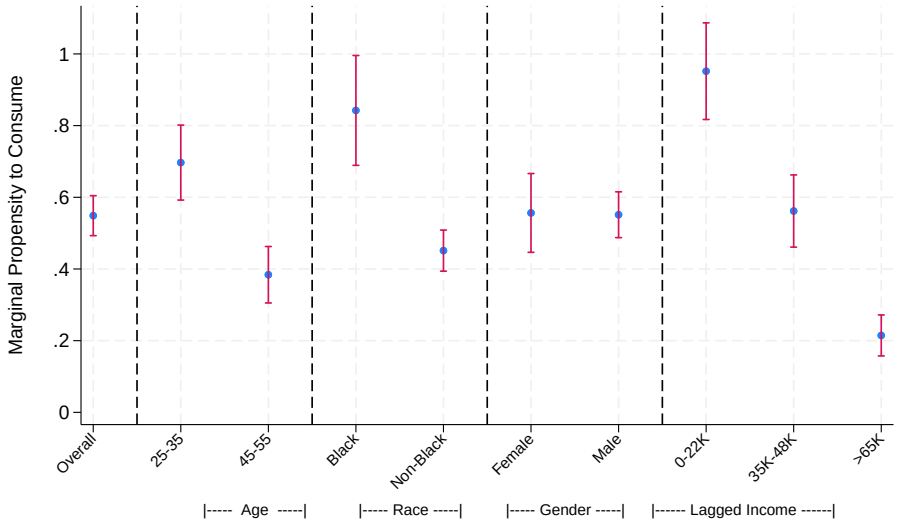


Fig. A1. Heterogeneity in estimated MPCs for total consumption across demographic groups.

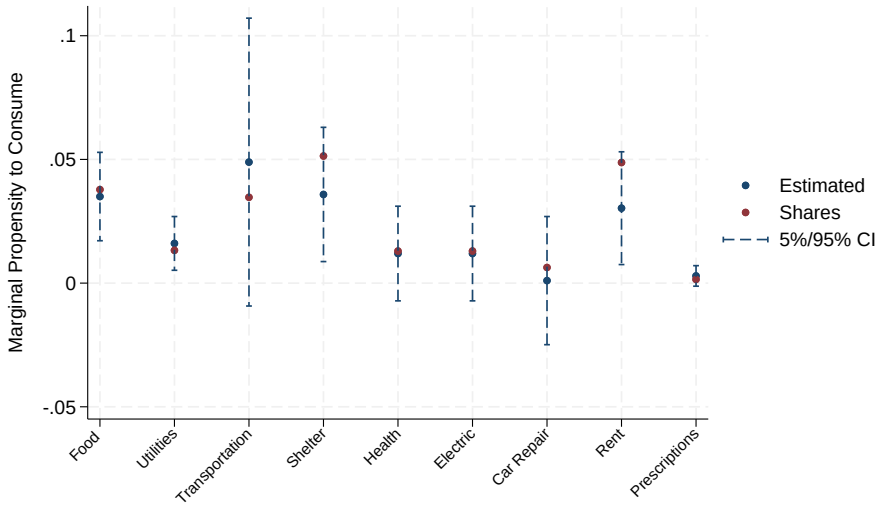


Fig. A2. Estimated Directed MPCs Vs. CEX basket-weighted MPCs

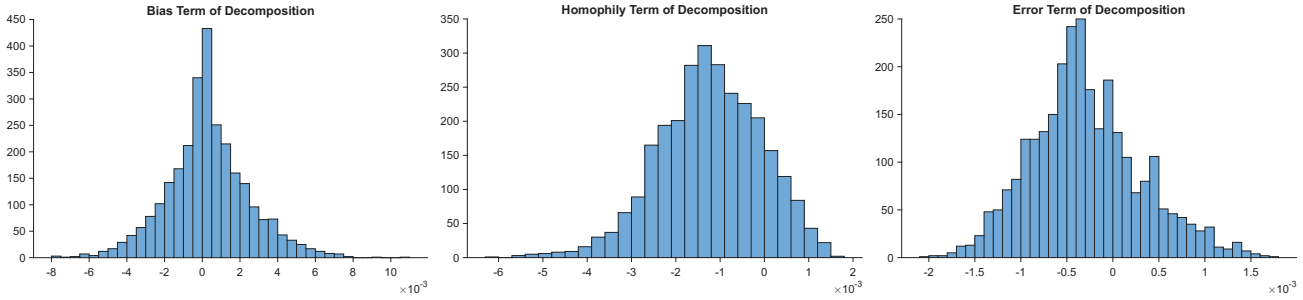


Fig. A3. Histograms of the bias term (left) and homophily term (middle) and overall error terms (right) from the decomposition in Proposition 2. For all subfigures, the distribution reflects a unit demand shock to each of the 2805 sector-region pairs, with baseline h^* given by the income incidence of a shock to demand proportional to 2012 state-industry GDP.

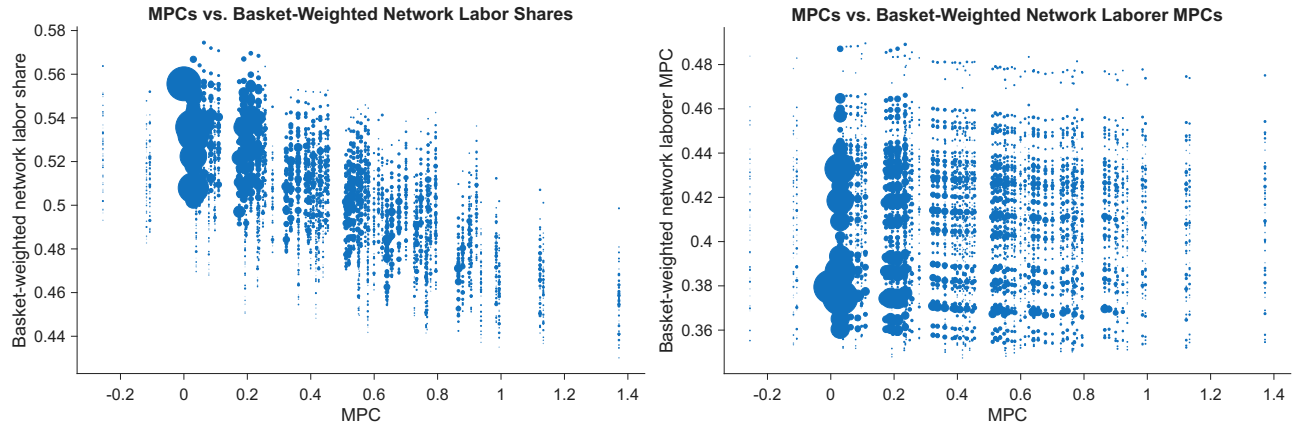


Fig. A4. The left panel shows the scatter plot of worker MPCs against the basket-weighted labor share of the sectors on which they consume. The right panel shows a scatter plot of worker MPCs against the basket-weighted MPCs of the labor employed in the sectors producing the goods they ultimately consume. In both plots, each dot corresponds to a region × demographic group level and its size corresponds to the incidence of a GDP-proportional shock onto the region-demographic's income.

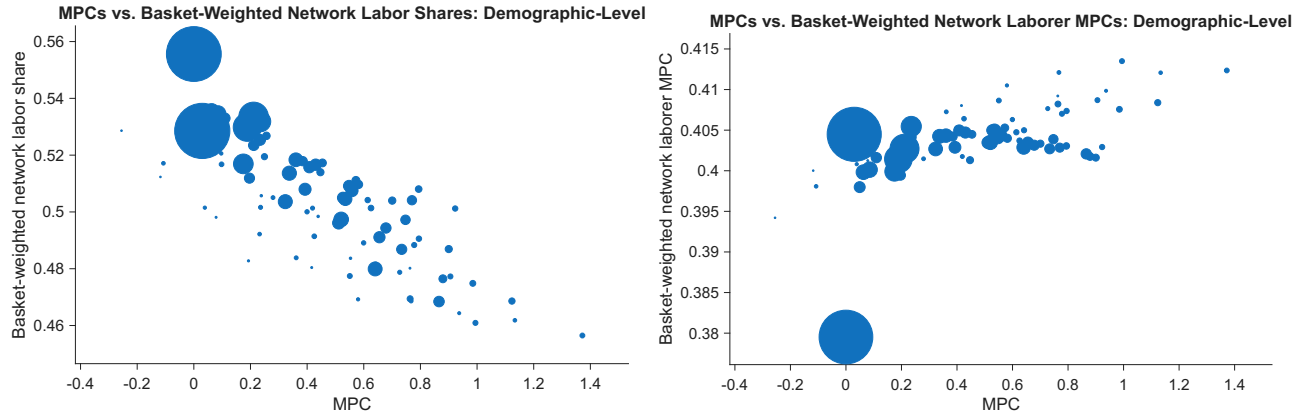


Fig. A5. The left panel shows the scatter plot of worker MPCs against the basket-weighted labor share of the sectors on which they consume. The right panel shows a scatter plot of worker MPCs against the basket-weighted MPCs of the labor employed in the sectors producing the goods they ultimately consume. In both plots, each dot corresponds to a demographic group level—we add group members across regions, weighting by the incidence of a GDP-proportional shock—and its size corresponds to the incidence of a GDP-proportional shock onto the demographic's income.

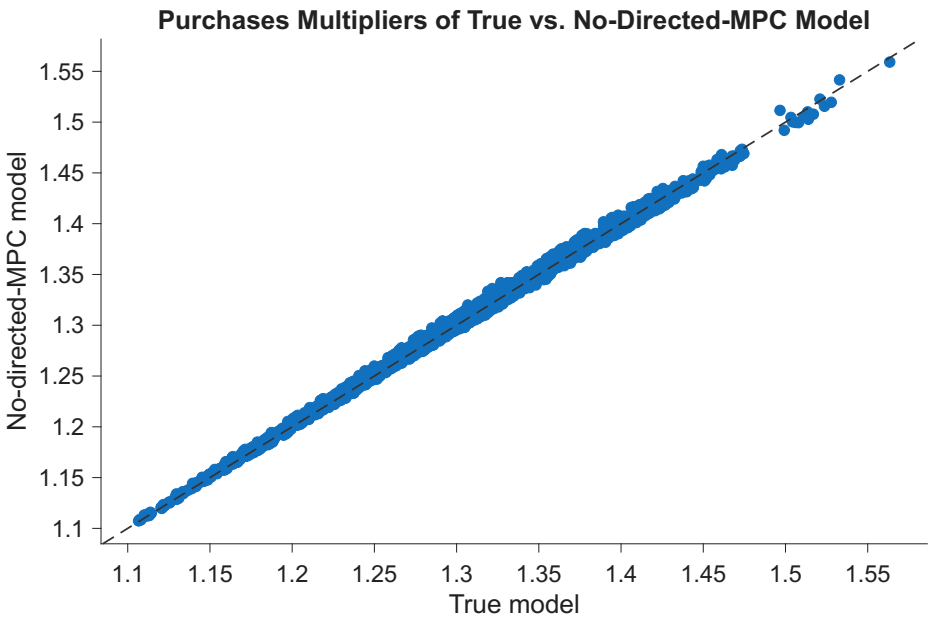


Fig. A6. Scatter plot of purchases multipliers for each of the 2805 industry-region pairs in the baseline model (x-axis) and the model in which all households have homogeneous consumption baskets in proportion to GDP (y-axis).

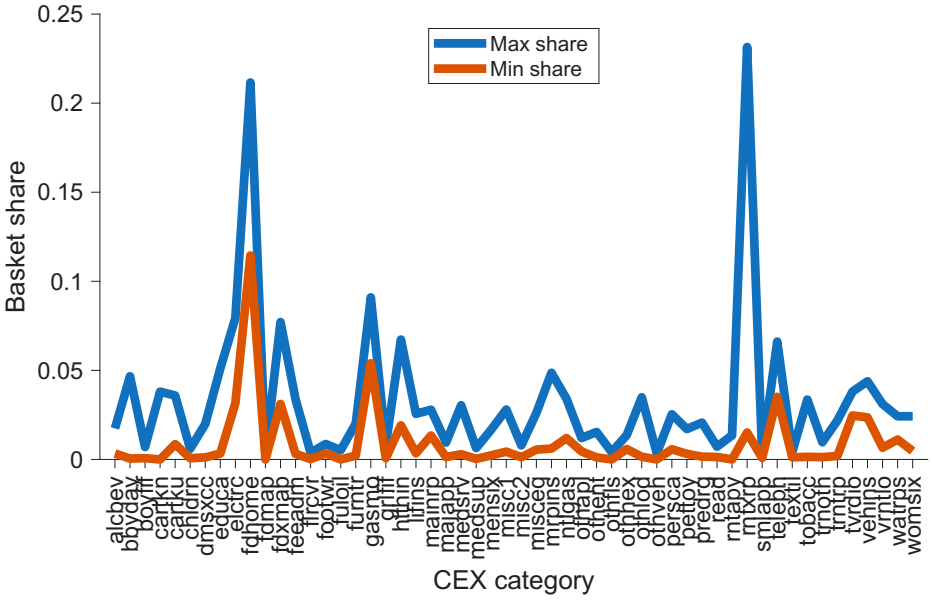


Fig. A7. Maximum and minimum consumption basket weights across all demographic groups in each CEX consumption category.

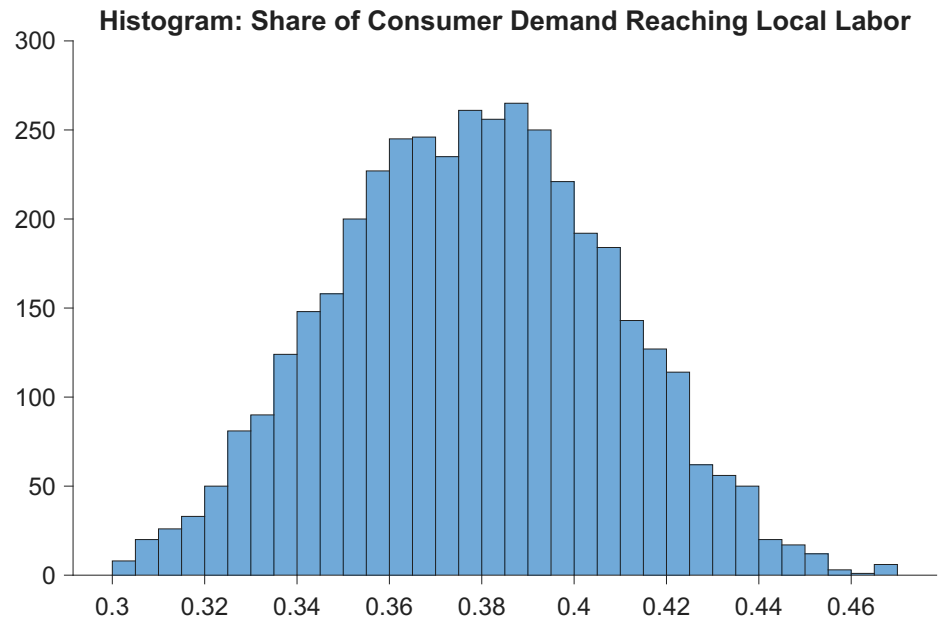


Fig. A8. Histogram of the fraction of consumer demand resulting in income for labor within the same state for each state-demographic pair, excluding "foreigners" demographic group.

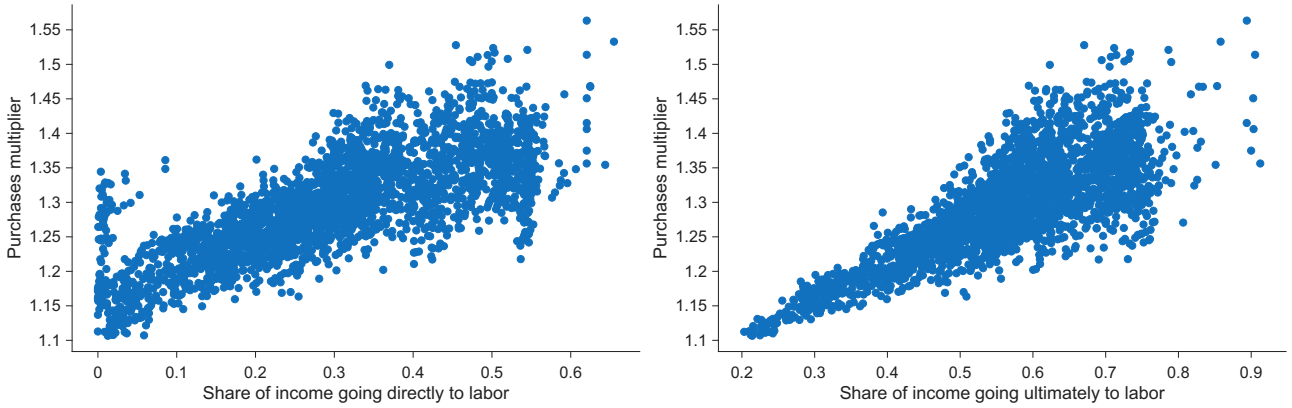


Fig. A9. Left Panel: Scatter plot comparing, for a $1 purchases shock to each industry-region, the change in aggregate consumption (y-axis) and the share of income from production that goes directly to labor (as opposed to capital, foreigners, or inputs) (x-axis). Right Panel: Scatter plot comparing, for a $1 purchases shock to each industry-region, the change in aggregate consumption (y-axis) and the ultimate labor share accounting for labor employed in the production of intermediates (x-axis).

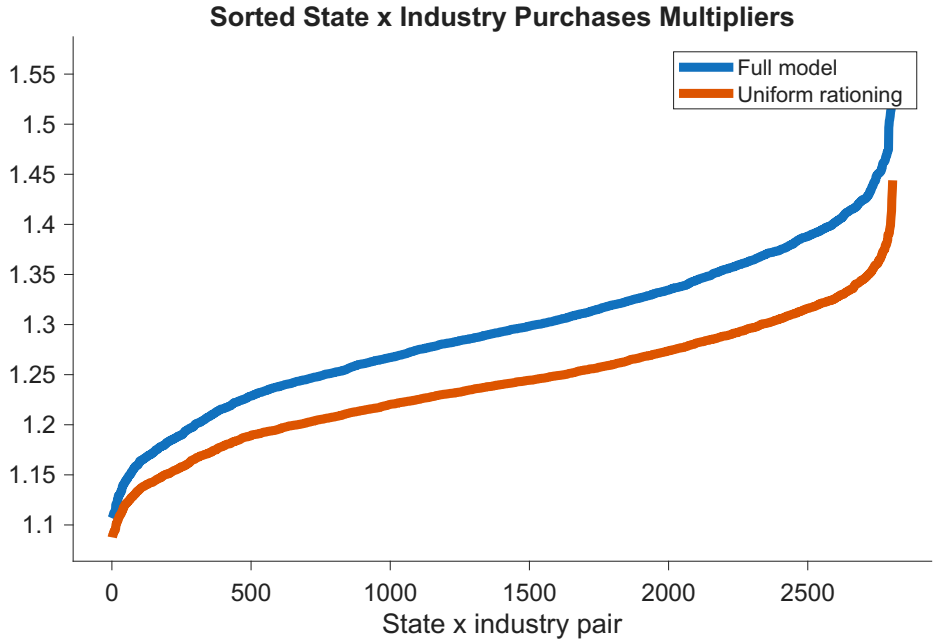


Fig. A10. Sorted change in aggregate consumption resulting from a one dollar demand shock to each industry-region pair. Full model is the baseline. Uniform rationing corresponds to all households' labor income being scaled in proportion to their labor income.

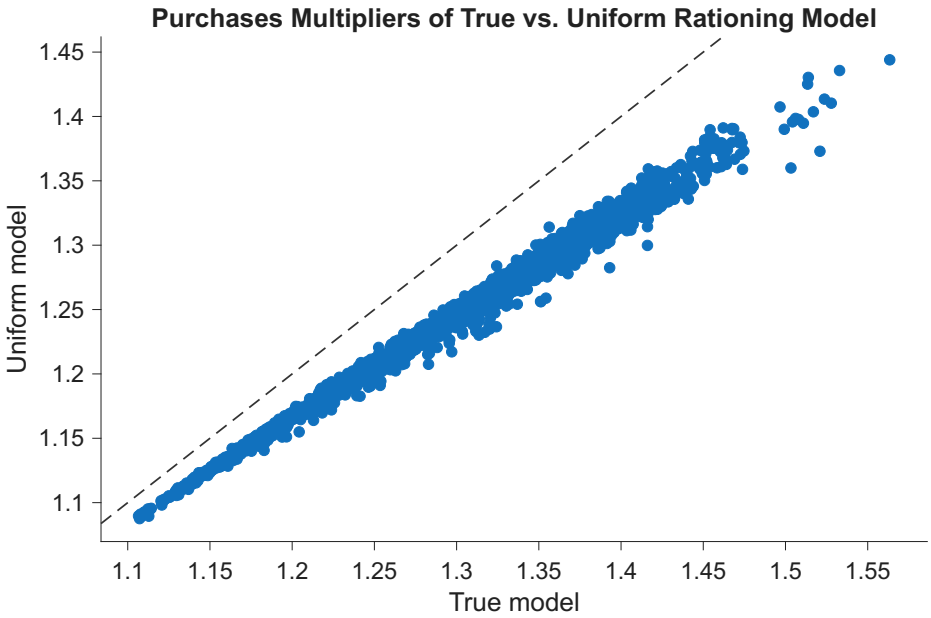


Fig. A11. Scatter plot of the change in aggregate consumption resulting from a one dollar demand shock to each industry-region pair. Full model is the baseline. Uniform rationing corresponds to all households' labor income being scaled in proportion to their income.

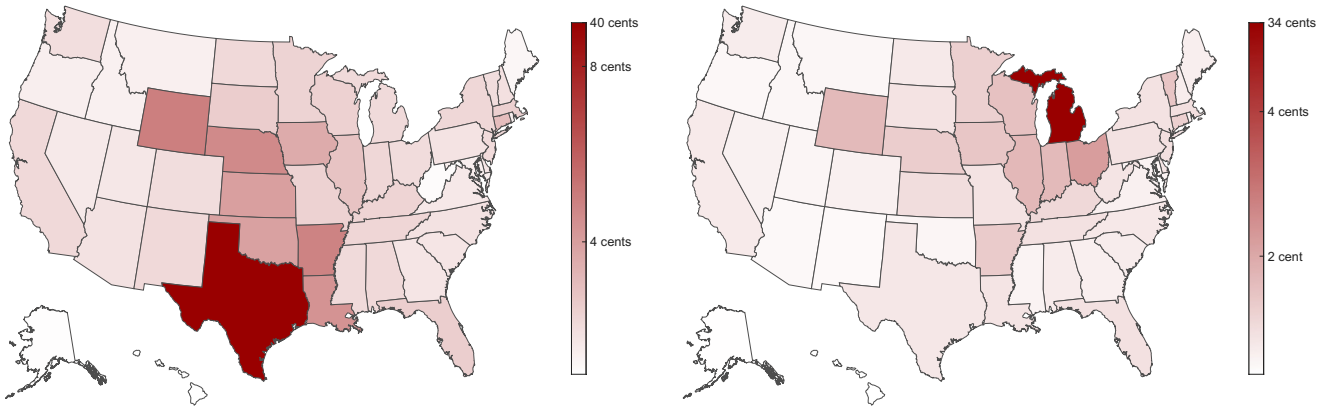


Fig. A12. Changes in state-level value added per worker following a uniform $1-per-capita transfer shock to households in Texas (left panel) and Michigan (right panel).

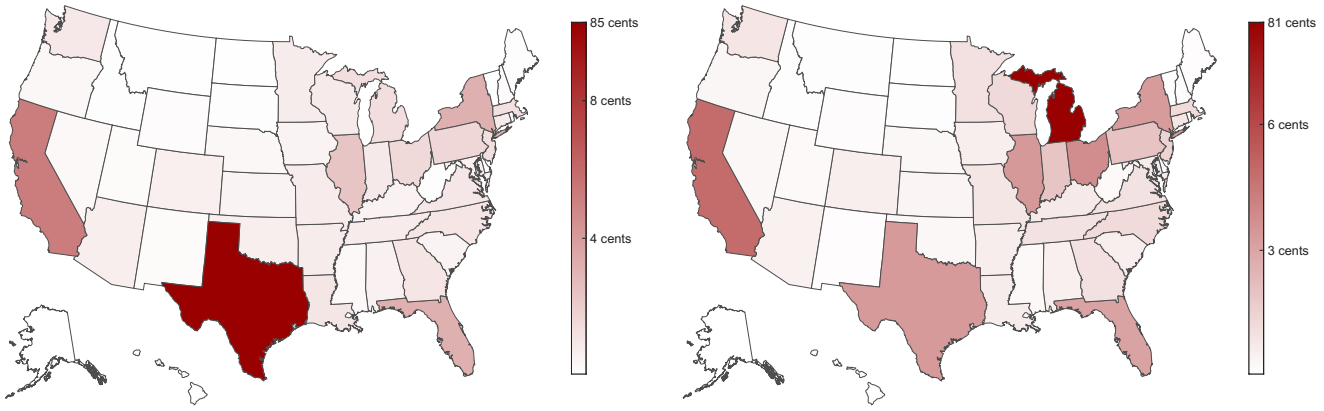


Fig. A13. Changes in state-level value added following a GDP-proportional $1 purchases shock to Texas (left panel) and Michigan (right panel).

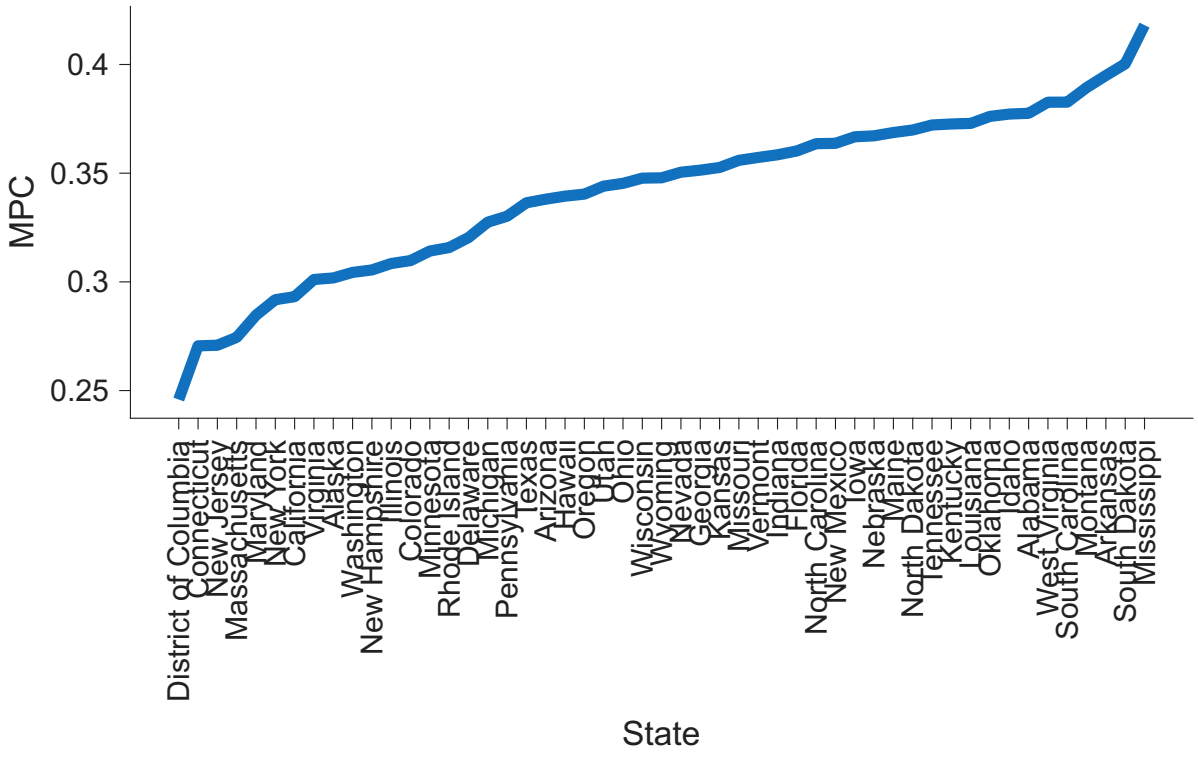


Fig. A14. Income-weighted average worker MPC by state.

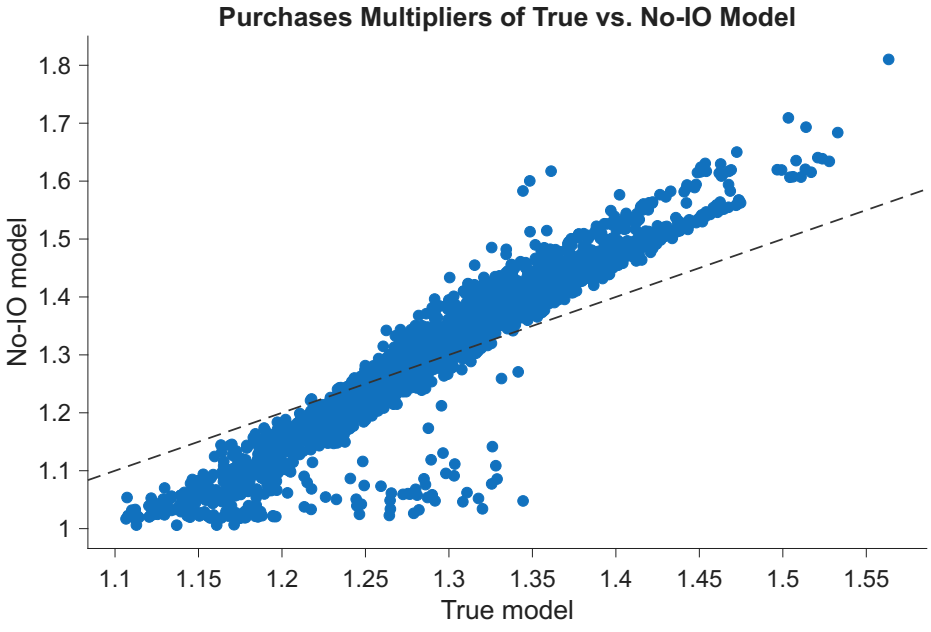


Fig. A15. Scatter plot of the change in aggregate consumption resulting from a one dollar demand shock to each industry-region pair. Full model is the baseline. No-IO corresponds to the case where firms do not purchase intermediate inputs and instead scale up their factor payments proportionally.

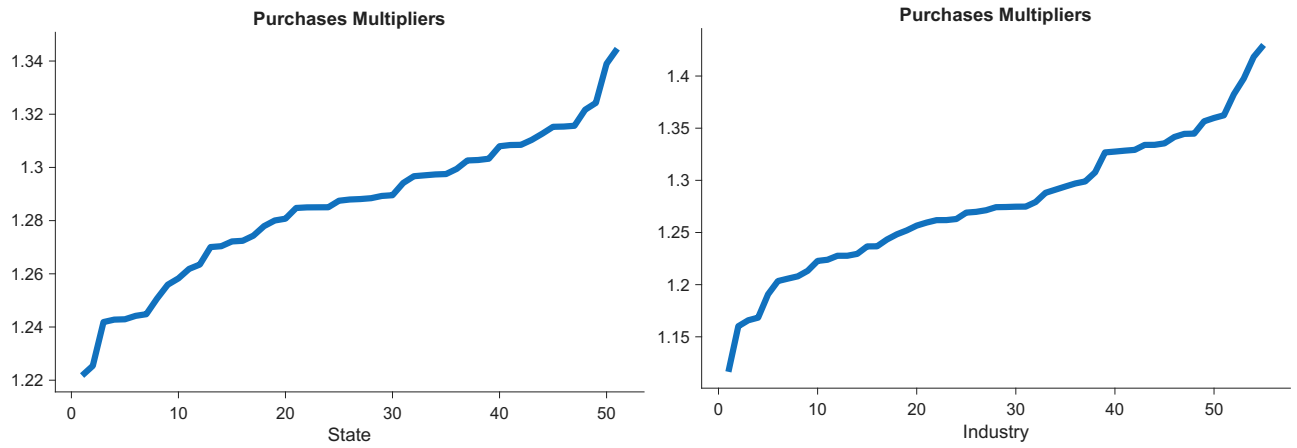


Fig. A16. Multipliers for state-level and industry level purchases shocks. Formally, for the state-level shock, we purchase from each state one dollar of output, in proportion to the sectoral composition of that state. For the industry-level shock, we purchase from each demographic group one dollar of output, in proportion to the state composition of that sector.

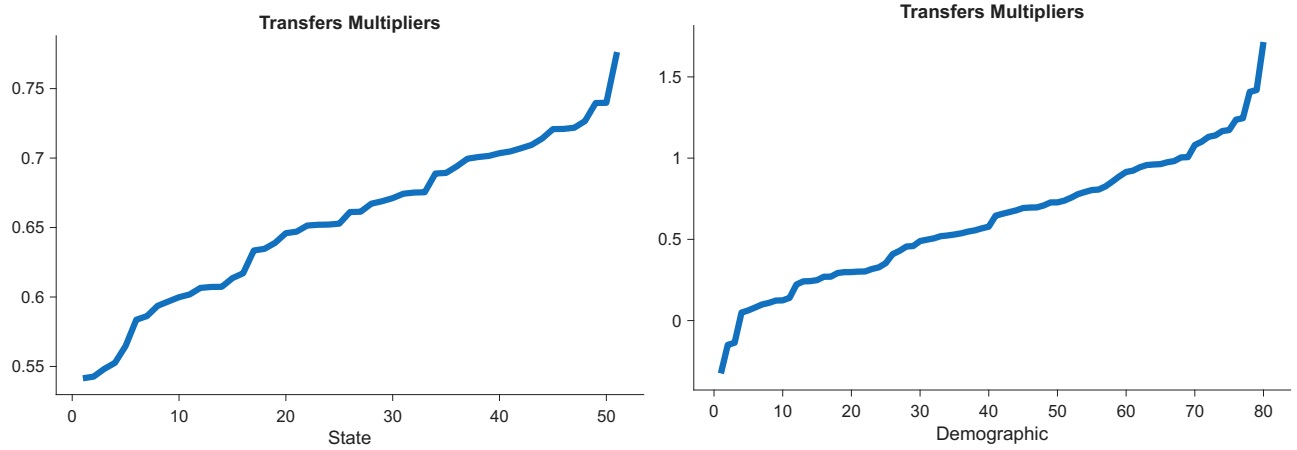


Fig. A17. Multipliers for state-level and demographic level transfers shocks. Formally, for the state-level shock, we transfer each state one dollar, in proportion to the demographic composition of that state. For the demographic-level shock, we transfer each demographic group one dollar, in proportion to the distribution of that demographic across states.