

For Online Publication - Appendix

Optimal Taxation of Inflation

Damien Capelle and Yang Liu

A Brief History of TIP and Literature

Early Proposals of TIP during the Great Inflation. In the context of high and accelerating inflation due to the combined food and oil price shocks of 1973 and 1979, and persistent wage-price spirals, [Wallich and Weintraub \(1971\)](#) formulated the first proposal for a permanent tax on wage increases. At this time, TIP stood for “Tax-based Incomes Policies” and according to their proposal it would be levied on wage increases in excess of a pre-announced target and it would be paid by employers.

This proposal started a literature analyzing the theoretical rationale for a TIP. [Kotowitz and Portes \(1974\)](#) build a microeconomic model in which a union sets wages and firms set prices and find that imposing such a tax does reduce the rate of change of wages. They show that this result is robust to assuming that the union is myopic or forward-looking. [Latham and Peel \(1977\)](#) show that the tax on wage increases is less effective when the firm is a monopsony. From a normative perspective, [Seidman \(1978\)](#) argues that when increasing their price, firms don’t take into account their economy-wide inflationary effects. Like a tax on pollution, TIP would signal to firms the social costs of their actions and would make them internalize the externalities of their price-setting behaviors.

A stream of papers studied other macroeconomic implications of TIP. In a macroeconomic model, [Peel \(1979\)](#) shows how the tax could reduce the likelihood of business cycles. [Scarth \(1983\)](#) finds that an employer TIP based on price increases, and an employee TIP based on wage increases are destabilizing, while an employer TIP based on wage increases is stabilizing. [Oswald \(1984\)](#) discusses the conditions for an inflation tax to be equivalent

to an employment subsidy and to result in higher employment. [Jackman and Layard \(1989\)](#) show how TIP could reduce the NAIRU and increase welfare even when the tax has an effect on workers' effort.

Contributions to the Literature on TIP. It was acknowledged at the time that a better understanding of price-setting behaviors and their macro implications was required to design TIP effectively ([Koford and Miller, 1992](#)). Our first contribution is to leverage improvements made in sticky price models since then to re-assess the effectiveness of TIP.

We share with the earlier literature the conclusion that TIP is an effective tool to control inflation, but while the earlier literature saw TIP as a substitute for MP, we find that MP and TIP are complementary instruments, each specializing in their area of comparative advantage. TIP should focus on markup shocks, and MP should focus on demand shocks. We also confirm the idea that a TIP in steady-state increases welfare by decreasing markup and increasing employment, but we find that TIP should vary over time. It should increase when inflation rises, and decrease as inflation reaches its long-term target.

On implementation, we contribute by analyzing the robustness of TIP to several alternative instruments (feebate and a market for inflation permits), to the use of targeting rules for MP and TIP and to alternative tax bases (tax on wage increases, or tax on large firms), which could be easier to administer. We discuss in conclusion a few other important issues of implementation that deserve more attention in future research.

Optimal Tax Policies. Our paper contributes to the rich literature on tax policies in New Keynesian models. [Correia et al. \(2008\)](#) shows that a sufficiently rich set of fiscal and monetary instruments, including labor taxes and subsidies, can implement the first-best allocation. When monetary policy is constrained by the ZLB, papers have found a welfare-enhancing role for tax increases aimed at restricting supply ([Eggertsson and Woodford, 2006](#)), tax cuts aimed by stimulating demand ([Eggertsson, 2011](#)), temporary gov-

ernment spending ([Woodford, 2011](#)), cuts in marginal labour tax rates that boost confidence ([Mertens and Ravn, 2014](#)), and well-designed paths of consumption tax and payroll subsidies, import and export tariffs ([Correia et al., 2013](#); [Farhi et al., 2014](#)). We generalize the set of fiscal instruments, showing that, like labor subsidies, TIP helps implement the first best. Importantly, we argue that TIP is much less costly for the government budget than payroll subsidies.

TIP in Practice. In the 1970s, versions of TIP were implemented. From 1974 to 1977, the French governments implemented the "prélèvement conjoncturel", which covered the largest 1500 firms, representing 60% of the economy, and was based on the excess increase in value-added in nominal terms relative to an announced threshold, with an adjustments for fast-growing firms. Other versions of TIP were implemented in Mexico, Belgium, Italy, as mentioned in [Paci \(1988\)](#), and in the Netherlands as explained in [OECD \(1975\)](#). More research is needed to analyze their institutional details and impacts.

TIP came close to being implemented in the U.S. as well. In 1978, the Carter administration proposed to Congress the "real wage insurance" to supplement the wage-price guidelines which included voluntary limits on nominal wage and price increases of 7% and 5.75% respectively. This program meant to give incentives to workers to enforce the guidelines: a worker belonging to an employee group whose earnings increased by less than 7% in a year would receive a tax-credit proportional to the difference between the realized inflation rate and 7% ([Colander, 1981](#)).

Why have discussions around TIP stopped in the U.S.? We see three causes. First, afraid of its uncertain costs for the Federal budget (the proposal was more a feebate than a tax), Congress didn't support the plan. Second, the plan combined TIP with price controls which were unpopular, especially since their use by Nixon in the early 1970s. Finally, and most importantly, with Volcker's successful anti-inflation policy in the 1980s, MP has emerged as the sole legitimate instrument for achieving price stability up until the

present day. While these may explain why TIP was not further pursued at the time, this paper reassesses its potential role as an additional stabilization tool.

In the context of the transition of formerly Soviet countries to market economies, TIP came back to the forefront of policy discussions and versions of TIP were implemented in Bulgaria, Poland and Romania. [Koford et al. \(1993\)](#) put forward an anti-inflation plan and incentive policies to stabilize prices and output in transition economies. [Bogetic and Fox \(1993\)](#) analyze the design, implementation and enforcement of these policies in Bulgaria and Romania and conclude that they helped stabilize output and prices. [Enev and Koford \(2000\)](#) find a fairly substantial inflation-reducing effect from the Bulgarian policy but no significant results from the Polish policy. Another analysis of the effects of the tax on inflation and on the employment behavior of Polish firms can be found in [Crombrugghe and de Walque \(2011\)](#). These examples suggest that TIP is implementable, effective, and not too costly to administer ([Paci, 1988](#)).

B Baseline Model with TIP

B.1 Market Clearing and Equilibrium Definition

Market Clearing We now show the market clearing conditions. In equilibrium, the markets for each intermediate good Y_{ti} and for the final good should clear

$$Y_t = C_t + \int_0^1 \frac{\theta}{2} \left(\frac{P_{ti}}{P_{t-1i}} - 1 \right)^2 Y_t di \quad (36)$$

The sum of labor hired in all firms should be equal to the supply of labor by households:

$$\left(\int N_{th}^{1-1/\sigma_N} dh \right)^{\frac{\sigma_N}{\sigma_N-1}} = N_t = \int_0^1 N_{ti} di \quad (37)$$

With no government debt, holdings of bonds by households are zero: $B_t = 0$. Finally, transfers received by households include profits and tax receipts:

$$T_t = \int_0^1 \Pi_{it} + \tau_t(P_{ti} - P_{t-1i})Y_{ti}di. \quad (38)$$

Equilibrium. An equilibrium is a path of output, labor, bonds, wages, price level, bond prices and TIP $\{\{C_{th}, N_{th}, B_{th}, W_{th}\}_h, \{Y_{ti}, N_{ti}, P_{ti}\}_i, W_t, P_t, Q_t, \tau_t\}_{t=0,1,2,\dots}$ such that

- Taking TIP as given, intermediate firms maximize their discounted sum of profits (11) subject to the definition of profits (10), the technology (8) and the demand schedule (7).
- Taking prices as given, final good firms maximize their profits subject to the technology (6).
- Taking prices and transfers (38) as given, households maximize their utility (1) subject to their budget constraint (2), no-ponzi condition (??) and demand for labor (3).
- The markets for final good (36), intermediate goods, labor (37) and bonds clear.

B.2 Linearization

Firms' First-Order Condition The first-order conditions associated with the firm's problem are:

$$\begin{aligned} & (1 - \epsilon_t) \left(\frac{P_{ti}}{P_t} \right)^{-\epsilon_t} Y_t - (1 - \epsilon_t) \left(\frac{P_{ti}}{P_t} \right)^{-\epsilon_t} Y_t \tau_t - \tau_t \epsilon_t \frac{P_{t-1i}}{P_{ti}} Y_t \left(\frac{P_{ti}}{P_t} \right)^{-\epsilon_t} \\ & + \frac{\epsilon_t}{1 - \alpha} \frac{W_t}{P_{ti}} \left[\left(\frac{P_{ti}}{P_t} \right)^{-\epsilon_t} \frac{Y_t}{A_t} \right]^{\frac{1}{1-\alpha}} - \frac{\theta}{P_{t-1i}} \left(\frac{P_{ti}}{P_{t-1i}} - 1 \right) P_t Y_t + EQ_t V'(P_{ti}) = 0 \end{aligned}$$

and

$$V'(P_{t-1i}) = \tau_t Y \left(\frac{P_{ti}}{P_t} \right)^{-\epsilon_t} + \frac{P_{ti}}{P_{t-1i}^2} \theta \left(\frac{P_{ti}}{P_{t-1i}} - 1 \right) P_t Y_t.$$

Assuming symmetry gives

$$\begin{aligned} & \left((1 - \epsilon_t) Y_t - (1 - \epsilon_t) Y_t \tau_t - \tau_t \epsilon_t \frac{P_{t-1}}{P_t} Y_t + \frac{\epsilon_t}{1 - \alpha} \frac{W_t}{P_t} \left[\frac{Y_t}{A_t} \right]^{\frac{1}{1-\alpha}} - \frac{\theta}{P_{t-1}} \pi_t P_t Y_t \right) \\ & \quad + E_t Q_t \left[\tau_{t+1} Y_{t+1} + (\pi_{t+1} + 1)^2 \theta \pi_{t+1} Y_{t+1} \right] = 0 \\ & \iff \left(1 - \epsilon_t - \tau_t \left(1 - \epsilon_t \frac{\pi_t}{1 + \pi_t} \right) + \epsilon_t MC_t - \theta \pi_t (\pi_t + 1) \right) \\ & \quad + E_t Q_t \left[\tau_{t+1} \frac{Y_{t+1}}{Y_t} + (\pi_{t+1} + 1)^2 \theta \pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] = 0 \\ & \iff \left((1 - \epsilon_t) \left(1 - \frac{\epsilon_t}{\epsilon_t - 1} MC_t \right) - \tau_t \left(1 - \epsilon_t \frac{\pi_t}{1 + \pi_t} \right) - \theta \pi_t (\pi_t + 1) \right) \\ & \quad + E_t Q_t \left[\tau_{t+1} \frac{Y_{t+1}}{Y_t} + (\pi_{t+1} + 1)^2 \theta \pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] = 0 \\ & \iff \frac{1}{\theta} \left((1 - \epsilon_t) (1 - \mathcal{M}_t MC_t) - \tau_t \left(1 - \epsilon_t \frac{\pi_t}{1 + \pi_t} \right) \right) - \pi_t (\pi_t + 1) \\ & \quad + E_t Q_t \left[\frac{\tau_{t+1}}{\theta} \frac{Y_{t+1}}{Y_t} + (\pi_{t+1} + 1)^2 \pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] = 0 \end{aligned}$$

$$\text{with } MC_t = \frac{W_t}{P_t(1 - \alpha)} \frac{Y_t^{\frac{\alpha}{1-\alpha}}}{A_t^{\frac{1}{1-\alpha}}} \quad \text{and} \quad \mathcal{M}_t = \frac{\epsilon_t}{\epsilon_t - 1}.$$

We denote the steady-state markup $\bar{\mathcal{M}}$. The next step is to linearize this optimality condition around a steady-state with no inflation, constant output, a *non-zero* tax on price changes, a price markup, $\pi = 0, \tau = \tau^{ss}, Y = Y', MC_t \mathcal{M} = 1$. Denoting mc the log of the real marginal cost MC and μ the log of $\mathcal{M}$, we obtain:

$$\begin{aligned}
& \frac{1}{\theta} [(1 - \epsilon) (1 - *(1 + mc_t - mc + \mu_t - \mu)) - \hat{\tau}_t - \tau^{ss} + \tau^{ss}\epsilon\pi_t] \\
& + \beta \frac{\tau^{ss}}{\theta} \left[1 + \hat{q}_t + \frac{\hat{\tau}_{t+1}}{\tau^{ss}} + \hat{y}_{t+1} - \hat{y}_t \right] + \beta\pi_{t+1} = \pi_t \\
& \frac{1}{\theta} [(1 - \epsilon) (1 - 1 * (1 + mc_t - mc + \mu_t - \mu)) - \hat{\tau}_t - \tau^{ss} + \tau^{ss}\epsilon\pi_t] \\
& + \beta \frac{\tau^{ss}}{\theta} \left[1 - \pi_{t+1} + \frac{\hat{\tau}_{t+1}}{\tau^{ss}} + (1 - \sigma)(\hat{c}_{t+1} - \hat{c}_t) \right] + \beta\pi_{t+1} = \pi_t \\
& \frac{1}{\theta} [(1 - \epsilon) (-1 * (\hat{m}c_t + \hat{\mu}_t)) - \hat{\tau}_t + \tau^{ss}\epsilon\pi_t] \\
& + \beta \frac{\tau^{ss}}{\theta} \left[-\pi_{t+1} + \frac{\hat{\tau}_{t+1}}{\tau^{ss}} + (1 - \sigma)(\hat{c}_{t+1} - \hat{c}_t) \right] + \beta\pi_{t+1} = \pi_t
\end{aligned}$$

$$\begin{aligned}
\pi_t \left(1 - \epsilon \frac{\tau^{ss}}{\theta} \right) &= \beta E_t \pi_{t+1} \left(1 - \frac{\tau^{ss}}{\theta} \right) + \frac{(\epsilon - 1)}{\theta} (\hat{m}c_t + \hat{\mu}_t) + \frac{1}{\theta} [\beta E_t \hat{\tau}_{t+1} - \hat{\tau}_t] \\
&+ \beta \frac{\tau^{ss}}{\theta} (1 - \sigma)(\hat{c}_{t+1} - \hat{c}_t)
\end{aligned}$$

where $\hat{m}c_t = mc_t - mc$ is the gap between the effective marginal cost and the desired marginal cost under flexible prices in steady-state. Later we will define the cost push shock $u_t = (\epsilon - 1)(\mu_t - \mu)$ is the markup shock.

From this we see why $\tau_t = \beta E_t \tau_{t+1} + u_t + u_t^w$ is the path of TIP that can stabilize inflation as shown in corollary 1.

With log consumption $\sigma = 1$, the expression simplifies slightly:

$$(\epsilon - 1)(\hat{m}c_t + \hat{\mu}_t) - \hat{\tau}_t + \beta E_t [\hat{\tau}_{t+1} + \pi_{t+1} (\theta - \tau^{ss})] = \pi_t (\theta - \epsilon \tau^{ss})$$

and dividing by θ on both sides

$$\pi_t \left(1 - \epsilon \frac{\tau^{ss}}{\theta} \right) = \beta E_t \pi_{t+1} \left(1 - \frac{\tau^{ss}}{\theta} \right) + \frac{(\epsilon - 1)}{\theta} (\hat{m}c_t + \hat{\mu}_t) + \frac{1}{\theta} [\beta E_t \hat{\tau}_{t+1} - \hat{\tau}_t + u_t]$$

Alternatively, if $\tau^{ss} = 0$ then the expression simplifies to

$$\begin{aligned}\pi_t &= \frac{1}{\theta} ((\epsilon - 1)\hat{m}c_t - \tau_t + u_t) + \beta E_t \left[\frac{\tau_{t+1}}{\theta} + \pi_{t+1} \right] \\ &= \beta E_t \pi_{t+1} + \frac{\epsilon - 1}{\theta} \hat{m}c_t + \frac{1}{\theta} [\beta E_t \tau_{t+1} - \tau_t + u_t]\end{aligned}$$

Equilibrium From the labor market clearing condition $N_t = \int_0^1 N_{ti} di$, we get

$$\begin{aligned}N_t &= \int_0^1 \left(\frac{Y_{ti}}{A_t} \right)^{\frac{1}{1-\alpha}} di \\ &= \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}}\end{aligned}$$

where we used the fact that all firms are ex post identical $Y_{ti} = Y_t$.

Phillips Curve and Euler Equation Since all firms are identical in equilibrium, the market clearing condition for the final goods market is given by

$$Y_t \left(1 - \frac{\theta}{2} \pi_t^2 \right) = C_t.$$

Taking logs

$$y_t + \log \left(1 - \frac{\theta}{2} \pi_t^2 \right) = c_t.$$

and approximating around the efficient equilibrium with zero inflation:

$$y_t^e + x_t + 0 = c_t^e + (c_t - c_t^e) \Rightarrow c_t = y_t.$$

We now turn to the first order conditions of the households given by

$$\begin{aligned} w_t - p_t &= \sigma c_t + \psi n_t \\ c_t &= E_t c_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \rho) \end{aligned}$$

with $i_t = -\log Q_t$. Combining them with the market clearing condition for the final goods gives

$$\begin{aligned} w_t - p_t &= \left(\sigma + \frac{\psi}{1-\alpha} \right) y_t - \frac{\psi}{1-\alpha} \log a_t \\ y_t &= E_t y_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \rho) \end{aligned}$$

From the definition of markup, we obtain

$$\begin{aligned} mc_t &= w_t - p_t + \frac{\alpha}{1-\alpha} y_t - \frac{1}{1-\alpha} a_t - \log(1-\alpha) \\ &= \left(\sigma + \frac{\psi}{1-\alpha} \right) y_t - \frac{\psi}{1-\alpha} \log a_t + \frac{\alpha}{1-\alpha} y_t - \frac{1}{1-\alpha} a_t - \log(1-\alpha) \\ &= \left(\sigma + \frac{\psi + \alpha}{1-\alpha} \right) y_t - \frac{1 + \psi}{1-\alpha} a_t - \log(1-\alpha) \end{aligned}$$

where the second line uses the first order condition of the households and the market clearing condition for labor.

Hence the deviation of the markup from steady-state is proportional to the deviation of output from its flexible price level

$$\hat{mc}_t = \left(\sigma + \frac{\psi + \alpha}{1-\alpha} \right) (y_t - y_t^n)$$

and its deviation from its efficient level is proportional to the deviation of output from its efficient level

$$mc_t - mc_t^e = \left(\sigma + \frac{\psi + \alpha}{1-\alpha} \right) (y_t - y_t^e)$$

where the flexible price and efficient (no markup) level of output are defined as follows

$$y_t^e = \frac{mc_t^e + \log(1 - \alpha)}{\left(\sigma + \frac{\psi + \alpha}{1 - \alpha}\right)} + \frac{1 + \psi}{(1 - \alpha) \left(\sigma + \frac{\psi + \alpha}{1 - \alpha}\right)} a_t$$

$$mc_t^e = mc$$

From the Euler equation, we obtain the flex-price rate of interest:

$$r_t^n = \rho + \sigma E_t(y_{t+1}^n - y_t^n)$$

$$r_t^n = \rho + \sigma E_t \left[\frac{1 + \psi}{(1 - \alpha) \left(\sigma + \frac{\psi + \alpha}{1 - \alpha}\right)} (a_{t+1} - a_t) \right]$$

Combining everything gives

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r_t^n)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t + \frac{1}{\theta} [\beta E_t \tau_{t+1} - \tau_t]$$

where $\tilde{y}_t$ denotes the deviation of output from its flex-price level. In addition, we have

$$x_t = E_t x_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r_t^e)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \frac{1}{\theta} [\beta E_t \tau_{t+1} - \tau_t + u_t]$$

where x_t is the deviation of output from its efficient level, with $\kappa = \frac{\epsilon - 1}{\theta} \left(\sigma + \frac{\psi + \alpha}{1 - \alpha}\right)$.

For future reference we also define the natural rate of output and the neutral interest rate, which are simply equal to their value in flexible price equilibrium:

$$\begin{aligned}
y_t^n &= \frac{mc^n + \log(1 - \alpha)}{\left(\sigma + \frac{\psi + \alpha}{1 - \alpha}\right)} + \frac{1 + \psi}{(1 - \alpha) \left(\sigma + \frac{\psi + \alpha}{1 - \alpha}\right)} a_t \\
r_t^n &= \rho + \sigma E_t(y_{t+1}^n - y_t^n) \\
r_t^n &= \rho + \sigma E_t \left[\frac{mc_{t+1} - mc_t}{\left(\sigma + \frac{\psi + \alpha}{1 - \alpha}\right)} + \frac{1 + \psi}{(1 - \alpha) \left(\sigma + \frac{\psi + \alpha}{1 - \alpha}\right)} (a_{t+1} - a_t) \right] \\
&= \rho + \sigma E_t \left[\frac{u_{t+1} - u_t}{(\epsilon - 1) \left(\sigma + \frac{\psi + \alpha}{1 - \alpha}\right)} + \frac{1 + \psi}{(1 - \alpha) \left(\sigma + \frac{\psi + \alpha}{1 - \alpha}\right)} (a_{t+1} - a_t) \right] \\
mc_t^n &= -\mu_t
\end{aligned}$$

Firms' profits. Real profits are given by

$$\frac{\Pi(P_{t-1i}, P_{ti})}{P_t} = \frac{1}{P_t} [P_{ti} Y_{ti} - W_t N_{ti} - \tau_t (P_{ti} - P_{t-1i}) Y_{ti} - C_t(P_{t-1i}, P_{ti})].$$

In a symmetric equilibrium, $P_t = P_{ti}$, and using the household's first-order condition $\frac{W_t}{P_t} = \mathcal{M}_t^w Y_t^{\left(\sigma + \frac{\psi}{1 - \alpha}\right)} A_t^{-\frac{\psi}{1 - \alpha}}$ gives:

$$\begin{aligned}
\frac{\Pi(P_{t-1}, P_t)}{P_t} &= Y_t - \frac{W_t}{P_t} N_t - \tau_t \frac{\pi_t}{1 + \pi_t} Y_t - \frac{\theta}{2} \pi_t^2 Y_t \\
&= Y_t - \mathcal{M}_t^w Y_t^\sigma \left(\frac{Y_t}{A_t}\right)^{\frac{\psi}{1 - \alpha}} N_t - \tau_t \frac{\pi_t}{1 + \pi_t} Y_t - \frac{\theta}{2} \pi_t^2 Y_t \\
&= Y_t - \mathcal{M}_t^w Y_t^\sigma \left(\frac{Y_t}{A_t}\right)^{\frac{\psi + 1}{1 - \alpha}} - \tau_t \frac{\pi_t}{1 + \pi_t} Y_t - \frac{\theta}{2} \pi_t^2 Y_t \\
&= Y_t \left[1 - \mathcal{M}_t^w Y_t^{\sigma - 1} \left(\frac{Y_t}{A_t}\right)^{\frac{\psi + 1}{1 - \alpha}} - \tau_t \frac{\pi_t}{1 + \pi_t} - \frac{\theta}{2} \pi_t^2 \right]
\end{aligned}$$

In the steady state, the efficient level of output is given by

$$\begin{aligned} y_t^e &= \frac{mc + \log(1 - \alpha)}{\left(\sigma + \frac{\psi + \alpha}{1 - \alpha}\right)} + \frac{1 + \psi}{(1 - \alpha) \left(\sigma + \frac{\psi + \alpha}{1 - \alpha}\right)} a_t \\ &= \frac{1}{\left(\sigma + \frac{\psi + \alpha}{1 - \alpha}\right)} \left[mc + \log(1 - \alpha) + \frac{1 + \psi}{(1 - \alpha)} a_t \right], \end{aligned}$$

which solves the steady-state labor share

$$\begin{aligned} \mathcal{M}^w (Y_t^e)^{\sigma-1} \left(\frac{Y_t^e}{A_t} \right)^{\frac{\psi+1}{1-\alpha}} &= \frac{\epsilon_N}{\epsilon_N - 1} e^{\left(\sigma-1 + \frac{\psi+1}{1-\alpha}\right) y_t^e - \frac{1+\psi}{(1-\alpha)} a_t} \\ &= \frac{\epsilon_N}{\epsilon_N - 1} e^{\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right) y_t^e - \frac{1+\psi}{(1-\alpha)} a_t} \\ &= \frac{\epsilon_N}{\epsilon_N - 1} e^{mc + \log(1-\alpha)} \\ &= \frac{\epsilon_N}{\epsilon_N - 1} \frac{\epsilon - 1}{\epsilon} (1 - \alpha). \end{aligned}$$

We can now derive the steady-state profit share:

$$\frac{\Pi}{PY} = 1 - \frac{\epsilon_N}{\epsilon_N - 1} \frac{\epsilon - 1}{\epsilon} (1 - \alpha)$$

It is natural to simplify the profit share $\frac{\Pi(P_{t-1}, P_t)}{P_t Y_t}$ by log-linearizing the labor share. Although TIP and the adjustment costs are negligible in the first-order approximation around the zero-inflation steady state, we keep

them in the expression in order to clarify the different mechanisms at play

$$\begin{aligned}
\frac{\Pi_t}{P_t Y_t} &= 1 - \mathcal{M}_t^w Y_t^{\sigma-1} \left(\frac{Y_t}{A_t} \right)^{\frac{\psi+1}{1-\alpha}} - \tau_t \frac{\pi_t}{1 + \pi_t} - \frac{\theta}{2} \pi_t^2 \\
&= 1 - \frac{\mathcal{M}_t^w}{\mathcal{M}^w} \left(\frac{Y_t}{Y_t^e} \right)^{\sigma-1 + \frac{\psi+1}{1-\alpha}} \left[\mathcal{M}^w (Y_t^e)^{\sigma-1} \left(\frac{Y_t^e}{A_t} \right)^{\frac{\psi+1}{1-\alpha}} \right] - \tau_t \frac{\pi_t}{1 + \pi_t} - \frac{\theta}{2} \pi_t^2 \\
&= 1 - \frac{\mathcal{M}_t^w}{\mathcal{M}^w} \left(\frac{Y_t}{Y_t^e} \right)^{\sigma-1 + \frac{\psi+1}{1-\alpha}} \left[\frac{\epsilon_N}{\epsilon_N - 1} \frac{\epsilon - 1}{\epsilon} (1 - \alpha) \right] - \tau_t \pi_t - \frac{\theta}{2} \pi_t^2 \\
&= 1 - \left[\frac{\epsilon_N}{\epsilon_N - 1} \frac{\epsilon - 1}{\epsilon} (1 - \alpha) \right] e^{\left(\sigma-1 + \frac{\psi+1}{1-\alpha} \right) x_t + \frac{u_t^w}{\epsilon-1}} - \tau_t \pi_t - \frac{\theta}{2} \pi_t^2 \\
&\approx 1 - \left[\frac{\epsilon_N}{\epsilon_N - 1} \frac{\epsilon - 1}{\epsilon} (1 - \alpha) \right] \left[1 + \left(\sigma - 1 + \frac{\psi + 1}{1 - \alpha} \right) x_t + \frac{u_t^w}{\epsilon - 1} \right] - \tau_t \pi_t - \frac{\theta}{2} \pi_t^2
\end{aligned}$$

The fourth equation uses the relationship between m_t^w ($\ln \frac{\mathcal{M}_t^w}{\mathcal{M}^w}$) and u_t^w : $u_t^w = (\epsilon - 1)m_t^w$. The approximated profit share allows us to analyze both the deviation of the profit share,

$$\frac{\Pi_t}{P_t Y_t} - \frac{\Pi}{PY} \approx - \left[\frac{\epsilon_N}{\epsilon_N - 1} \frac{\epsilon - 1}{\epsilon} (1 - \alpha) \right] \left[\left(\sigma - 1 + \frac{\psi + 1}{1 - \alpha} \right) x_t + \frac{u_t^w}{\epsilon - 1} \right] - \tau_t \pi_t - \frac{\theta}{2} \pi_t^2$$

and the deviation of the real profit level normalized by steady-state output

$$\left(\frac{\Pi_t}{P_t} - \frac{\Pi}{P} \right) Y^{-1} = \frac{\Pi_t}{P_t Y_t} \frac{Y_t}{Y} - \frac{\Pi}{PY} \approx \frac{\Pi_t}{P_t Y_t} (1 + x_t) - \frac{\Pi}{PY}.$$

B.3 Calibration of the three equation model.

Welfare is evaluated using the following *ad hoc* welfare function:

$$\mathcal{L} = \sum_{t=0}^{\infty} E_0 \beta^t \left[\pi_t^2 + \eta_y (x_t)^2 + \eta_i i_t^2 \right] \quad (39)$$

for some $\eta_y, \eta_i \geq 0$. This welfare function is *ad hoc* because in the calibration, we will choose $\eta_i > 0$ so that the optimal MP implied by equation (39) is

consistent with the standard Taylor rule observed empirically. Note however that a second-order approximation of the household welfare loss around the efficient steady-state would lead to a similar expression of this objective function for $\eta_i = 0$ as shown in [Rotemberg and Woodford \(1999\)](#). One could assume that there is a steady-state production subsidy in the background.

In our calibration, a period is a quarter. The list of parameters is in Table B1. We follow [Galí \(2015\)](#) to calibrate the elasticity of output to labor $(1 - \alpha)$, the discount factor β , the elasticity of intertemporal substitution σ , the Frisch elasticity ψ and the elasticity of substitution ϵ .

Parameters	Description	Value
α	One minus the elasticity of output to labor	0.25
β	Time discount factor	0.99
σ	Elasticity of intertemporal substitution	1
ψ	Inverse Frish elasticity of labor	5
ϵ	Elasticity of substitution across varieties	9
ϵ_N	Elasticity of substitution across labor types	∞
θ	Adjustment cost	372.8
ρ_a	Autocorrelation of productivity shock	0.5
ρ_u	Autocorrelation of markup shock	0.5
ρ_{mp}	Autocorrelation of monetary shock	0.5
η_y	Preference for output stability	0.113
η_i	Preference for interest rate stability	0.687

Table B1: Model parameters

We choose the adjustment cost parameter θ such that the slope of the linearized Phillips curve, κ , in our model is equal to the slope in [Galí \(2015\)](#). If $\bar{\phi}$ denotes the Calvo parameter, the slope with Calvo pricing is given by $\kappa = \frac{(1-\bar{\phi})(1-\bar{\phi}\beta)}{\bar{\phi}} \frac{1-\alpha}{1-\alpha+\alpha\epsilon} \left(\sigma + \frac{\psi+\alpha}{1-\alpha} \right)$. In our model, the slope of the Phillips curve is instead given by $\kappa = \frac{\epsilon-1}{\theta} \left(\sigma + \frac{\psi+\alpha}{1-\alpha} \right)$. Equating the two gives:

$$\theta = (\epsilon - 1) \frac{\bar{\phi}}{(1 - \bar{\phi})(1 - \bar{\phi}\beta)} \frac{1 - \alpha + \alpha\epsilon}{1 - \alpha} \quad (40)$$

A Calvo parameter of 0.75 which corresponds to an average price dura-

tion of one year implies $\kappa = 0.17$, and a Rotemberg parameter of 372.8.

To calibrate the parameters in the welfare loss function η_y, η_i , we choose η_y and η_i such that the Taylor rule with coefficients $\phi_y = .125$ and $\phi_\pi = 1.5$ minimizes welfare losses (39) in the absence of TIP. Both η_y, η_i depend on the persistence of the shocks, and for an interior solution to exist, the persistence of the shocks cannot be too high (Giannoni and Woodford, 2003; Giannoni, 2014). Therefore, we set all persistence parameters to 0.5.²⁰

B.4 Fiscal cost of subsidies

We first log-linearize equation 26,

$$\begin{aligned} \frac{\epsilon - 1}{\theta} (mc_t - mc + \mu_t - \mu - a_t^w) + E_t Q_t \pi_{t+1} &= \pi_t \\ \frac{\epsilon - 1}{\theta} (\hat{m}c_t - a_t^w) + \frac{u_t}{\theta} + \beta E_t \pi_{t+1} &= \pi_t \end{aligned}$$

To derive the fiscal cost of a payroll subsidy, we assume that its rate, a_t^w , follows a rule targeting inflation, similar to equation 23:

$$a_t^w = \varphi_\pi^w \pi_t.$$

We choose φ_π^w so that the impulse response functions of inflation and the output gap match the ones in Figure 1 after a markup shock.

Figures C4, C5, and C6 report the effects of payroll subsidies following a markup shock, a productivity shock, and a monetary policy shock. To achieve the same macroeconomic outcome after a markup shock as a strong TIP ($\varphi_\pi = 300$) does in Figure 1, payroll subsidies amount to 4% of total payrolls or 3.5% of output in the first period. In sharp contrast, the revenues collected with TIP amount to about 0.15% of output in the same period. The persistence of the fiscal cost also differs across the two instruments. The fiscal cost of payroll subsidies, which are proportional to π_t , decreases much

²⁰We have checked that the impulse response functions and welfare implications of TIP are similar when ρ is larger, even when there is no interior solution for the optimal MP.

slower than the revenues from TIP, because they are proportional to π_t^2 instead.

Finally, as we increase the strength of the instrument, payroll subsidies grow linearly while the fiscal revenues implied by TIP increase at a decreasing speed. This is because a stronger TIP can significantly reduce its tax base by moderating price changes, while a higher payroll subsidy rate marginally raises its base by increasing the output gap.

B.5 Determinacy

In this Appendix we analyze the conditions ensuring the uniqueness of the equilibrium path. When there are multiple equilibria, the economy is subject to coordination failures. For example, if all firms expect high inflation and high output gap, the economy could shift to a self-fulfilling equilibrium with excessive inflation. These coordination failures are a potential source of excessive inflation.

To ensure determinacy, it is well-known that in the baseline New Keynesian model, MP should implement the Taylor principle according to which the policy rate reacts strongly to inflation. A natural question is whether TIP could guarantee the uniqueness of the equilibrium path.

Proposition 6. *Assume $\tau^{ss} = 0$. The equilibrium path is unique if one of the following conditions holds*

- $\phi_\pi > 1$ and $\varphi_y < \min \left(\frac{\theta\kappa}{1-\beta}, \frac{\theta\phi_\pi\kappa + \theta(1-\beta)(\sigma(1-\beta) + \phi_y)(1 + \frac{\varphi_\pi}{\theta})}{\phi_\pi - \beta} \right)$
- $\beta < \phi_\pi < 1$ and $\frac{\theta\kappa}{1-\beta} < \varphi_y < \frac{\theta\phi_\pi\kappa + \theta(1-\beta)(\sigma(1-\beta) + \phi_y)(1 + \frac{\varphi_\pi}{\theta})}{\phi_\pi - \beta}$
- $\phi_\pi < \beta$ and $\frac{\theta\kappa}{1-\beta} < \varphi_y$

The first bullet point corresponds to the traditional Taylor principle according to which the reaction of monetary policy to inflation is strong enough so that the path of inflation is always unique. In this case, the reaction of TIP to the output gap can't be too strong. The second and third cases correspond

to a new principle according to which the reaction of the monetary policy to inflation is weak but the reaction of TIP to the output gap is strong.

This finding that a high enough φ_y can ensure determinacy is intuitive and the mechanism is analogous to the Taylor principle. Assume that agents expect the economy to jump to a situation of high inflation and high output gap. Following the TIP rule, policymakers set a very high tax on inflation which leads to deflation, which in turn triggers a recession. This outcome contradicts the initial expectation. This ensures that the economy always stays on a unique equilibrium path.

However, in practice, only the Taylor principle can realistically be implemented for any reasonable calibration of the model's parameters. Following the calibration laid out in the quantitative section, the value of the coefficient φ_y necessary to ensure determinacy, $\frac{\theta\kappa}{1-\beta}$, is slightly above 6300. This coefficient implies a reaction function that is too strong to be realistically implemented. For this reason, in the rest of the paper, we restrict our attention to targeting rules where the Taylor principle applies and the reaction of TIP to the output gap is not too strong.

We now provide a formal proof for the theorem. Substituting out the Taylor rule (23) into the Euler equation (15) gives

$$\begin{aligned} x_t &= E_t x_{t+1} - \frac{1}{\sigma} (\rho + \phi_\pi \pi_t + \phi_y \hat{y}_t - E_t \pi_{t+1} - r_t^e), \\ x_t &= E_t x_{t+1} - \frac{1}{\sigma} (\rho + \phi_\pi \pi_t + \phi_y x_t - E_t \pi_{t+1} + \phi_y (y_t^e - y^{ss}) - r_t^e), \end{aligned}$$

and substituting out the rule for TIP (24) into the Phillips curve (16) gives

$$\left(1 - \epsilon \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right) \pi_t = \beta \left(1 - \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right) E_t \pi_{t+1} + \left(\kappa - \frac{\varphi_y}{\theta}\right) x_t + \frac{\varphi_y}{\theta} \beta E_t x_{t+1} + \frac{1}{\theta} u_t.$$

To analyze the uniqueness of the solution to this system of two difference equations, we compute its eigenvalues. This system with two non predetermined variables is determinate if and only if both eigenvalues are inside the unit circle (Blanchard and Kahn, 1980). We derive the necessary and sufficient conditions for this to hold in appendix, and show sufficient

conditions in the following proposition which turn out to be more intuitive.

We first rewrite this system in matrix form

$$A \begin{pmatrix} \pi_t \\ x_t \end{pmatrix} = B \begin{pmatrix} E_t \pi_{t+1} \\ E_t x_{t+1} \end{pmatrix} + C \begin{pmatrix} \tilde{u}_t \\ r_t^e - \rho - \phi_y(y_t^e - y^{ss}) \end{pmatrix}$$

with $A = \begin{pmatrix} \phi_\pi & \sigma + \phi_y \\ 1 - \epsilon \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta} & \frac{\varphi_y}{\theta} - \kappa \end{pmatrix}$, $B = \begin{pmatrix} 1 & \sigma \\ \beta \left(1 - \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right) & \frac{\varphi_y}{\theta} \beta \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\theta} \end{pmatrix}$

We now solve for the eigenvalues of this system. We first invert A

$$A^{-1} = \frac{1}{\phi_\pi \left(\frac{\varphi_y}{\theta} - \kappa\right) - (\sigma + \phi_y) \left(1 - \epsilon \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right)} \begin{pmatrix} \frac{\varphi_y}{\theta} - \kappa & -\sigma - \phi_y \\ \epsilon \frac{\tau^{ss}}{\theta} - 1 - \frac{\varphi_\pi}{\theta} & \phi_\pi \end{pmatrix}$$

and then multiply it by B :

$$\begin{aligned} A^{-1}B &= \frac{1}{\phi_\pi \left(\frac{\varphi_y}{\theta} - \kappa\right) - (\sigma + \phi_y) \left(1 - \epsilon \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right)} \begin{pmatrix} \frac{\varphi_y}{\theta} - \kappa & -\sigma - \phi_y \\ \epsilon \frac{\tau^{ss}}{\theta} - 1 - \frac{\varphi_\pi}{\theta} & \phi_\pi \end{pmatrix} \begin{pmatrix} 1 & \sigma \\ \beta \left(1 - \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right) & \frac{\varphi_y}{\theta} \beta \end{pmatrix} \\ &= \Omega \begin{pmatrix} \frac{\varphi_y}{\theta} - \kappa - (\sigma + \phi_y) \beta \left(1 - \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right) & \sigma \left(\frac{\varphi_y}{\theta} - \kappa\right) - (\sigma + \phi_y) \beta \frac{\varphi_y}{\theta} \\ \left(1 + \frac{\varphi_\pi}{\theta}\right) [\beta \phi_\pi - 1] + \frac{\tau^{ss}}{\theta} (\epsilon - \phi_\pi \beta) & -\sigma \left(1 - \epsilon \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right) + \phi_\pi \frac{\varphi_y}{\theta} \beta \end{pmatrix} \\ &= -\Omega \begin{pmatrix} -\frac{\varphi_y}{\theta} + \kappa + (\sigma + \phi_y) \beta \left(1 - \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right) & \sigma \left(\kappa - \frac{\varphi_y}{\theta}\right) + (\sigma + \phi_y) \beta \frac{\varphi_y}{\theta} \\ \left(1 + \frac{\varphi_\pi}{\theta}\right) [1 - \beta \phi_\pi] + \frac{\tau^{ss}}{\theta} (\phi_\pi \beta - \epsilon) & \sigma \left(1 - \epsilon \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right) - \phi_\pi \frac{\varphi_y}{\theta} \beta \end{pmatrix} \end{aligned}$$

with $\Omega = \frac{1}{\phi_\pi \left(\frac{\varphi_y}{\theta} - \kappa\right) - (\sigma + \phi_y) \left(1 - \epsilon \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right)}$. We denote this matrix A' , and we now compute its trace and determinant.

$$\begin{aligned} \text{Tr} A' &= -\Omega \left[-\frac{\varphi_y}{\theta} + \kappa + (\sigma + \phi_y) \beta \left(1 - \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right) + \sigma \left(1 - \epsilon \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right) - \phi_\pi \frac{\varphi_y}{\theta} \beta \right] \\ &= \frac{1}{(\sigma + \phi_y) \left(1 - \epsilon \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right)} \\ &\quad \left[-\frac{\varphi_y}{\theta} (1 + \beta \phi_\pi) + \kappa + (\sigma(\beta + 1) + \phi_y \beta) \left(1 + \frac{\varphi_\pi}{\theta}\right) - \frac{\tau^{ss}}{\theta} ((\sigma + \phi_y) \beta + \epsilon \sigma) \right] \end{aligned}$$

$$\begin{aligned}
\det A' &= \left(\frac{1}{(\sigma + \phi_y) \left(1 - \epsilon \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right)} \right)^2 \\
&\times \left[\left[-\frac{\varphi_y}{\theta} + \kappa + (\sigma + \phi_y) \beta \left(1 - \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right) \right] \left[\sigma \left(1 - \epsilon \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right) - \phi_\pi \frac{\varphi_y}{\theta} \beta \right] \right. \\
&\quad \left. - \left[\sigma \left(\kappa - \frac{\varphi_y}{\theta}\right) + (\sigma + \phi_y) \beta \frac{\varphi_y}{\theta} \right] \left[\left(1 + \frac{\varphi_\pi}{\theta}\right) [1 - \beta \phi_\pi] + \frac{\tau^{ss}}{\theta} (\phi_\pi \beta - \epsilon) \right] \right] \\
&= \left(\frac{1}{(\sigma + \phi_y) \left(1 - \epsilon \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right)} \right)^2 \\
&\times \left(\left[\left[-\frac{\varphi_y}{\theta} + \kappa + (\sigma + \phi_y) \beta \left(1 + (\epsilon - 1) \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi - \epsilon \tau^{ss}}{\theta}\right) \right] \left[\sigma \left(1 + \frac{\varphi_\pi - \epsilon \tau^{ss}}{\theta}\right) - \phi_\pi \frac{\varphi_y}{\theta} \beta \right] \right. \right. \\
&\quad \left. \left. - \left[\sigma \left(\kappa - \frac{\varphi_y}{\theta}\right) + (\sigma + \phi_y) \beta \frac{\varphi_y}{\theta} \right] \left[\left(1 + \frac{\varphi_\pi - \epsilon \tau^{ss}}{\theta}\right) [1 - \beta \phi_\pi] + \frac{\tau^{ss}}{\theta} \beta \phi_\pi (1 - \epsilon) \right] \right] \right) \\
&= \left(\frac{1}{(\sigma + \phi_y) \left(1 + \frac{\varphi_\pi - \epsilon \tau^{ss}}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right)} \right)^2 \\
&\times \left(\beta \left[(\sigma + \phi_y) \left(1 + \frac{\varphi_\pi - \epsilon \tau^{ss}}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right) \right] \left[\sigma \left(1 + \frac{\varphi_\pi - \epsilon \tau^{ss}}{\theta}\right) - \frac{\varphi_y}{\theta} \right] \right. \\
&\quad \left. + \frac{\tau^{ss}}{\theta} (\epsilon - 1) \left((\sigma + \phi_y) \beta \left[\sigma \left(1 + \frac{\varphi_\pi - \epsilon \tau^{ss}}{\theta}\right) - \phi_\pi \frac{\varphi_y}{\theta} \beta \right] + \beta \phi_\pi \left[\sigma \left(\kappa - \frac{\varphi_y}{\theta}\right) + (\sigma + \phi_y) \beta \frac{\varphi_y}{\theta} \right] \right) \right) \\
&= \left(\frac{1}{(\sigma + \phi_y) \left(1 + \frac{\varphi_\pi - \epsilon \tau^{ss}}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right)} \right)^2 \\
&\times \beta \left(\left[(\sigma + \phi_y) \left(1 + \frac{\varphi_\pi - \epsilon \tau^{ss}}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right) \right] \left[\sigma \left(1 + \frac{\varphi_\pi - \epsilon \tau^{ss}}{\theta}\right) - \frac{\varphi_y}{\theta} \right] \right. \\
&\quad \left. + \frac{\tau^{ss}}{\theta} (\epsilon - 1) \sigma \left((\sigma + \phi_y) \left(1 + \frac{\varphi_\pi - \epsilon \tau^{ss}}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
\det A' &= \left(\frac{1}{(\sigma + \phi_y) \left(1 + \frac{\varphi_\pi - \epsilon \tau^{ss}}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right)} \right)^2 \\
&\times \beta \left[(\sigma + \phi_y) \left(1 + \frac{\varphi_\pi - \epsilon \tau^{ss}}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right) \right] \left[\sigma \left(1 + \frac{\varphi_\pi - \epsilon \tau^{ss}}{\theta}\right) - \frac{\varphi_y}{\theta} + \frac{\tau^{ss}}{\theta} (\epsilon - 1) \sigma \right] \\
&= \left(\frac{1}{(\sigma + \phi_y) \left(1 + \frac{\varphi_\pi - \epsilon \tau^{ss}}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right)} \right) \beta \left[\sigma \left(1 + \frac{\varphi_\pi - \tau^{ss}}{\theta}\right) - \frac{\varphi_y}{\theta} \right]
\end{aligned}$$

Let's assume $\varphi_y, \varphi_\pi, \phi_\pi, \phi_y \geq 0$ and restrict our analysis to the case where the determinant is positive (both eigenvalues have the same sign and are non-imaginary), i.e. assume that

$$\left(\sigma \left(1 + \frac{\varphi_\pi - \tau^{ss}}{\theta}\right) - \frac{\varphi_y}{\theta} \right) \left((\sigma + \phi_y) \left(1 + \frac{\varphi_\pi - \epsilon \tau^{ss}}{\theta}\right) + \phi_\pi \kappa - \phi_\pi \frac{\varphi_y}{\theta} \right) > 0$$

. These restrictions are consistent with any empirically reasonable parametrization.

This system with two non predetermined variables is determinate if and only if both eigenvalues are within the unit circle ([Blanchard and Kahn, 1980](#)). There are then two sufficient and necessary conditions for both eigenvalues to be within the unit circle: $\det A' < 1$ and $\text{Tr} A < 1 + \det A'$. The latter condition can be derived from the condition that both eigenvalues are strictly below 1 $(1 - \lambda_1)(1 - \lambda_2) > 0$ when both are positive, or both strictly above -1 $(-1 - \lambda_1)(-1 - \lambda_2) > 0$ when both are negative. The condition $\det A' < 1$ gives:

$$\frac{\tau^{ss}}{\theta} ((\sigma + \phi_y) \epsilon + \beta \sigma) + \frac{\varphi_y}{\theta} (\phi_\pi - \beta) < \phi_\pi \kappa + (\sigma(1 - \beta) + \phi_y) \left(1 + \frac{\varphi_\pi}{\theta}\right)$$

This condition is that φ_y is small enough:

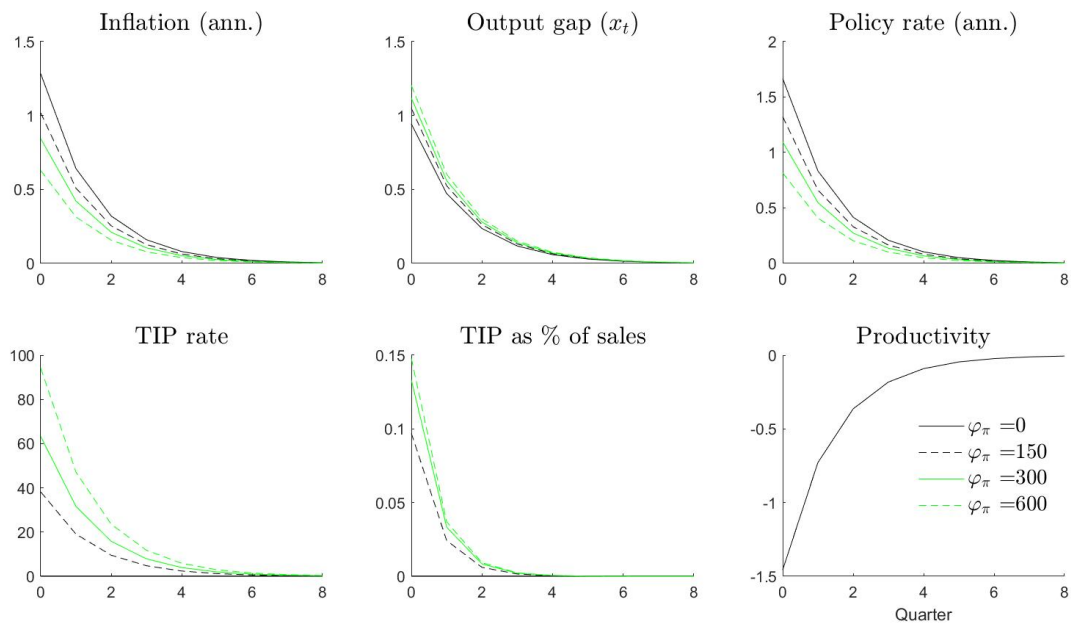
$$\varphi_y < \frac{\theta\phi_\pi\kappa + \theta(\sigma(1 - \beta) + \phi_y) \left(1 + \frac{\varphi_\pi}{\theta}\right) - \tau^{ss}((\sigma + \phi_y)\epsilon + \beta\sigma)}{\phi_\pi - \beta}$$

The second necessary and sufficient condition is that $Tr A' < 1 + det A'$ which gives

$$\begin{aligned} & \frac{\left[-\frac{\varphi_y}{\theta}(1 + \beta\phi_\pi) + \kappa + (\sigma(\beta + 1) + \phi_y\beta) \left(1 + \frac{\varphi_\pi}{\theta}\right) - \frac{\tau^{ss}}{\theta}((\sigma + \phi_y)\beta + \epsilon\sigma)\right]}{(\sigma + \phi_y) \left(1 + \frac{\varphi_\pi - \epsilon\tau^{ss}}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right)} < \\ & 1 + \frac{\beta \left[\sigma \left(1 + \frac{\varphi_\pi - \tau^{ss}}{\theta}\right) - \frac{\varphi_y}{\theta}\right]}{(\sigma + \phi_y) \left(1 + \frac{\varphi_\pi - \epsilon\tau^{ss}}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right)} \\ & - \frac{\varphi_y}{\theta}(1 + \beta\phi_\pi) + \kappa + (\sigma(\beta + 1) + \phi_y\beta) \left(1 + \frac{\varphi_\pi}{\theta}\right) - \frac{\tau^{ss}}{\theta}((\sigma + \phi_y)\beta + \epsilon\sigma) < \\ & (\sigma + \phi_y) \left(1 + \frac{\varphi_\pi - \epsilon\tau^{ss}}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right) + \beta \left[\sigma \left(1 + \frac{\varphi_\pi - \tau^{ss}}{\theta}\right) - \frac{\varphi_y}{\theta}\right] \\ & - \frac{\varphi_y}{\theta}(1 + \beta\phi_\pi) + \kappa - \left(1 + \frac{\varphi_\pi}{\theta}\right) (1 - \beta) \phi_y \\ & < \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right) - \beta \frac{\varphi_y}{\theta} + \frac{\tau^{ss}}{\theta}((\sigma + \phi_y)\beta + \epsilon\sigma - (\sigma + \phi_y)\epsilon - \beta\sigma) \\ & (\phi_\pi - 1)\kappa + \frac{\varphi_y}{\theta} [(1 + \beta\phi_\pi) - \beta - \phi_\pi] + \left(1 + \frac{\varphi_\pi}{\theta}\right) (1 - \beta) \phi_y + \frac{\tau^{ss}}{\theta} \phi_y (\beta - \epsilon) > 0 \\ & (\phi_\pi - 1) \left[\kappa - \frac{\varphi_y}{\theta}(1 - \beta)\right] + \left(1 + \frac{\varphi_\pi}{\theta}\right) (1 - \beta) \phi_y > \frac{\tau^{ss}}{\theta} \phi_y (\epsilon - \beta) \end{aligned}$$

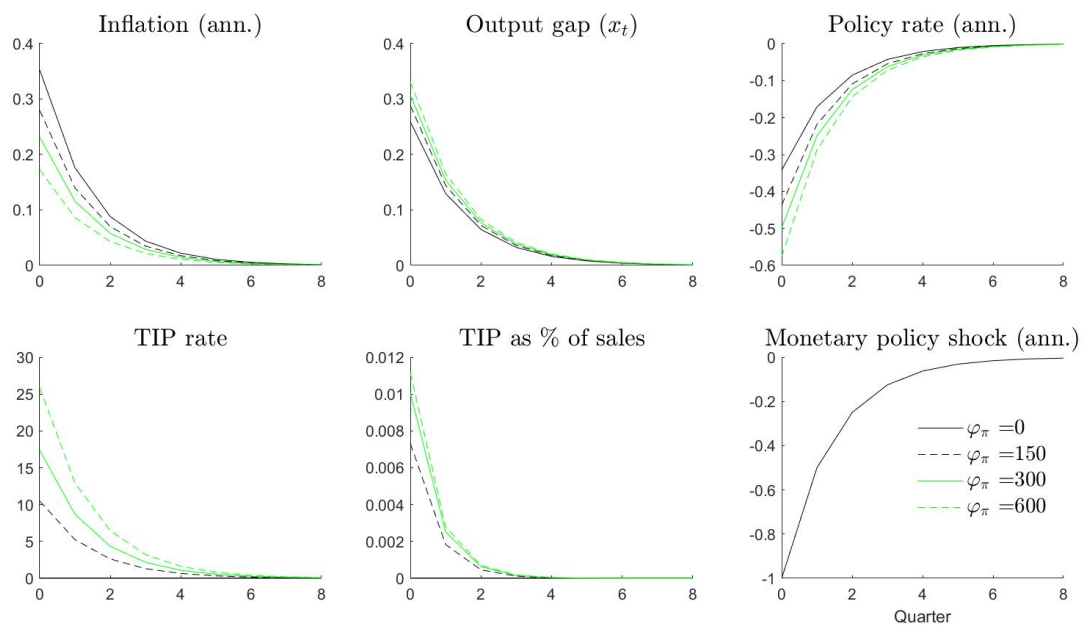
This ends the proof.

C Additional Graphs - Simulations



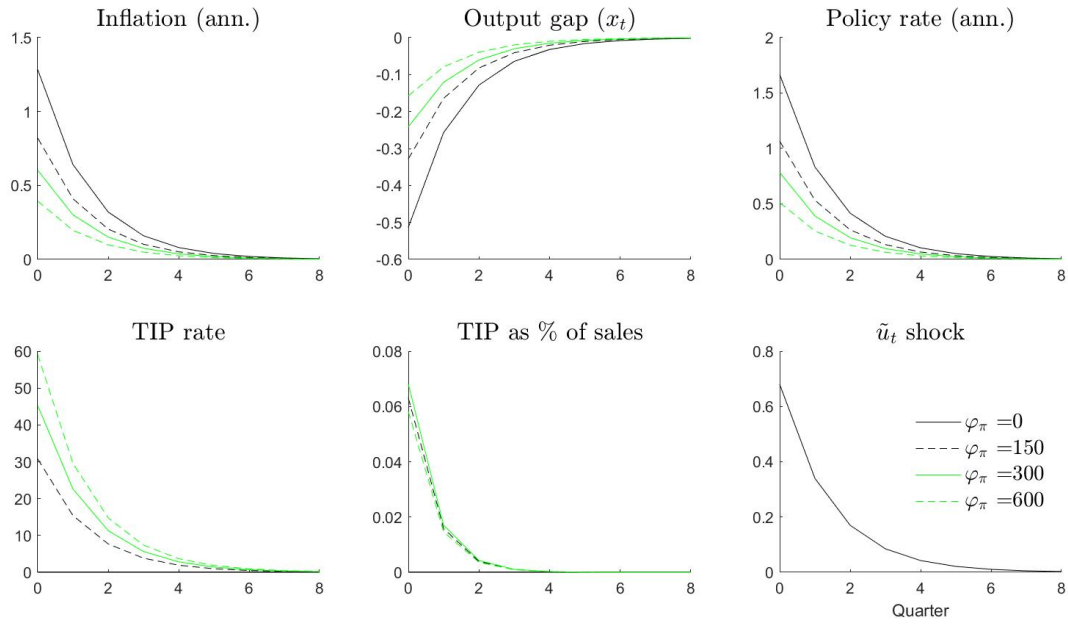
Notes: The initial productivity shock is such that the response of inflation in the baseline without TIP is the same as in the case of a markup shock. Inflation and the policy rate are annualized. Output gap refers to the deviation of output from its efficient level, x_t .

Figure C1: Effects of TIP following a negative productivity shock



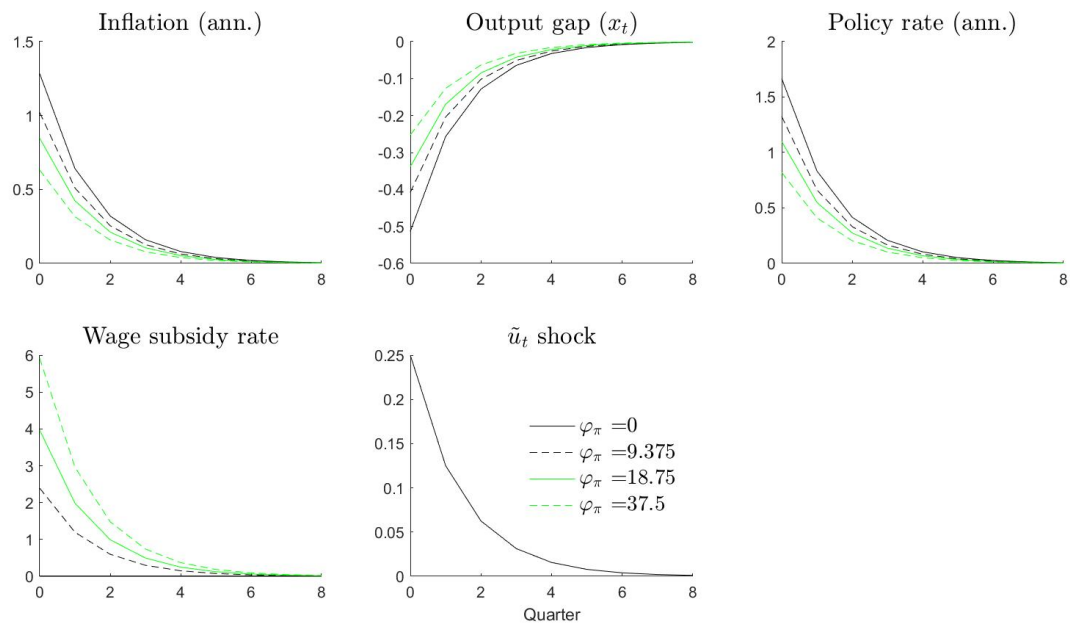
Notes: The initial monetary policy shocks is 0.25p.p. (or 1p.p. annualized). Inflation and the policy rate are annualized. Output gap refers to the deviation of output from its efficient level, x_t .

Figure C2: Effects of TIP following a monetary policy shock



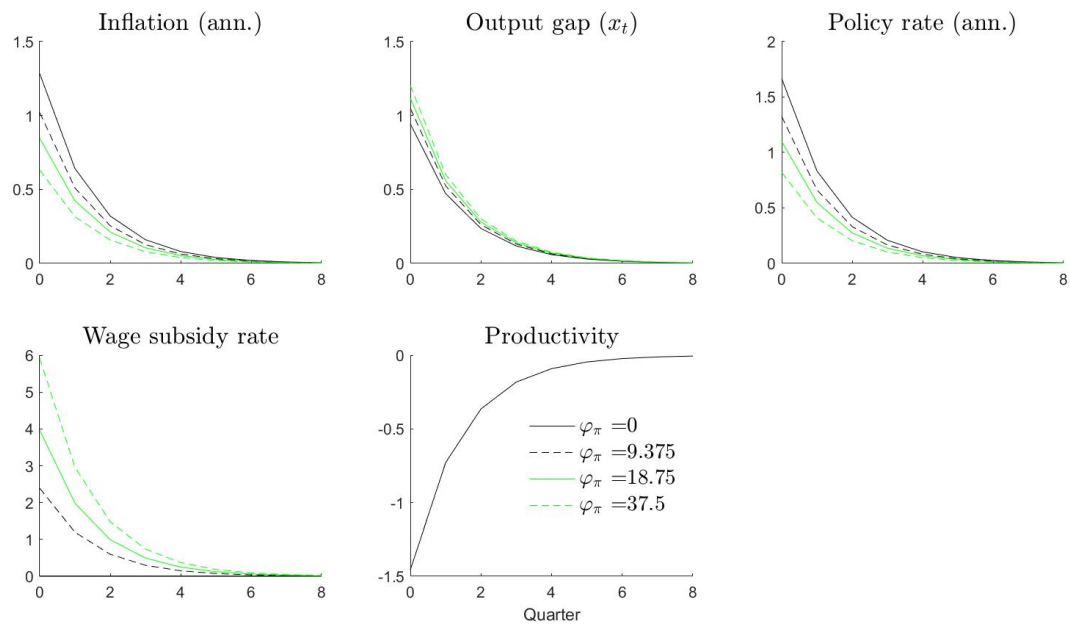
Notes: This figure corresponds to an economy with smaller adjustment costs. We set the Rotemberg parameter such that the equivalent Calvo parameter is equal to 0.5 instead of 0.75. Inflation and the policy rate are annualized. Output gap refers to the deviation of output from its efficient level, x_t .

Figure C3: Effects of TIP following a markup shock with lower price stickiness



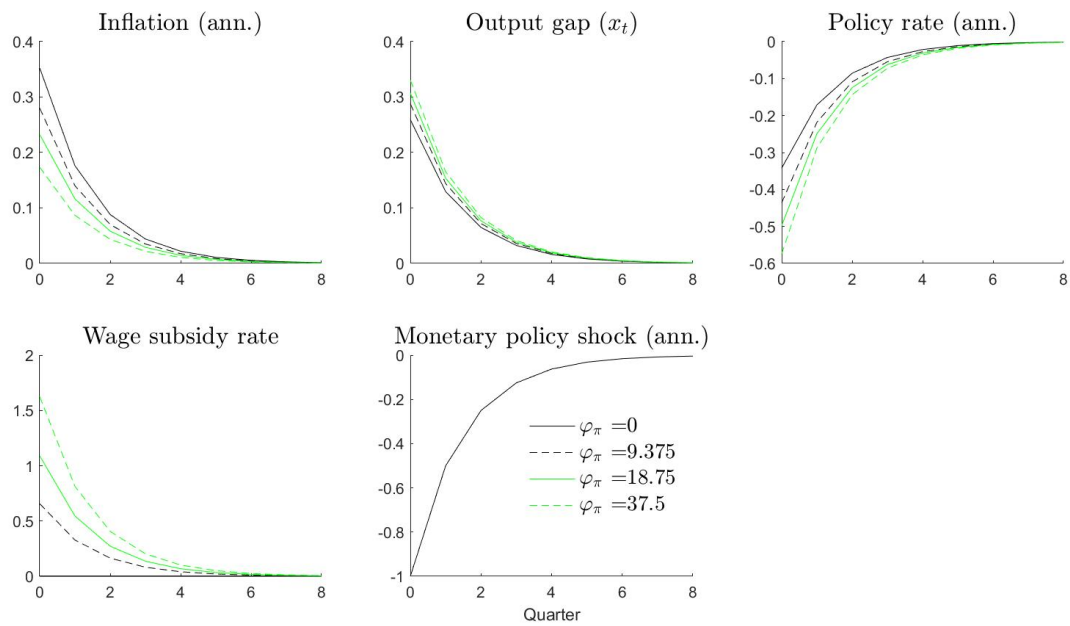
Notes: This figure corresponds to an economy with time-varying payroll subsidies instead of TIP. The shock is the same as in Figure 1. Inflation and the policy rate are annualized. Output gap refers to the deviation of output from its efficient level, x_t .

Figure C4: Effects of payroll subsidies following a markup shock



Notes: This figure corresponds to an economy with time-varying payroll subsidies instead of TIP. The shock is the same as in Figure C1. Inflation and the policy rate are annualized. Output gap refers to the deviation of output from its efficient level, x_t .

Figure C5: Effects of payroll subsidies following a productivity shock



Notes: This figure corresponds to an economy with time-varying payroll subsidies instead of TIP. The shock is the same as in Figure C2. Inflation and the policy rate are annualized. Output gap refers to the deviation of output from its efficient level, x_t .

Figure C6: Effects of payroll subsidies following a monetary policy shock

	No TIP	Moderate TIP	Strong TIP	Extreme TIP
φ_π	0	150	300	600
ϕ_π	1.5	1.5	1.5	1.5
ϕ_y	0.125	0.125	0.125	0.125
$\sigma(\pi_t^{ann})$	0.74	0.59	0.49	0.36
$\sigma(x_t)$	1.09	1.21	1.29	1.39
$\sigma(i_t^{ann})$	0.96	0.76	0.63	0.47
$\sigma(\tau_t)$	0.00	44.06	73.02	108.77
$E(\pi_t \tau_t)$	0.00	0.13	0.18	0.20
$\mathcal{L}^* \times 10^4$	1.32	0.91	0.70	0.50

Notes: Productivity shocks only. The setting is the same as in Figure C1.

Table C2: Evaluation of policy rules under productivity shocks

	No TIP	Moderate TIP	Strong TIP	Extreme TIP
φ_π	0	150	300	600
ϕ_π	1.5	1.5	1.5	1.5
ϕ_y	0.125	0.125	0.125	0.125
$\sigma(\pi_t^{ann})$	0.20	0.16	0.13	0.10
$\sigma(x_t)$	0.30	0.33	0.35	0.38
$\sigma(i_t^{ann})$	0.20	0.25	0.29	0.33
$\sigma(\tau_t)$	0.00	12.10	20.06	29.88
$E(\pi_t \tau_t)$	0.00	0.01	0.01	0.01
$\mathcal{L}^* \times 10^4$	0.08	0.08	0.09	0.10

Notes: Monetary policy shocks only. The setting is the same as in Figure C2.

Table C3: Evaluation of policy rules under monetary policy shocks

D Distortion of Relative Prices

D.1 Model

Setting. We extend the model to include a continuum of sectors indexed by $s \in [0, 1]$. There is also a continuum of firms within each sector which are in monopolistic competition. These firms produce differentiated goods which are used in the production of a new type of goods: the sector goods, which we denote C_s . Sector goods are used for the production of final goods. They are also used by intermediate firms to pay for price increases in their relevant sector. The final good firms have the following technology:

$$\ln Y_t = \int \gamma_s \ln C_{ts} ds$$

where $\int \gamma_s = 1$. There are also aggregator firms in each sector with the following technology:

$$Y_{ts} = \left(\int y_{ts}^{1-1/\epsilon} ds \right)^{\frac{\epsilon}{\epsilon-1}}$$

Denoting $\mathcal{C}(P_{tis}, P_{t-1is})$ the nominal adjustment costs of firm i in sector s at time t , the market clearing condition in each sector is given by:

$$Y_{ts} = C_{ts}$$

The adjustment cost needs to be paid in terms of final goods.

$$Y_t = C_t + \int_s \frac{\theta_s}{2} \pi_s^2 Y_{ts}$$

Firms' First-Order Condition Consider firms in sector s . The first-order conditions associated with these firm's problem are:

$$(1 - \epsilon_{st}) \left(\frac{P_{ti}}{P_{st}} \right)^{-\epsilon_{st}} Y_{st} - (1 - \epsilon_{st}) \left(\frac{P_{ti}}{P_{st}} \right)^{-\epsilon_{st}} Y_{st} \tau_t - \tau_t \epsilon_{st} \frac{P_{t-1i}}{P_{ti}} Y_{st} \left(\frac{P_{ti}}{P_{st}} \right)^{-\epsilon_{st}} \\ + \frac{\epsilon_{st}}{1 - \alpha} \frac{W_t}{P_{ti}} \left[\left(\frac{P_{ti}}{P_{st}} \right)^{-\epsilon_{st}} \frac{Y_{st}}{A_{st}} \right]^{\frac{1}{1-\alpha}} - \frac{\theta_s}{P_{t-1i}} \left(\frac{P_{ti}}{P_{t-1i}} - 1 \right) P_t Y_{st} + E Q_t V'(P_{ti}) = 0$$

and

$$V'(P_{t-1i}) = \tau_t Y_{st} \left(\frac{P_{ti}}{P_{st}} \right)^{-\epsilon_{st}} + \frac{P_{ti}}{P_{t-1i}^2} \theta_s \left(\frac{P_{ti}}{P_{t-1i}} - 1 \right) P_t Y_{st}.$$

Assuming symmetry gives

$$\left((1 - \epsilon_{st}) Y_{st} - (1 - \epsilon_{st}) Y_{st} \tau_t - \tau_t \epsilon_{st} \frac{P_{t-1}}{P_{st}} Y_{st} + \frac{\epsilon_{st}}{1 - \alpha} \frac{W_t}{P_{st}} \left[\frac{Y_{st}}{A_{st}} \right]^{\frac{1}{1-\alpha}} - \frac{\theta_s}{P_{t-1,s}} \pi_{st} P_t Y_{st} \right) \\ + E_t Q_t \left[\tau_{t+1} Y_{t+1s} + (\pi_{t+1s} + 1) \frac{P_{t+1}}{P_{ts}} \theta_s \pi_{t+1s} Y_{t+1s} \right] = 0 \\ \iff \left(1 - \epsilon_{st} - \tau_t \left(1 - \epsilon_{st} \frac{\pi_{st}}{1 + \pi_{st}} \right) + \epsilon_{st} MC_{st} - \theta_s \pi_{st} \frac{(\pi_{st} + 1)}{\tilde{P}_{st}} \right) \\ + E_t Q_t \left[\tau_{t+1} \frac{Y_{t+1s}}{Y_{st}} + \frac{(\pi_{t+1s} + 1)^2}{\tilde{P}_{st+1}} \theta_s \pi_{t+1s} \frac{Y_{t+1s}}{Y_{st}} \right] = 0 \\ \iff \left((1 - \epsilon_{st}) \left(1 - \frac{\epsilon_{st}}{\epsilon_{st} - 1} MC_{st} \right) - \tau_t \left(1 - \epsilon_{st} \frac{\pi_{st}}{1 + \pi_{st}} \right) - \theta_s \pi_{st} \frac{(\pi_{st} + 1)}{\tilde{P}_{st}} \right) \\ + E_t Q_t \left[\tau_{t+1} \frac{Y_{t+1s}}{Y_{st}} + \frac{(\pi_{t+1s} + 1)^2}{\tilde{P}_{st+1}} \theta_s \pi_{t+1s} \frac{Y_{t+1s}}{Y_{st}} \right] = 0 \\ \iff \frac{1}{\theta_s} \left((1 - \epsilon_{st}) (1 - \mathcal{M}_{st} MC_{st}) - \tau_t \left(1 - \epsilon_{st} \frac{\pi_{st}}{1 + \pi_{st}} \right) \right) - \pi_{st} \frac{(\pi_{st} + 1)}{\tilde{P}_{st}} \\ + E_t Q_t \left[\frac{\tau_{t+1}}{\theta_s} \frac{Y_{t+1s}}{Y_{st}} + \frac{(\pi_{t+1s} + 1)^2}{\tilde{P}_{st+1}} \pi_{t+1s} \frac{Y_{t+1s}}{Y_{st}} \right] = 0$$

with $MC_{st} = \frac{W_t}{P_{st}(1-\alpha)} \frac{Y_{st}^{\frac{\alpha}{1-\alpha}}}{A_{st}^{\frac{1}{1-\alpha}}}$ and $\mathcal{M}_{st} = \frac{\epsilon_{st}}{\epsilon_{st}-1}$ and $\tilde{P}_{ts} = \frac{P_{st}}{P_t}$.

We denote the steady-state markup $\bar{\mathcal{M}}$. The next step is to linearize this optimality condition around a steady-state with no inflation, constant output, a *non-zero tax* on price changes, a price markup, $\pi = 0, \tau = \tau^{ss}, Y_s = Y'_s, MC_{st}\mathcal{M} = 1$. Denoting mc the log of the real marginal cost MC and μ the log of $\mathcal{M}$, we obtain:

$$\begin{aligned}
& \frac{1}{\theta_s} [(1-\epsilon)(1 - *(1 + mc_{st} - mc + \mu_t - \mu)) - \hat{\tau}_t - \tau^{ss} + \tau^{ss}\epsilon\pi_{st}] \\
& + \beta \frac{\tau^{ss}}{\theta_s} \left[1 + \hat{q}_t + \frac{\hat{\tau}_{t+1}}{\tau^{ss}} + \hat{y}_{t+1} - \hat{y}_t \right] + \beta\pi_{st+1} = \pi_{st} \\
& \frac{1}{\theta_s} [(1-\epsilon)(1 - 1*(1 + mc_{st} - mc + \mu_t - \mu)) - \hat{\tau}_t - \tau^{ss} + \tau^{ss}\epsilon\pi_{st}] \\
& + \beta \frac{\tau^{ss}}{\theta_s} \left[1 - \pi_{st+1} + \frac{\hat{\tau}_{t+1}}{\tau^{ss}} + (1-\sigma)(\hat{c}_{t+1} - \hat{c}_t) \right] + \beta\pi_{st+1} = \pi_{st} \\
& \frac{1}{\theta_s} [(1-\epsilon)(-1*(\hat{m}c_{st} + \hat{\mu}_t)) - \hat{\tau}_t + \tau^{ss}\epsilon\pi_{st}] \\
& + \beta \frac{\tau^{ss}}{\theta_s} \left[-\pi_{st+1} + \frac{\hat{\tau}_{t+1}}{\tau^{ss}} + (1-\sigma)(\hat{c}_{t+1} - \hat{c}_t) \right] + \beta\pi_{st+1} = \pi_{st} \\
\\
& \pi_{st} \left(1 - \epsilon \frac{\tau^{ss}}{\theta_s} \right) = \beta E_t \pi_{st+1} \left(1 - \frac{\tau^{ss}}{\theta_s} \right) + \frac{(\epsilon-1)}{\theta_s} (\hat{m}c_{st} + \hat{\mu}_t) + \frac{1}{\theta_s} [\beta E_t \hat{\tau}_{t+1} - \hat{\tau}_t] \\
& \quad + \beta \frac{\tau^{ss}}{\theta_s} (1-\sigma)(\hat{c}_{t+1} - \hat{c}_t)
\end{aligned}$$

Final goods optimal demand for sector goods From the FOC of final goods firms,

$$Y_{ts} = \gamma_s \frac{P_t Y_t}{P_{ts}}.$$

we take the logs and obtain

$$\begin{aligned} y_{ts} &= \log \gamma_s + p_t + y_t - p_{ts} \\ \Rightarrow y_{ts} - y_t &= \log \gamma_s - (p_{ts} - p_t) \\ \Rightarrow \tilde{y}_{ts} &= \log \gamma_s - \tilde{p}_{ts} \end{aligned}$$

where $\tilde{x}_s = x_s - x$ and

$$\begin{aligned} y_t &= \int \gamma_s y_{ts} ds \\ p_t &= \int \gamma_s (p_{ts} - \log \gamma_s) ds \end{aligned}$$

We can obtain the log change in the aggregate consumer price index by taking the time difference of the last equation

$$\pi_t = p_t - p_{t-1} = \int \gamma_s \pi_{ts} ds$$

Market clearing Since all firms are identical within each sector in equilibrium, the market clearing condition for the sectorial goods market is given by

$$Y_{st} = C_{st}.$$

where C_{st} denotes the consumption of sectorial good s by final goods firms. Taking logs

$$c_{st} = y_{st}$$

The market clearing for the final goods market is simply given by

$$Y_t = C_t + \int_s \frac{\theta_s}{2} \pi_{st}^2 Y_s$$

At first order the approximation is given by

$$c_t = y_t.$$

and at second order the approximation is given by

$$c_t = y_t - \frac{1}{2}E[\theta_s \pi_{st}^2].$$

From the labor market clearing condition $N_t = \int_s \int_{i \in s} N_{ti} di ds$, we get

$$\begin{aligned} N_t &= \int_s \int_{i \in s} \left(\frac{Y_{ti}}{A_{st}} \right)^{\frac{1}{1-\alpha}} di ds \\ &= \int_s \left(\frac{Y_{ts}}{A_{st}} \right)^{\frac{1}{1-\alpha}} ds \\ &= \int_s \left(\frac{\gamma_s P_t Y_t}{P_{st} A_{st}} \right)^{\frac{1}{1-\alpha}} ds \\ &= \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_s \left(\frac{\gamma_s P_t A_t}{P_{st} A_{st}} \right)^{\frac{1}{1-\alpha}} ds \\ &= \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \Delta_t \end{aligned}$$

where $\Delta = \int_s \left(\frac{\gamma_s P_t A_t}{P_{st} A_{st}} \right)^{\frac{1}{1-\alpha}} ds$ captures the costs entailed by price distortions and misallocation of sector goods, and where we used the fact that all firms are ex post identical $Y_{ti} = Y_{st}$ in each sector and the FOC of the final goods firms $Y_{st} = \gamma_s \frac{P_t Y_t}{P_{st}}$, and where we define A_t the geometric mean of sectorial TFPs, $A_{st}: a_t = \int_s \gamma_s a_{st} ds$.

We then approximate Δ_t . We need to first obtain an expression of relative prices under flexible prices. We start from the optimal markup of a firm in

sector s given by

$$\mathcal{M}_{st}^{-1} = \frac{W_t^f}{P_{st}^f} \frac{(Y_{st}^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)A_{st}^{\frac{1}{1-\alpha}}} = \frac{W_t^f}{P_t^f} \frac{P_t^f}{P_{st}^f} \frac{(\gamma_s P_t^f Y_t^f / P_{st}^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)A_{st}^{\frac{1}{1-\alpha}}}$$

which implies

$$\frac{P_t^f A_t}{P_{st}^f A_{st}} = \left[\left(\frac{\mathcal{M}_{st} W_t^f}{P_t^f} \right) \frac{(Y_t^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)A_t^{\frac{1}{1-\alpha}}} \right]^{\alpha-1} \gamma_s^{-\alpha}$$

We next take this ratio at the power γ_s , take the product over all sectors:

$$\begin{aligned} \Pi_s^S \left(\frac{P_t^f A_t}{P_{st}^f A_{st}} \right)^{\gamma_s} &= \Pi_s^S \left(\left[\left(\frac{\mathcal{M}_{st} W_t^f}{P_t^f} \right) \frac{(Y_t^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)A_t^{\frac{1}{1-\alpha}}} \right]^{\alpha-1} \gamma_s^{-\alpha} \right)^{\gamma_s} \\ \frac{P_t^f A_t}{\Pi_s^S \gamma_s^{\gamma_s} \Pi_s^S (P_{st}^f / \gamma_s)^{\gamma_s} \Pi_s^S A_{st}^{\gamma_s}} &= \left[\left(\frac{W_t^f}{P_t^f} \right) \frac{(Y_t^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)A_t^{\frac{1}{1-\alpha}}} \right]^{\alpha-1} \Pi_s^S \mathcal{M}_{st}^{\gamma_s(\alpha-1)} \gamma_s^{-\alpha \gamma_s} \\ \frac{P_t^f A_t}{\Pi_s^S \gamma_s^{\gamma_s} P_t^f A_t} &= \left[\left(\frac{W_t^f}{P_t^f} \right) \frac{(Y_t^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)A_t^{\frac{1}{1-\alpha}}} \right]^{\alpha-1} \Pi_s^S \mathcal{M}_{st}^{\gamma_s(\alpha-1)} \gamma_s^{-\alpha \gamma_s} \\ 1 &= \left[\left(\frac{\mathcal{M}_{st} W_t^f}{P_t^f} \right) \frac{(Y_t^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)A_t^{\frac{1}{1-\alpha}}} \right]^{\alpha-1} \end{aligned}$$

with $\mathcal{M}_t = \Pi_s^S \left(\frac{\mathcal{M}_{st}}{\gamma_s} \right)^{\gamma_s}$. Hence

$$\frac{P_t^f A_t}{P_{st}^f A_{st}} = \left[\left(\frac{\mathcal{M}_{st} \mathcal{M}_t W_t^f}{\mathcal{M}_t P_t^f} \right) \frac{(Y_t^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha) A_t^{\frac{1}{1-\alpha}}} \right]^{\alpha-1} \gamma_s^{-\alpha} = \left[\frac{\mathcal{M}_{st}}{\mathcal{M}_t} \right]^{\alpha-1} \gamma_s^{-\alpha}$$

$$\Rightarrow \tilde{p}_{st}^f = -\tilde{a}_{st} + (1-\alpha)\tilde{\mu}_{st} + \alpha \log \gamma_s$$

We can now go back to Δ and linearize it around the steady-state with $\frac{PA}{P_s A_s} = \left[\frac{\mathcal{M}_s}{\mathcal{M}} \right]^{\alpha-1} \gamma_s^{-\alpha} = [\Pi_s^S \gamma_s^{\gamma_s}]^{\alpha-1} \gamma_s^{-\alpha}$ where $\mathcal{M}_s$ are equal for all s .

$$\Delta_t = \int_s \left(\frac{\gamma_s P_t A_t}{P_{st} A_{st}} \right)^{\frac{1}{1-\alpha}} ds$$

$$\Delta \exp(\hat{\delta}_t) = \int_s \left(\frac{\gamma_s P A}{P_s A_s} \right)^{\frac{1}{1-\alpha}} \exp \left(-\frac{1}{1-\alpha} (\hat{p}_{st} + \hat{a}_{st}) \right) ds$$

$$= [\Pi_s^S \gamma_s^{-\gamma_s}] \int_s \gamma_s \exp \left(-\frac{1}{1-\alpha} (\hat{p}_{st} + \hat{a}_{st}) \right) ds$$

$$\hat{\delta}_t = -\frac{1}{1-\alpha} \int_s \gamma_s (\hat{p}_{st} + \hat{a}_{st}) ds$$

where $\hat{x}_{ts}$ denotes the log deviation of "relative x" from its steady state. Importantly, $\hat{\delta}_t$ is zero at first order

$$\hat{\delta}_t = -\frac{1}{1-\alpha} \int_s \gamma_s (\hat{p}_{st} + \hat{a}_{st}) ds = -\frac{1}{1-\alpha} \int_s \gamma_s (\tilde{p}_{st} - \tilde{p}_s + \tilde{a}_{st} - \tilde{a}_s) ds$$

$$= -\frac{1}{1-\alpha} \int_s \gamma_s (\tilde{p}_{st} - \tilde{p}_s - \tilde{p}_{st}^e + \tilde{p}_s^e) ds = 0$$

where the last line uses the definition of the price index $p = \int \gamma_s (p_s - \log \gamma_s) ds$ which implies $\int \gamma_s (\tilde{p} - p) = \int \gamma_s \log \gamma_s ds$ which is the same for the four components, hence they cancel out. For future reference we define the log deviation of relative prices from its efficient level

$$\hat{\tilde{p}}_{st} = \tilde{p}_{st} - \tilde{p}_{st}^e.$$

Importantly, relative prices in the steady-state are the same as in the efficient

steady-state. Hence we have

$$\hat{p}_{st} = \tilde{p}_{st} - \tilde{p}_s - \tilde{p}_{st}^e + \tilde{p}_s^e \quad \text{and} \quad \int \gamma_s \hat{p}_{st} = 0$$

which will be useful later.

Going back to $\hat{\delta}$, a second order approximation around the steady state gives

$$\begin{aligned} \hat{\delta}_t &= \int_s \gamma_s \left(-\frac{1}{1-\alpha} (\hat{p}_{st} + \hat{a}_{st}) + \frac{1}{2(1-\alpha)^2} (\hat{p}_{st} + \hat{a}_{st})^2 \right) ds \\ &= \frac{1}{(1-\alpha)} \left[-E_\gamma (\hat{p}_{st} + \hat{a}_{st}) + \frac{E \left((\tilde{p}_{st} - \tilde{p}_{st}^e)^2 \right)}{2(1-\alpha)} \right] \\ &= \frac{E \left((\tilde{p}_{st} - \tilde{p}_{st}^e)^2 \right)}{2(1-\alpha)^2} \end{aligned}$$

where we used $\hat{p}_{st} + \hat{a}_{st} = \hat{p}_{st}$, the fact that $\hat{\delta}$ is zero at first order and the expectation is taken across sectors under the weights γ_s .

Phillips Curve and Euler Equation We now turn to the first order conditions of the households given by

$$\begin{aligned} w_t - p_t &= \sigma c_t + \psi n_t \\ c_t &= E_t c_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \rho) \end{aligned}$$

with $i_t = -\log Q_t$. Combining them with the market clearing condition for the final goods gives

$$\begin{aligned} w_t - p_t &= \left(\sigma + \frac{\psi}{1-\alpha} \right) y_t - \frac{\psi}{1-\alpha} \log a_t + \psi \delta_t \\ y_t &= E_t y_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \rho) \end{aligned}$$

with $\delta_t = \log \Delta_t$. From the definition of markup, we obtain

$$\begin{aligned}
mc_{st} &= w_t - p_t + p_t - p_{st} + \frac{\alpha}{1-\alpha} y_{st} - \frac{1}{1-\alpha} a_{st} - \log(1-\alpha) \\
&= \left(\sigma + \frac{\psi}{1-\alpha} \right) y_t + \psi \delta_t + p_t - p_{st} - \frac{\psi}{1-\alpha} a_t + \frac{\alpha}{1-\alpha} y_{st} - \frac{1}{1-\alpha} a_{st} - \log(1-\alpha) \\
&= \left(\sigma + \frac{\psi + \alpha}{1-\alpha} \right) y_t + \psi \delta_t - (p_{st} - p_t) - \frac{1+\psi}{1-\alpha} a_t - \frac{1}{1-\alpha} (a_{st} - a_t) \\
&\quad + \frac{\alpha}{1-\alpha} (y_{st} - y_t) - \log(1-\alpha) \\
&= \left(\sigma + \frac{\psi + \alpha}{1-\alpha} \right) y_t + \psi \delta_t - \tilde{p}_{st} - \frac{1+\psi}{1-\alpha} a_t - \frac{1}{1-\alpha} \tilde{a}_{st} + \frac{\alpha}{1-\alpha} \tilde{y}_{st} - \log(1-\alpha) \\
&= mc_t + \psi \delta_t - \tilde{p}_{st} - \frac{1}{1-\alpha} \tilde{a}_{st} + \frac{\alpha}{1-\alpha} \tilde{y}_{st}
\end{aligned}$$

where the second line uses the first order condition of households and the market clearing condition for labor, where the third line uses $\tilde{x}_s = x_s - x$ and the last line uses $mc_t = \left(\sigma + \frac{\psi + \alpha}{1-\alpha} \right) y_t - \frac{1+\psi}{1-\alpha} a_t - \log(1-\alpha)$. Finally we can use the first order condition of the final goods firms to substitute for $\tilde{y}_{st}$:

$$mc_{st} = mc_t + \psi \delta_t - \frac{1}{1-\alpha} \tilde{a}_{st} - \left(\frac{1}{1-\alpha} \right) \tilde{p}_{st}$$

We next take the difference with the steady-state:

$$\begin{aligned}
\hat{mc}_{st} &= \hat{mc}_t + \psi \hat{\delta}_t - \left(\frac{1}{1-\alpha} \right) (\tilde{p}_{st} - \tilde{p}_s + \tilde{a}_{st} - \tilde{a}_s) \\
\hat{mc}_{st} &= \hat{mc}_t + \psi \hat{\delta}_t - \left(\frac{1}{1-\alpha} \right) \hat{\tilde{p}}_{st}^e.
\end{aligned}$$

Using this to substitute for the marginal cost in the sector-level Phillips curve:

$$\begin{aligned}
\pi_{st} \left(1 - \epsilon \frac{\tau^{ss}}{\theta_s} \right) &= \beta E_t \pi_{t+1s} \left(1 - \frac{\tau^{ss}}{\theta_s} \right) + \frac{\epsilon - 1}{\theta_s} \left[\hat{mc}_t - \left(\frac{1}{1-\alpha} \right) \hat{\tilde{p}}_{st}^e \right] \\
&\quad + \frac{1}{\theta_s} [\beta E_t \hat{\tau}_{t+1} - \hat{\tau}_t + u_t] + \beta \frac{\tau^{ss}}{\theta_s} (1 - \sigma) (\hat{c}_{t+1} - \hat{c}_t) \quad (41)
\end{aligned}$$

where we used the fact that at first order $\hat{\delta}_t = 0$.

Taking the (γ_s -weighted) integral of the Phillips curves over all sectors gives

$$\begin{aligned} \pi_t \left(1 - \epsilon \frac{\tau^{ss}}{\theta_s}\right) &= \beta E_t \pi_{t+1} \left(1 - \frac{\tau^{ss}}{\theta_s}\right) + \hat{m} c_t \int_s \frac{\epsilon - 1}{\theta_s} \gamma_s - \frac{1}{1 - \alpha} \int_s \frac{(\epsilon - 1) \gamma_s}{\theta_s} \hat{p}_{st}^e \\ &\quad + [u_t + \beta E_t \hat{\tau}_{t+1} - \hat{\tau}_t] \int_s \frac{\gamma_s}{\theta_s} ds + \beta \frac{\tau^{ss}}{\theta_s} (1 - \sigma) (\hat{c}_{t+1} - \hat{c}_t) \end{aligned} \quad (42)$$

D.2 Second-order approximation of welfare

We next show that the welfare loss function of the household depends on the dispersion of relative prices around their efficient levels and on the average inflation rates across sectors.

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\sigma + \frac{\alpha + \psi}{1 - \alpha} \right) (\hat{x}_{st})^2 + \frac{1}{2} E[\theta_s \pi_{st}^2] + \frac{1}{2} \frac{E((\tilde{p}_{st} - \tilde{p}_{st}^e)^2)}{1 - \alpha} - \Phi \hat{x}_t \right]$$

The log deviation of the representative household is given by

$$\frac{U_t - U}{U_c C} = \left(\hat{c}_t + \frac{1 - \sigma}{2} \hat{c}_t^2 \right) + \frac{U_N N}{U_c C} \left(\hat{n}_t + \frac{1 + \psi}{2} \hat{n}_t^2 \right)$$

.

We next use a second order approximation of the goods market clearing condition to get an approximation for the first bracket

$$\hat{c}_t = \hat{y}_t - \frac{1}{2} E[\theta_s \pi_{st}^2].$$

We next use the labor market clearing condition to get an approximation for the second bracket:

$$\hat{n}_t = \frac{1}{1 - \alpha} (\hat{y}_t - \hat{a}_t) + \hat{\delta}_t$$

Combining both, and keeping only the terms of first and second order, we

obtain

$$\begin{aligned} \frac{U_t - U}{U_c C} = & \left(\hat{y}_t - \frac{1}{2} E[\theta_s \pi_{st}^2] + \frac{1 - \sigma}{2} (\hat{y}_t)^2 \right) \\ & + \frac{U_N N}{U_c C} \left(\frac{1}{1 - \alpha} (\hat{y}_t - \hat{a}_t) + \hat{\delta}_t + \frac{1 + \psi}{2} \left(\frac{1}{1 - \alpha} \right)^2 (\hat{y}_t - \hat{a}_t)^2 \right) \end{aligned}$$

Denote Φ the steady-state distortion implicitly defined by $-\frac{U_N N}{U_c C(1 - \alpha)} = 1 - \Phi$. We obtain:

$$\begin{aligned} \frac{U_t - U}{U_c C} = & \left(\hat{y}_t - \frac{1}{2} E[\theta_s \pi_{st}^2] + \frac{1 - \sigma}{2} (\hat{y}_t)^2 \right) \\ & - (1 - \Phi) \left((\hat{y}_t - \hat{a}_t) + (1 - \alpha) \hat{\delta}_t + \frac{1 + \psi}{2} \left(\frac{1}{1 - \alpha} \right) (\hat{y}_t - \hat{a}_t)^2 \right) \end{aligned}$$

Under the “small distortion” assumption (so that the product of Φ with a second-order term can be ignored as negligible)

$$\begin{aligned} \frac{U_t - U}{U_c C} &= \Phi \hat{y}_t - \frac{1}{2} E[\theta_s \pi_{st}^2] + \left(\frac{1 - \sigma}{2} (\hat{y}_t)^2 \right) - \left(-\hat{a}_t + \frac{1 + \psi}{2(1 - \alpha)} (\hat{y}_t - \hat{a}_t)^2 \right) - (1 - \alpha) \hat{\delta} + t.i.p \\ &= \Phi \hat{y}_t - \frac{1}{2} E[\theta_s \pi_{st}^2] - \frac{1}{2} \left(\sigma + \frac{\alpha + \psi}{1 - \alpha} \right) (\hat{y}_t)^2 + \frac{1 + \psi}{(1 - \alpha)} \hat{y}_t \hat{a}_t - (1 - \alpha) \hat{\delta} + t.i.p \\ &= \Phi \hat{x}_t + \Phi \hat{y}_t^e - \frac{1}{2} E[\theta_s \pi_{st}^2] - \frac{1}{2} \left(\sigma + \frac{\alpha + \psi}{1 - \alpha} \right) [(\hat{y}_t)^2 - 2 \hat{y}_t \hat{y}_t^e] - (1 - \alpha) \hat{\delta} + t.i.p \\ &= \Phi \hat{x}_t - \frac{1}{2} E[\theta_s \pi_{st}^2] - \frac{1}{2} \left(\sigma + \frac{\alpha + \psi}{1 - \alpha} \right) (\hat{x}_t)^2 - (1 - \alpha) \hat{\delta} + t.i.p \end{aligned}$$

note that $\hat{a}_t$, $\hat{a}_t^2$ and $\hat{y}_t^e$ are in t.i.p. because they depend only on exogenous TFP shocks. We used the definition of y_t^e , x_t , the fact that $\hat{y}_t^e = y_t^e - y^e$, $x_t = y_t - y^e$, $\hat{x}_t = x_t - x = \hat{y}_t - \hat{y}_t^e$ where x is the steady-state welfare-relevant output gap.

Substituting the second order approximation of $\hat{\delta}_t$

$$\hat{\delta}_t = \frac{E\left((\tilde{p}_{st} - \tilde{p}_{st}^e)^2\right)}{2(1-\alpha)^2}$$

into the previous expression gives the result

$$\frac{U_t - U}{U_c C} = \Phi \hat{x}_t - \frac{1}{2} E[\theta_s \pi_{st}^2] - \frac{1}{2} \left(\sigma + \frac{\alpha + \psi}{1 - \alpha} \right) (\hat{x}_t)^2 - \frac{E\left((\tilde{p}_{st} - \tilde{p}_{st}^e)^2\right)}{2(1-\alpha)} + t.i.p.$$

Finally the term $E[\theta_s \pi_{st}^2]$ can be decomposed into an aggregate and sector-specific components:

$$\begin{aligned} E[\theta_s \pi_{st}^2] &= E[\theta_s (\pi_t + (\pi_{st} - \pi_t))^2] \\ &= E\left[\theta_s \pi_t^2 + \theta_s (\pi_{st} - \pi_t)^2 + 2\theta_s \pi_t (\pi_{st} - \pi_t)\right] \\ &= \left(E\left[\theta_s \pi_t^2\right] + E\left[\theta_s \tilde{\pi}_{st}^2\right] + 2cov[\pi_t, \theta_s \tilde{\pi}_{st}]\right) \\ &= \pi_t^2 E[\theta_s] + E\left[\theta_s \tilde{\pi}_{st}^2\right] \end{aligned}$$

where the last line uses the fact that in the cross-section π_t is constant.

D.3 Proof of Proposition (5).

Substituting $\theta_s = \theta$ in the sector-s Phillips curve just derived we obtain

$$\begin{aligned} \pi_{st} \left(1 - \epsilon \frac{\tau^{ss}}{\theta}\right) &= \beta E_t \pi_{t+1s} \left(1 - \frac{\tau^{ss}}{\theta}\right) + \frac{\epsilon - 1}{\theta} \left[\hat{m}c_t - \left(\frac{1}{1-\alpha}\right) \hat{p}_{st}^e \right] \\ &\quad + \frac{1}{\theta} [\beta E_t \tau_{t+1} - \tau_t + u_t] + \beta \frac{\tau^{ss}}{\theta} (1 - \sigma) (\hat{c}_{t+1} - \hat{c}_t) \end{aligned} \quad (43)$$

Taking the difference between equations (43) and (42) to get an expression

for relative price inflation $\tilde{\pi}_{st} = \pi_{st} - \pi_t$ we obtain:

$$\tilde{\pi}_{st} \left(1 - \epsilon \frac{\tau^{ss}}{\theta}\right) = \beta E_t \tilde{\pi}_{t+1s} \left(1 - \frac{\tau^{ss}}{\theta}\right) - \frac{\epsilon - 1}{\theta} \frac{1}{1 - \alpha} (\tilde{p}_{st} + \tilde{a}_{st})$$

which gives

$$\left(1 - \epsilon \frac{\tau^{ss}}{\theta}\right) (\tilde{p}_{st} - \tilde{p}_{st-1}) = \left(1 - \frac{\tau^{ss}}{\theta}\right) \beta E_t (\tilde{p}_{st+1} - \tilde{p}_{st}) - \frac{\epsilon - 1}{\theta} \frac{1}{1 - \alpha} (\tilde{p}_{st} + \tilde{a}_{st}).$$

This is a linear difference equation in $\tilde{p}_{st}$ which depends only on θ , τ^{ss} , ϵ , β , α and the stochastic process for $\tilde{a}_{st}$. Importantly, it is independent of the deviation of TIP from the steady-state $\hat{\tau}_t$. We see that the condition $\theta_s = \theta$ is important to get this result, otherwise we wouldn't be able to subtract the aggregate and sector-specific Phillips curve.

If in addition, $\tau^{ss} = 0$ then the difference equation—hence relative prices—is completely independent of TIP:

$$(\tilde{p}_{st} - \tilde{p}_{st-1}) = \beta E_t (\tilde{p}_{st+1} - \tilde{p}_{st}) - \frac{\epsilon - 1}{\theta} \frac{1}{1 - \alpha} (\tilde{p}_{st} + \tilde{a}_{st})$$

Looking now at average (squared) price changes $E[\theta_s \pi_{st}^2] = \pi_t^2 E[\theta_s] + E[\theta_s \tilde{\pi}_{st}^2]$, under the same conditions, the second term $E[\theta_s \tilde{\pi}_{st}^2]$ is independent of aggregates and of TIP. Therefore $E[\theta_s \pi_{st}^2]$ varies with TIP only through its effect on aggregate inflation π_t^2 .

Note that if we had initially allowed for heterogeneous elasticities of substitution, ϵ , they would have had to be the same across sectors as well for the proof to go through.

D.4 Stationary distribution of price distortions in the linearized model

Using the following difference equation

$$\left(1 - \epsilon \frac{\tau^{ss}}{\theta}\right) (\tilde{p}_{st} - \tilde{p}_{st-1}) = \left(1 - \frac{\tau^{ss}}{\theta}\right) \beta E_t (\tilde{p}_{st+1} - \tilde{p}_{st}) - \frac{\epsilon - 1}{\theta} \frac{1}{1 - \alpha} (\tilde{p}_{st} + \tilde{a}_{st})$$

we can compute the stationary distribution of relative prices $\tilde{p}_{st}$ and distortions, $\tilde{p}_{st} + \tilde{a}_{st}$ for a stochastic process for $\tilde{a}_{st}$. We assume it is drawn from a normal distribution with mean 0 and standard deviation $\sigma_a = 1$ and i.i.d. over time and over sectors:

$$\tilde{a}_{st} \sim i.i.d. \mathcal{N}(0, 1)$$

To solve the previous difference equation, we guess the following AR(1) process for relative prices:

$$\tilde{p}_{st} = \chi_p \tilde{a}_{st-1} + \chi_a \tilde{a}_{st}$$

Replacing in the previous equation, we get:

$$\left(1 - \epsilon \frac{\tau^{ss}}{\theta} + (1 - \chi_p) \beta \left(1 - \frac{\tau^{ss}}{\theta}\right) + \frac{\epsilon - 1}{\theta} \frac{1}{1 - \alpha}\right) \tilde{p}_{st} = \left(1 - \epsilon \frac{\tau^{ss}}{\theta}\right) \tilde{p}_{st-1} - \frac{\epsilon - 1}{\theta} \frac{1}{1 - \alpha} \tilde{a}_{st}$$

We then solve for χ_p and χ_a using:

$$\chi_p = \frac{\left(1 - \epsilon \frac{\tau^{ss}}{\theta}\right)}{\left(1 - \epsilon \frac{\tau^{ss}}{\theta} + (1 - \chi_p) \beta \left(1 - \frac{\tau^{ss}}{\theta}\right) + \frac{\epsilon - 1}{\theta} \frac{1}{1 - \alpha}\right)}$$

$$\chi_a = -\chi_p \frac{\frac{\epsilon - 1}{\theta} \frac{1}{1 - \alpha}}{\left(1 - \epsilon \frac{\tau^{ss}}{\theta}\right)}$$

This gives

$$\chi_p = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

$$A = -\beta \left(1 - \frac{\tau^{ss}}{\theta}\right)$$

$$B = \left(1 - \epsilon \frac{\tau^{ss}}{\theta} + \beta \left(1 - \frac{\tau^{ss}}{\theta}\right) + \frac{\epsilon - 1}{\theta} \frac{1}{1 - \alpha}\right)$$

$$C = -\left(1 - \epsilon \frac{\tau^{ss}}{\theta}\right)$$

Note that we keep only the positive solution to the quadratic equation because the other one gives a negative AR(1) coefficient, which isn't economically sensible.

With this we can solve for the stationary distribution of price distortions:
 $\tilde{a}_{st} + \tilde{p}_{st}$

$$\begin{aligned}\tilde{a}_{st} + \tilde{p}_{st} &= \chi_p(\tilde{a}_{st-1} + \tilde{p}_{st-1}) + (1 + \chi_a)\tilde{a}_{st} - \chi_p\tilde{a}_{st} + \tilde{a}_{st-1} \\ \Rightarrow E(\tilde{a}_{st} + \tilde{p}_{st}) &= 0 \quad \text{and} \quad V(\tilde{a}_{st} + \tilde{p}_{st}) = \frac{(1 + \chi_a)^2 + \chi_p^2}{1 - \chi_p^2}\end{aligned}$$

where we used the i.i.d. assumption across sectors and time periods.

Using the same calibration described in Appendix B.3, we can compute the sum of squared price deviations $V(\tilde{a}_{st} + \tilde{p}_{st})$ for different values of τ^{ss} ranging from 0 to 100%. Results are reported below. We see that a positive steady-state TIP τ^{ss} has no meaningful quantitative effect on price distortions. If anything it slightly decreases price distortions by lowering the persistence of relative prices χ_p and by increasing the response of relative prices to sector-specific TFP shocks $|\chi_a|$. We see that this result remains true when decreasing β from .99 to .8 and when decreasing the adjustment cost parameter θ from 372 to 200.

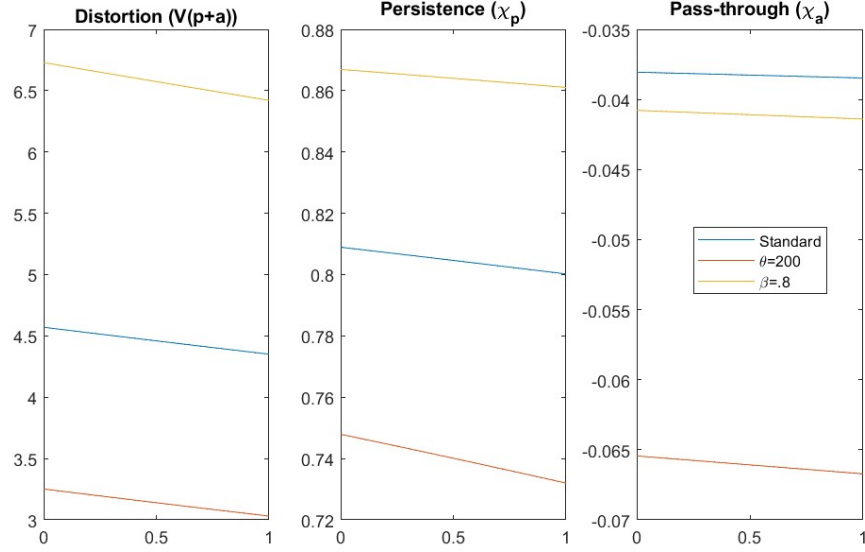


Figure D7: Steady-state distortions for different values of τ^{ss}

D.5 Calibration

Our data used to calibrate the model is a panel of Personal Consumption Expenditures (PCE) price indices for different types of product from the Bureau of Economic Analysis, NIPA Table 2.3.4 and 2.3.5 ([U.S. Bureau of Economic Analysis, 2025b](#)). Following the BEA classification, we group products into 13 sectors, which are listed in the first column of Table D4. We focus on core inflation and exclude food and fuel. We also exclude services provided by non-profit institutions servicing households. We do so to be consistent with the model's assumptions that firms have monopolistic power to set prices and seek to maximize profits. Our sample covers the period 1960Q1 to 2019Q4. We exclude the COVID-19 years because the large drop and rebound of economic activity could bias our estimates.

For each sector, we compute the standard deviation and autocorrelation coefficients of their relative price changes. We construct relative price changes in the following way. Let's denote P_{st} the price index of sector s at the time t . We compute the log difference of the price index of each product

Sector	PCE Weight	$\hat{\sigma}_s^{PCE} \times 100$	$\hat{\rho}_s^{PCE}$	Simulated $\sigma_s^{PCE} \times 100$	Simulated ρ_s^{PCE}	θ_s	$\sigma_s \times 100$
Durable goods							
Motor vehicles and parts	5.2%	0.72	0.32	0.71	0.32	465.6	45.85
Furnishings and durable household equipment	3.2%	0.53	0.26	0.51	0.24	146.8	7.99
Recreational goods and vehicles	3.6%	0.48	0.23	0.48	0.24	110.8	5.93
Other durable goods	1.8%	0.72	0.33	0.71	0.32	465.9	45.85
Clothing and footwear	4.4%	0.55	0.02	0.55	0.01	14.2	1.53
Other nondurable goods	9.1%	0.48	0.36	0.49	0.34	711.6	45.87
Housing and utilities	21.1%	0.37	0.33	0.38	0.33	538.2	17.32
Health care	17.7%	0.39	0.34	0.40	0.35	683.3	24.09
Transportation services	3.9%	0.58	0.12	0.59	0.12	32.5	2.84
Recreation services	4.4%	0.33	0.13	0.33	0.13	37.3	1.79
Food services and accommodations	7.4%	0.38	0.10	0.39	0.10	26.6	1.63
Financial services and insurance	8.6%	1.21	-0.05	1.20	-0.06	7.8	2.39
Other services	9.7%	0.42	0.14	0.42	0.14	39.7	2.36

Table D4: Empirical Targets, Model-implied Moments and Parameter Values

category, $\log(P_{st}/P_{st-1})$, and then take the difference with the overall price index: $\log(P_{st}/P_{st-1}) - \log(P_t/P_{t-1})$. Next we remove category-specific trends by applying an HP filter. We use the resulting panel of de-trended relative price log changes to compute both moments: the standard deviation $\hat{\sigma}_s^{PCE}$ and the autocorrelation coefficient $\hat{\rho}_s^{PCE}$.

Using a simulated method of moments, for each sector we look for the pair of parameters (σ_s, θ_s) that generates the standard deviation and autocorrelation coefficient $(\sigma_s^{PCE}, \rho_s^{PCE})$ that best match their empirical counterparts. Before doing so, we first need to set the persistence of productivity ρ . We then draw random samples of θ_s and σ_s . We simulate the stationary distribution of price changes and production in each sector. We compute the model-implied moments $(\sigma_s^{PCE}, \rho_s^{PCE})$. We then use this initial set of moments to update our guess for the parameters. We iterate until the simulated moments match the target. Once convergence is achieved for all sectors, we manually update our guess for the persistence parameter ρ until the weighted average of θ_s across all sectors is of the same order of magnitude as in the representative model in Section 3.3. In practice we obtain $\rho = .425$ and a weighted average θ_s of 350.5.

Table D4 displays the empirical targets, the model-implied moments and the resulting parameter values for each sector.

D.6 Numerical simulations of non-linear model

We now describe how we solve numerically for the steady state and the transitional dynamics. We start with describing the equilibrium conditions used in the algorithm.

Policy function of firms. In each sector s , we solve for the firms' pricing rule $\tilde{P}_{s,t}(A_{s,t}, \tilde{P}_{s,t-1})$ given the paths of real wages $\{RW^t\}$, aggregate outputs $\{Y^t\}$, aggregate inflation rates $\{\pi^t\}$ (normalize $P_{-1} = 1$ such that $P_t = \exp \sum_{h=1}^t \pi_h$), aggregate markup shocks $\{\mu^t\}$, and TIP rates $\{\tau^t\}$ using the following first order condition:

$$\begin{aligned} & \left((1 - \epsilon_s) (1 - \mu_t \mathcal{M}_s MC_{s,t}) - \tau_t \left(1 - \epsilon_s \frac{\pi_{s,t}}{1 + \pi_{s,t}} \right) \right) - \theta_s \pi_{s,t} \frac{(\pi_{s,t} + 1)}{\tilde{P}_{s,t}} \\ & + E_t Q \left[\tau_{t+1} \frac{Y_{s,t+1}}{Y_{s,t}} + \theta_s \frac{(\pi_{s,t+1} + 1)^2}{\tilde{P}_{s,t+1}} \pi_{s,t+1} \frac{Y_{s,t+1}}{Y_{s,t}} \right] = 0, \end{aligned} \quad (44)$$

with $\tilde{P}_{s,t} = \frac{P_{s,t}}{P_t}$, $\pi_{s,t} = \log(\tilde{P}_{s,t}) - \log(\tilde{P}_{s,t-1}) + \pi_t$ and

$$MC_{s,t} = \frac{RW_t}{\tilde{P}_{s,t}(1 - \alpha)} \frac{Y_{s,t}^{\frac{\alpha}{1-\alpha}}}{A_{s,t}^{\frac{1}{1-\alpha}}}, \quad Y_{s,t} = \frac{\gamma_s Y_t}{\tilde{P}_{s,t}}, \quad \mathcal{M}_s = \frac{\epsilon_s}{\epsilon_s - 1} \quad (45)$$

and subject to the following exogenous stochastic process for the sector-specific TFP, $A_{s,t}$,

$$\log(A_{s,t}) = (1 - \rho) \log A_s + \rho \log(A_{s,t-1}) + v_{s,t}$$

where $v_{s,t}$ is drawn from a normal distribution with standard deviation σ_s and where A_s is chosen such that in the stationary equilibrium each sector's average relative prices is equal to one.

Aggregate output and inflation. We next aggregate all sectors and define aggregate output. It is given by

$$Y_t = (A_t)^{(1+\psi)k} (1-\alpha)^{(1-\alpha)k} (\Gamma_t)^{-\sigma(1-\alpha)k} (\Delta_t)^{-\psi(1-\alpha)k} (\mathcal{M}_t)^{-(1-\alpha)k}, \quad (46)$$

with $k = \frac{1}{\psi + (1-\alpha)\sigma + \alpha}$ and

$$\Gamma_t = 1 - \int \gamma_s \frac{1}{\bar{P}_s} \frac{\theta_s}{2} \pi_{s,t}^2 ds \quad (47)$$

$$\Delta_t = \int_s \left(\frac{\gamma_s A}{\tilde{P}_{s,t} A_{s,t}} \right)^{\frac{1}{1-\alpha}} ds \quad (48)$$

$$\mathcal{M}_t = \exp \left(\int \gamma_s \log(\mathcal{M}_{s,t}) ds \right) \quad (49)$$

$$\mathcal{M}_{s,t} = \frac{1}{MC_{s,t}} \quad (50)$$

$$A_t = \exp \left(\int_s \gamma_s \log(A_{s,t}) ds \right) \quad (51)$$

$$RW_t = \frac{W_t}{P_t} = (Y_t)^{\sigma + \frac{\psi}{1-\alpha}} A_t^{-\frac{\psi}{1-\alpha}} (\Delta_t)^\psi (\Gamma_t)^\sigma \quad (52)$$

The aggregate price level and inflation are given by

$$P_t = \exp p_t \quad (53)$$

$$p_t = \int \gamma_s (p_{s,t} - \log \gamma_s) ds \quad (54)$$

$$\pi_t = \int \gamma_s \pi_{s,t} ds \quad (55)$$

$$\tilde{p}_t = \int \gamma_s \tilde{p}_{s,t} ds \quad (56)$$

Aggregate demand, monetary policy and TIP. We close the model with the Euler Equation, the targeting rules for monetary policy and TIP:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \zeta_t) \quad (57)$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t \quad (58)$$

$$\tau_t = \phi_\pi \pi_t \quad (59)$$

where ζ_t is a generic demand shock centered around ρ , and the Taylor rule targets deviations of inflation and output from the steady state, $\hat{y}_t$ and TIP targets inflation.

Steady state. In steady-state, we normalize the aggregate price level to 1. The algorithm is then as follows: (i) start from a guess for steady-state output Y , (ii) using equation (44) solve for the firms' pricing rule $\tilde{P}_s(A_s, \tilde{P}_{s,-1})$ given aggregate output and the price index; (iii) using equation (46) update the aggregate output Y' ; (iv) iterate until output $Y = Y'$ converges.

Impulse responses. We next solve for the transition path of the economy following a persistent shock to markup demand or productivity, starting from and going back to the steady-state equilibrium. A transition is a set of vectors $x^t = (x_1, x_2, \dots, x_T)$ where T is the horizon of the transition and where $x \in \{Y, RW, \Gamma, \Delta, \mathcal{M}, i, P, \pi, \tilde{p}, \dots\}$. The algorithm is as follows:

1. *Guess a sequence of inflation rates.* Let $\pi^{t(i)}$ be the guess for π^t in the beginning of the i -th iteration. We make use of the sequence-space Jacobian approach developed by [Auclert et al. \(2021\)](#) to make an accurate first guess $\pi^{t(1)}$. We give more details below.
2. *Compute the path of output, interest rate and TIP.* Using the Euler equation and the targeting rules for monetary policy and TIP we compute the implied paths for $y^{t(i)}$, $i^{t(i)}$ and $\tau^{t(i)}$.

3. *Update the path of real wages, $rw^{(i)}$.* We next use the equilibrium expression of real wages

$$rw^{t(i)} = \left(\sigma + \frac{\psi}{1-\alpha} \right) y^{t(i)} - \left(\frac{\psi}{1-\alpha} \right) a^t + \psi \ln \Delta^{t(i-1)} + \sigma \ln \Gamma^{t(i-1)}. \quad (60)$$

where we use $\Gamma^{t(i-1)}$ and $\Delta^{t(i-1)}$, which are already known from the last iteration, instead of guessing a new sequence for $\Gamma^{t(i)}$ and $\Delta^{t(i)}$.

4. *Compute the implied policy rules.* Use the paths of $\{a^t, y^{t(i)}, \pi^{t(i)}, rw^{t(i)}, u^t, \tau^{t(i)}\}$ to solve for the firms' policy rules recursively $\tilde{P}_{s,t}(A_{s,t}, \tilde{P}_{s,t-1})$ for all periods $t = 1 \dots T$ using equation (44).
5. *Compute the paths of aggregate variables.* Use the paths of $\{a^t, y^{t(i)}, \pi^{t(i)}, rw^{t(i)}, u^t, \tau^{t(i)}\}$ and the updated policy rules to compute the paths of aggregate price index. This gives a new path of inflation rates $\pi^{t(i)*}$.
6. *Update guess of inflation path, $\pi^{t(i+1)}$ and iterate until convergence.* We update the path of inflation $\pi^{t(i+1)}$ based on the previous guess $\pi^{t(i)}$ and the new path of inflation rates. Our updating rule is explained below and uses the sequence-space Jacobian. We iterate until the path of inflation and output are consistent with their guesses, $\pi^{t*} = \pi^t$. This implies that the path of output $y^{t(i)}$ converges.

Sequence-space Jacobian. We make use of the sequence-space Jacobian approach developed by [Auclert et al. \(2021\)](#) for two purposes. First we use the sequence space Jacobian to compute the impulse responses of the aggregate variables of the linearized economy. We use this response as our initial guess of the path of inflation $\pi^{t(1)}$ in the loop to compute the solution to the non-linear model. It turns out to be a very accurate first guess, suggesting that the model is close to linear at the aggregate level.

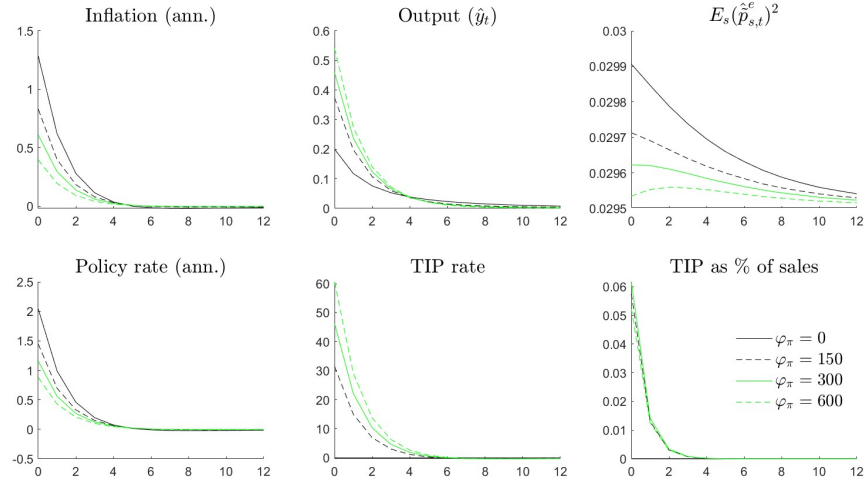
Second we use the sequence space Jacobian to update our guess of inflation $\pi^{t(i)}$ at every iteration of the loop. Recall that the implied path of

inflation at the end of iteration (i) is denoted $\pi^{t(i)*}$. We denote $\mathbf{A}$ the matrix encoding the total derivative of $\pi^{t(i)*}$ with respect to $\pi^{t(i)}$: $d\pi^{t(i)*} \approx \mathbf{A}d\pi^{t(i)}$. Our updating rule is given by

$$\pi^{t(i+1)} = \pi^{t(i)} + \iota(\mathbf{I} - \mathbf{A})^{-1}(\pi^{t(i)*} - \pi^{t(i)}), \quad (61)$$

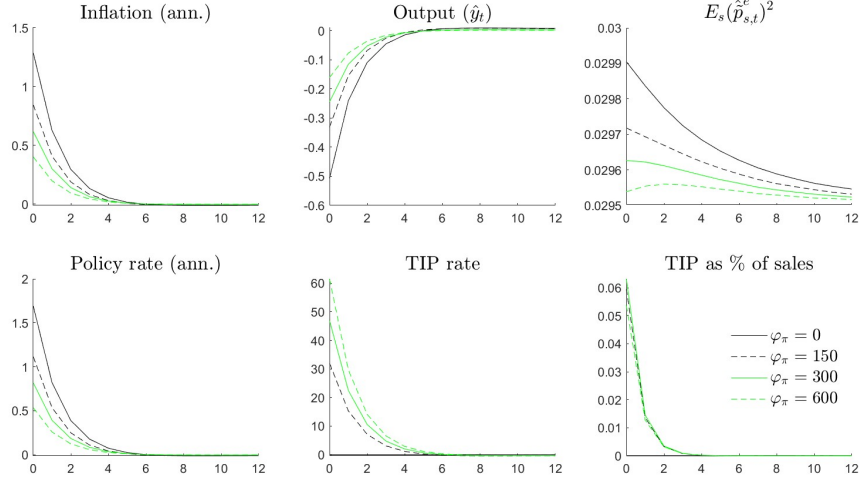
where ι is a parameter controlling the speed of adjustment and $\mathbf{I}$ is the identify matrix.

D.7 Additional Graphs



Notes: The initial markup shock is calibrated to match the initial inflation response in Figure 1. Inflation and the policy rate are annualized. Output refers to the deviation of output from its steady-state level, $\hat{y}_t$.

Figure D8: Effects of TIP following a demand shock in the multi-sector economy



Notes: The initial markup shock is calibrated to match the initial inflation response in Figure 1. Inflation and the policy rate are annualized. Output refers to the deviation of output from its steady-state level, $\hat{y}_t$.

Figure D9: Effects of TIP following a TFP shock in the multi-sector economy

E Time-dependent Calvo pricing

In this appendix we derive the TIP that can implement the first best in a setting with time-dependent Calvo frictions and we show that the Phillips curve implied by Calvo pricing is the same at first order as in the economy with Rotemberg adjustment costs. Following the notations in Galí (2015), the maximization problem of a firm having the opportunity to reset its price is given by

$$\begin{aligned} \max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t^f \left\{ Q_{t,t+k} (P_{ti}^* Y_{t+k|ti} - \Psi_{t+k}(Y_{t+k|ti})) \right\} - \tau_t (P_{ti}^* - P_{t-1i}) Y_{t|ti} \\ - \sum_{k=1}^{\infty} \theta^{k-1} (1 - \theta) E_t^f \left\{ Q_{t,t+k} \tau_{t+k} (P_{t+ki}^{**} - P_{ti}^*) Y_{t+k|t+ki} \right\} \end{aligned}$$

where we denote $\Psi()$ the cost function, P_{t+ki}^{**} the optimal price of firm i when it gets the chance to reset the price in the future, and $Y_{t+k|ti}$ is the output at period $t+k$ of firm i which resets its price in period t . Note that contrary to

the setting without TIP, the optimal price for a firm that gets to reset at t may differ across firms, depending on when they last reset their price. We thus need to keep the i -subscript. The F.O.C. is given by

$$\sum_{k=0}^{\infty} \theta^k E_t^f \left\{ Q_{t,t+k} \left((1 - \epsilon_t) Y_{t+k|t} + \epsilon_t \Psi'_{t+k}(Y_{t+k|ti}) \right) \right\} - \tau_t \left(1 - \epsilon_t \frac{P_{ti}^* - P_{t-1i}}{P_{ti}^*} \right) Y_{t|ti} \\ + \sum_{k=1}^{\infty} \theta^{k-1} (1 - \theta) E_t^f \left\{ Q_{t,t+k} \tau_{t+k} Y_{t+k|t+ki} \right\} = 0$$

Dividing by $(1 - \epsilon_t)$, and multiplying by P_{ti}^*/P_{t-1} we obtain

$$\sum_{k=0}^{\infty} \theta^k E_t^f \left\{ Q_{t,t+k} Y_{t+k|ti} \left(\frac{P_{ti}^*}{P_{t-1}} - \mathcal{M}_t MC_{t+k|ti} \Pi_{t-1,t+k} \right) \right\} + \frac{\tau_t}{\epsilon_t - 1} \left(1 - \epsilon_t \frac{P_{ti}^* - P_{t-1i}}{P_{ti}^*} \right) \frac{P_{ti}^*}{P_{t-1}} Y_{t|ti} \\ - \sum_{k=1}^{\infty} \theta^{k-1} (1 - \theta) E_t^f \left\{ Q_{t,t+k} \frac{\tau_{t+k}}{\epsilon_t - 1} \frac{P_{ti}^*}{P_{t-1}} Y_{t+k|t+ki} \right\}$$

with $MC_{t+k|ti} = \psi_{t+k|ti}/P_{t+k}$ the real marginal cost in period $t+k$ for firm i which last reset its price in period t .

First-best implementation. Together with monetary policy TIP can implement the first best. TIP should follow

$$\tau_t = \frac{\epsilon_t - 1}{Y_t} \left[\sum_{k=0}^{\infty} \theta^k E_t^f \left\{ Q_{t,t+k}^e Y_{t+k} (\mathcal{M}_t \mathcal{M}_t^w - 1) \right\} + \sum_{k=1}^{\infty} \theta^{k-1} (1 - \theta) E_t^f \left\{ Q_{t,t+k}^e \frac{\tau_{t+k}}{\epsilon_t - 1} Y_{t+k} \right\} \right]$$

and monetary policy should target the neutral rate of interest

$$i_t = (Q_t^e)^{-1} - 1 \quad \text{with} \quad Q_t^e = E_t \left[\beta_t \left(\frac{A_{t+1}}{A_t} \right)^{-\frac{\sigma(1+\psi)}{(1-\alpha)\sigma+\psi+\alpha}} \right].$$

First-order approximation. Going back to the first-order condition of firms that get to reset their price, we then linearize around $\frac{P_t^*}{P_{t-1}} = 1$, steady-state

output Y and $\tau = 0$.

$$\begin{aligned} \sum_{k=0}^{\infty} \theta^k \beta^k Y E_t \left\{ p_{ti}^* - p_{t-1} - \hat{m} c_{t+k|ti} + p_{t+k} - p_{t-1} \right\} + \frac{\tau_t}{\epsilon - 1} Y \\ - \sum_{k=1}^{\infty} \theta^{k-1} (1 - \theta) \beta^k E_t \left\{ \frac{\tau_{t+k}}{\epsilon - 1} Y \right\} = 0 \end{aligned}$$

We then show that the marginal cost is given by

$$\begin{aligned} mc_{t+k|ti} &= mc_{t+k} - \frac{\alpha}{1 - \alpha} (y_{t+k|ti} - y_{t+k}) \\ &= mc_{t+k} - \frac{\epsilon \alpha}{1 - \alpha} (p_{ti}^* - p_{t+k}) \end{aligned}$$

and substituting this expression into the optimal pricing decision yields

$$\begin{aligned} \sum_{k=0}^{\infty} \theta^k \beta^k E_t \left\{ p_{ti}^* - p_{t-1} - \Theta \hat{m} c_{t+k} - (p_{t+k} - p_{t-1}) \right\} + \Theta \frac{\tau_t}{\epsilon - 1} \\ - \sum_{k=1}^{\infty} \theta^{k-1} (1 - \theta) \Theta \beta^k E_t \left\{ \frac{\tau_{t+k}}{\epsilon - 1} \right\} = 0. \end{aligned}$$

Rearranging the previous equation gives

$$\begin{aligned} p_{ti}^* - p_{t-1} &= (1 - \beta \theta) \sum_{k=0}^{\infty} (\theta \beta)^k E_t \left\{ \Theta \hat{m} c_{t+k} + (p_{t+k} - p_{t-1}) \right\} - (1 - \beta \theta) \Theta \frac{\tau_t}{\epsilon - 1} \\ &\quad + (1 - \beta \theta) \sum_{k=1}^{\infty} \theta^{k-1} (1 - \theta) \Theta \beta^k E_t \left\{ \frac{\tau_{t+k}}{\epsilon - 1} \right\} \end{aligned}$$

where $\Theta = \frac{1 - \alpha}{1 - \alpha + \alpha \epsilon}$. We then rewrite the sum as a difference equation

$$\begin{aligned}
p_{ti}^* - p_{t-1} &= (1 - \beta\theta)\Theta\hat{m}c_t + (1 - \beta\theta) \sum_{k=0}^{\infty} (\theta\beta)^k (p_t - p_{t-1}) - (1 - \beta\theta)\Theta \frac{\tau_t}{\epsilon - 1} \\
&\quad + \beta\theta(1 - \beta\theta)\Theta \frac{\tau_{t+1}}{\epsilon - 1} \\
&\quad + (1 - \beta\theta)\theta\beta \sum_{k=1}^{\infty} (\theta\beta)^{k-1} E_t \{ \Theta\hat{m}c_{t+k} + (p_{t+k} - p_t) \} - \beta\theta(1 - \beta\theta)\Theta \frac{\tau_{t+1}}{\epsilon - 1} \\
&\quad + (1 - \beta\theta)(1 - \theta)\Theta\beta E_t \left\{ \frac{\tau_{t+1}}{\epsilon - 1} \right\} \\
&\quad + (1 - \beta\theta)\beta\theta \sum_{k=2}^{\infty} \theta^{k-2}(1 - \theta)\Theta\beta^{k-1} E_t \left\{ \frac{\tau_{t+k}}{\epsilon - 1} \right\} \\
p_{ti}^* - p_{t-1} &= (1 - \beta\theta)\Theta\hat{m}c_t + \pi_t - (1 - \beta\theta)\Theta \frac{\tau_t}{\epsilon - 1} + \beta\theta(1 - \beta\theta)\Theta \frac{\tau_{t+1}}{\epsilon - 1} \\
&\quad + \beta\theta E_t(p_{t+1}^* - p_t) \\
&\quad + (1 - \beta\theta)(1 - \theta)\Theta\beta E_t \left\{ \frac{\tau_{t+1}}{\epsilon - 1} \right\} \\
p_{ti}^* - p_{t-1} &= \beta\theta E_t \{ p_{t+1i}^* - p_t \} + (1 - \beta\theta)\Theta\hat{m}c_t + \pi_t - (1 - \beta\theta)\Theta \frac{\tau_t}{\epsilon - 1} \\
&\quad + (1 - \beta\theta)\beta\Theta E_t \left\{ \frac{\tau_{t+1}}{\epsilon - 1} \right\}
\end{aligned}$$

We then use $\pi_t = (1 - \theta)(p_t^* - p_{t-1})$ to get

$$\begin{aligned}
\frac{\pi_t}{1 - \theta} &= \beta\theta E_t \frac{\tau_{t+1}}{1 - \theta} + (1 - \beta\theta)\Theta\hat{m}c_t + \pi_t - (1 - \beta\theta)\Theta \frac{\tau_t}{\epsilon - 1} + (1 - \beta\theta)\beta\Theta E_t \left\{ \frac{\tau_{t+1}}{\epsilon - 1} \right\} \\
\pi_t &= \beta E_t \pi_{t+1} + \frac{(1 - \beta\theta)(1 - \theta)}{\theta} \Theta\hat{m}c_t - \frac{(1 - \beta\theta)(1 - \theta)}{\theta(\epsilon - 1)} \Theta\tau_t \\
&\quad + \frac{(1 - \beta\theta)(1 - \theta)}{\theta(\epsilon - 1)} \beta\Theta E_t \{ \tau_{t+1} \} \\
\pi_t &= \beta E_t \pi_{t+1} + \lambda \hat{m}c_t - \zeta [\tau_t - \beta E_t \{ \tau_{t+1} \}]
\end{aligned}$$

with $\lambda = \frac{(1 - \beta\theta)(1 - \theta)}{\theta} \Theta$ and $\zeta = \frac{\lambda}{(\epsilon - 1)}$.

Finally, using $\hat{m}c_t = \left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right) \hat{y}_t$ we get

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{y}_t - \zeta [\tau_t - \beta E_t \{\tau_{t+1}\}]$$

with $\kappa = \lambda \left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)$.

This shows that using time-dependent frictions instead of state-dependent frictions leads to the same representation and macroeconomic dynamics at first order as with Rotemberg-type frictions. Therefore the results on optimal policies derived in the linearized model—corollary 1, section 3.3 and proposition 5—are robust to using Calvo-type frictions. The following paragraph gives more details on proposition 5.

Relative price distortions and independence from TIP. In a setting with heterogeneous sectors, we can follow the same reasoning as for Rotemberg adjustment cost and derive the following Phillips curve for each sector:

$$\pi_{st} = \beta E_t \pi_{t+1s} + \lambda_s \left[\hat{m}c_t - \left(\frac{1}{1-\alpha}\right) \hat{p}_{st}^e \right] + \zeta_s [\beta E_t \tau_{t+1} - \tau_t] + u_t$$

with $\lambda_s = \frac{(1-\beta\theta_s)(1-\theta_s)}{\theta_s} \Theta$ and $\zeta_s = \frac{\lambda_s}{(\epsilon-1)}$. Taking the difference between any two sectors, we see that the sufficient conditions for proposition 5 to hold, i.e. for relative prices to be independent of aggregate and on TIP, are the same, namely $\lambda_s = \lambda$ and $\zeta_s = \zeta$ so ultimately $\theta_s = \theta$. The second bullet point of the proposition is a direct corollary of the first.

F TIP in a Medium-Scale DSGE Model

In this Appendix we explain how we embed TIP into a [Smets and Wouters \(2007\)](#)-type medium scale model (SW hereafter), estimate it and conduct counterfactual analysis.

F.1 Incorporating TIP into SW

NKPC with TIP. To make comparison with SW easier, in this section, we follow their notation unless otherwise noted. Adding TIP to the SW model changes only the price Phillips Curve. The original profit maximization problem for intermediate goods producers (page 2 of SW's online Appendix) is given by:²¹

$$\max_{\tilde{P}_t(i)} E_t \sum_{s=0}^{\infty} \xi_P^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} [\tilde{P}_t(i) X_{t,s} - MC_{t+s}] Y_{t+s}(i), \quad (62)$$

s.t.

$$Y_{t+s}(i) = Y_{t+s} G'^{-1} \left(\frac{P_t(i) X_{t,s}}{P_{t+s}} \tau_{t+s} \right), \quad (63)$$

$$X_{t,s} = \begin{cases} 1 & \text{if } s = 0 \\ \Pi_{l=1}^s (\pi_{t+l-1}^{\iota_p} \bar{\pi}^{1-\iota_p}) & \text{if } s > 0 \end{cases}. \quad (64)$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate; $\tilde{P}_t(i)$ denotes the newly set price; $\Pi_{l=1}^s \pi_{t+l-1}^{\iota_p} \bar{\pi}^{1-\iota_p}$ captures partial price indexation, where the weights on past inflation (π_{t+l-1}) and trend inflation ($\bar{\pi}$) are ι_p and $1 - \iota_p$, respectively; ξ_P is the Calvo probability of resetting prices; $\frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}}$ is the nominal discount factor; $\tau_t = \int_0^1 G' \left(\frac{Y_t(i)}{Y_t} \right) \frac{Y_t(i)}{Y_t} di$, where $G(\cdot)$ corresponds to the Kimball aggregator; and MC_{t+s} is the marginal cost, which is common to all firms.

Since the model has a constant trend inflation, we adjust the TIP rule such that it is centered around trend inflation. Firms pay TIP taxes in two cases. First, when they reset their price $\tilde{P}_t(i)$; second, when aggregate inflation fluctuates around the trend due to partial indexation. However, as we shall see below, the second one has no effect at the first order. Denoting the TIP rate τ_t^{TIP} (since τ_t is already taken in the original SW notation), the maximization

²¹https://assets.aeaweb.org/asset-server/articles-attachments/aer/data/june07/20041254_app.pdf

problem with TIP is given by

$$\begin{aligned}
\max_{\tilde{P}_t(i)} E_t \sum_{s=0}^{\infty} \zeta_P^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} & [\tilde{P}_t(i) X_{t,s} - MC_{t+s}] Y_{t+s}(i) - \tau_t^{TIP} (\tilde{P}_t(i) - \bar{\pi} P_{t-1}(i)) Y_t(i) \\
& - E_t \sum_{s=1}^{\infty} \zeta_P^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \tau_{t+s}^{TIP} [\tilde{P}_t(i) X_{t,s} - \tilde{P}_t(i) \bar{\pi} X_{t,s-1}] Y_{t+s}(i) \\
& - E_t \sum_{s=1}^{\infty} \zeta_P^{s-1} (1 - \zeta_P) \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \tau_{t+s}^{TIP} [\tilde{P}_{t+s}(i) - \tilde{P}_t(i) \bar{\pi} X_{t,s-1}] Y_{t+s|\tilde{P}_{t+s}}(i).
\end{aligned} \tag{65}$$

As in Section E, the last term for output $Y_{t+s|\tilde{P}_{t+s}}(i)$ is a function of the new reset price and is thus unaffected by the current reset price $\tilde{P}_t(i)$. The F.O.C. is given by

$$\begin{aligned}
0 = E_t \sum_{s=0}^{\infty} \zeta_P^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} & \left[X_{t,s} Y_{t+s}(i) + (\tilde{P}_t(i) X_{t,s} - MC_{t+s}) \frac{\partial Y_{t+s}(i)}{\partial \tilde{P}_t(i)} \right] \\
& - \tau_t^{TIP} \left[Y_t(i) + (\tilde{P}_t(i) - \bar{\pi} P_{t-1}(i)) \frac{\partial Y_t(i)}{\partial \tilde{P}_t(i)} \right] \\
& - E_t \sum_{s=1}^{\infty} \zeta_P^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \tau_{t+s}^{TIP} (X_{t,s} - \bar{\pi} X_{t,s-1}) \left(Y_{t+s}(i) + \tilde{P}_t(i) \frac{\partial Y_{t+s}(i)}{\partial \tilde{P}_t(i)} \right) \\
& + E_t \sum_{s=1}^{\infty} \zeta_P^{s-1} (1 - \zeta_P) \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \tau_{t+s}^{TIP} \bar{\pi} X_{t,s-1} Y_{t+s|\tilde{P}_{t+s}}(i).
\end{aligned} \tag{66}$$

Define $\mathcal{G}_{t+s} = \frac{1}{G'^{-1}(z_{t+s})} \frac{G'(x_{t+s})}{G''(x_{t+s})} = \frac{\tilde{P}_t(i)}{Y_{t+s}(i)} \frac{\partial Y_{t+s}(i)}{\partial \tilde{P}_t(i)}$, $x_t = G'^{-1}(z_t)$, and $z_t =$

$\frac{P_t(i)}{P_t} \tau_t$. Multiply both sides by $\tilde{P}_t(i)$:

$$\begin{aligned}
0 = & E_t \sum_{s=0}^{\infty} \xi_P^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} Y_{t+s}(i) \left[(1 + \mathcal{G}_{t+s}) \tilde{P}_t(i) X_{t,s} - \mathcal{G}_{t+s} MC_{t+s} \right] \\
& - \tau_t^{TIP} \tilde{P}_t(i) Y_t(i) \left[1 + \frac{\tilde{P}_t(i) - \bar{\pi} P_{t-1}(i)}{\tilde{P}_t(i)} \mathcal{G}_t \right] \\
& - E_t \sum_{s=1}^{\infty} \xi_P^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \tau_{t+s}^{TIP} \tilde{P}_t(i) Y_{t+s}(i) (X_{t,s} - \bar{\pi} X_{t,s-1}) (1 + \mathcal{G}_{t+s}) \\
& + E_t \sum_{s=1}^{\infty} \xi_P^{s-1} (1 - \xi_P) \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \tau_{t+s}^{TIP} \tilde{P}_t(i) \bar{\pi} X_{t,s-1} Y_{t+s|\tilde{P}_{t+s}}(i). \quad (67)
\end{aligned}$$

Following SW's online appendix (equation 32), we detrend this FOC. Let lower case variables denote de-trended real variables, e.g., $mc_t = \frac{MC_t}{P_t}$, $\xi_t = \Xi_t \gamma^{\sigma_c t}$, $y_{t+s}(i) = \frac{Y_{t+s}(i)}{\gamma^s}$, $\bar{\beta}^t = \beta^t \gamma^{-\sigma_c t}$, $\tilde{p}_t(i) = \frac{\tilde{P}_t(i)}{P_t}$, $x_{t,s}^p = X_{t,s} \frac{P_t}{P_{t+s}}$. The de-trended FOC is given by:

$$\begin{aligned}
0 = & E_t \sum_{s=0}^{\infty} \xi_P^s \bar{\beta}^s \gamma^s \frac{\xi_{t+s}}{\xi_t} y_{t+s}(i) \left[(1 + \mathcal{G}_{t+s}) \tilde{p}_t(i) x_{t,s}^p - \mathcal{G}_{t+s} mc_{t+s} \right] \\
& - \tau_t^{TIP} \tilde{p}_t(i) y_t(i) \left[1 + \frac{\tilde{P}_t(i) - \bar{\pi} P_{t-1}(i)}{\tilde{P}_t(i)} \mathcal{G}_t \right] \\
& - E_t \sum_{s=1}^{\infty} \xi_P^s \bar{\beta}^s \gamma^s \frac{\xi_{t+s}}{\xi_t} \tau_{t+s}^{TIP} \tilde{p}_t(i) y_{t+s}(i) \left(x_{t,s}^p - x_{t,s}^p \left(\frac{\bar{\pi}}{\pi_{t+s-1}} \right)^{l_p} \right) (1 + \mathcal{G}_{t+s}) \\
& + E_t \sum_{s=1}^{\infty} \xi_P^{s-1} (1 - \xi_P) \bar{\beta}^s \gamma^s \frac{\xi_{t+s}}{\xi_t} \tau_{t+s}^{TIP} \tilde{p}_t(i) x_{t,s}^p \left(\frac{\bar{\pi}}{\pi_{t+s-1}} \right)^{l_p} y_{t+s|\tilde{P}_{t+s}}(i). \quad (68)
\end{aligned}$$

We next linearize this FOC around $\tau_t^{TIP} = 0$:

$$\begin{aligned}
0 = & E_t \sum_{s=0}^{\infty} \xi_P^s (\bar{\beta} \gamma)^s \left[(1 + \mathcal{G}_{t+s}) \tilde{p}_t(i) x_{t,s}^p - \mathcal{G}_{t+s} mc_{t+s} \right] \\
& - \left[\tau_t^{TIP} - E_t \sum_{s=1}^{\infty} \xi_P^{s-1} (1 - \xi_P) (\bar{\beta} \gamma)^s \tau_{t+s}^{TIP} \right] \quad (69)
\end{aligned}$$

$$= E_t \sum_{s=0}^{\infty} \xi_P^s (\bar{\beta} \gamma)^s \mathcal{G} \left[\frac{1 + \mathcal{G}_{t+s}}{\mathcal{G}_{t+s}} \tilde{p}_t(i) x_{t,s}^p - mc_{t+s} - \frac{\tau_{t+s}^{TIP} - (\bar{\beta} \gamma) \tau_{t+s+1}^{TIP}}{\mathcal{G}} \right] \quad (70)$$

At first order, the denominator in the last term $\frac{\tau_{t+s}^{TIP} - (\bar{\beta}\gamma)\tau_{t+s+1}^{TIP}}{\mathcal{G}}$ is simply $\mathcal{G}$ because we linearize around $\tau^{TIP} = 0$. Define $mc_t^* = mc_t + \frac{\tau_t^{TIP} - (\bar{\beta}\gamma)E_t\tau_{t+1}^{TIP}}{\mathcal{G}}$ the marginal cost including current and future TIP. Using this definition into equation (70), we obtain an equation identical to equation 49 in SW's appendix with mc_{t+s}^* instead of mc_{t+s} . Therefore, following the same approach as SW, we obtain the following fully linearized equation:

$$(1 + \bar{\beta}\gamma\iota_p)\hat{\pi}_t = \iota_p\hat{\pi}_{t-1} + \bar{\beta}\gamma E_t\hat{\pi}_{t+1} + A \frac{(1 - \xi_p\bar{\beta}\gamma)(1 - \xi_p)}{\xi_p}(\hat{mc}_t^* + \hat{\lambda}_{p,t}), \quad (71)$$

where $\hat{\pi}_t$ is the deviation of inflation from trend inflation $\bar{\pi}$, and $A = \frac{1}{1 + \lambda_p\epsilon}$ captures the effect of strategic complementarities stemming from the Kimball aggregator. The parameter λ_p is the steady state markup, $\hat{\lambda}_{p,t}$ is the markup shock and $\epsilon = \frac{d\left(\frac{\mathcal{G}''}{x\mathcal{G}'}\right)}{d(x)}$.

$$\hat{mc}_t^* = \frac{dmc_t^*}{mc} = \frac{dmc_t}{mc} + \frac{\tau_t^{TIP} - (\bar{\beta}\gamma)E_t\tau_{t+1}^{TIP}}{\mathcal{G}mc}. \quad (72)$$

In steady state, $\mathcal{G} = -\epsilon(1)$ and $mc = \frac{\epsilon(1)-1}{\epsilon(1)}$, therefore,

$$\hat{mc}_t^* = \hat{mc}_t - \frac{1}{\epsilon(1) - 1}(\tau_t^{TIP} - \bar{\beta}\gamma E_t\tau_{t+1}^{TIP}). \quad (73)$$

Therefore, the NKPC with TIP in the SW model is given by

$$(1 + \bar{\beta}\gamma\iota_p)\hat{\pi}_t = \iota_p\hat{\pi}_{t-1} + \bar{\beta}\gamma E_t\hat{\pi}_{t+1} + A \frac{(1 - \xi_p\bar{\beta}\gamma)(1 - \xi_p)}{\xi_p} \left[\hat{mc}_t - \frac{\tau_t^{TIP} - \bar{\beta}\gamma E_t\tau_{t+1}^{TIP}}{\epsilon(1) - 1} + \hat{\lambda}_{p,t} \right]. \quad (74)$$

Targeting rule for TIP. Since trend inflation $\bar{\pi}$ is strictly positive, we modify the inflation-targeting rule 24 in Section 3.3 to center it around $\bar{\pi}$:

$$\tau_t^{TIP} = \varphi_{\pi} \hat{\pi}_t = \varphi_{\pi} (\pi_t - \bar{\pi}). \quad (75)$$

The trend-neutral targeting rule ensures the consistency between the non-zero trend inflation and zero TIP when the economy is on its trend.

F.2 Data and Estimation

We extend the original time series from FRED used in SW to the most recent available data point (U.S. Bureau of Economic Analysis, 2025a). Our final quarterly sample covers the period from 1960Q1 to 2024Q3. We closely follow their procedures to clean the data. Relative to SW, we make two modifications before estimating the model.

To address both zero-lower bound periods in our sample during which the Federal Funds rate was at zero and the Fed deployed unconventional tools, we replace the Federal Funds rate with the shadow rate by Wu and Xia (2016), as in Anderson et al. (2017), Wu and Zhang (2019), Avdjiev et al. (2020). We also tried alternative measures for robustness, including the new shadow rate by Jones et al. (2021), the 3-year Treasury yields, and keeping the Federal Funds rate unchanged. Our counterfactual results are virtually unchanged with these alternative measures of the monetary policy stance. Results are available upon request.

We restrict the sample to 1960Q1-2019Q4 to exclude the COVID pandemic whose extreme recession and rebound could bias our parameters' estimates. The model may indeed not be well suited to rationalize the dramatic shocks that occurred during the COVID-19 pandemic, especially in 2020Q2, when quarterly GDP collapsed by 31.6% (annualized) relative to 2020Q1 and then rebounded by 31.3% in 2020Q3. We then back out the seven structural shocks from the entire sample period, from 1960Q1 to 2024Q3.

We estimate the parameters and structural shocks using the Dynare

replication code in Johannes Pfeifer’s Github repository.²² Our priors follow SW’s original priors. The only exception is the trend inflation $\bar{\pi}$: we replace the original gamma distribution with a normal distribution $N(0.5, 0.05^2)$ to bring the posterior trend closer to 2% per annum. This is an important estimate in our counterfactual analysis because we assume that TIP follows an inflation-targeting rule centered around detrended inflation, and a trend of 2% per annum is a more realistic policy objective. Table F5 reports the priors and posteriors from the baseline estimation.

F.3 Counterfactual analysis.

We now used the estimated parameters and shocks from Section F.2, as well as the extended model with TIP to run counterfactuals when $\varphi_{\pi} = 300$. Results are reported below.

Historical Counterfactual with TIP. We find that TIP has significant stabilizing effects on inflation, particularly during high-inflation periods such as the 1970s and the early 2020s. The top panel of Figure 5 shows the actual path of inflation in blue, measured by the GDP deflator, and its counterfactual path under a TIP rule with $\varphi_{\pi} = 300$ in red. Both are the 4-quarter moving averages of annualized log changes. When inflation peaked in 1974Q4 at 10.4% per annum, TIP would have cut it by 2.9%, equivalent to 36% of the deviation from the trend inflation of 2.2%. When inflation peaked again in 1980Q4, TIP would have lowered it by 2.3%, equivalent to 31% of the deviation from trend. During the recent COVID-19 pandemic, TIP would have lowered the peak inflation in 2022Q1 by 2.6%, which is almost 50% of the deviation from the trend. During lower-inflation periods, TIP turns out to be highly effective at reducing the volatility of inflation too. Finally, despite its significant effect on inflation, TIP barely affects output growth in the simulations. The actual output growth and the counterfactual path show little difference in the middle panel of Figure 5.

²² https://github.com/johannespfeifer/dsge_mod

Table F5: Priors and posteriors

	Dist.	Prior		Posterior	
		Mean	Stdev	Mode	Stdev
ρ_a	beta	0.500	0.2000	0.9909	0.0041
ρ_b	beta	0.500	0.2000	0.8546	0.0339
ρ_g	beta	0.500	0.2000	0.9760	0.0075
ρ_i	beta	0.500	0.2000	0.8217	0.0681
ρ_r	beta	0.500	0.2000	0.1677	0.0614
ρ_p	beta	0.500	0.2000	0.9655	0.0316
ρ_w	beta	0.500	0.2000	0.9833	0.0090
μ_p	beta	0.500	0.2000	0.8992	0.0462
μ_w	beta	0.500	0.2000	0.9664	0.0119
φ	norm	4.000	1.5000	4.2620	1.0924
σ_c	norm	1.500	0.3750	1.4598	0.2039
λ	beta	0.700	0.1000	0.5072	0.0639
ξ_w	beta	0.500	0.1000	0.8006	0.0435
σ_l	norm	2.000	0.7500	1.5198	0.6019
ξ_p	beta	0.500	0.1000	0.7747	0.0453
ι_w	beta	0.500	0.1500	0.6198	0.1353
ι_p	beta	0.500	0.1500	0.3006	0.0926
ψ	beta	0.500	0.1500	0.6627	0.1133
ϕ_p	norm	1.250	0.1250	1.5042	0.0746
r_π	norm	1.500	0.2500	1.9852	0.1473
ρ	beta	0.750	0.1000	0.8395	0.0211
r_y	norm	0.125	0.0500	0.0977	0.0202
$r_{\Delta y}$	norm	0.125	0.0500	0.2340	0.0252
$100(\beta^{-1} - 1)$	gamm	0.250	0.1000	0.1026	0.0434
$\bar{\pi}$	norm	0.500	0.0500	0.5591	0.0503
$\bar{l}$	norm	0.000	2.0000	3.5819	1.2550
$\bar{\gamma}$	norm	0.400	0.1000	0.3770	0.0366
ρ_{ga}	norm	0.500	0.2500	0.5234	0.0637
α	norm	0.300	0.0500	0.1769	0.0165
η^a	invg	0.100	2.0000	0.4795	0.0261
η^b	invg	0.100	2.0000	0.0946	0.0110
η^g	invg	0.100	2.0000	0.4522	0.0215
η^i	invg	0.100	2.0000	0.3171	0.0301
η^m	invg	0.100	2.0000	0.2137	0.0119
η^p	invg	0.100	2.0000	0.1277	0.0110
η^w	invg	0.100	2.0000	0.3571	0.0197

This large reduction in the volatility of inflation is achieved by setting high TIP rates in times of high inflation. The peak TIP rate would be as high as 300% in the 1970s and 200% in 2022. By contrast, TIP would be largely

negative during the Great Moderation because inflation was constantly below the estimated trend of 2.2%. However, since TIP is applied only to price changes, the implied tax burden remains low. As shown in the last panel, TIP costs would be negligible relative to sales for most of the time. TIP costs would have briefly exceeded 3% of sales when inflation reached its highest level in 1974 and 1980. During the COVID period, the average (peak) cost of TIP would have been 0.85% and 1.9% of sales in 2021 and 2022 respectively, while the average reduction in inflation would have been 1.9% per annum.²³

The stabilization effects of TIP are robust to alternative ways to dealing with the zero-lower bound. We have run the same analysis using the [Jones et al. \(2021\)](#) shadow rate, the 3-year Treasury yields, and the Federal Funds rate. We find virtually the same results, which are available upon request.

Impulse response functions. We now examine the impulse response functions of the economy to the seven structural shocks using the estimated DSGE model. Figure [F10](#) shows the responses of inflation, output, and the TIP rate. Figure [F12](#) shows the responses of more variables to all seven shocks.

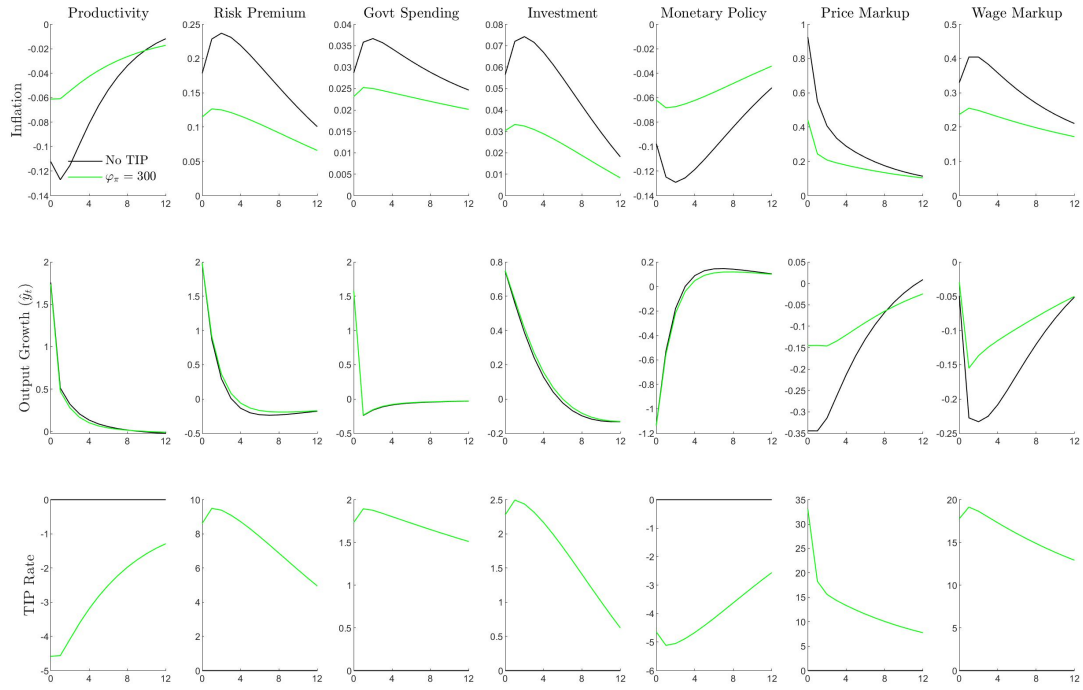
We find that an inflation-targeting TIP rule with φ_π dampens the response of inflation after all seven shocks by 30 to 50% depending on the shock, consistent with the IRFs implied by the small-scale model in Section [3.3](#). More importantly, TIP remarkably attenuates output losses after price and wage markup shocks, reaffirming the divine coincidence with markup shocks in Section [3.3.1](#). After all other five non-markup shocks, TIP has only very limited effects on output, suggesting that trade-offs are quantitatively negligible in the medium-scale DSGE model. The main reason is that inflation responses are an order of magnitude smaller than output responses after the five non-markup shocks. While lowering inflation qualitatively decreases the need for monetary policy tightening, quantitatively the magnitude is

²³ TIP would have been quite effective to dampen inflation during this period because price markup shocks were, according to the model, important in driving inflation.

small.

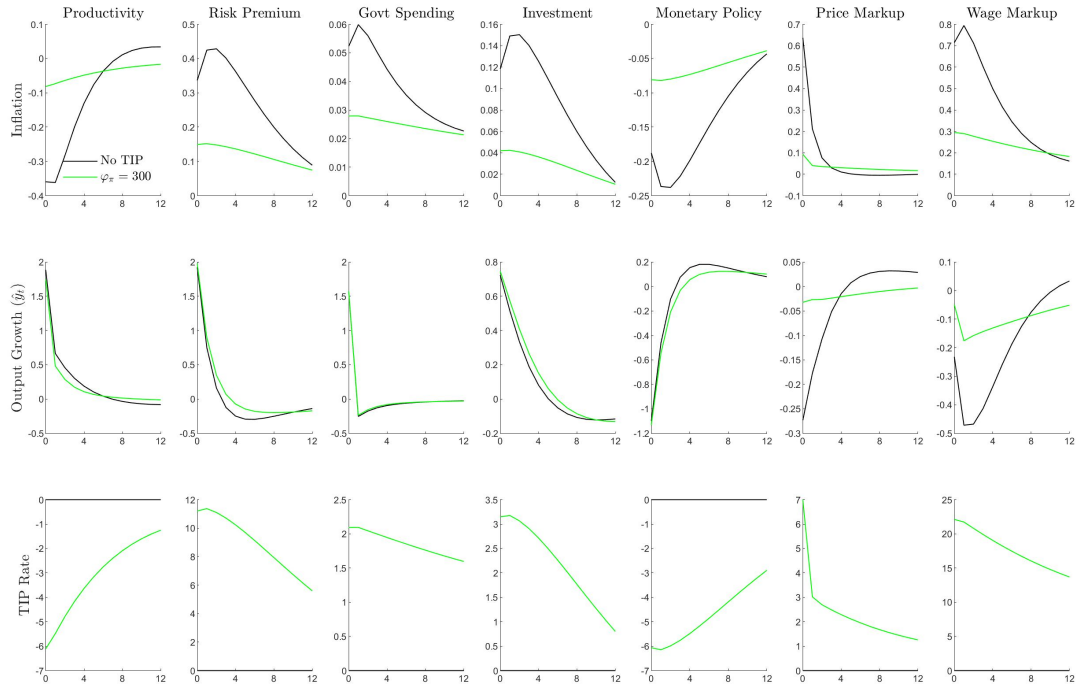
Next, we look at the robustness of our conclusions when prices become more flexible and the Phillips Curve becomes "steeper" as suggested in recent research. To do so, we lower the Calvo pricing parameter ξ_p by 0.25 percentage points, from 0.7747 to 0.5247 as in the exercise in Section 3.3.1. Other parameters remain unchanged. The IRFs are shown in Figure F11.

After a price markup shock, without TIP, the response of inflation on impact is stronger when price are more flexible, from 0.23% in Figure F12 to 0.65% in Figure F11. With TIP, however, the response inflation on impact is slightly lower, below 0.1% instead of around 0.4%. Even though inflation subsides slightly more slowly with TIP after 1 year than without, the cumulative impact on the price level in the medium run remains significant. When prices are more flexible, TIP is able to lower the cumulative inflation response by more than 40% compared to 30% when price are more sticky. For all other shocks, TIP also becomes more effective at attenuating inflation when prices become more flexible (see Figure F11).



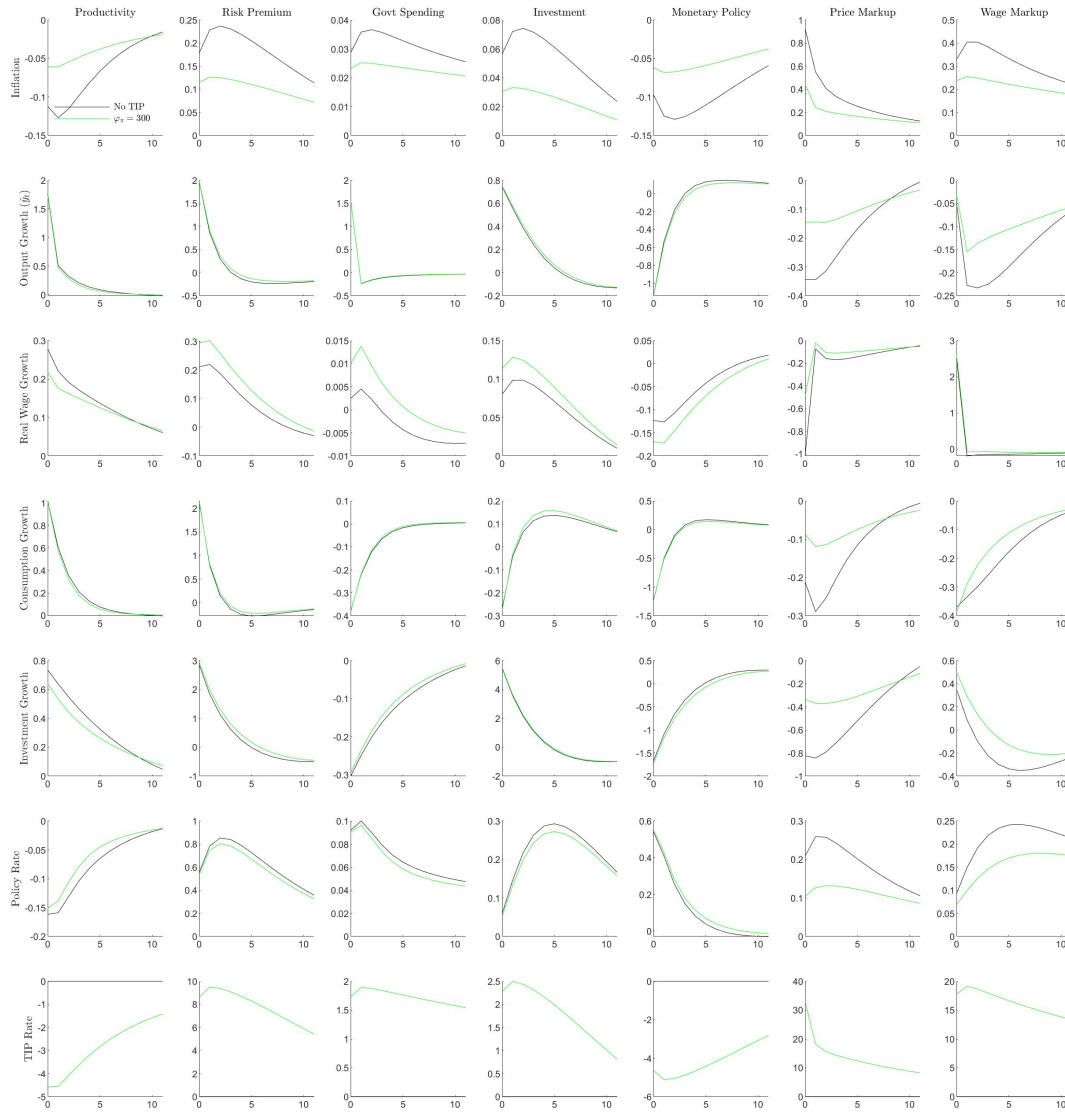
Notes: The initial shock size is one standard deviation. Inflation and the output growth rate are annualized. Output refers to the deviation of output from its steady-state level, $\hat{y}_t$.

Figure F10: Impulse response functions



Notes: The initial shock size is one standard deviation. Inflation and the output growth rate are annualized. Output refers to the deviation of output from its steady-state level, $\hat{y}_t$.

Figure F11: Impulse response functions with lower price stickiness



Notes: The initial shock size is one standard deviation. Inflation, all growth rate, and the policy rate are annualized. Output refers to the deviation of output from its steady-state level, $\hat{y}_t$.

Figure F12: Impulse response functions (continued)

References

- Anderson, E., Malin, B. A., Nakamura, E., Simester, D., and Steinsson, J. (2017). Informational rigidities and the stickiness of temporary Sales. *Journal of Monetary Economics*, 90:64–83.
- Auclert, A., Bardóczy, B., Rognlie, M., and Straub, L. (2021). Using the sequence-space jacobian to solve and estimate heterogeneous-agent models. *Econometrica*, 89(5):2375–2408.
- Avdjiev, S., Gambacorta, L., Goldberg, L. S., and Schiaffi, S. (2020). A shadow rate New Keynesian model. *Journal of International Economics*, 125(103324).
- Blanchard, O. J. and Kahn, C. M. (1980). The Solution of Linear Difference Models under Rational Expectations. *Econometrica*, 48(5):1305–1311.
- Bogetic, Z. and Fox, L. (1993). Incomes Policy During Stabilization: A Review and Lessons from Bulgaria and Romania. *Comparative Economic Studies*, 35(1):39–57.
- Colander, D. (1981). *Incentive Anti-Inflation Plans: A Study for the Use of the Joint Economic Committee Congress of the United States*. U.S. government printing office.
- Correia, I., Farhi, E., Nicolini, J. P., and Teles, P. (2013). Unconventional Fiscal Policy at the Zero Bound. *American Economic Review*, 103(4):1172–1211.
- Correia, I., Nicolini, J. P., and Teles, P. (2008). Optimal fiscal and monetary policy: equivalence results. Staff Report 403, Federal Reserve Bank of Minneapolis.
- Crombrugghe, A. D. and de Walque, G. (2011). Wage and employment effects of a wage norm : The Polish transition experience. Working Paper Research 209, National Bank of Belgium.

- Eggertsson, G. B. (2011). What Fiscal Policy Is Effective at Zero Interest Rates? In *NBER Macroeconomics Annual 2010, volume 25*, NBER Chapters, pages 59–112. National Bureau of Economic Research, Inc.
- Eggertsson, G. B. and Woodford, M. (2006). Optimal Monetary and Fiscal Policy in a Liquidity Trap. In *NBER International Seminar on Macroeconomics 2004*, NBER Chapters, pages 75–144. National Bureau of Economic Research, Inc.
- Enev, T. and Koford, K. (2000). The Effect of Incomes Policies on Inflation in Bulgaria and Poland. *Economic Change and Restructuring*, 33(3):141–169.
- Farhi, E., Gopinath, G., and Itskhoki, O. (2014). Fiscal Devaluations. *Review of Economic Studies*, 81(2):725–760.
- Galí, J. (2015). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications Second edition*. Number 10495 in Economics Books. Princeton University Press.
- Giannoni, M. P. (2014). Optimal Interest-Rate Rules and Inflation Stabilization versus Price-Level Stabilization. *Journal of Economic Dynamics and Control*, 41:110–129.
- Giannoni, M. P. and Woodford, M. (2003). Optimal Interest-Rate Rules: II. Applications. *NBER Working Paper*, (9420).
- Jackman, R. and Layard, R. (1989). The Real Effects Of Tax-Based Incomes Policies. Papers 341, London School of Economics - Centre for Labour Economics.
- Jones, C., Kulish, M., and Morley, J. (2021). A Structural Measure of the Shadow Federal Funds Rate. *Federal Reserve Board Finance and Economics Discussion Series*, (2021-064).
- Koford, K. J. and Miller, J. B. (1992). Macroeconomic Market Incentive Plans: History and Theoretical Rationale. *American Economic Review*, 82(2):330–334.

- Koford, K. J., Miller, J. B., and Colander, D. C. (1993). Application of Market Anti-inflation Plans in the Transition to a Market Economy. *Eastern Economic Journal*, 19(3):379–393.
- Kotowitz, Y. and Portes, R. (1974). The ‘Tax on Wage Increases’ : A Theoretical Analysis. *Journal of Public Economics*, 3(2):113–132.
- Latham, R. and Peel, D. (1977). The Tax on Wage Increases when the Firm is a Monopsonist. *Journal of Public Economics*, 8(2):247–253.
- Mertens, K. R. S. M. and Ravn, M. O. (2014). Fiscal Policy in an Expectations-Driven Liquidity Trap. *The Review of Economic Studies*, 81(4 (289)):1637–1667.
- OECD (1975). *Socially responsible wage policies and inflation : a review of four countries’ experience* . OECD.
- Oswald, A. J. (1984). Three Theorems on Inflation Taxes and Marginal Employment Subsidies. *Economic Journal*, 94(375):599–611.
- Paci, P. (1988). Tax-based incomes policies: will they work? have they worked? *Fiscal Studies*, 9(2):81–94.
- Peel, D. A. (1979). The Dynamic Behaviour of a Simple Macroeconomic Model with a Tax-based Incomes Policy. *Economics Letters*, 3(2):139–143.
- Rotemberg, J. J. and Woodford, M. (1999). Interest Rate Rules in an Estimated Sticky Price Model. In *Monetary Policy Rules*, NBER Chapters, pages 57–126. National Bureau of Economic Research, Inc.
- Scarth, W. M. (1983). Tax-related Incomes Policies and Macroeconomic Stability. *Journal of Macroeconomics*, 5(1):91–103.
- Seidman, L. S. (1978). Tax-Based Incomes Policies. *Brookings Papers on Economic Activity*, 9(2):301–361.
- Smets, F. and Wouters, R. (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, 97(3):586–606.

U.S. Bureau of Economic Analysis (2025a). "Real Gross Domestic Product", "Gross Domestic Product: Implicit Price Deflator", "Personal Consumption Expenditures", "Fixed Private Investment Federal Funds Effective Rate", "Employment Level Civilian Labor Force Level", "Population Level", "Nonfarm Business Sector: Average Weekly Hours for All Workers", "Nonfarm Business Sector: Hourly Compensation for All Workers", "Market Yield on U.S. Treasury Securities at 3-Year Constant Maturity, Quoted on an Investment Basis." . <https://fred.stlouisfed.org/series/GDPC1>. Retrieved from FRED, Federal Reserve Bank of St. Louis, in May 2025.

U.S. Bureau of Economic Analysis (2025b). Table 2.3.4. Price Indexes by Major Type of Product and Table 2.3.5. Personal Consumption Expenditures by Major Type of Product. <https://www.bea.gov/itable/national-gdp-and-personal-income>. Accessed in May 2025.

Wallich, H. C. and Weintraub, S. (1971). A Tax-Based Incomes Policy. *Journal of Economic Issues*, 5(2):1–19.

Woodford, M. (2011). Simple Analytics of the Government Expenditure Multiplier. *American Economic Journal: Macroeconomics*, 3(1):1–35.

Wu, J. C. and Xia, F. D. (2016). Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound. *Journal of Money, Credit, and Banking*, 48(2-3):253–291.

Wu, J. C. and Zhang, J. (2019). A shadow rate New Keynesian model. *Journal of Economic Dynamics and Control*, 107(103728).