

Supplemental Appendix

“Markups and Inflation in Oligopolistic Markets: Evidence from
Wholesale Price Data”

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Disclaimer: The views expressed herein are those of the authors and not necessarily those of the Bank of Canada.

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A Data and Measurement

A.1 Data cleaning process

Our empirical analysis relies on two raw micro datasets. The first is the monthly WSPI micro data file. This includes information on monthly wholesale purchase prices and selling prices for individual firm-products. It also includes other information we use in our analysis and to inform the cleaning process. The second dataset is the weights file, which is developed to be merged with the WSPI for the purpose of providing a representative sample to construct the WSPI. We provide more details about both files below.

The raw micro data have not been cleaned prior to our receiving the data, and none of the prices in our data are imputed. Prior to constructing the WSPI, Statistics Canada conducts various error detection tests and excludes some outliers and anomalies in the data. More generally, Statistics Canada dedicates resources to ensuring any reported price changes in the survey are not contaminated by structural changes to the product definition, including product reclassifications or changes in units.

The survey is stratified by NAICS5 codes. The largest establishments in a given NAICS industry are selected as “take-all” (100% probability), with remaining establishments selected with probabilities that are proportional to their revenue. To construct the index, individual respondents are assigned weights based on establishment revenues and industry gross margins to arrive at a representative sample of wholesale sector prices. New survey participants are introduced to the survey through telephone calls, where respondents are guided through a process of selecting representative products. The data are updated (and revised) quarterly, where respondents are asked to answer the survey based on information from the preceding three months. The sample of respondents for the price file is updated roughly every five years. The weights file is updated, along with the sample update, roughly every five years. The sample used for this analysis was last updated in May 2023.

The price file includes several variables that correspond in some fashion to firm/establishment/firm-product identifiers: (1) the “PID” is intended to uniquely identify each firm-product in the data. It is assigned by the Producer Price Division (PPD) based on the Generic Processing System (GPS) and serves as our primary unit of analysis in the paper; (2) the “PPDID” is assigned by the PPD and is intended to correspond one-to-one with the establishment level from Statistics Canada’s Business Register (BR); the PPDID is the sampling unit for the WSPI survey; (3) the “Operating Entity Number” is also intended to correspond one-to-one with the establishment level and is taken directly from the value reported in the BR; and (4) the “Enterprise Number” is intended to correspond one-to-one with the enterprise level and is taken directly from the value reported in the BR.¹ Also included are classification codes for the NAICS5 industry that the reporting firm is associated with and the NAPCS7 product code that the firm-product is associated with.

¹An “enterprise” (also referred to as a “firm”) is defined as an institutional unit that directs and controls the allocation of resources relating to business operations and for which financial statements are maintained. An “establishment” (also referred to as a “plant”) is below the enterprise in the statistical hierarchy and is defined as the most homogeneous unit of production for which the business maintains accounting records.

The price file also includes information on the country of origin of the product, the currency that product prices are reported in, and several flags that help to ascertain the quality of the data. There are two variables that provide information on the country of origin of the product. One is an “imported” binary variable that takes the value of 1 if imported and 0 if not imported. The second is an “origin” variable that takes different values that each correspond to a specific country of origin. In terms of currency, there are separate “currency of purchase price” and “currency of selling price” variables included in the data. In terms of flags, there is a “product change” flag that identifies cases where the product name appears to change, a “non comparable” flag that identifies where the series appears to have changed in a significant way that suggests a break in the series, and an “exclude” flag that identifies outlier observations based on the patterns observed in that particular firm-product over the periods around that observation. The “non comparable” and “exclude” flags also have associated variables that provide the reason for these flags based on a defined set of possible reasons. All these flags are introduced by analysts in the PPD and not entered by the survey respondents.

The price file sample that we use begins in January 2013. The data are currently available up to 2024, but we drop all observations past December 2019 to exclude the COVID-19 pandemic period.

The weights file includes two sub-files: one for the 2013 reference period (released in the second quarter of 2016) and one for the 2020 reference period (released in the third quarter of 2022). Each weight in a given file is intended to correspond uniquely to a single PPDID in the corresponding WSPI file. However, there are cases where a single PPDID is reported in both sampling periods, so that one PPDID has a weight reported in each weight file. In these cases, we use the weight from the older weight sample. In some cases the weight is missing in one weight sample but is available in the other. In these cases, we use the weight that is available. These files include several weights that we use. The first is a “revenue weight,” derived from establishment revenue data based on Statistics Canada’s BR and industry gross margins based on the Annual Wholesale Trade Survey micro data.² The second is a “design weight,” equal to the inverse of the firm’s selection probability, induced by the sample design. This weight can be interpreted as the number of times that each sampled establishment should be replicated to represent the entire population. Finally, a “sampling revenue weight” is equal to the product of the revenue weight and the design weight and represents the relative importance of the establishment in the industry. It is used to construct an index that is representative of the aggregate.

Once the WSPI file is merged with the weights file, we initially apply a few small cleaning procedures. In terms of hierarchical structure, a single enterprise might nest several establishments, but a single establishment should not nest several enterprises. So we drop cases where multiple Enterprise Numbers are associated with a single Operating Entity Number or PPDID. The PPDID and the Operating Entity Number are supposed to map one-to-one with one another, so we also drop cases where this mapping is not one-to-one. Once that procedure is applied, we are left with

²The revenue weight corresponds one-to-one with the wholesale establishment level.

roughly 420,000 observations.

From there, we identify cases where the “imported” variable is 0 but the “origin” variable indicates a foreign origin. We assume the “origin” variable is correct and re-code the “imported” variable to 1. Also, in cases where the “imported” variable is missing, we assume the good is not imported and re-code the variable to 0. We drop observations where the currency of the reported selling or purchase price is in a currency other than Canadian dollars, US dollars, or euros.³ In cases where the currency reported is either US dollars or euros, we apply the bilateral monthly exchange rate and convert the price into Canadian dollars. We also drop cases where the firm-product margin is less than 1, indicating that the selling price is lower than the purchase price, and drop observations where the selling price or purchase price changes by more than 100% between consecutive months. We drop cases where a single PID is reported for only one period in the data.

For cases where the “product change” flag is 1, we reclassify the product so that a new PID is assigned. We drop observations where there is an “exclude” flag and cases where there is a “non comparable” but not a “product change” flag (since these are reclassified as new products). We also drop observations where the selling price or purchase price is either missing or zero, and where the establishment revenue weight is either missing or non-positive.

After all of these changes, we are left with roughly 280,000 observations in the cleaned sample.⁴

A.2 Additional descriptions of the data

The breakdown of the average number of products per firm across periods in our cleaned sample is reported in Table A1. We produce this by constructing a new variable equal to the number of PIDs associated with each PPDID per period, and then tabulating the share of this variable in the total sample that falls under 1, 2, 3, 4, and 5+.

Table A1: Number of products per firm in cleaned WSPI sample

	1	2	3	4	5+
Share of firms	9%	15%	67%	3%	6%

Our sample covers 90 months, from January 2013 to December 2019. Table A2 reports the number of observations per firm-product, calculated by creating a new variable that is equal to the number of observations per PID in the sample and then classifying each PID according to which bin this number falls into (e.g., 1–20, 20–40, etc.). As depicted in the table, 75% of products include more than 20 observation months. This feature of the data is attractive in that most of our analysis will rely on product-level cross-time variation, and so a long sample period at the product level is desirable.

Survey respondents are encouraged to report all of their prices in Canadian dollars, regardless of the actual currency of invoicing. Table A3 reports the share of observations in the cleaned dataset

³Note that this only affects a small number of observations.

⁴Most of the roughly 140,000 observations that are dropped are removed due to missing prices.

Table A2: Number of observation months per firm-product in cleaned WSPI sample

	1–20	21–40	41–60	61–80	80–100
Share of observations	25%	24%	26%	20%	6%

that are reported in Canadian dollars and US dollars. Roughly 96% (97%) of respondents report purchase (selling) prices in Canadian dollars, and nearly all the rest report in US dollars. In cases where the currency reported is either US dollars or euros, we apply the bilateral monthly exchange rate and convert the price into Canadian dollars.

Table A3: Currency of prices reported in cleaned WSPI sample, shares

	Purchase prices	Selling prices
Canadian dollar	0.96	0.97
US dollar	0.04	0.03

In terms of the origin of the products reported in the data, we can group products into three different types: domestic goods, goods imported from the US, and goods imported from non-US countries. In the aggregate sample, 44% of goods originate from the domestic economy, 32% originate from the US, and the remaining 25% (rounded) originate from other countries (non-US). This breakdown, however, is different from industry to industry. Figure A1 reports the breakdown separately for each NAICS3 industry, where the heterogeneity is clear. For example, the domestic economy is the top source of goods in most industries, but the US is the most common origin for goods in the “Machinery, equipment and supplies” industry, and non-US foreign economies are the most common origin for goods in the “Personal and household goods” industry. This heterogeneity provides one foundation for heterogeneity in exposure to shocks across industries. For example, industries that are more reliant on imported goods would be more exposed to exchange rate shocks and foreign common shocks.

Figure A2 reports the average size of purchase and selling price changes in the sample for each NAICS3 industry. The cross-industry heterogeneity in this figure is fairly similar to the pattern observed in Figure 2, which reports the average fraction of price changes across industries. The average size of price change is equal to the average fraction of price changes times the size of the price change conditional on adjustment, so the positive correlation here indicates that the fraction of price adjustments plays a large role in determining average price changes.

Figure A3 reports a histogram for the (log) markup across firm-products in our cleaned sample. The important thing to note is that the distribution is far from uniform, indicating a high degree of heterogeneity.

Figure A4 reports scatter plots for the correlation between selling and purchase price stickiness across industries and products for NAICS4 and NAPCS7 classifications. The figure is very similar to Figure 3 except, in this case, prices within each industry or product group are weighted by

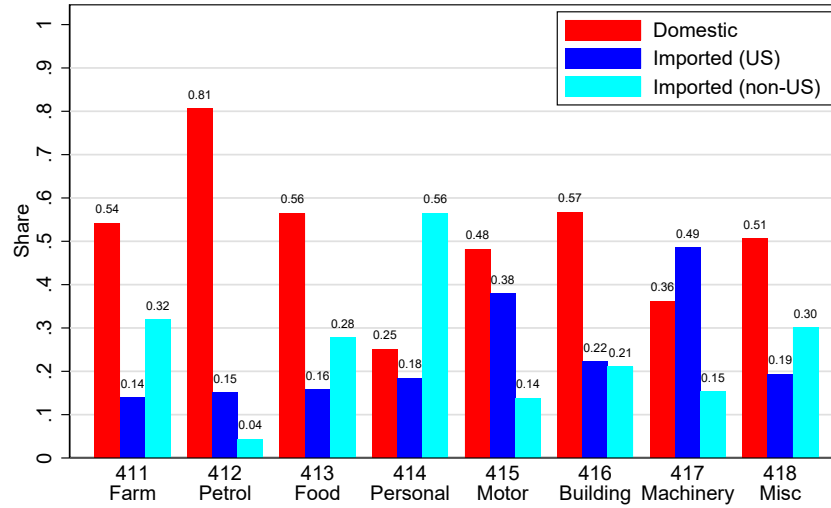


Figure A1: Origin of products by 3-digit NAICS wholesale industry

Notes: Reports the design-weighted mean of product origins across all observations in the sector.

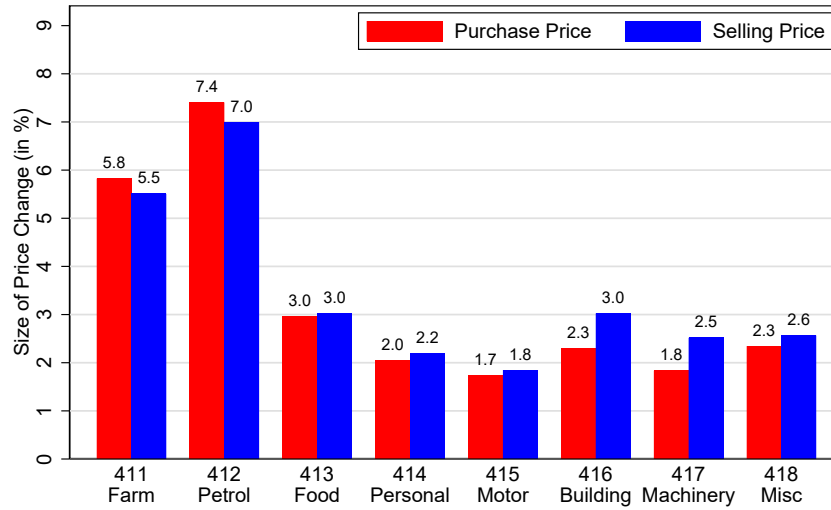


Figure A2: Average size of price changes by 3-digit NAICS wholesale industry

Notes: Reports the design-weighted mean size of price changes across all observations in the sector.

sampling revenue.

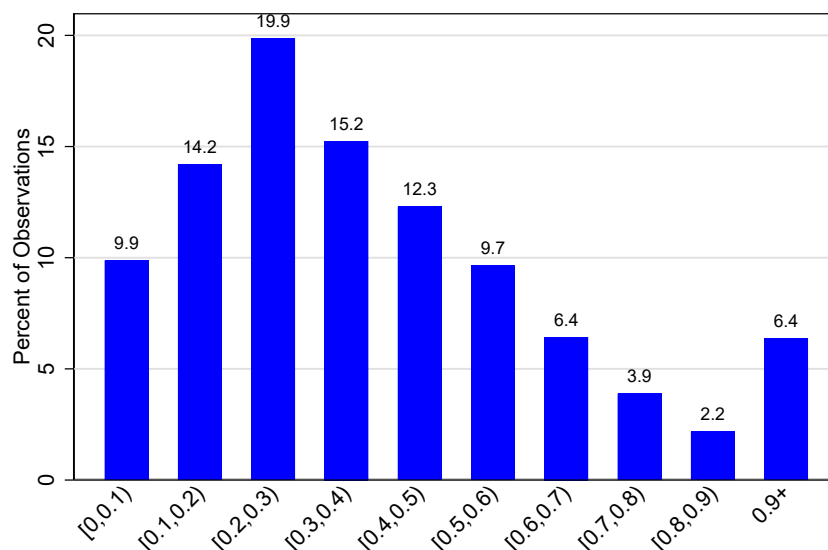
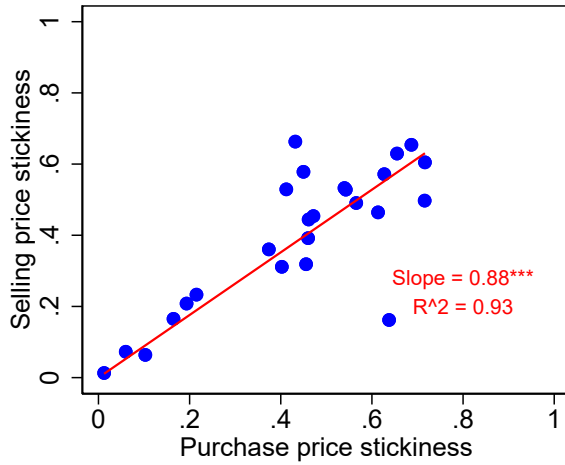


Figure A3: Histogram of markup across firm-products, pooled sample

Notes: Reports the design-weighted distribution of markups across firm-products in the sample.

Our analysis relies on firm and product classifications according to NAICS4 industry codes and NAPCS7 product codes. The firms surveyed for the WSPI are each classified to a single NAICS code under the 2-digit “wholesale trade” industry (NAICS 41). The complete list of 25 NAICS4 codes under NAICS 41 (i.e., the set of codes assigned to the firms in the WSPI survey) is reported in Table A4. Each firm-product is assigned to a single NAPCS7 product code under the 3-digit product group “Wholesale services (except commissions)” (NAPCS 551). Our cleaned dataset includes 166 NAPCS7 codes under NAPCS 551 (i.e., the set of codes assigned to the products in the WSPI survey). See [this link](#) for the complete list of NAPCS product codes.

(a) 4-digit NAICS wholesale industry level



(b) 7-digit NAPCS wholesale product level

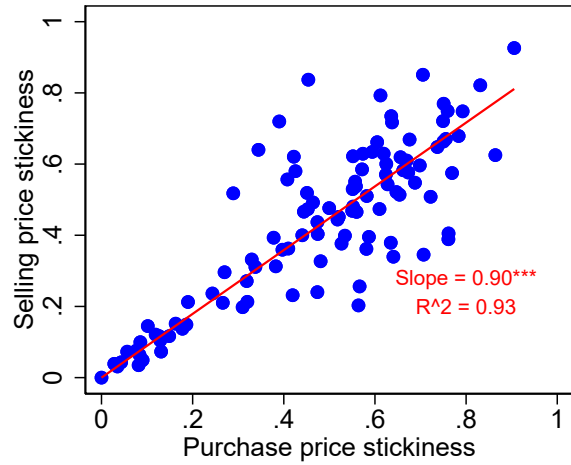


Figure A4: Selling and purchase price synchronization at the industry and product levels (weighted)

Table A4: 4-digit NAICS wholesale industries

NAICS	Industry Description
4111	Farm product merchant wholesalers
4121	Petroleum and petroleum products merchant wholesalers
4131	Food merchant wholesalers
4132	Beverage merchant wholesalers
4133	Cigarette and tobacco product merchant wholesalers
4134	Cannabis merchant wholesalers
4141	Textile, clothing and footwear merchant wholesalers
4142	Home entertainment equipment and household appliance merchant wholesalers
4143	Home furnishings merchant wholesalers
4144	Personal goods merchant wholesalers
4145	Pharmaceuticals, toiletries, cosmetics and sundries merchant wholesalers
4151	Motor vehicle merchant wholesalers
4152	New motor vehicle parts and accessories merchant wholesalers
4153	Used motor vehicle parts and accessories merchant wholesalers
4161	Electrical, plumbing, heating and air-conditioning equipment and supplies merchant wholesalers
4162	Metal service centres
4163	Lumber, millwork, hardware and other building supplies merchant wholesalers
4171	Farm, lawn and garden machinery and equipment merchant wholesalers
4172	Construction, forestry, mining, and industrial machinery, equipment and supplies merchant wholesalers
4173	Computer and communications equipment and supplies merchant wholesalers
4179	Other machinery, equipment and supplies merchant wholesalers
4181	Recyclable material merchant wholesalers
4182	Paper, paper product and disposable plastic product merchant wholesalers
4183	Agricultural supplies merchant wholesalers
4184	Chemical (except agricultural) and allied product merchant wholesalers
4189	Other miscellaneous merchant wholesalers

A.3 Measures of price stickiness and market power

Measures of the average degree of price stickiness are constructed as follows:⁵

$$\begin{aligned}\lambda_j &= 1 - \frac{1}{2} \frac{\sum_{i \in I_j} \sum_{t \in T_{ij}} \omega_{ij}^E \{ \mathbb{1} [\Delta \ln(P_{ijt}) \neq 0] + \mathbb{1} [\Delta \ln(Q_{ijt}) \neq 0] \}}{\sum_{i \in I_j} \sum_{t \in T_{ij}} \omega_{ij}^E}, & (\text{Sector price stickiness}) \\ \lambda_{fj} &= 1 - \frac{1}{2} \frac{\sum_{i \in I_f} \sum_{t \in T_{ij}} \omega_{ij}^E \{ \mathbb{1} [\Delta \ln(P_{ijt}) \neq 0] + \mathbb{1} [\Delta \ln(Q_{ijt}) \neq 0] \}}{\sum_{i \in I_f} \sum_{t \in T_{ij}} \omega_{ij}^E}, & (\text{Firm price stickiness}) \\ \lambda_{ij} &= 1 - \frac{1}{2} \frac{\sum_{t \in T_{ij}} \{ \mathbb{1} [\Delta \ln(P_{ijt}) \neq 0] + \mathbb{1} [\Delta \ln(Q_{ijt}) \neq 0] \}}{\sum_{t \in T_{ij}} 1}, & (\text{Firm-product price stickiness})\end{aligned}$$

where I_j and I_f denote the sets of firm-product observations in industry j and in firm f , respectively; T_{ij} denotes the set of months for which a price change from the previous month is observed for firm-product i in industry j ; and ω_{ij}^E is the economic weight of the firm, calculated as the establishment revenue of the firm divided by the probability of selection. Intuitively, price stickiness is equal to 1 minus the average monthly fraction of adjusting prices at a sector, firm, or product level. As discussed in Section 3, the selling price stickiness is very similar to the purchase price stickiness for most sectors and products. We take the average of the two measures to account for the small discrepancy in some industries.

Unlike price stickiness, the market power of a firm is not directly observed in the data. According to most models of imperfect competition, the price markup is a suitable proxy for market power. In our model, market power, summarized by strategic complementarity φ_{ij} , is linear in the steady-state price markup $\mu_{ij} \equiv \frac{\vartheta_{ij}}{\vartheta_{ij}-1} = \frac{\theta}{\theta-1} \frac{1}{1-s_{ij}}$ for any given θ under the assumption of Cournot competition:

$$\varphi_{ij} = \left(\frac{\theta-1}{\theta} \mu_{ij} - 1 \right) (\theta - 1).$$

We exploit the distributor's margin as the proxy for the price markup to construct two dummies that capture the variation in market power across and within sectors:

$$\begin{aligned}D_j &= \begin{cases} 1 & \text{if } \mu_j \in \text{upper quartile of } \{\mu_j\} \text{ across all sectors,} \\ 0 & \text{otherwise,} \end{cases}, \\ D_{ij} &= \begin{cases} 1 & \text{if } \mu_{ij} \in \text{upper quartile of } \{\mu_{ij}\} \text{ among all } i \in I_j, \\ 0 & \text{otherwise,} \end{cases},\end{aligned}$$

where $\mu_j = \frac{\sum_{i \in I_j} \sum_{t \in T_{ij}} \omega_{ij}^E \mu_{ijt}}{\sum_{i \in I_j} \sum_{t \in T_{ij}} \omega_{ij}^E}$ is the weighted mean margin across all firm-product observations in sector j ,⁶ and μ_{ij} is the average margin of firm-product i . $D_j = 1$ identifies the top quartile of high-markup sectors, and $D_{ij} = 1$ defines the top quartile of high-markup firms in sector j .

⁵As discussed in Section 3, the degree of selling price stickiness is highly correlated with that of purchase price stickiness. Using the purchase price stickiness measures yields similar estimates for our pass-through results.

⁶Conditioning on observations with price changes yields similar average sector markups.

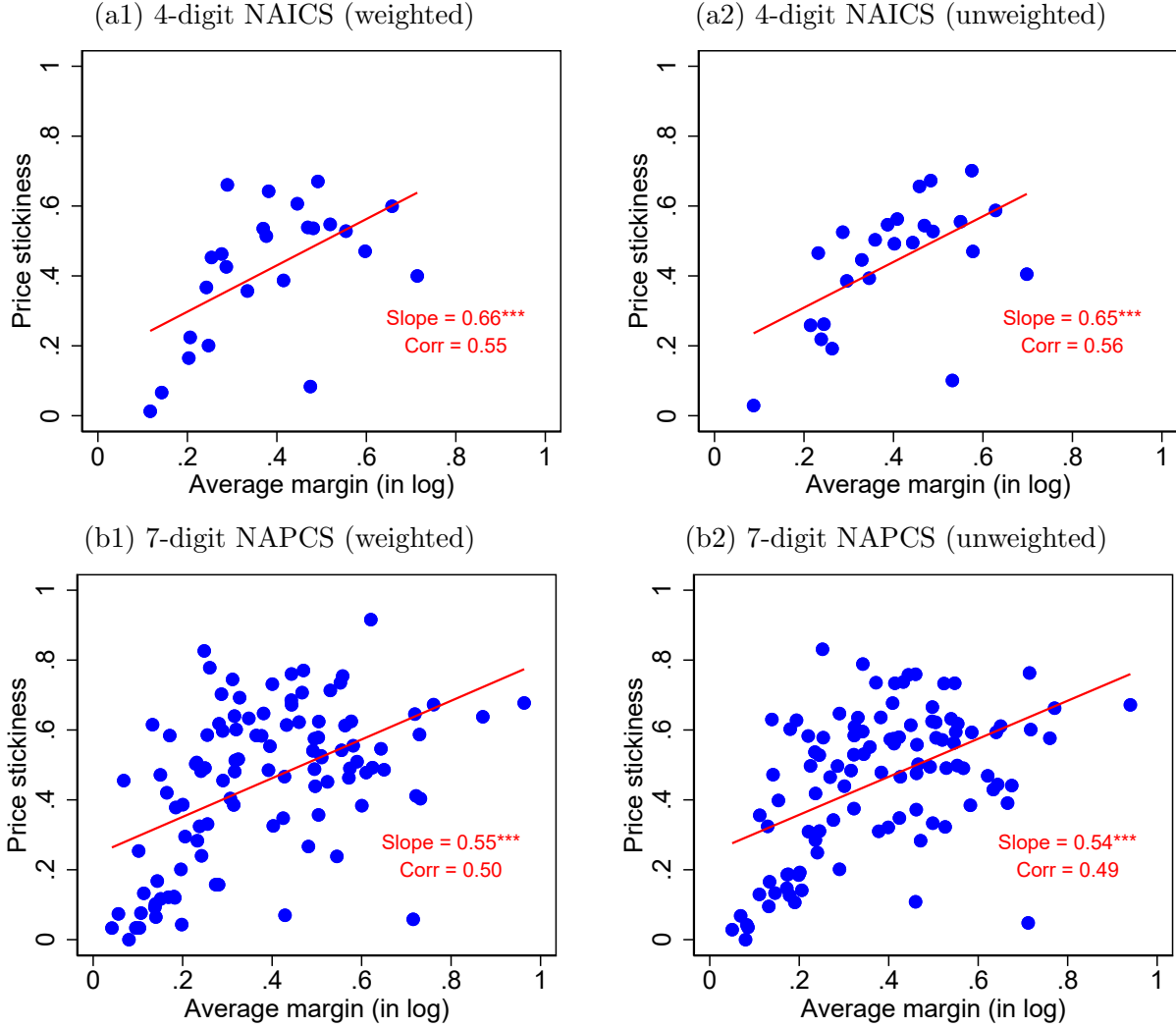


Figure A5: Correlation between price stickiness and average markup

Notes: The figures illustrate the cross-sector correlation between the average price stickiness λ_j and the average markup μ_j , with measures calculated at NAICS4 and NAPCS7 levels, respectively. The weighted measures constructed use the economic weight ω_{ij}^E .

A.4 Approximation of the marginal cost by the purchase price

Our benchmark analysis assumes that the observed purchase price is a good proxy for the true marginal cost of the product sold. In the context of the wholesale industry, we believe this is a reasonable assumption. Nonetheless, in this subsection we discuss the implications when this assumption no longer holds.

Consider a more general setting where the marginal cost of firm-product i in sector j , MC_{ijt} , consists of two components: (1) the observed purchase price Q_{ijt} and (2) the unobserved marginal cost component X_{ijt} :

$$MC_{ijt} = Q_{ijt}^\gamma X_{ijt}^{1-\gamma} \quad \text{with } \gamma \in (0, 1].$$

Note that if the unobserved cost component X_{ijt} is highly correlated with the observed component, then the change in the observed purchase price $\hat{Q}_{ijt}$ remains a good proxy for the change in the marginal cost $\widehat{MC}_{ijt}$. Taking the extreme case where these two variables are perfectly correlated, we have

$$\widehat{MC}_{ijt} = \gamma \hat{Q}_{ijt} + (1 - \gamma) \hat{X}_{ijt} = \hat{Q}_{ijt}.$$

The potential problem arises when the two cost components are not perfectly correlated. In the case where $\text{Corr}(\hat{Q}_{ijt}, \hat{X}_{ijt}) = a$, we have

$$\widehat{MC}_{ijt} = \gamma \hat{Q}_{ijt} + (1 - \gamma) \hat{X}_{ijt} = [\gamma + (1 - \gamma)a] \hat{Q}_{ijt}. \quad (\text{A.1})$$

If the actual change in marginal cost is smaller than the observed purchase price change (i.e., $\widehat{MC}_{ijt} < \hat{Q}_{ijt}$ or $\gamma + (1 - \gamma)a < 1$), then the estimated price pass-through to the cost shocks measured by the purchase price changes may be downward biased. For example, one concern is that our estimated pass-through to the common cost shock is incomplete not because of the interaction between price stickiness and market power but simply because the costs are not precisely measured. For example, in the context of a monopolistic competition Calvo model, the estimated reset price pass-through rate using $\hat{Q}_{ijt}$ as the regressor will be $\gamma + (1 - \gamma)a$, smaller than theoretical 100% obtained using the true marginal cost $\widehat{MC}_{ijt}$ if $a < 1$.

We note that this hypothesis is rejected by our estimates of the price pass-through to common cost shocks in a flexible price sector. Under the assumptions of our oligopolistic competition Calvo model and the cost process of (A.1), the pass-through to a common purchase price shock in the *flexible price sector* is

$$\Psi = \gamma + (1 - \gamma)a.$$

Our empirical estimate of Ψ^{Est} is very close to 1, which implies that $\gamma + (1 - \gamma)a \approx 1$. In other words, our empirical estimates implicitly suggest that the observed purchase price is a good proxy for the unobserved marginal cost in the wholesale industry.

B Supplementary Estimates of Contemporaneous Pass-through

B.1 Estimation results using product classification

In the main text, sectors are defined using a 4-digit industry classification (NAICS4). In this section, we present the results using 7-digit product classification (NAPCS7) to define sectors. Figure B1 provides scatter plots of the estimated pass-through coefficients against the price stickiness and the average markup of the sector. The plots include the fitted line to summarize the relationship. Table B1 provides estimated pass-through coefficients capturing variation in price stickiness and market power both across and within sectors.

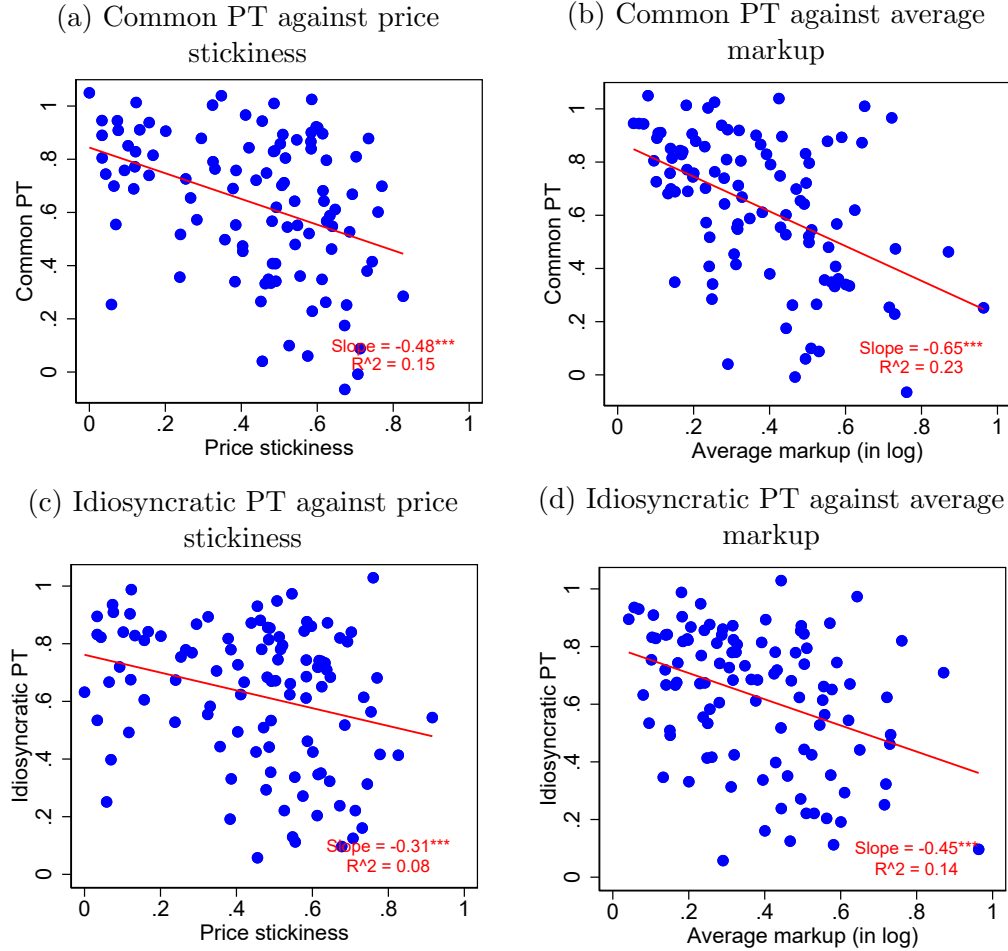


Figure B1: Estimates at the 7-digit NAPCS wholesale product level

Notes: The figures plot the estimated selling price pass-through to common and idiosyncratic cost shocks against the average price stickiness and markup measured at the NAPCS7 product level. Specifically, we estimate $\Delta \ln(P_{ijt}) = \Psi_j \epsilon_{jt}^{Est} + \psi_j \epsilon_{ijt}^{Est} + FE_{ij} + \nu_{ijt}$ separately for each product. For this graphical presentation, we have included only the products with estimated pass-through rates in the range of $[-0.1, 1.1]$ and an average markup $\mu_j < 1$. The red line in each figure represents the fitted line obtained by regressing the estimated coefficients (Ψ_j^{Est} , ψ_j^{Est}) on the price stickiness λ_j or the average markup μ_j . The slope and the R^2 of the fitted line are reported in the bottom right corner of each figure.

Table B1: Pass-through estimates, 7-digit NAPCS wholesale products

	(1)	(2)	(3)	(4)	(5)	(6)
Common cost	0.79*** (0.038)	0.86*** (0.063)	0.86*** (0.063)	0.86*** (0.063)	0.89*** (0.044)	0.93*** (0.032)
Idio. cost	0.69*** (0.028)	0.70*** (0.065)	0.70*** (0.065)	0.70*** (0.065)	0.75*** (0.042)	0.84*** (0.037)
Common cost $\times$ Sector stickiness		-0.32* (0.149)	-0.30* (0.151)	-0.26 (0.146)	-0.23 (0.167)	-0.21 (0.143)
Idio. cost $\times$ Sector stickiness		-0.02 (0.137)	0.06 (0.14)	0.04 (0.142)	0.04 (0.102)	0.01 (0.092)
Common cost $\times$ Firm stickiness			-0.03 (0.134)			
Idio. cost $\times$ Firm stickiness			-0.23** (0.074)			
Common cost $\times$ Firm-product stickiness				-0.14 (0.131)		
Idio. cost $\times$ Firm-product stickiness				-0.22** (0.074)		
Common cost $\times$ High-markup industry					-0.22 (0.145)	-0.21 (0.126)
Idio. cost $\times$ High-markup industry					-0.23** (0.085)	-0.22* (0.087)
Common cost $\times$ High-markup firm						-0.18* (0.08)
Idio. cost $\times$ High-markup firm						-0.28*** (0.034)
Observations	133,620	133,620	133,620	133,620	133,620	133,620
Firm-product fixed effects	✓	✓	✓	✓	✓	✓
R^2	0.54	0.54	0.54	0.54	0.55	0.57

Notes: This table presents estimates for pass-through of common shocks and idiosyncratic shocks, interacted with indicators of sector/firm/firm-product stickiness and high sector/firm markups. The dependent variable is the firm-product selling price. Estimates are based on monthly price data, are weighted using sampling revenue weights, and are conditional on selling price adjustment (cases where the selling price is unchanged between periods are excluded). Common costs are identified via a first-stage regression of the firm-product purchase price on a product-time fixed effect, where product is defined as the firm-product's NAPCS7 product code. Idiosyncratic shocks are defined as the residual of this first-stage regression. Statistical significance, based on robust standard errors clustered at the firm level is reported at the 1, 5, or 10 percent level, which is indicated by ***, **, or *, respectively.

B.2 Estimation results for firm- and firm-product-level shocks

We decomposed suppliers' price variation into *three* shock components (instead of two components in (16)): sectoral (common), firm and firm-product specific components. We then re-estimated equation (17) with interactions for all three components. Tables B2 and B3 provide the estimates for NAICS4 and NAPCS7 classifications. We find very similar responses to firm and firm-product specific shocks. Also the estimates for sectoral and firm-product shocks remain very similar to those in Table 2 in the main text.

Table B2: Three-shock pass-through estimates, 4-digit NAICS industries

	(1)	(2)	(3)	(4)	(5)	(6)
Common cost shock	0.82*** (0.09)	1.01*** (0.11)	1.00*** (0.11)	1.00*** (0.11)	1.09*** (0.11)	1.05*** (0.05)
Firm cost shock	0.66*** (0.04)	0.72*** (0.09)	0.72*** (0.09)	0.72*** (0.09)	0.74*** (0.08)	0.88*** (0.04)
Firm-product cost shock	0.65*** (0.03)	0.72*** (0.06)	0.72*** (0.06)	0.72*** (0.06)	0.76*** (0.04)	0.89*** (0.04)
Sector stickiness $\times$ common cost shock		-1.16*** (0.32)	-1.04*** (0.31)	-1.02*** (0.31)	-0.97*** (0.34)	-0.69*** (0.25)
Sector stickiness $\times$ firm cost shock		-0.18 (0.19)	-0.11 (0.19)	-0.11 (0.19)	0.04 (0.17)	-0.03 (0.10)
Sector stickiness $\times$ firm-product cost shock		-0.19 (0.14)	-0.15 (0.16)	-0.15 (0.15)	0.01 (0.12)	-0.05 (0.12)
Firm stickiness $\times$ common cost shock			-0.16 (0.29)			
Firm stickiness $\times$ firm cost shock			-0.23** (0.09)			
Firm stickiness $\times$ firm-product cost shock			-0.06 (0.11)			
Firm-product stickiness $\times$ common cost shock				-0.17 (0.27)		
Firm-product stickiness $\times$ firm cost shock				-0.26*** (0.09)		
Firm-product stickiness $\times$ firm-product cost shock				-0.09 (0.10)		
High ave margin ind. $\times$ common cost shock					-0.29*** (0.11)	-0.30*** (0.10)
High ave margin ind. $\times$ firm cost shock					-0.24*** (0.05)	-0.23*** (0.04)
High ave margin ind. $\times$ firm-product cost shock					-0.27*** (0.05)	-0.26*** (0.05)
Large firm $\times$ common cost shock						-0.05 (0.18)
Large firm $\times$ firm cost shock						-0.35*** (0.05)
Large firm $\times$ firm-product cost shock						-0.31*** (0.05)
Observations	136,085	136,085	136,085	136,085	136,085	136,085
Firm-product fixed effects	✓	✓	✓	✓	✓	✓
R^2	0.49	0.49	0.49	0.49	0.50	0.52

Notes: Standard errors clustered at the firm level are in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table B3: Three-shock pass-through estimates, 7-digit NAPCS products

	(1)	(2)	(3)	(4)	(5)	(6)
Common cost shock	0.79*** (0.04)	0.86*** (0.06)	0.86*** (0.06)	0.86*** (0.06)	0.89*** (0.04)	0.93*** (0.03)
Firm cost shock	0.65*** (0.03)	0.65*** (0.07)	0.65*** (0.07)	0.65*** (0.07)	0.70*** (0.05)	0.80*** (0.04)
Firm-product cost shock	0.71*** (0.03)	0.73*** (0.07)	0.74*** (0.07)	0.73*** (0.07)	0.79*** (0.05)	0.87*** (0.04)
Sector stickiness $\times$ common cost shock		-0.30* (0.15)	-0.28* (0.15)	-0.24 (0.15)	-0.22 (0.17)	-0.20 (0.15)
Sector stickiness $\times$ firm cost shock		0.03 (0.14)	0.10 (0.14)	0.09 (0.14)	0.09 (0.10)	0.06 (0.09)
Sector stickiness $\times$ firm-product cost shock		-0.05 (0.16)	0.02 (0.17)	-0.00 (0.17)	0.00 (0.12)	-0.02 (0.12)
Firm stickiness $\times$ common cost shock			-0.04 (0.14)			
Firm stickiness $\times$ firm cost shock			-0.25*** (0.08)			
Firm stickiness $\times$ firm-product cost shock			-0.22** (0.09)			
Firm-product stickiness $\times$ common cost shock				-0.14 (0.14)		
Firm-product stickiness $\times$ firm cost shock				-0.29*** (0.08)		
Firm-product stickiness $\times$ firm-product cost shock				-0.17* (0.09)		
High ave margin ind. $\times$ common cost shock					-0.21 (0.15)	-0.20 (0.13)
High ave margin ind. $\times$ firm cost shock					-0.29*** (0.06)	-0.30*** (0.05)
High ave margin ind. $\times$ firm-product cost shock					-0.21** (0.10)	-0.18* (0.11)
Large firm $\times$ common cost shock						-0.18** (0.07)
Large firm $\times$ firm cost shock						-0.29*** (0.04)
Large firm $\times$ firm-product cost shock						-0.28*** (0.04)
Observations	133,620	133,620	133,620	133,620	133,620	133,620
Firm-product fixed effects	✓	✓	✓	✓	✓	✓
R^2	0.54	0.54	0.55	0.55	0.55	0.57

Notes: Standard errors clustered at the firm level are in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

B.3 Non-parametric estimation and nonlinearity

The theoretical model in Section 2 predicts the effects of changes in price stickiness and market power on pass-through level, slope, and curvature. The empirical analysis in the main text focuses on the level and the slope: the empirical specification (17) in Section 4.1 includes linear interactions of shocks with the sector-, firm- and firm-product-level measures of price stickiness and linear interactions with dummies for top quartiles of the markup distribution across sectors and across firms within each sector.

In this section, we employ non-parametric estimation to assess the degree of nonlinearity in the estimated pass-through. To this end, we extend empirical analysis using two complementary approaches: (1) visualization of the pass-through over the discretized distribution of markups and price stickiness, and (2) test for nonlinearities in the effect of stickiness and market power on the pass-through.

We partition the distributions of sector-level markups (margins) and price stickiness into 5 quintile bins in each dimension. Sectors are assigned to quintiles according to their ranking for 1) weighted mean log margin of firms in the sector, and 2) sector price stickiness (i.e., average of purchase and selling price stickiness). For firm-products in each bin, we estimate the common and idiosyncratic shock pass-through using specification (17), conditioning on observations with price changes. The regression includes firm-product fixed effects and clustered standard errors at the firm level.

Figure B2 provides the estimated pass-through coefficients as functions of sector-level margins and price stickiness. In each figure, we overlay the estimates from both the NAICS4 (4-digit NAICS sector) and NAPCS7 (7-digit product category) classifications.

We also plot predictions from the calibrated model with strategic complementarities. Theoretical idiosyncratic pass-through is given by

$$\text{IC Pass-Through} = \frac{1}{1 + \varphi},$$

where $\varphi = \left(\frac{\theta-1}{\theta}\mu - 1\right)(\theta - 1)$ captures the degree of strategic complementarity, θ is the demand elasticity, and $\mu = \exp(\log \text{ markup})$ is the level markup. Theoretical common pass-through is

$$\text{CC Pass-Through} = \frac{1}{1 + \varphi} + \frac{\varphi}{1 + \varphi} \cdot \frac{\rho - \Lambda}{1 - \beta\lambda\Lambda},$$

where ρ is the persistence of common shocks, β is the discount factor, λ is price stickiness (probability of not adjusting), and Λ is defined as

$$\Lambda = \frac{1}{2} \left(\lambda + \frac{1-b}{\beta\lambda} - \sqrt{\left(\lambda + \frac{1-b}{\beta\lambda} \right)^2 - \frac{4}{\beta}} \right),$$

with $b = \frac{\varphi}{1+\varphi}(1 - \lambda\beta)(1 - \lambda)$.

For plotting theoretical pass-through, we set $\theta = 4.8$, $\beta = 0.97^{1/12}$ (monthly discount factor), and $\rho = 0.92$. Panels (a)–(b) plot pass-throughs as a function of the markup (μ) keeping price stickiness fixed at the sample average $\lambda = 0.45$. Panels (c)–(d) plot pass-throughs as a function of stickiness (λ) holding the markup fixed at the sample average $\mu = 1.46$ (log markup of 0.38). The theoretical line is not plotted where the implied idiosyncratic pass-through exceeds 1. This occurs at low markup values where $\mu < \theta/(\theta - 1) = 1.26$.

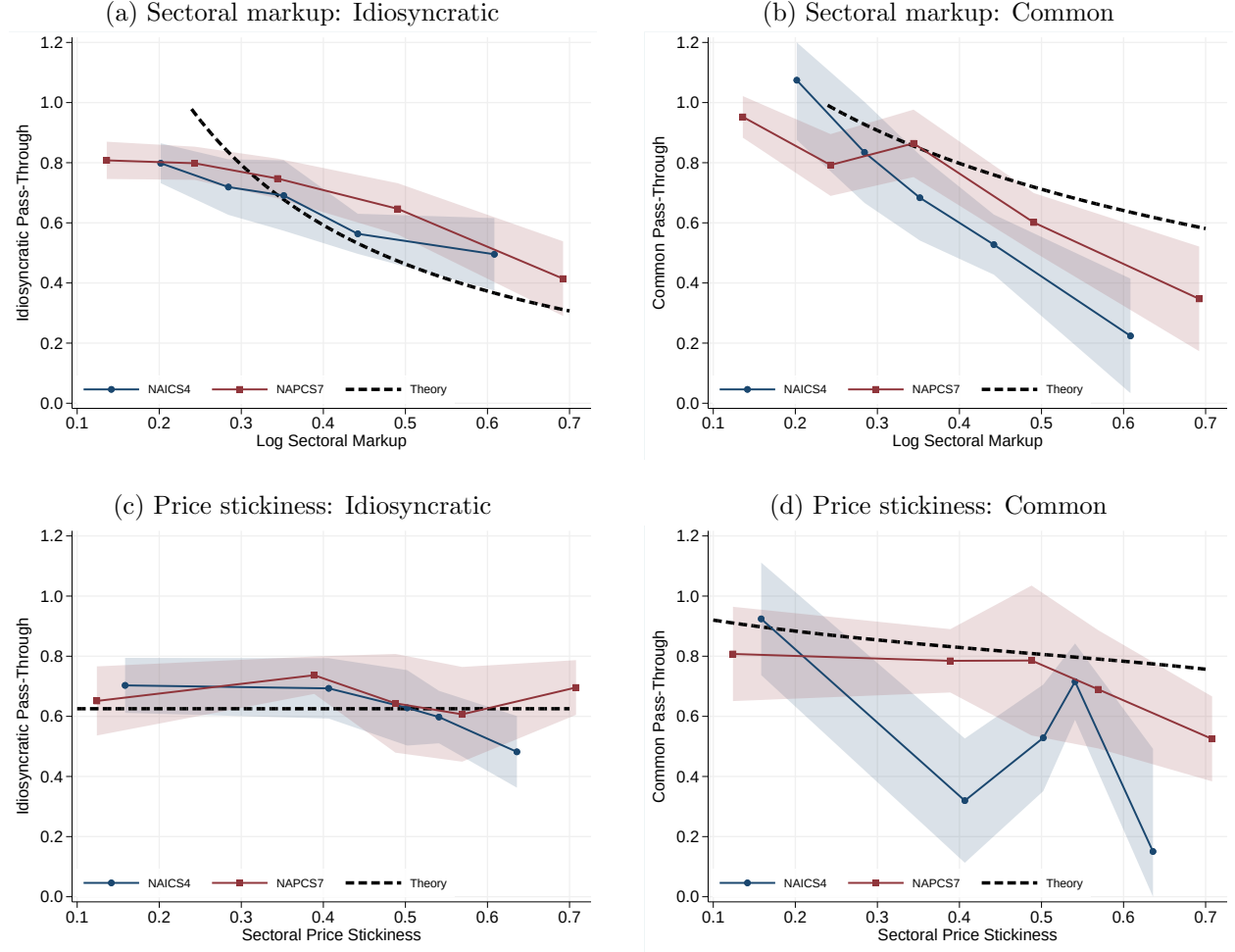


Figure B2: Non-parametric pass-through estimates by sectoral markup and price stickiness

Notes: Non-parametric pass-through estimates with 95% confidence intervals for NAICS4 (navy) and NAPCS7 (maroon). Dashed line shows theoretical predictions. X-axis shows bin means.

As reported in the main text, the empirical estimates broadly align with model predictions for pass-through level and slope: both pass-through measures decline with markups, and idiosyncratic pass-through shows little variation with stickiness. Regarding the curvature of the pass-through, the estimates do not provide sufficient evidence in favor or against the convexity.

Alternatively, we ranked markups across firm-products (regardless of the sector) instead of across sectors. Figure B3 provides non-parametric estimates of the pass-through in this case, again

finding decreasing pass-through as in Figure B2 (a)–(b).

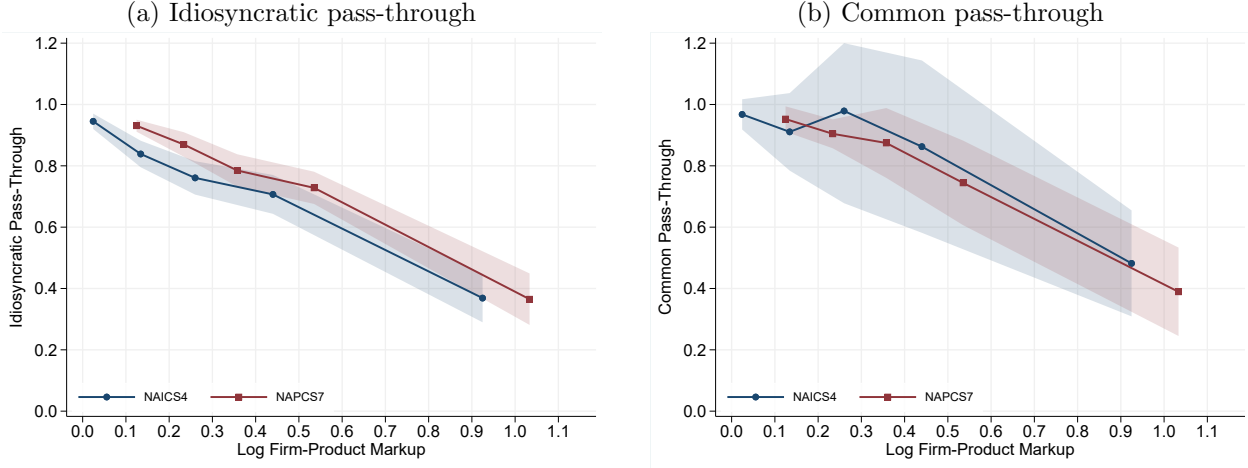


Figure B3: Non-parametric pass-through estimates by firm-product markup

Notes: Each panel shows pass-through estimates with 95% confidence intervals for NAICS4 (navy circles) and NAPCS7 (maroon squares) sector classifications. X-axis shows bin means for each quintile from firm-product markup ranking across all firms in the sample.

In a complementary approach, we directly test for nonlinearities by adding interaction terms with squared levels of sector-, firm- and firm-product price stickiness, and squared levels of sector- and firm-level margins in regressions corresponding to columns (2)–(6) in Table 2 in the main text. The results are provided in Tables B4 and B5, where shaded rows point to interaction terms with squared parameters. The estimates do not reject linearity with respect to market power at sector or firm levels. For price stickiness, some of the nonlinear terms have significant coefficients, but they are not robust across sector classifications. For example, the coefficient for Sector stickiness² interacted with common shock pass-through is 4.99** for NAICS4 but only 0.48 for NAPCS7. And the coefficients for Sector stickiness² interacted with idiosyncratic shock pass-through are significant, but at odds with theory where the pass-through is constant.

Table B4: Nonlinear pass-through estimates, 4-digit NAICS industries

	(1)	(2)	(3)	(4)	(5)
Common cost shock	1.15*** (0.13)	1.01*** (0.11)	1.00*** (0.11)	1.38*** (0.23)	1.35*** (0.14)
Idio. cost shock	0.62*** (0.12)	0.73*** (0.07)	0.73*** (0.07)	0.91*** (0.15)	1.01*** (0.06)
Sector stickiness $\times$ common cost shock	-3.95*** (1.28)	-0.96*** (0.29)	-0.96*** (0.30)	-0.34 (0.26)	-0.16 (0.20)
Sector stickiness $\times$ idio. cost shock	0.88 (0.77)	-0.09 (0.16)	-0.09 (0.15)	0.39*** (0.15)	0.34*** (0.12)
Sector stickiness ² $\times$ common cost shock	4.91** (2.06)				
Sector stickiness ² $\times$ idio. cost shock	-1.70 (1.06)				
Firm stickiness $\times$ common cost shock		-0.72 (0.71)			
Firm stickiness $\times$ idio. cost shock		-0.79*** (0.25)			
Firm stickiness ² $\times$ common cost shock		0.58 (0.98)			
Firm stickiness ² $\times$ idio. cost shock		0.88*** (0.29)			
Firm-product stickiness $\times$ common cost shock			-0.26 (0.70)		
Firm-product stickiness $\times$ idio. cost shock			-1.09*** (0.23)		
Firm-product stickiness ² $\times$ common cost shock			-0.24 (1.01)		
Firm-product stickiness ² $\times$ idio. cost shock			1.28*** (0.26)		
Industry markup $\times$ common cost shock				-2.07* (1.14)	-1.36*** (0.29)
Industry markup $\times$ idio. cost shock				-0.95 (0.89)	-0.80*** (0.16)
Industry markup ² $\times$ common cost shock				0.91 (1.36)	
Industry markup ² $\times$ idio. cost shock				-0.22 (1.05)	
Firm markup $\times$ common cost shock					-0.06*** (0.02)
Firm markup $\times$ idio. cost shock					-0.10*** (0.02)
Firm markup ² $\times$ common cost shock					-0.00 (0.00)
Firm markup ² $\times$ idio. cost shock					-0.00 (0.00)
Observations	136,085	136,085	136,085	136,085	136,085
Firm-product fixed effects	✓	✓	✓	✓	✓
R ²	0.49	0.49	0.49	0.51	0.52

Notes: Standard errors clustered at the firm level are in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table B5: Nonlinear pass-through estimates, 7-digit NAPCS products

	(1)	(2)	(3)	(4)	(5)
Common cost shock	0.87*** (0.07)	0.88*** (0.06)	0.87*** (0.06)	0.99*** (0.05)	1.02*** (0.04)
Idio. cost shock	0.66*** (0.09)	0.72*** (0.07)	0.72*** (0.06)	0.86*** (0.05)	0.91*** (0.04)
Sector stickiness $\times$ common cost shock	-0.58 (0.38)	-0.29** (0.14)	-0.26* (0.14)	-0.03 (0.20)	-0.10 (0.18)
Sector stickiness $\times$ idio. cost shock	0.32 (0.47)	0.07 (0.14)	0.04 (0.14)	0.17 (0.10)	0.11 (0.10)
Sector stickiness ² $\times$ common cost shock	0.48 (0.52)				
Sector stickiness ² $\times$ idio. cost shock	-0.49 (0.55)				
Firm stickiness $\times$ common cost shock		-0.70 (0.44)			
Firm stickiness $\times$ idio. cost shock		-0.72*** (0.26)			
Firm stickiness ² $\times$ common cost shock		0.98* (0.53)			
Firm stickiness ² $\times$ idio. cost shock		0.66** (0.31)			
Firm-product stickiness $\times$ common cost shock			-0.61* (0.36)		
Firm-product stickiness $\times$ idio. cost shock			-0.98*** (0.26)		
Firm-product stickiness ² $\times$ common cost shock			0.64 (0.52)		
Firm-product stickiness ² $\times$ idio. cost shock			1.07*** (0.31)		
Industry markup $\times$ common cost shock				-0.56 (0.36)	-0.35 (0.27)
Industry markup $\times$ idio. cost shock				-0.49* (0.29)	-0.33* (0.18)
Industry markup ² $\times$ common cost shock				-0.01 (0.32)	
Industry markup ² $\times$ idio. cost shock				-0.12 (0.18)	
Firm markup $\times$ common cost shock					-0.05** (0.02)
Firm markup $\times$ idio. cost shock					-0.07*** (0.02)
Firm markup ² $\times$ common cost shock					-0.00 (0.00)
Firm markup ² $\times$ idio. cost shock					0.00 (0.00)
Observations	133,620	133,620	133,620	133,620	133,620
Firm-product fixed effects	✓	✓	✓	✓	✓
R ²	0.54	0.55	0.55	0.56	0.57

Notes: Standard errors clustered at the firm level are in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

B.4 Shock persistence and the contemporaneous pass-through

This section examines whether contemporaneous pass-through varies with the persistence of the underlying cost shock. Proposition 2 and Figure 6 in the main text show that, under an AR(1) cost process, the common-cost pass-through should increase with shock persistence, whereas the idiosyncratic-cost pass-through should not depend on persistence. We explore these predictions empirically.

To proxy for persistence in the data, we estimate a sector-level AR(1) coefficient ρ_j for each 4-digit NAICS sector by fitting an AR(1) process to the observed series for purchase prices, conditional on price changes (i.e., using only non-zero adjustments). As discussed in Section C.1, to the degree that the AR(1) cost shock process in the model is a good approximation to the cost dynamics in the data, the resulting $\hat{\rho}_j$ is an approximation of shock persistence. We classify sectors into quartiles of $\hat{\rho}_j$ and define a high-persistence dummy equal to one for sectors in the top quartile, which we then interact with the shock variables in an augmented version of our baseline specification.

Column (1) of Table B6 suggests that sectors with higher shock persistence tend to exhibit somewhat larger contemporaneous pass-through of common-cost shocks, which is qualitatively consistent with the model. However, the evidence is not uniformly strong across alternative specifications reported in Table B6, and several interacted terms are imprecisely estimated or are sensitive to the controls included. For idiosyncratic shocks, the interaction terms are generally small and often not statistically significant, in line with theoretical prediction that their pass-through does not depend on persistence. Overall, the empirical patterns provide tentative evidence for the model’s persistence-based comparative statics.

Table B6: Persistence in pass-through estimates, 4-digit NAICS industries

	(1)	(2)	(3)	(4)	(5)	(6)
Common cost shock	0.66*** (0.04)	0.86*** (0.08)	0.86*** (0.08)	0.86*** (0.08)	0.80*** (0.09)	0.96*** (0.09)
Idio. cost shock	0.69*** (0.03)	0.88*** (0.04)	0.88*** (0.04)	0.88*** (0.04)	0.82*** (0.04)	0.96*** (0.05)
Common cost shock $\times \rho$ dummy	0.28** (0.13)	0.22 (0.14)	0.24* (0.14)	0.24* (0.14)	0.43** (0.18)	0.14 (0.10)
Idio. cost shock $\times \rho$ dummy	-0.16 (0.10)	-0.30** (0.13)	-0.30** (0.13)	-0.30** (0.13)	-0.07 (0.15)	-0.08 (0.09)
Sector stickiness $\times$ common cost shock		-0.56*** (0.21)	-0.47** (0.21)	-0.49** (0.21)	-0.23 (0.32)	-0.35 (0.29)
Sector stickiness $\times$ idio. cost shock		-0.49*** (0.11)	-0.45*** (0.12)	-0.45*** (0.12)	-0.17 (0.12)	-0.28** (0.13)
Sector stick. $\times$ common cost $\times \rho$ dummy		-1.22*** (0.44)	-2.36*** (0.60)	-1.98*** (0.56)	-1.58*** (0.56)	-0.78** (0.39)
Sector stick. $\times$ idio. cost $\times \rho$ dummy		0.21 (0.32)	0.08 (0.35)	0.16 (0.34)	-0.12 (0.26)	0.03 (0.19)
Firm stickiness $\times$ common cost shock			-0.19 (0.20)			
Firm stickiness $\times$ idio. cost shock			-0.13* (0.08)	-0.12 (0.08)		
Firm stick. $\times$ common cost $\times \rho$ dummy			1.40** (0.63)			
Firm stick. $\times$ idio. cost $\times \rho$ dummy			0.22 (0.22)			
Firm-product stickiness $\times$ common cost shock				-0.14 (0.20)		
Firm-prod. stick. $\times$ common cost $\times \rho$ dummy				0.90* (0.53)		
Firm-prod. stick. $\times$ idio. cost $\times \rho$ dummy				0.12 (0.20)		
High ave margin industry $\times$ common cost shock					-0.11 (0.13)	-0.14 (0.12)
High ave margin industry $\times$ idio. cost shock					-0.17*** (0.06)	-0.17*** (0.05)
High ave margin ind. $\times$ common cost $\times \rho$ dummy					-0.40** (0.20)	-0.32* (0.17)
High ave margin ind. $\times$ idio. cost $\times \rho$ dummy					-0.15 (0.15)	-0.08 (0.10)
Large firm (margin based) $\times$ common cost shock						-0.31** (0.15)
Large firm (margin based) $\times$ idio. cost shock						-0.28*** (0.04)
Large firm $\times$ common cost $\times \rho$ dummy						0.41* (0.23)
Large firm $\times$ idio. cost $\times \rho$ dummy						-0.18* (0.10)
Observations	136,085	136,085	136,085	136,085	136,085	136,085
Firm-product fixed effects	✓	✓	✓	✓	✓	✓
R^2	0.49	0.50	0.50	0.50	0.51	0.53

Notes: Standard errors clustered at the firm level are in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively. The ρ dummy equals one if the persistence parameter is in the top quartile (highest persistence).

B.5 Results using orthogonalized shocks

Tables B7 and B8 present estimated pass-through coefficients for our baseline specification (17), but using orthogonalized shocks constructed from specification (19) rather than shocks constructed using (16) the main text. In sum, the contemporaneous pass-through coefficients are very similar to those in the baseline tables (i.e., Tables 2 and B1).

Table B7: Pass-through estimates (orthogonalized shocks), 4-digit NAICS industries

	(1)	(2)	(3)	(4)	(5)	(6)
Common cost shock	0.71*** (0.06)	0.98*** (0.09)	0.97*** (0.09)	0.97*** (0.09)	1.01*** (0.09)	1.06*** (0.06)
Idio. cost shock	0.68*** (0.03)	0.73*** (0.07)	0.73*** (0.07)	0.73*** (0.07)	0.76*** (0.06)	0.88*** (0.04)
Sector stickiness $\times$ common cost shock		-0.88*** (0.21)	-0.72*** (0.21)	-0.71*** (0.21)	-0.82*** (0.23)	-0.63*** (0.17)
Sector stickiness $\times$ idio. cost shock		-0.17 (0.14)	-0.11 (0.15)	-0.11 (0.15)	0.04 (0.13)	-0.01 (0.10)
Firm stickiness $\times$ common cost shock			-0.26* (0.14)			
Firm stickiness $\times$ idio. cost shock			-0.16** (0.08)			
Firm-product stickiness $\times$ common cost shock				-0.26** (0.13)		
Firm-product stickiness $\times$ idio. cost shock				-0.18** (0.07)		
High ave margin industry $\times$ common cost shock					-0.11 (0.09)	-0.14** (0.06)
High ave margin industry $\times$ idio. cost shock					-0.25*** (0.04)	-0.24*** (0.04)
Large firm (margin based) $\times$ common cost shock						-0.31*** (0.12)
Large firm (margin based) $\times$ idio. cost shock						-0.33*** (0.04)
Observations	134,960	134,960	134,960	134,960	134,960	134,960
Firm-product fixed effects	✓	✓	✓	✓	✓	✓
R^2	0.50	0.51	0.51	0.51	0.52	0.54

Notes: Standard errors clustered at the firm level are in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table B8: Pass-through estimates (orthogonalized shocks), 7-digit NAPCS products

	(1)	(2)	(3)	(4)	(5)	(6)
Common cost shock	0.77*** (0.03)	0.88*** (0.06)	0.88*** (0.06)	0.88*** (0.06)	0.91*** (0.04)	0.96*** (0.03)
Idio. cost shock	0.71*** (0.03)	0.72*** (0.06)	0.72*** (0.06)	0.72*** (0.06)	0.76*** (0.04)	0.85*** (0.03)
Sector stickiness $\times$ common cost shock		-0.36*** (0.12)	-0.32*** (0.12)	-0.30** (0.12)	-0.26** (0.11)	-0.23** (0.10)
Sector stickiness $\times$ idio. cost shock		-0.04 (0.13)	0.04 (0.13)	0.02 (0.14)	0.06 (0.10)	0.03 (0.09)
Firm stickiness $\times$ common cost shock			-0.10 (0.09)			
Firm stickiness $\times$ idio. cost shock			-0.24*** (0.08)			
Firm-product stickiness $\times$ common cost shock				-0.16* (0.09)		
Firm-product stickiness $\times$ idio. cost shock				-0.21*** (0.07)		
High ave margin industry $\times$ common cost shock					-0.23** (0.10)	-0.22** (0.09)
High ave margin industry $\times$ idio. cost shock					-0.23*** (0.08)	-0.22** (0.09)
Large firm (margin based) $\times$ common cost shock						-0.23*** (0.04)
Large firm (margin based) $\times$ idio. cost shock						-0.29*** (0.04)
Observations	131,650	131,650	131,650	131,650	131,650	131,650
Firm-product fixed effects	✓	✓	✓	✓	✓	✓
R^2	0.55	0.56	0.56	0.56	0.57	0.58

Notes: Standard errors clustered at the firm level are in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

C Supplementary Estimates of the Dynamic Pass-through

This section provides supplementary estimates of the dynamic pass-through of cost shocks to prices. Section C.1 estimates the persistence of the cost (purchase price) process. We compare these responses to those for an AR(1) cost process (constant rate of decay), and for a process with decreasing rate of decay. Section C.2 compares the empirical estimates of cost pass-through into prices with their theoretical counterparts. Section C.3 estimates the dynamic responses of the frequency of price adjustment. We find some evidence of state-dependent adjustment, but argue that the Calvo model remains a reasonable approximation for aggregate dynamics. Section C.4 reports estimates using identified monetary policy shocks. Section C.5 validates our estimation strategy using simulated data.

C.1 Persistence of cost shocks

This subsection estimates the cost shock process directly from the data and evaluates how well it is approximated by the AR(1) process assumed in the main analysis.

Estimation. We estimate the dynamic response of purchase prices to identified cost shocks using local projections. In the first step, we estimate orthogonalized cost shocks η_{ijt} as residuals in regression

$$\Delta \ln(Q_{ijt}) = \delta \ln(Q_{ijt-1}) + \eta_{ijt}. \quad (\text{C.1})$$

In the second step, we estimate local projections using the estimated orthogonalized shocks $\hat{\eta}_{ijt}$, conditioning on purchase price adjustment at horizon τ (i.e., $I_{ijt+\tau} = 1$):

$$\Delta_\tau \ln(Q_{ijt+\tau}) = \beta_\tau \hat{\eta}_{ijt} + FE_{ij} + \nu_{ijt+\tau}, \quad (\text{C.2})$$

where $\Delta_\tau \ln(Q_{ijt+\tau}) \equiv \ln(Q_{ijt+\tau}) - \ln(Q_{ijt-1})$ denotes the purchase price change from $t-1$ to $t+\tau$, FE_{ij} are firm-product fixed effects, and $\nu_{ijt+\tau}$ is the residual. The coefficient β_τ captures the impact of a cost innovation at t on the purchase price at $t+\tau$. We estimate the AR(1) persistence parameter ρ by minimizing the sum of squared differences between $\hat{\beta}_\tau$ and ρ^τ over the first 12 months after the impulse.

Estimated shock process. Estimates of equation (C.2) at different horizons, based on our data, are presented in Figure C1. For comparison, we show the corresponding estimates obtained from an AR(1) process, with the persistence calibrated to give the best match to the estimated cost shock coefficients. The figure shows that the estimated cost dynamics are highly persistent, e.g., fitting an AR(1) yields average persistence of 0.931 (NAICS4, panel a) and 0.924 (NAPCS7, panel b). Furthermore, the fitted persistence does not vary when we split samples by high/low markups or high/low price stickiness, suggesting that the differences for estimated pass-through to selling prices do not stem from the differences in cost processes.

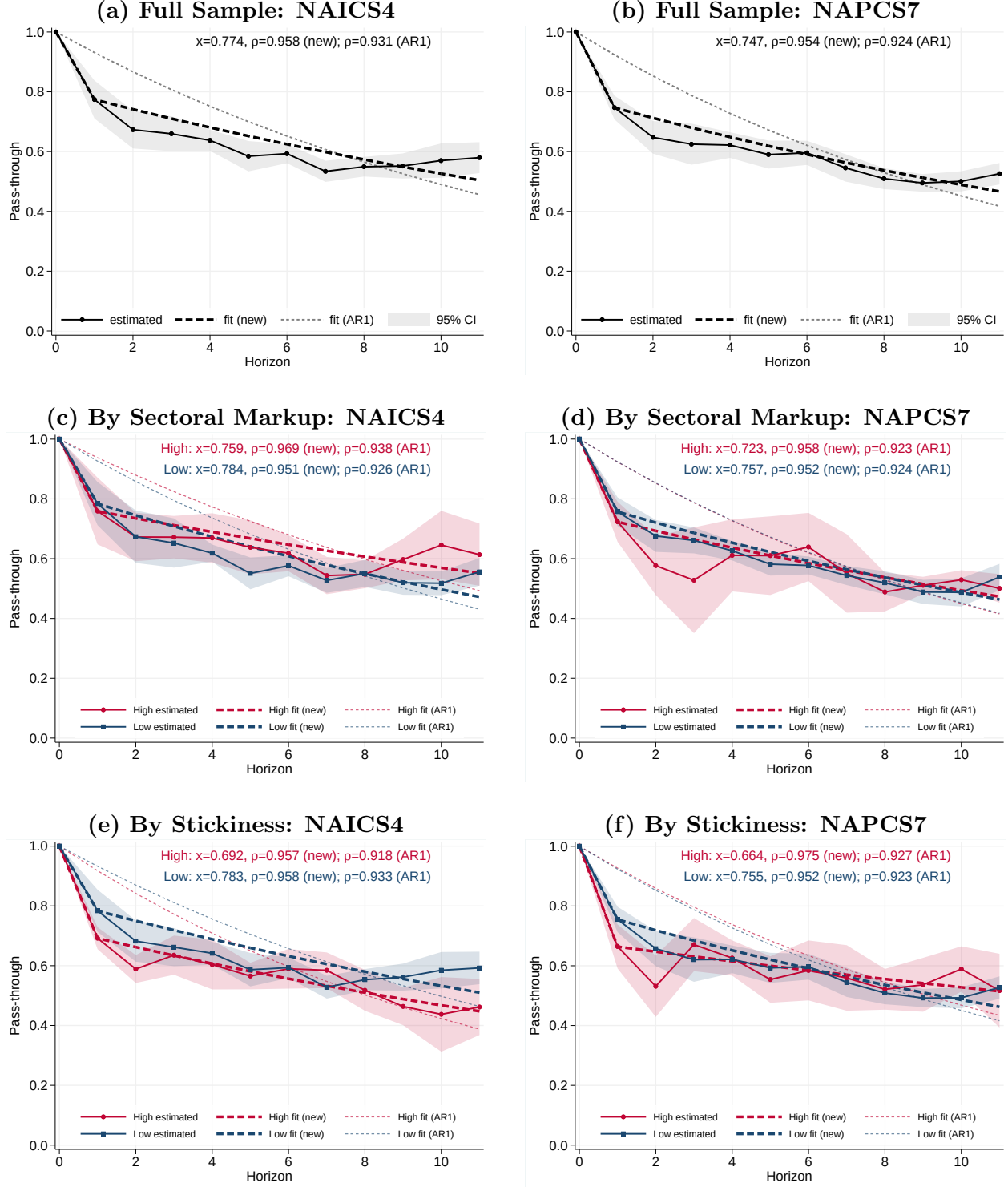


Figure C1: Underlying cost shock process

Notes: This figure shows the estimated underlying cost shock process across different sample splits. Panels (a)–(b) present estimates for the full sample. Panels (c)–(d) split the sample by sectoral average markup. Panels (e)–(f) split by sectoral average price stickiness. In each panel, the solid line with confidence bands shows the empirical local projection estimates, while dashed lines show fitted theoretical paths specified in (C.3). The AR(1) case imposes $x = \rho$, while the generalized case (marked with “new”) allows separate estimation of the initial drop parameter x and the subsequent decay rate ρ . All parameters (x, ρ) are estimated separately for each subsample.

A notable feature of the estimated cost process is that it decays faster than the AR(1) process, which decays at a constant rate. For example, $\hat{\beta}_\tau$ (solid line, panel a) drops sharply from 1.0 at horizon 0 to approximately 0.77 at horizon 1, and then decays slower at longer horizons. Panels (c)–(f) show that this pattern is present when we estimate cost responses for sub-samples split by sectoral markup and price stickiness. (Sectors are classified as “high markup” if the sales-weighted average log margin is in the top quartile across sectors. Sectors are classified as “high stickiness” if the average of purchase and selling price stickiness is in the top quartile across sectors.)

Relaxing the AR(1) assumption. To achieve a better match with the empirical cost process in our data, we generalize the AR(1) assumption by allowing the effect of a cost shock ϵ_{ijt} on costs at horizon τ to follow a more flexible path $\tilde{q}(\tau; x, \rho)$ rather than ρ^τ . Specifically, if the original cost shock at t is ϵ_{ijt} , its contribution to the log cost at $t + \tau$ is $\tilde{q}(\tau; x, \rho) \cdot \epsilon_{ijt}$, where:

$$\tilde{q}(\tau; x, \rho) \equiv \begin{cases} 1, & \tau = 0, \\ x, & \tau = 1, \\ x \rho^{\tau-1}, & \tau \geq 2, \end{cases} \quad (\text{C.3})$$

where $x \in (0, 1]$ governs the immediate pass-through in period 1 and $\rho \in (0, 1]$ governs the subsequent AR(1) decay. When $x = \rho$, this specification reduces to the standard AR(1) with $\tilde{q}(\tau; \rho) = \rho^\tau$. To fit this shock process to the empirical pass-through in Figure C1, we separately estimate the initial drop parameter x (between $\tau = 0$ and $\tau = 1$) and the subsequent decay rate ρ (for $\tau \geq 2$) by fitting the piecewise function to the estimated coefficients $\hat{\beta}_\tau$. The thick dashed lines (marked with “new”) in Figure C1 show that this modified process substantially improves the match with the data.

What might explain the sharp initial drop ($x < \rho$) observed in the data? We conjecture that such a response may reflect within-sector heterogeneity in the degree of persistence. For example, a mixture of two AR(1) processes (transient and persistent) achieves a better fit with the empirical responses than a single AR(1) process. Another interpretation is that the estimated response profile may reflect behaviors not captured in the model (e.g., inventory adjustments by wholesalers).

C.2 Dynamic pass-through for the alternative shock process

Section C.1 documents that the empirical cost shock process departs from the standard AR(1) assumption, exhibiting a faster initial decay between impact and period one ($x < \rho$). This section incorporates the alternative cost shock process described in (C.3) into the model and constructs the corresponding theoretical pass-through using the estimated data moments. We show that allowing for this alternative cost shock process notably improves the match between the pass-through rates estimated from the data and those predicted by the model.

We construct two sets of theoretical predictions. “Theory (new)” uses the generalized shock path $\tilde{q}(\tau; x, \rho)$ with parameters (x, ρ) estimated in Section C.1 by fitting the piecewise function (C.3)

to the local projection coefficients $\widehat{\beta}_\tau$. “Theory (AR1)” imposes the standard restriction $x = \rho$. Comparing these predictions against the empirical estimates isolates the improvement from relaxing the AR(1) assumption. The pass-through formulas under the generalized shock process are derived in Appendix D.5; setting $x = \rho$ recovers the baseline AR(1) formulas in Appendix D.4.

Constructing theoretical predictions. We combine the shock process parameters (x, ρ) estimated in Section C.1 with structural parameters (λ, φ) . The Calvo stickiness λ is the average non-adjustment probability within each subsample. The strategic complementarity φ is inferred from the contemporaneous idiosyncratic pass-through: $\varphi^{\text{Est}} = 1/\psi_0^{\text{Est}} - 1$ (see the expression of $\psi_\tau^{\text{theory}}$ below; note that $\tilde{q}(0; x, \rho) = 1$). We set the monthly discount factor to $\beta = 0.995$.

Define the following auxiliary functions (see Appendix D.5 for details):

$$\begin{aligned} b(\lambda, \varphi) &= \frac{\varphi(1 - \beta\lambda)(1 - \lambda)}{1 + \varphi}, & a(\lambda, \varphi) &= \frac{(1 - \beta\lambda)(1 - \lambda)}{1 + \varphi}, \\ \Lambda(\lambda, \varphi) &= \frac{1}{2} \left[\lambda + \frac{1 - b(\lambda, \varphi)}{\beta\lambda} - \sqrt{\left(\lambda + \frac{1 - b(\lambda, \varphi)}{\beta\lambda} \right)^2 - \frac{4}{\beta}} \right], \\ D(\lambda, \varphi, \rho) &= \rho[1 - b(\lambda, \varphi)] + \lambda[\beta\rho(\lambda - \rho) - 1], & E(\lambda, \varphi, \rho) &= (1 - b) + \beta\lambda(\lambda - \rho), \\ \tilde{H}(\tau; x, \rho, \lambda, \varphi) &= \varkappa \left[x\rho^\tau + \tilde{\Xi}(x) \frac{\Lambda(\lambda, \varphi)^{\tau+1}}{1 - \beta\lambda\Lambda(\lambda, \varphi)} \right], \end{aligned}$$

where $\varkappa = a/D$ and $\tilde{\Xi}(x) = \Xi(x) + \beta\lambda x = (D - xE + \beta\lambda^2 x)/\lambda$ is the synchronized feedback coefficient, with $\Xi(x) = (D - xE)/\lambda$. With estimated parameters $(x^{\text{Est}}, \rho^{\text{Est}}, \lambda^{\text{Est}}, \varphi^{\text{Est}})$, we evaluate theoretical kernels using the synchronized formulas (D.44) and (D.47):

$$\psi_\tau^{\text{theory}} = \frac{1}{1 + \varphi^{\text{Est}}} \tilde{q}(\tau; x^{\text{Est}}, \rho^{\text{Est}}), \quad \Psi_\tau^{\text{theory}} = \psi_\tau^{\text{theory}} + \frac{\varphi^{\text{Est}}}{1 + \varphi^{\text{Est}}} \tilde{H}(\tau; x^{\text{Est}}, \rho^{\text{Est}}, \lambda^{\text{Est}}, \varphi^{\text{Est}}).$$

For “Theory (AR1),” we impose $x = \rho$ using $\rho_{\text{AR1}}^{\text{Est}}$ from Section C.1. All parameters $(x, \rho, \lambda, \varphi)$ are estimated separately for each subsample, and the theoretical predictions are constructed accordingly.

Results. Figures C2 and C3 compare the empirical estimates (solid line with circular markers) with theoretical predictions: “Theory (new)” (thick dashed line) and “Theory (AR1)” (gray short-dashed line). The generalized model captures the sharp initial decline from $\tau = 0$ to $\tau = 1$ that the AR(1) model misses, improving the fit at intermediate horizons. Panels (c)–(f) confirm that the model captures cross-sectional differences by markup and stickiness.

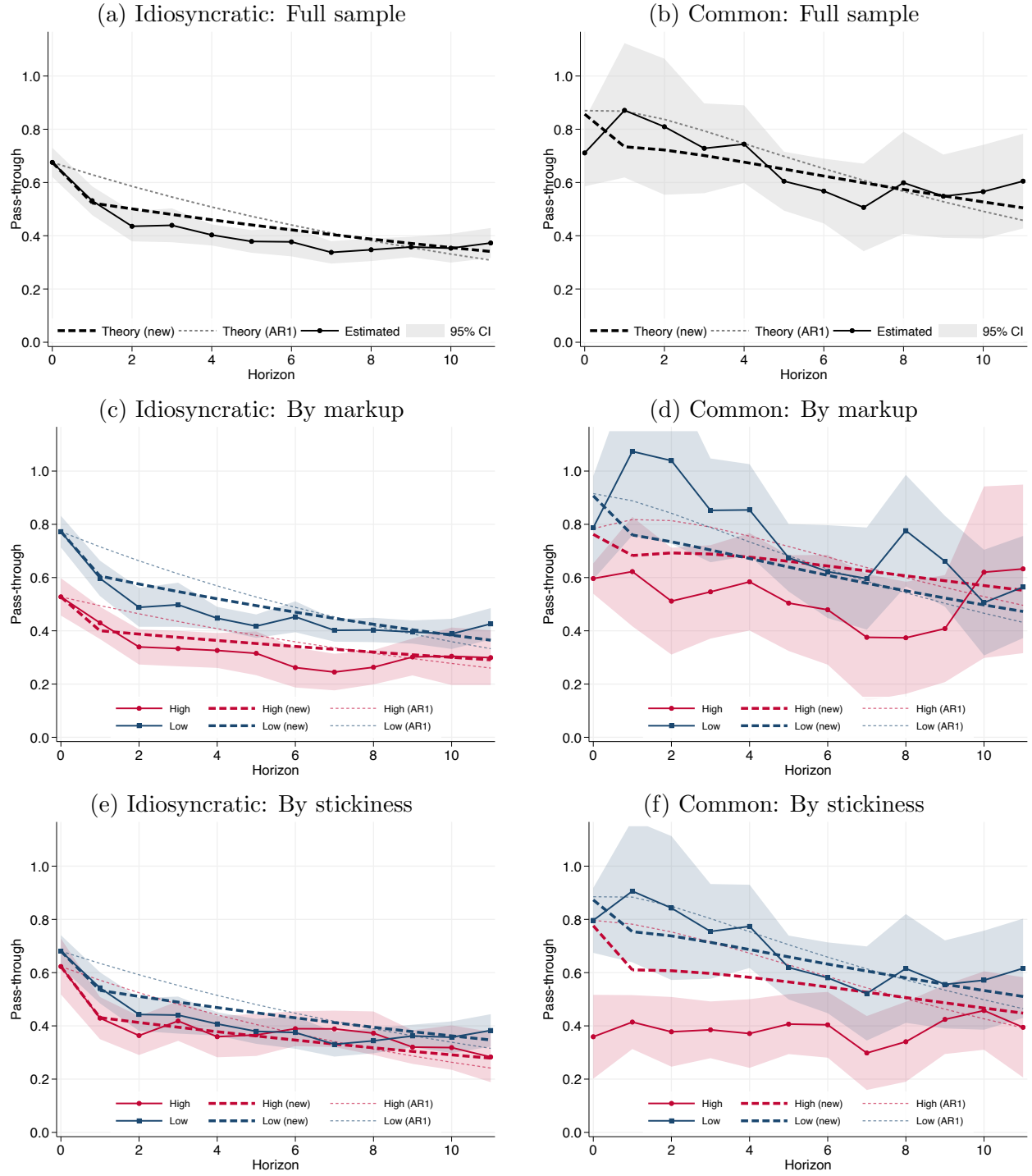


Figure C2: Dynamic pass-through: Theory comparison (NAICS4)

Notes: Comparison of estimated dynamic pass-through with theoretical predictions. Thick dashed line: Theory (new); gray short-dashed line: Theory (AR1); solid line with markers: estimates; shaded areas: 95% CI (firm-clustered SE). Left column: idiosyncratic kernel ψ_τ ; right column: common kernel Ψ_τ . All parameters ($x, \rho, \lambda, \varphi$) are estimated separately for each subsample. 4-digit NAICS classification.

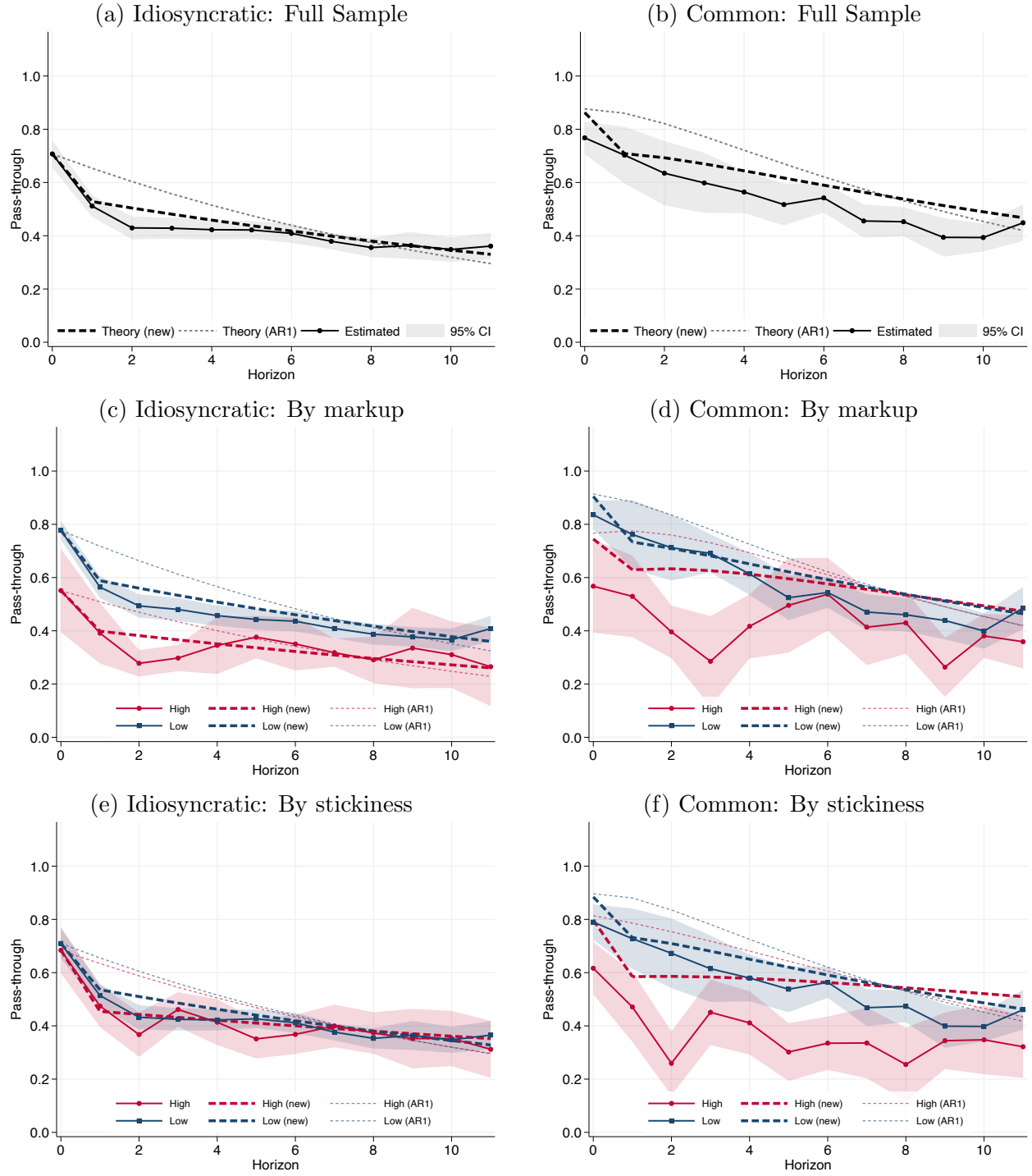


Figure C3: Dynamic pass-through: Theory comparison (NAPCS7)

Notes: Comparison of estimated dynamic pass-through with theoretical predictions. Thick dashed line: Theory (new); gray short-dashed line: Theory (AR1); solid line with markers: estimates; shaded areas: 95% CI (firm-clustered SE). Left column: idiosyncratic kernel ψ_τ ; right column: common kernel Ψ_τ . All parameters ($x, \rho, \lambda, \varphi$) are estimated separately for each subsample. 7-digit NAPCS classification.

C.3 Extensive margin adjustments

The baseline analysis assumes Calvo pricing, where firms adjust prices with an exogenous probability independent of shocks. This section tests this assumption by examining whether the frequency of price adjustment responds to cost shocks.

We estimate the extensive margin response by replacing the dependent variable in the local projection specification (20) with a dummy variable indicating whether there is a selling price change from $t - 1$ to $t + \tau$:

$$\Delta_\tau I_{ijt+\tau} = \underbrace{\gamma_\tau^C}_{\text{common PT}} \cdot \hat{\eta}_{jt} + \underbrace{\gamma_\tau^I}_{\text{idiosyncratic PT}} \cdot \hat{\eta}_{ijt} + FE_{ij} + \nu_{ijt+\tau}, \quad (\text{C.4})$$

where $\Delta_\tau I_{ijt+\tau} = 1$ if $\ln(P_{ijt+\tau}) \neq \ln(P_{ijt-1})$ and $= 0$ otherwise. Unlike specification (20), which conditions on price adjustment at $t + \tau$, here we use the full sample using all price observations. Under the Calvo model, the coefficients γ_τ^I and γ_τ^C are zero since the probability of price adjustment is exogenous. Deviations from zero indicate state-dependent pricing behavior.

Figure C4 reports the estimated extensive margin responses. In both the NAICS4 and NAPCS7 samples, the probability of price adjustment increases in response to both common and idiosyncratic cost shocks, with larger responses to common shocks. These results reject the Calvo assumption of exogenous adjustment probabilities, providing evidence of state-dependent pricing. However, the quantitative importance of these extensive margin responses is relatively small. For example, in NAICS4 sectors, a one-standard-deviation increase in the common cost shock ($\sigma_{\hat{\eta}_{jt}} = 0.005$) raises the probability of price adjustment on impact by only $\sigma_{\hat{\eta}_{jt}} \times \hat{\gamma}_0^C \times 100 = 0.005 \times 0.12 \times 100 = 0.06$ percentage points, where $\hat{\gamma}_0^C \approx 0.12$ is the estimated coefficient from Figure C4(b). Similarly, a one-standard-deviation increase in the idiosyncratic cost shock ($\sigma_{\hat{\eta}_{ijt}} = 0.09$) raises the probability of adjustment by $\sigma_{\hat{\eta}_{ijt}} \times \hat{\gamma}_0^I \times 100 = 0.09 \times 0.02 \times 100 = 0.18$ percentage points, where $\hat{\gamma}_0^I \approx 0.02$ is the estimated coefficient from Figure C4(a).

Quantifying contribution of the extensive margin on price dynamics. To further assess the quantitative importance of the state-dependent extensive margin adjustments on sectoral price dynamics, we compare the estimated pass-through rates from two approaches:

- **“Estimated” (direct approach):** We estimate the average price responses, $\gamma_\tau^{I,\text{Est}}$ and $\gamma_\tau^{C,\text{Est}}$, via local projections conditioning on an adjustment at time t while leaving the adjustment status at $t + \tau$ unrestricted (i.e., we do *not* condition on adjustment at $t + \tau$):

$$\Delta_\tau \ln(P_{ijt+\tau}) = \underbrace{\gamma_\tau^C}_{\text{common PT}} \cdot \hat{\eta}_{jt} + \underbrace{\gamma_\tau^I}_{\text{idiosyncratic PT}} \cdot \hat{\eta}_{ijt} + FE_{ij} + \nu_{ijt+\tau},$$

where $\hat{\eta}_{ijt}$ is the idiosyncratic cost shock and $\hat{\eta}_{jt}$ is the common cost shock. This approach captures the combined effect of the extensive margin (which prices adjust) and the intensive margin (by how much prices adjust).

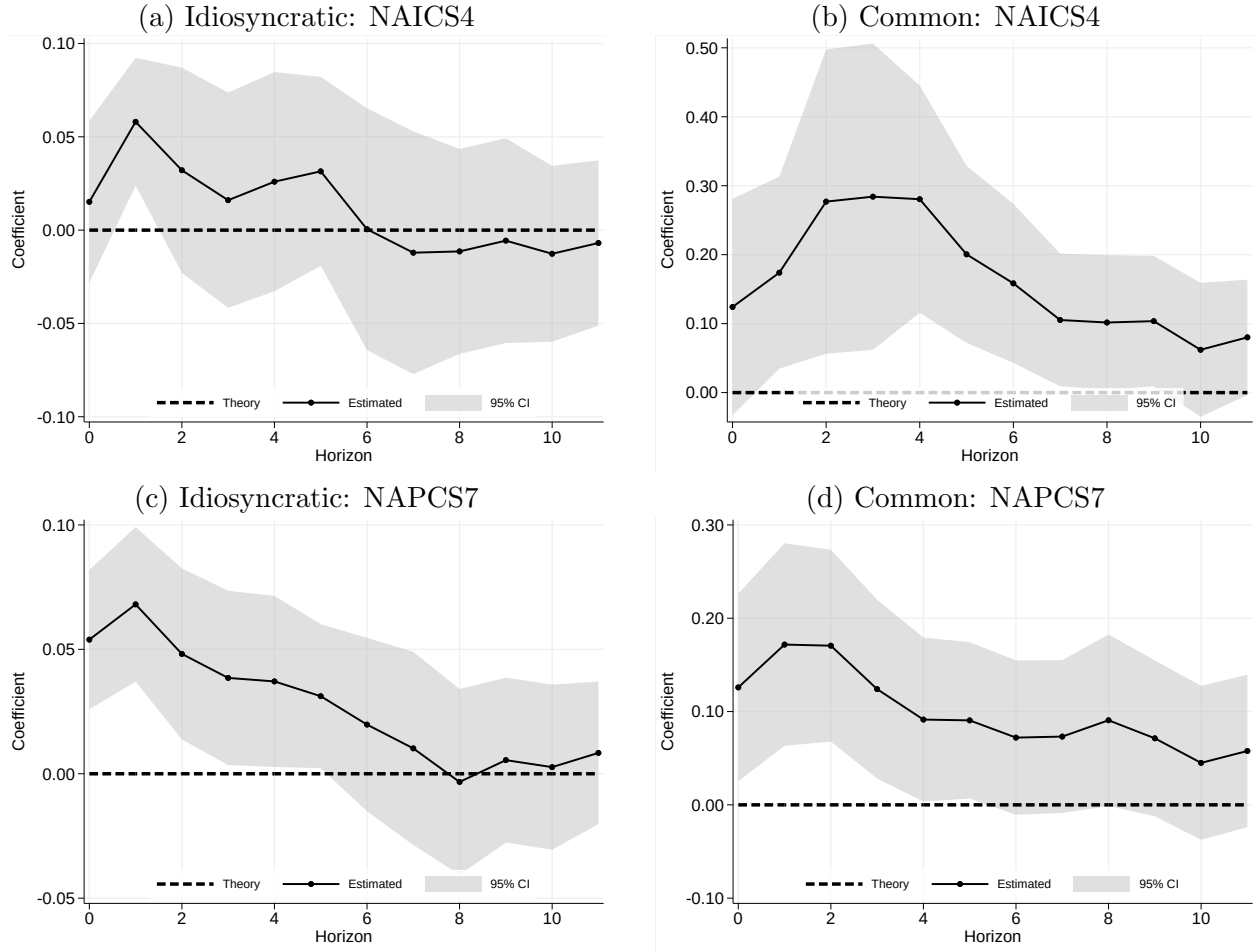


Figure C4: Responses of the cumulative probability of selling price changes.

Notes: Responses are estimated via local projections based on equation (C.4) for cumulative probability of selling price changes. Left column: idiosyncratic shock response; right column: common shock response. Dashed line: theory (zero under Calvo); Solid line with markers: estimates; shaded areas: 95% CI (firm-clustered SE).

- **“Theory (Calvo)” (indirect approach):** We construct counterfactual coefficients $\tilde{\gamma}_\tau^C$ and $\tilde{\gamma}_\tau^I$ that would prevail in a Calvo model, i.e., in which prices adjust exogenously (not in response to shocks). For this, we combine the estimated reset-price pass-through ψ_τ^{Est} and Ψ_τ^{Est} from the baseline local projection specification (20) with the estimated average price stickiness λ^{Est} . Under Calvo pricing, a fraction $(1 - \lambda)$ of firms adjust each period, and adjustment timing is random. This implies a recursive relationship between the reset-price pass-through responses and the cumulative responses:

$$\tilde{\gamma}_0^I = \psi_0^{\text{Est}}, \quad \tilde{\gamma}_\tau^I = \lambda^{\text{Est}} \tilde{\gamma}_{\tau-1}^I + (1 - \lambda^{\text{Est}}) \psi_\tau^{\text{Est}} \quad (\tau \geq 1),$$

and analogously for $\tilde{\gamma}_\tau^C$ using Ψ_τ^{Est} . The intuition is that at each horizon, the cumulative response is a weighted average of (i) the previous response carried forward by the fraction λ of prices that do not adjust, and (ii) the current response of the fraction $(1 - \lambda)$ of prices that do adjust.

By design, if state-dependent extensive margin adjustments play an important quantitative role, we would expect the estimated average pass-through rates for combined intensive/extensive price adjustments ($\gamma_\tau^{I,\text{Est}}$ and $\gamma_\tau^{C,\text{Est}}$) to differ from the counterfactual pass-through rates under exogenous (Calvo) adjustment timing ($\tilde{\gamma}_\tau^I$ and $\tilde{\gamma}_\tau^C$).

Figures C5 and C6 show that the “Theory (Calvo)” and “Estimated” lines align closely across horizons, for both idiosyncratic and common shocks, and across sample splits by markup and stickiness. This alignment indicates that the Calvo model provides a reasonable approximation of the empirical price dynamics.

Hazard rate analysis. As a complementary test of the Calvo assumption, we examine whether the conditional probability of price adjustment depends on the time since the last adjustment. The hazard rate is defined as the probability of a price adjustment in period t , conditional on the price having remained unchanged for k periods (i.e., the probability of adjustment as a function of price spell duration). Figure C7 presents the estimated hazard rate function for selling price changes. Under Calvo pricing, the hazard rate is constant across all price spell durations. The estimated hazard rates are relatively flat, consistent with the Calvo assumption. Combined with the modest extensive margin responses, flat hazard rates suggest that the Calvo model provides a reasonable first-order approximation.

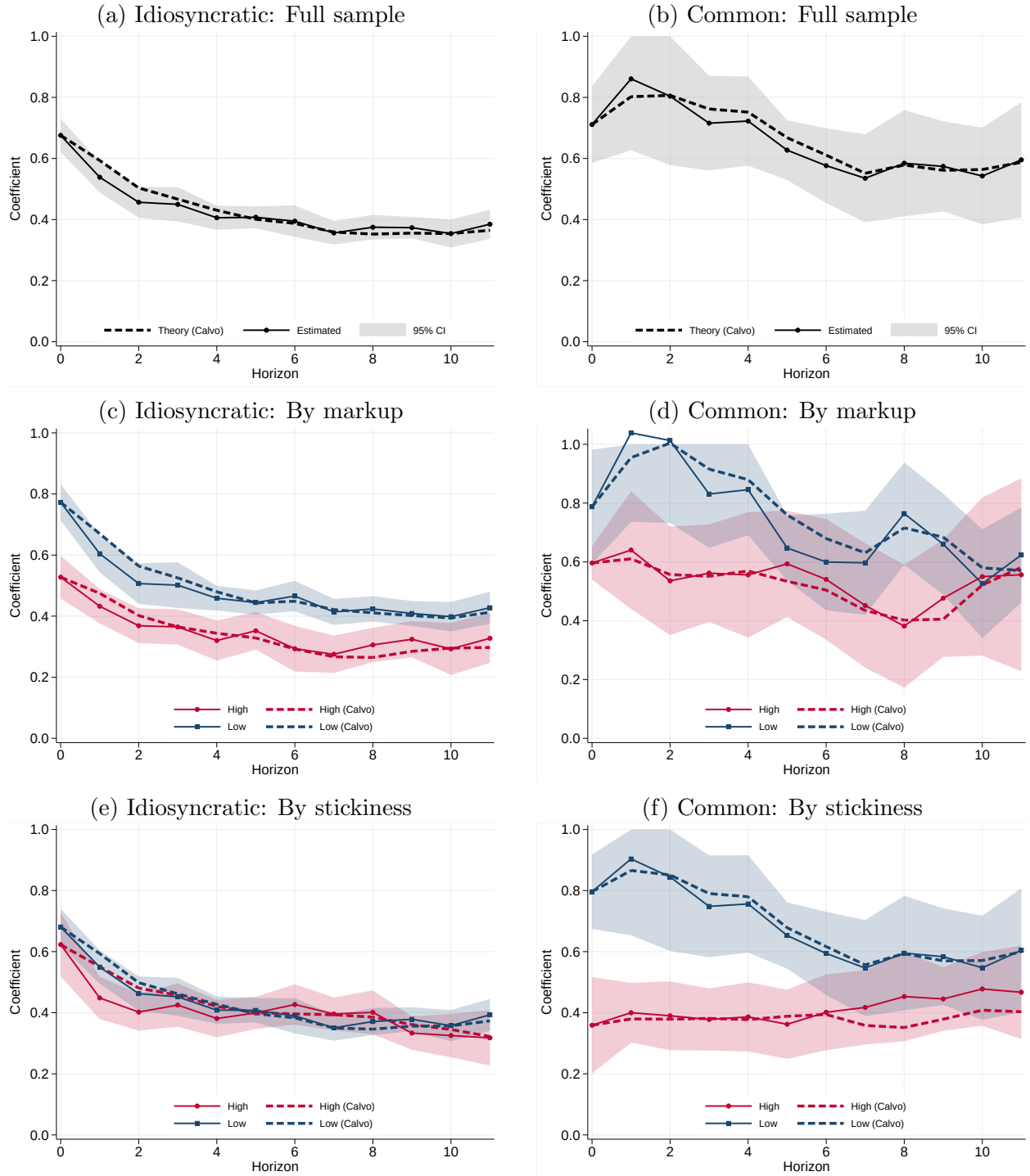


Figure C5: Combined intensive and extensive price adjustments (NAICS4)

Notes: Comparison of estimated cumulative price responses (“Estimated”) with counterfactual Calvo responses (“Theory (Calvo)”). The “Estimated” line shows LP estimates that do not condition on adjustment at $t + \tau$, with 95% confidence intervals (shaded area) constructed using firm-clustered standard errors. The “Theory (Calvo)” line constructs counterfactual responses using estimated reset-price pass-through and average stickiness. Close alignment indicates the Calvo model provides a reasonable approximation. Left column: idiosyncratic shocks; right column: common shocks. 4-digit NAICS classification.

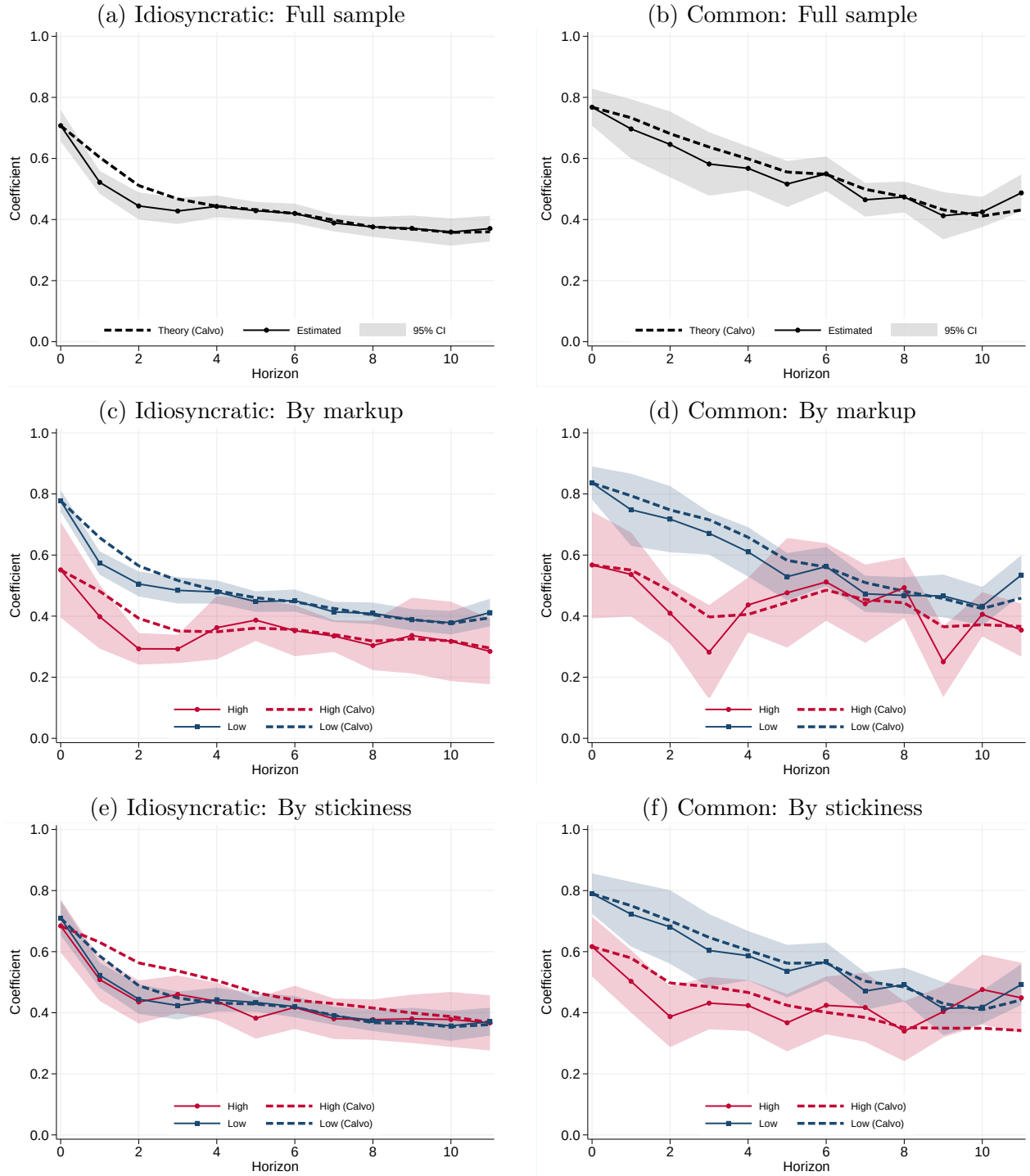


Figure C6: Combined intensive and extensive price adjustments (NAPCS7)

Notes: Comparison of estimated cumulative price responses (“Estimated”) with counterfactual Calvo responses (“Theory (Calvo)”). The “Estimated” line shows LP estimates that do not condition on adjustment at $t + \tau$, with 95% confidence intervals (shaded area) constructed using firm-clustered standard errors. The “Theory (Calvo)” line constructs counterfactual responses using estimated reset-price pass-through and average stickiness. Close alignment indicates the Calvo model provides a reasonable approximation. Left column: idiosyncratic shocks; right column: common shocks. 7-digit NAPCS classification.

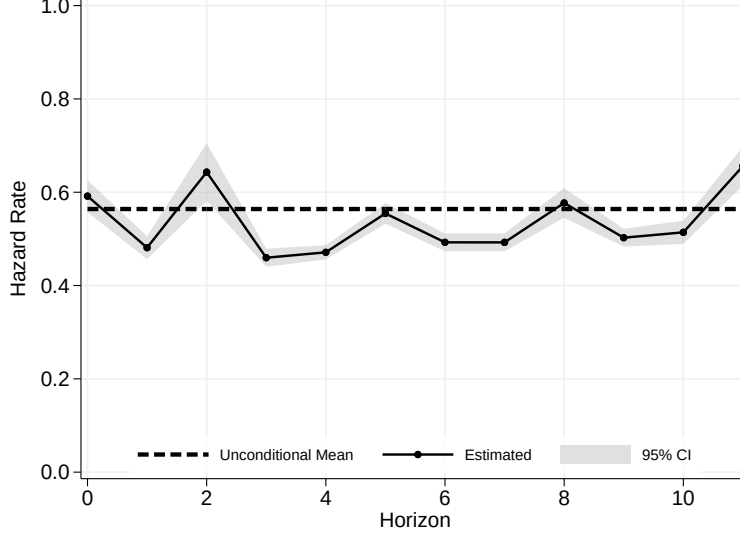


Figure C7: Hazard rates for selling price adjustments.

Notes: Estimated hazard rate is the probability of selling price change as a function of months since last price change. Dashed line: unconditional mean hazard rate; solid line with markers: estimated hazard by price age; shaded area: 95% CI. Under Calvo pricing, the hazard rate is constant across all price ages.

C.4 Responses to monetary policy shocks

This section presents the estimated responses of purchase prices and selling prices to identified monetary policy shocks (Sekkel, Stern and Zhang, 2025). Specifically, we regress the selling and purchase price changes on the identified monetary policy shocks ε_t^M conditional on a price change at $t + \tau$:

$$\Delta_\tau \ln(y_{ijt+\tau}) = \gamma_\tau \cdot \varepsilon_t^M + FE_{ij} + \nu_{ijt+\tau} \quad \text{with} \quad y_{ijt+\tau} \in \{P_{ijt+\tau}, Q_{ijt+\tau}\}, \quad (\text{C.5})$$

where $\Delta_\tau \ln(y_{ijt+\tau}) \equiv \ln(y_{ijt+\tau}) - \ln(y_{ijt-1})$ denotes the cumulative selling or purchase price change from $t - 1$ to $t + \tau$, FE_{ij} are firm-product fixed effects, and $\nu_{ijt+\tau}$ is the residual.

The estimated coefficients $\hat{\gamma}_\tau$ do not have a direct economic interpretation because the monetary policy shock ε_t^M is identified only up to scale. To interpret the estimated responses to monetary policy shocks, we normalize the estimated coefficients and standard errors by the first coefficient of the purchase price response, $\hat{\gamma}_0^Q$:

$$\hat{\gamma}_\tau^{Q,norm} = \hat{\gamma}_\tau^Q / \hat{\gamma}_0^Q \quad \text{and} \quad \hat{\gamma}_\tau^{P,norm} = \hat{\gamma}_\tau^P / \hat{\gamma}_0^Q. \quad (\text{C.6})$$

The normalized coefficients can be interpreted as the effect from a monetary policy shock that generates a 1 percent change in the purchase price on impact (at $\tau = 0$).

Figure C8 shows the corresponding normalized coefficients for NAICS4 sectors and NAPCS7 products. Theoretical values ψ_τ and Ψ_τ are constructed assuming that monetary policy follows a random walk process, i.e., $M_t = M_{t-1} + \varepsilon_t^M$. Overall, the estimated coefficients are imprecise and

exhibit wide confidence intervals compared to theoretical pass-through values, which range from 0 to 1. We conjecture that this imprecision is driven by the fact that monetary policy shocks account for only a small fraction of the variation in purchase price changes. Combined with the short time span of our sample ($T < 80$ for most firm-products; see Table A2), this makes it difficult to obtain precise estimates. We verify this hypothesis using model-simulated data in Section C.5, where we find similarly imprecise estimates in model simulations.

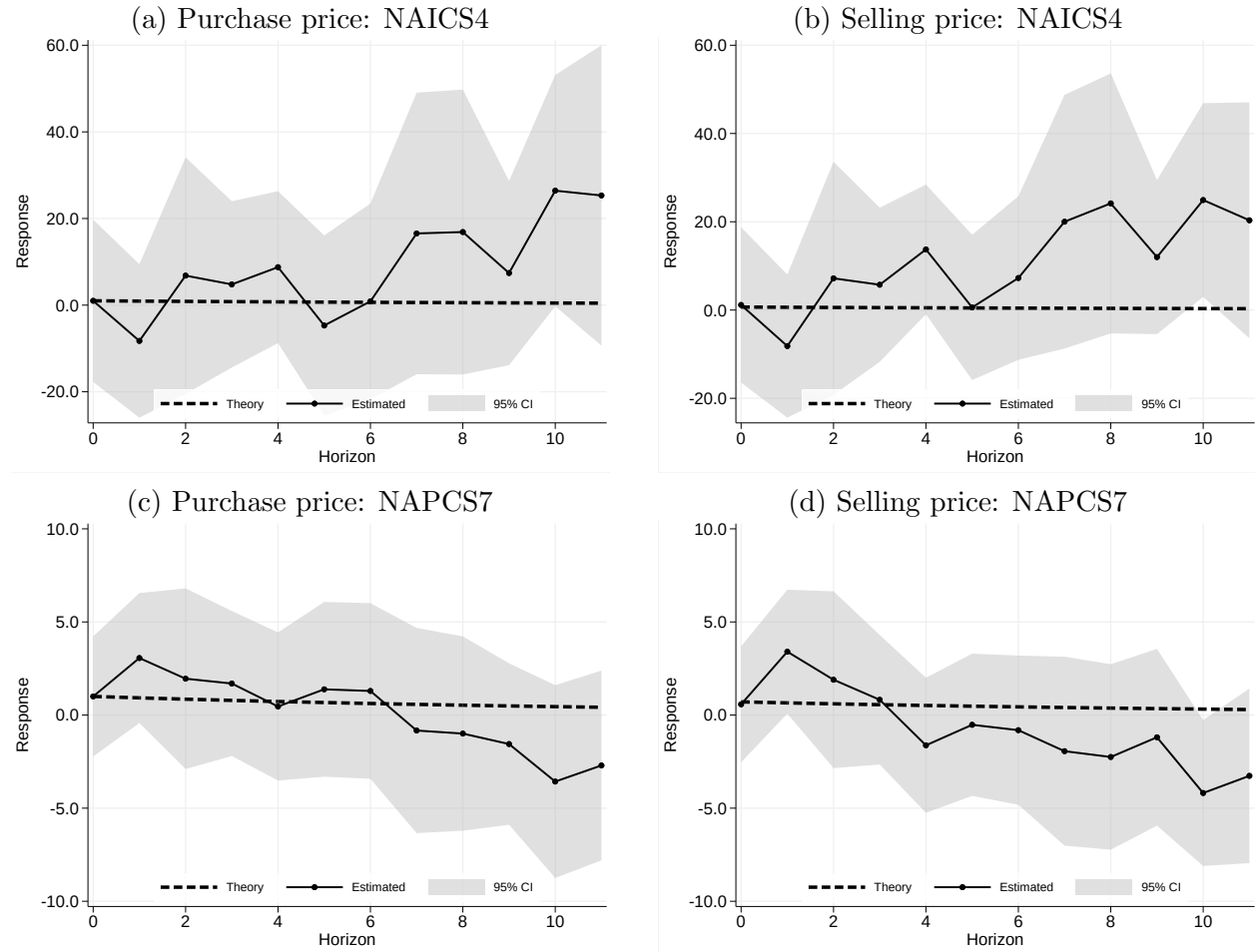


Figure C8: Response to monetary policy shocks

Notes: Confidence intervals are constructed using firm-clustered standard errors. The same specification (C.5) is applied to both NAICS4 and NAPCS7 estimations; the differences between the top and bottom panels reflect differences in the sampling revenue weights used for NAICS4 sectors versus NAPCS7 products.

C.5 Validation of estimation strategy using simulated data

This section applies our empirical procedures to model-simulated data constructed to mirror the constraints and features of the empirical setting, allowing us to assess how well the proposed methods recover the assumed theoretical parameters. We simulate a multi-sector panel within a Calvo price-setting environment. Each sector is characterized with sector-specific Calvo stickiness λ_j , average markup μ_j and market-power φ_j , calibrated to reproduce the empirical moments derived from the 4-digit NAICS data. The simulated economy contains $N=4$ firms per sector and spans $T=96$ monthly periods (2012–2019).

We incorporate the generalized shock path $\tilde{q}(\tau; x, \rho)$ introduced in Section C.1. For idiosyncratic shocks ε_{ijt} , we set $(x, \rho) = (0.75, 0.95)$, capturing both temporary and persistent components. For common shocks—economy-wide monetary-policy shocks ε_t^{MP} and sector-specific shocks ε_{jt}^v —we set $x=\rho=1$.

The structural parameters are set as follows: the discount factor is $\beta=0.995$; the monetary-policy shock scaling parameter is $\delta=1000$, chosen to match the empirical value of $1/\hat{\gamma}_0^Q$ in equation (C.5); and the innovation standard deviations are $\sigma_\varepsilon=1.0$ for idiosyncratic shocks ε_{ijt} , $\sigma_{MP}=0.01$ for monetary policy shocks ε_t^{MP} , and $\sigma_v=0.01$ for sector-specific shocks ε_{jt}^v . The relatively small monetary-policy shock variance reflects our empirical finding that monetary disturbances explain only a very small share of purchase-price variation.

Optimal prices. The flexible cost (purchase price) of firm i in sector j (in logs) is the sum of all previous shocks:

$$q_{ijt}^* = \underbrace{\sum_{\tau \geq 0} \tilde{q}(\tau; x, \rho) \varepsilon_{ijt-\tau}}_{\text{idiosyncratic}} + \underbrace{\frac{1}{\delta} \sum_{\tau \geq 0} (\varepsilon_{t-\tau}^{MP} + \varepsilon_{jt-\tau}^v)}_{\text{common (random walk)}}.$$

Let $\bar{\varepsilon}_{jt} \equiv \frac{1}{N_j} \sum_{k=1}^{N_j} \varepsilon_{kjt}$ denote the sector average of idiosyncratic innovations. The desired selling price (in logs) separates into idiosyncratic and common components using the synchronized pass-through coefficients $\psi_\tau^{\text{sync}}(x, \rho)$ and $\Psi_\tau^{\text{sync}}(x, \rho)$ (derived in Section D.5.2, equations (D.44) and (D.47)):

$$p_{ijt}^* = \underbrace{\sum_{\tau \geq 0} \psi_\tau(x, \rho) (\varepsilon_{ijt-\tau} - \bar{\varepsilon}_{jt-\tau})}_{\text{idiosyncratic response}} + \underbrace{\sum_{\tau \geq 0} \Psi_\tau(x, \rho) \bar{\varepsilon}_{jt-\tau}}_{\text{common response to sector avg}} + \underbrace{\frac{1}{\delta} \sum_{\tau \geq 0} \Psi_\tau(1, 1) (\varepsilon_{t-\tau}^{MP} + \varepsilon_{jt-\tau}^v)}_{\text{common response to MP and sector shocks}}.$$

Infrequent adjustment. Let $I_{ijt} \in \{0, 1\}$ indicate an adjustment opportunity with $\Pr(I_{ijt} = 1) = 1 - \lambda_j$ (Calvo), independent over i, j, t and of shocks. The *observed* prices follow the following process:

$$p_{ijt} = I_{ijt} p_{ijt}^* + (1 - I_{ijt}) p_{ijt-1}, \quad q_{ijt} = I_{ijt} q_{ijt}^* + (1 - I_{ijt}) q_{ijt-1}.$$

Results. We apply the same estimation equations used in the empirical analysis to simulated data and assess the extent to which our estimation strategy can recover the assumed parameters of the model. The simulation produces three sets of verification figures. In each case, theoretical reference lines are constructed using the assumed calibrated parameters from the data-generating process ($x=0.75, \rho=0.95$), allowing a direct comparison between the estimated and theoretical pass-through.

- *Dynamic pass-through (Figure C9):* Six panels report the estimated idiosyncratic (ψ_τ) and common (Ψ_τ) pass-through coefficients for the full sample and for subsamples defined by concentration or stickiness. Each panel shows the estimated coefficients with 95% confidence intervals alongside the corresponding theoretical kernels.
- *Estimated cost process (Figure C10):* This figure displays the estimated propagation of idiosyncratic cost shocks over time. The response begins at 1 (full impact pass-through) and decays according to the persistence parameter. Theoretical reference lines show both the generalized model structure and the AR(1) benchmark implied by the calibrated parameters.
- *Monetary policy response (Figure C11):* We estimate the response of purchase and selling prices to monetary policy shocks by regressing cumulative price changes on the monetary policy shock ε_t^{MP} , conditioning on $I_{ijt+h} = 1$. Similar to our empirical results in Section C.4, the estimated pass-through of monetary policy shocks is imprecisely measured and exhibits large standard errors.

Overall, the close correspondence between the empirical estimates and estimates from simulated data supports the validity of our estimation strategy. These results indicate that the LP-based approach can successfully recover the underlying pass-through coefficients from panel data generated by a Calvo price-setting environment.

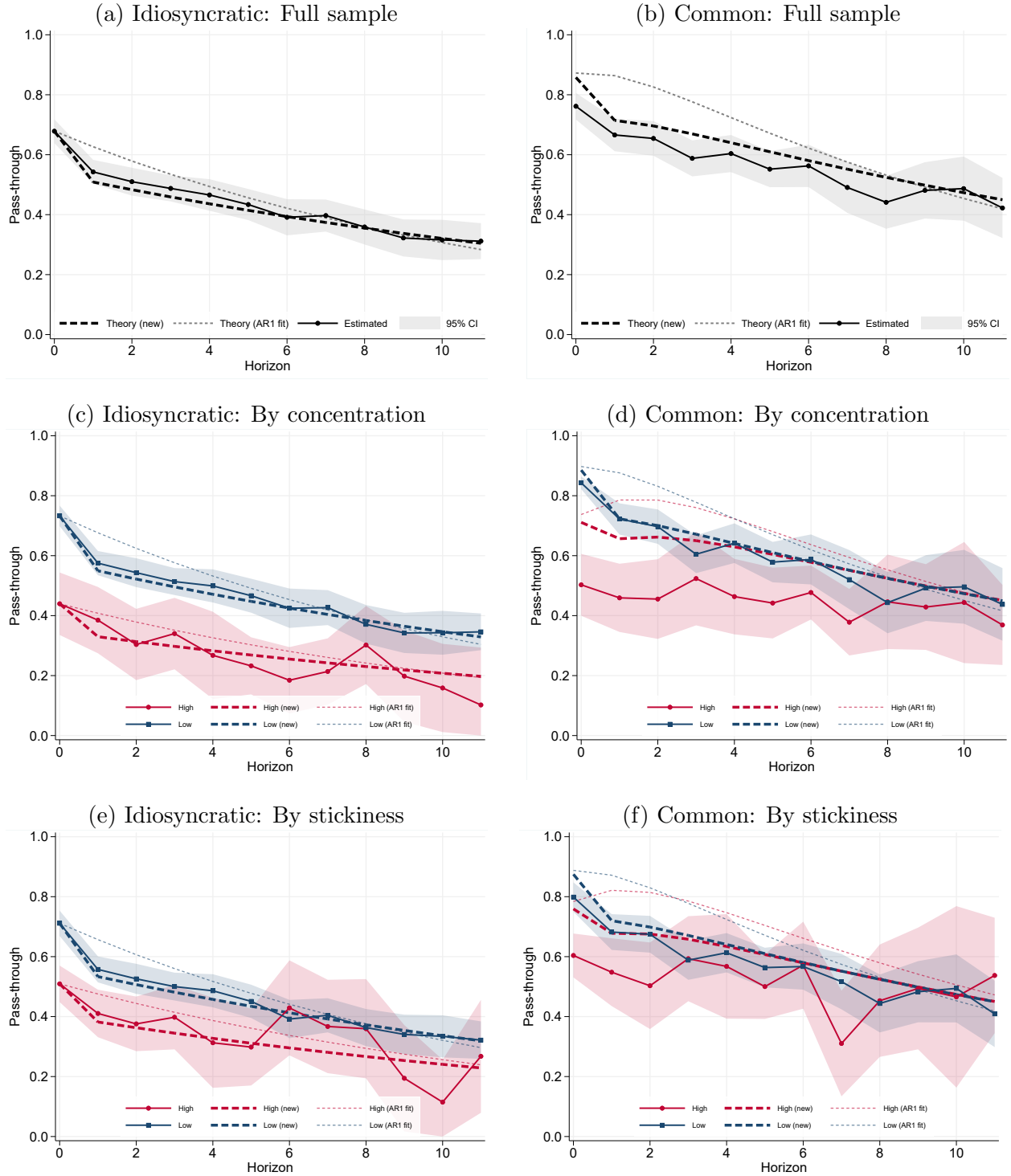


Figure C9: Dynamic pass-through (Simulated Data)

Notes: Estimated pass-through kernels from simulated panel data compared to theoretical predictions in model calibrated to NAICS4 sectors. Solid lines with markers show LP estimates with 95% confidence intervals (shaded). Thick dashed lines show Theory (new) computed using the true calibrated parameters ($x=0.75, \rho=0.95$); short-dashed lines show Theory (AR1 fit) with estimated $\hat{\rho}$. Panels (a)–(b): full sample; (c)–(d): split by market concentration; (e)–(f): split by price stickiness.

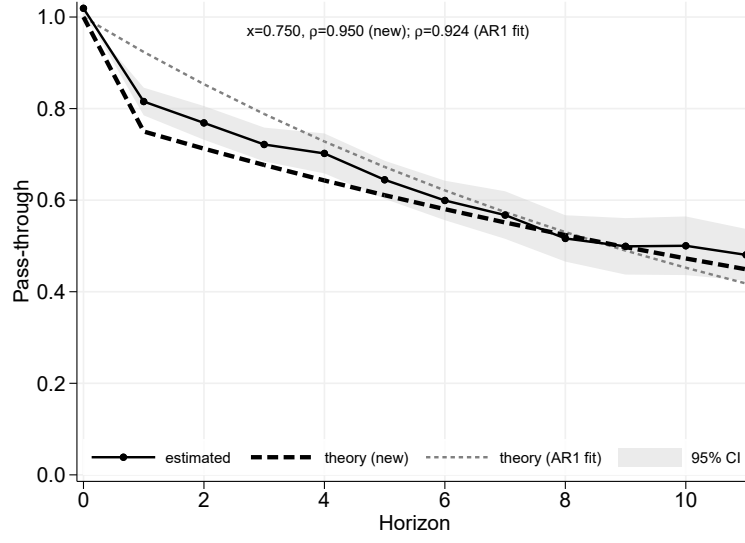


Figure C10: Estimated cost process (Simulated Data)

Notes: Estimated cost process from simulated data using model calibrated to NAICS4 sectors. The cost pass-through starts at 1 (full pass-through at impact) and decays according to the underlying shock persistence. Dashed line shows theory (new) computed using the true calibrated parameters ($x=0.75, \rho=0.95$); short-dashed line shows theory (AR1 fit) with estimated $\hat{\rho}$.

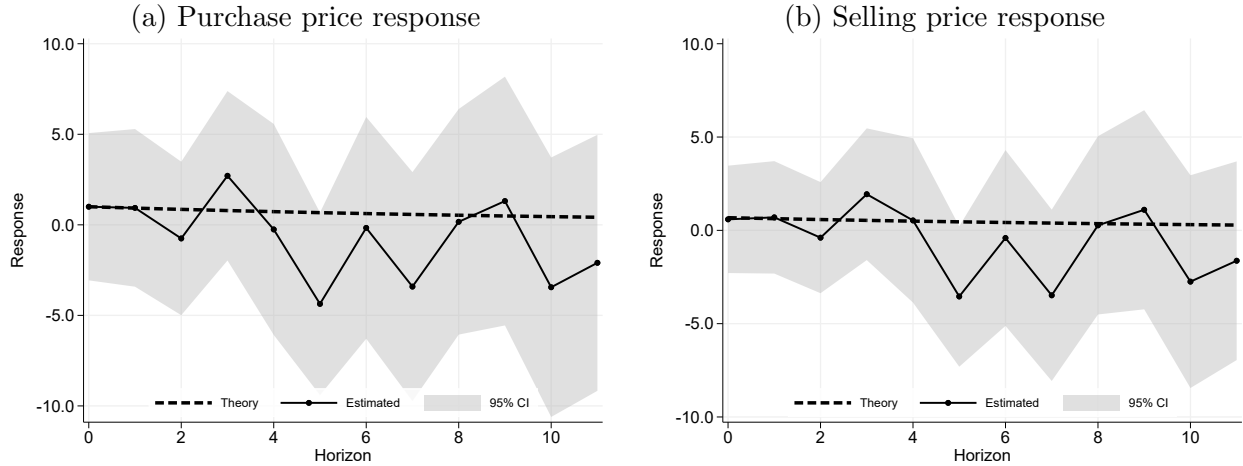


Figure C11: Response to monetary policy shocks (Simulated Data)

Notes: Estimated responses of purchase and selling prices to monetary policy shocks from simulated data using model calibrated to NAICS4 sectors. Solid lines with markers show LP estimates with 95% confidence intervals (shaded). Dashed lines show theoretical pass-through computed using the calibrated parameters. Similar to the empirical results in Section C.4, the estimated responses are imprecise due to the small variance of monetary policy shocks relative to idiosyncratic shocks.

D Model Details and Supplementary Theoretical Results

D.1 Producer's problem

This section provides a more detailed exposition of the producer's problem, including the production technology, microfoundation of cost shocks, and price adjustment mechanism.

Production technology. Producer i in sector j uses a constant returns to scale (CRS) production function to produce output y_{ijt} using various inputs. Under CRS technology, the marginal cost does not depend on the quantity produced and can be expressed as:

$$MC_{ijt} = \frac{c(W_t, R_t, \dots)}{A_{ijt}},$$

where $c(W_t, R_t, \dots)$ is the unit cost function that depends on input prices (such as wages W_t , capital rental rates R_t , prices of intermediate inputs, etc.), and A_{ijt} is a productivity shifter.

Shocks to the producer's marginal cost can originate from various sources:

- *Productivity shocks:* Changes in firm-specific productivity A_{ijt} represent idiosyncratic shocks, while sector-wide or economy-wide technology shifts constitute common shocks.
- *Input price shocks:* Changes in wages W_t or other input prices. In our model, the wage is tied to monetary policy through the labor supply condition $W_t = M_t$ (equation (1) in the main text), making wage shocks common across all firms.
- *Imported input price shocks:* For firms using imported intermediate inputs, exchange rate movements or foreign price changes can affect marginal costs. These can be idiosyncratic (firm-specific suppliers) or common (aggregate exchange rate shocks).

We abstract from the specific microfoundation of marginal cost shocks, as we do not directly observe the underlying sources in our data. Instead, we assume that the marginal cost shocks ϵ_{ijt} follow an AR(1) process with persistence $\rho_j \in [0, 1]$ and the realized marginal cost in (log) levels is:

$$\ln(MC_{ijt}) = \rho_j \ln(MC_{ijt-1}) + \epsilon_{ijt}, \quad \text{or equivalently,} \quad \ln(MC_{ijt}) = \sum_{\tau \geq 0} \rho_j^\tau \epsilon_{ijt-\tau}. \quad (\text{D.1})$$

Notably, we do not require the marginal cost shocks ϵ_{ijt} to be i.i.d. They may be correlated both within and across sectors. For example, the variation in ϵ_{ijt} may arise from three components: $\epsilon_{ijt} = \varepsilon_{ijt} + \varepsilon_{jt} + \varepsilon_t$, where ε_{ijt} , ε_{jt} , and ε_t denote firm-specific, sector-specific, and economy-wide shocks, respectively. We provide a simulated example of this setting in Section C.5.

In Section D.5, we relax the baseline AR(1) assumption and allow for a more general cost process, as described in Section C.1, deriving the corresponding closed-form solutions.

Common vs idiosyncratic shock components. Our theoretical framework decomposes the cost shocks into two components:

$$\text{Idiosyncratic component: } \epsilon_{ijt} - \epsilon_{jt} \quad \text{and} \quad \text{Common component: } \epsilon_{jt} = \sum_i s_{ij} \epsilon_{ijt}. \quad (\text{D.2})$$

The idiosyncratic component captures changes in the relative competitiveness of firm i compared to other firms in the sector, whereas the common component captures changes in average sectoral cost (the overall competitiveness of the sector). The common component reflects two sources of variation: (a) sector-specific ϵ_{jt} or economy-wide ϵ_t shocks, and (b) the non-zero aggregation of idiosyncratic ϵ_{ijt} shocks due to firm concentration in each sector (i.e., the granularity hypothesis; Gabaix, 2011; Acemoglu et al., 2012; di Giovanni, Levchenko and Méjean, 2014).

These two sources of variations naturally distinguish themselves through their different loading onto the idiosyncratic and common components. Sector-wide or economy-wide shocks that affect all firms symmetrically load entirely onto the common component, leaving the idiosyncratic component at zero. By contrast, when only firm i receives a shock of size $\epsilon_{ijt} = 1$, the common component equals s_{ij} , while the idiosyncratic component equals $1 - s_{ij}$ for firm i and $-s_{ij}$ for all other firms. Notably, the idiosyncratic component changes even for firms not receiving the shock, as they become relatively more competitive (due to lower relative cost) and thus may want to increase their markups. Our theoretical propositions show that the idiosyncratic and common cost components are sufficient statistics for capturing firms' price responses.

Optimal reset price. Varieties are supplied to distributors by producers competing in monopolistically competitive markets. The optimal price of the producer is given by

$$Q_{ijt}^* = \frac{\eta}{\eta - 1} MC_{ijt},$$

where η is the elasticity of substitution among producers' products. It follows from D.1 that:

$$\hat{Q}_{ijt}^* = \rho_j \hat{Q}_{ijt-1}^* + \epsilon_{ijt}.$$

Price adjustments. We assume a producer's price, Q_{ijt} , is sticky, changing according to a Poisson process with probability $1 - \lambda_j^Q$: when the price adjusts, the producer resets it to the frictionless optimal price Q_{ijt}^* , equal to the constant markup over its marginal cost; otherwise the price remains equal to the last period's price, Q_{ijt-1} . Under this assumption, in each period a fraction $1 - \lambda_j^Q$ of producers in sector j receive the opportunity to adjust their prices, while the remaining fraction λ_j^Q keep their prices unchanged.

As we document in Section 3 of the main text, in the wholesale price data, producer and distributor price adjustments are highly synchronized. We therefore assume that the distributor and its corresponding producer face an *identical* Poisson process for price adjustments, i.e., their price adjustments are perfectly synchronized. This assumption effectively collapses two layers of

nominal rigidity—one at the producer level and the other at the distributor level—into a single layer, making our model more comparable to models with only one layer of nominal rigidity (Wang and Werning, 2022; Mongey, 2021). See Section E.2.1 for more details.

D.2 Steady state equilibrium

The deterministic steady state of our dynamic model with sticky prices corresponds to the equilibrium of a static oligopolistic competition model without nominal rigidities and without shocks. In steady state, all prices are set at their optimal flexible levels. The key steady-state properties are derived from the static optimization problem where firms set prices as a markup over marginal cost, taking into account strategic interactions with competitors.

Steady-state prices and markups. In steady state, firm i in sector j sets its price P_{ij} according to the static profit-maximization condition:

$$P_{ij} = \mu_{ij} \cdot Q_{ij},$$

where $\mu_{ij} = \frac{\vartheta_{ij}}{\vartheta_{ij}-1}$ is the steady-state markup, and ϑ_{ij} is the perceived demand elasticity. Under Cournot competition with CES demand, the effective demand elasticity depends on the firm's steady-state market share s_{ij} :

$$\vartheta_{ij} = \left[\frac{1}{\theta}(1 - s_{ij}) + s_{ij} \right]^{-1},$$

which implies:

$$\mu_{ij} = \frac{\theta}{\theta - 1} \cdot \frac{1}{1 - s_{ij}}.$$

This relationship shows that, for a given elasticity of substitution θ , firms with larger market shares s_{ij} charge higher markups.

Steady-state market shares. The steady-state market share of firm i in sector j is determined by the sector price index and the firm's relative price:

$$s_{ij} = \frac{P_{ij}c_{ij}}{P_jc_j} = \frac{(P_{ij})^{1-\theta}}{\sum_{k=1}^{N_j}(P_{kj})^{1-\theta}},$$

where $P_j = \left[\sum_{k=1}^{N_j} (P_{kj})^{1-\theta} \right]^{\frac{1}{1-\theta}}$ is the steady-state sector price index, and c_{ij} and c_j are the steady-state quantities consumed.

Strategic complementarity. The strength of strategic pricing complementarity in steady state, which governs the degree to which firms respond to competitors' price changes, is given by:

$$\varphi_{ij} = \frac{s_{ij}}{1 - s_{ij}}(\theta - 1).$$

This parameter is strictly increasing in the firm's market share s_{ij} , reflecting the fact that larger firms have greater incentive to absorb cost shocks into their markups rather than fully passing them through to prices. Strategic complementarity is zero under monopolistic competition (where $s_{ij} \rightarrow 0$) and increases with market concentration.

Connection to static models. In steady state, our dynamic model reduces to the static oligopolistic competition framework. With no shocks and all prices set at optimal levels, the steady-state pricing conditions are identical to those in a static model where firms set prices as a markup over marginal cost, taking competitors' prices as given.

The mapping between our dynamic model's steady state and the static framework of [Atkeson and Burstein \(2008\)](#) and [Amiti, Itskhoki and Konings \(2019\)](#) (AIK) is detailed in Online Appendix D.8. In the static framework, the pass-through to an idiosyncratic cost shock is given by $\varphi_{ijt} = \frac{1-s_{ijt}}{1-s_{ijt}+\Gamma_{ijt}}$, where Γ_{ijt} is the markup elasticity with respect to the firm's own price. Our model's strategic complementarity parameter φ_{ij} is related to AIK's Γ_{ij} through the market share: $\varphi_{ij} = \frac{s_{ij}}{1-s_{ij}}(\theta - 1)$ under Cournot competition.

In the static framework (which corresponds to our model's steady state), the pass-through to an idiosyncratic cost shock depends on market power, while the pass-through to a common cost shock is always 100%. Our dynamic model adds an additional layer: price stickiness interacts with market power to reduce the pass-through to common shocks below 100%, with the magnitude of this reduction depending on both λ_j (price stickiness) and φ_{ij} (strategic complementarity). This is the key contribution of our dynamic framework relative to static models.

D.3 Closed-form solutions for contemporaneous reset prices

In this subsection, we first solve for the firm's optimal reset price in response to both common and idiosyncratic cost shocks, when costs are flexible (i.e., $\hat{Q}_{ijt} = \hat{Q}_{ijt}^*$) to prove Proposition 1. We then show that Proposition 2 can be solved following similar steps.

D.3.1 Proof of Proposition 1

We begin by characterizing how the expected sector price reacts to an arbitrary set of firm-level cost shocks, which follow the same AR(1) process:

$$\hat{Q}_{ijt}^* = \rho_j \hat{Q}_{ijt-1}^* + \epsilon_{ijt},$$

where ϵ_{ijt} are ex-ante mean zero shocks that can be correlated across firms. For example, a common (sector) shock occurs when $\epsilon_{ijt} = 1 \forall i \in j$, and an idiosyncratic shock occurs when $\epsilon_{ijt} = 1$ and $\epsilon_{kjt} = 0 \forall k \neq i \in j$.

We can re-express the sector NKPC (10) as a second-order difference equation in price levels:

$$\mathbb{E}_t \left[\hat{P}_{jt+\tau} - \lambda_j \hat{P}_{jt+\tau-1} - \beta \lambda_j (\hat{P}_{jt+\tau+1} - \lambda_j \hat{P}_{jt+\tau}) \right] = \mathbb{E}_t \sum_i s_{ij} \frac{(1 - \beta \lambda_j)(1 - \lambda_j)}{(1 + \varphi_{ij})} (\hat{Q}_{ijt+\tau}^* + \varphi_{ij} \hat{P}_{jt+\tau}).$$

For any arbitrary set of realized shocks $\{\epsilon_{ijt}\}$ at t , the expected sector price at $t + \tau$ can be solved as

$$\mathbb{E}_t \hat{P}_{jt+\tau} = \frac{\rho_j^{\tau+1} - \Lambda_j^{\tau+1}}{\rho_j(1-b_j) + \lambda_j[\beta\rho_j(\lambda_j - \rho_j) - 1]} a_j \hat{Q}_{jt}^* \quad \forall \tau \geq 0, \quad (\text{D.3})$$

where

$$\Lambda_j \equiv \frac{1}{2} \left[\lambda_j + \frac{1-b_j}{\beta\lambda_j} - \sqrt{\left(\lambda_j + \frac{1-b_j}{\beta\lambda_j} \right)^2 - \frac{4}{\beta}} \right], \quad (\text{D.4})$$

$$a_j \equiv \left(\sum_i \frac{(1-\beta\lambda_j)(1-\lambda_j)}{(1+\varphi_{ij})} s_{ij} \hat{Q}_{ijt}^* \right) / \hat{Q}_{jt}^* \quad \text{with} \quad \hat{Q}_{jt}^* \equiv \sum_i s_{ij} \hat{Q}_{ijt}^*, \quad (\text{D.5})$$

$$b_j \equiv \sum_i s_{ij} \frac{\varphi_{ij}(1-\beta\lambda_j)(1-\lambda_j)}{(1+\varphi_{ij})}. \quad (\text{D.6})$$

Plugging the expected sector prices back into the firm's reset price (7), we have

$$\hat{P}_{ijt}^* = \frac{1-\beta\lambda_j}{1+\varphi_{ij}} \sum_{\tau=0}^{\infty} (\beta\lambda_j)^\tau \left[\rho_j^\tau \hat{Q}_{ijt} + \varphi_{ij} \frac{\rho_j^{\tau+1} - \Lambda_j^{\tau+1}}{\rho_j(1-b_j) + \lambda_j[\beta\rho_j(\lambda_j - \rho_j) - 1]} a_j \hat{Q}_{jt}^* \right].$$

Solving the geometric series gives

$$\hat{P}_{ijt}^* = \frac{1}{1+\varphi_{ij}} \frac{1-\beta\lambda_j}{1-\beta\lambda_j\rho_j} \hat{Q}_{ijt}^* + \frac{\varphi_{ij}}{1+\varphi_{ij}} \frac{\rho_j - \Lambda_j}{1-\beta\lambda_j\Lambda_j} \frac{1-\beta\lambda_j}{1-\beta\lambda_j\rho_j} \varkappa_j \hat{Q}_{jt}^*, \quad (\text{D.7})$$

where

$$\varkappa_j \equiv \frac{a_j}{\rho_j(1-b_j) + \lambda_j[\beta\rho_j(\lambda_j - \rho_j) - 1]}. \quad (\text{D.8})$$

The parameter $\varkappa_j$ reappears in Section D.4 when deriving future reset prices.

We can rewrite (D.7) as responses to idiosyncratic and common (or average) cost shocks:

$$\hat{P}_{ijt}^* = \underbrace{\frac{1}{1+\varphi_{ij}} \frac{1-\beta\lambda_j}{1-\beta\lambda_j\rho_j}}_{\text{PT to idiosyncratic cost changes}} \left(\hat{Q}_{ijt}^* - \hat{Q}_{jt}^* \right) + \underbrace{\left[\frac{1}{1+\varphi_{ij}} + \frac{\varphi_{ij}}{1+\varphi_{ij}} \frac{\rho_j - \Lambda_j}{1-\beta\lambda_j\Lambda_j} \varkappa_j \right] \frac{1-\beta\lambda_j}{1-\beta\lambda_j\rho_j}}_{\text{PT to common (or average) cost changes}} \hat{Q}_{jt}^* \quad (\text{D.9})$$

Due to strategic interactions, the effects of a cost shock on a firm's optimal reset price can be decomposed into two distinct components: (1) the impact of deviations of the firm's cost from the average cost change; and (2) the impact of average cost change. ■

D.3.2 Price and cost synchronization and proof of Proposition 2

When the timing of price adjustment is perfectly synchronized with that of the cost changes, the firm's expected future cost during the periods that its price remains fixed is just its current

cost:

$$\mathbb{E}_t \hat{Q}_{ijt+\tau}^* = \hat{Q}_{ijt}^* \quad \forall \tau \geq 0.$$

In this case, the optimal equations for individual (7) and sector (8) reset prices become

$$\begin{aligned} \hat{P}_{ijt}^* &= \frac{1}{1 + \varphi_{ij}} \hat{Q}_{ijt}^* + \frac{\varphi_{ij}(1 - \beta\lambda_j)}{(1 + \varphi_{ij})} \sum_{\tau=0}^{\infty} (\beta\lambda_j)^\tau \mathbb{E}_t \hat{P}_{jt+\tau} \\ \mathbb{E}_t \hat{P}_{jt}^* &= \mathbb{E}_t \sum_i s_{ij} \hat{P}_{ijt}^* = \sum_i s_{ij} \frac{1}{1 + \varphi_{ij}} \hat{Q}_{ijt}^* + \sum_i \left\{ s_{ij} \frac{\varphi_{ij}(1 - \beta\lambda_j)}{(1 + \varphi_{ij})} \sum_{\tau=0}^{\infty} (\beta\lambda_j)^\tau \mathbb{E}_t \hat{P}_{jt+\tau} \right\}. \end{aligned} \quad (\text{D.10})$$

Using the relationship $\mathbb{E}_t \hat{P}_{jt+\tau} = (1 - \lambda_j) \mathbb{E}_t \hat{P}_{jt+\tau}^* + \lambda_j \mathbb{E}_t \hat{P}_{jt+\tau-1}$, we have

$$\begin{aligned} &\mathbb{E}_t \left[\hat{P}_{jt+\tau} - \lambda_j \hat{P}_{jt+\tau-1} - \beta\lambda_j (\hat{P}_{jt+\tau+1} - \lambda_j \hat{P}_{jt+\tau}) \right] \\ &= (1 - \lambda_j) \mathbb{E}_t \left(\hat{P}_{jt+\tau}^* - \beta\lambda_j \hat{P}_{jt+\tau+1}^* \right) \\ &= \mathbb{E}_t \sum_i s_{ij} \frac{1 - \lambda_j}{1 + \varphi_{ij}} \left(\hat{Q}_{ijt+\tau}^* - \beta\lambda_j \hat{Q}_{ijt+\tau+1}^* \right) + \mathbb{E}_t \sum_i s_{ij} \frac{\varphi_{ij}(1 - \beta\lambda_j)(1 - \lambda_j)}{(1 + \varphi_{ij})} \hat{P}_{jt+\tau} \\ &= \sum_i s_{ij} \frac{(1 - \lambda_j)(1 - \beta\lambda_j \rho_j)}{1 + \varphi_{ij}} \hat{Q}_{ijt+\tau}^* + \mathbb{E}_t \sum_i s_{ij} \frac{\varphi_{ij}(1 - \beta\lambda_j)(1 - \lambda_j)}{(1 + \varphi_{ij})} \hat{P}_{jt+\tau} \\ &= \sum_i s_{ij} \frac{(1 - \lambda_j)(1 - \beta\lambda_j \rho_j)}{1 + \varphi_{ij}} \rho_j^\tau \hat{Q}_{ijt}^* + \mathbb{E}_t \sum_i s_{ij} \frac{\varphi_{ij}(1 - \beta\lambda_j)(1 - \lambda_j)}{(1 + \varphi_{ij})} \hat{P}_{jt+\tau} \end{aligned}$$

Rearranging the above equation, we obtain a second order difference equation for $\mathbb{E}_t \hat{P}_{jt+\tau+1}$, $\mathbb{E}_t \hat{P}_{jt+\tau}$, and $\mathbb{E}_t \hat{P}_{jt+\tau-1}$ with the exogenous (shock) term given by $\sum_i s_{ij} \frac{(1 - \lambda_j)(1 - \beta\lambda_j \rho_j)}{1 + \varphi_{ij}} \rho_j^\tau \hat{Q}_{ijt}^*$. Solving the dynamics gives

$$\mathbb{E}_t \hat{P}_{jt+\tau} = \frac{\rho_j^{\tau+1} - \Lambda_j^{\tau+1}}{\rho_j(1 - b_j) + \lambda_j[\beta\rho_j(\lambda_j - \rho_j) - 1]} \frac{1 - \beta\lambda_j \rho_j}{1 - \beta\lambda_j} a_j \hat{Q}_{jt}^* \quad \forall \tau \geq 0, \quad (\text{D.11})$$

where Λ_j , a_j , and b_j are defined in (D.4), (D.5), (D.6), respectively.

Finally, plugging the expected sector prices back into the firm's reset price (D.10) and solving the geometric series gives

$$\hat{P}_{ijt}^* = \frac{1}{1 + \varphi_{ij}} \hat{Q}_{ijt}^* + \frac{\varphi_{ij}}{1 + \varphi_{ij}} \frac{\rho_j - \Lambda_j}{1 - \beta\lambda_j \Lambda_j} \varkappa_j \hat{Q}_{jt}^*, \quad (\text{D.12})$$

■

D.3.3 Extension: Two underlying shock processes

In this subsection, we show that a similar expression to (D.9) can be obtained when there are two underlying shock processes. Specifically, we allow for two different AR(1) processes for the common $\hat{Q}_{jt}^C$ and idiosyncratic $\hat{Q}_{ijt}^I$ components of the cost process $\hat{Q}_{ijt}^*$, and arbitrary serial

correlations in the residual terms ϵ_{jt} and ϵ_{ijt} of these two AR(1) processes:

$$\begin{aligned}\widehat{Q}_{ijt}^* &= \widehat{Q}_{ijt}^I + \widehat{Q}_{jt}^C \\ \widehat{Q}_{ijt}^I &= \rho_j^I \widehat{Q}_{ijt-1}^I + \epsilon_{ijt} \\ \widehat{Q}_{jt}^C &= \rho_j^C \widehat{Q}_{jt-1}^C + \epsilon_{jt}\end{aligned}$$

Note that, due to the limited number of firms in a sector, the idiosyncratic shocks ϵ_{ijt} may not sum to zero. As a result, we need to keep track of the evolution of both idiosyncratic and common shocks in deriving the expected change in sector prices. Under these conditions, the expression for expected sector price becomes more complicated:

$$\mathbb{E}_t \widehat{P}_{jt+\tau} = \frac{\Lambda_{1,j} \Lambda_{2,j}}{\lambda_j (\Lambda_{1,j} - \rho_j^I) (\Lambda_{2,j} - \rho_j^I) (\Lambda_{1,j} - \rho_j^C) (\Lambda_{2,j} - \rho_j^C)} K_{jt+\tau} \quad (\text{D.13})$$

with

$$\begin{aligned}K_{jt+\tau} \equiv & -d_j \widehat{Q}_{jt}^C (\rho_j^C)^{\tau+1} (\Lambda_{1,j} - \rho_j^I) (\Lambda_{2,j} - \rho_j^I) - a_j \widehat{Q}_{jt}^I (\rho_j^I)^{\tau+1} (\Lambda_{1,j} - \rho_j^C) (\Lambda_{2,j} - \rho_j^C) \\ & + \Lambda_{1,j}^{\tau+1} \left[d_j \widehat{Q}_{jt}^C (\Lambda_{1,j} - \rho_j^I) (\Lambda_{2,j} - \rho_j^I) + a_j \widehat{Q}_{jt}^I (\Lambda_{1,j} - \rho_j^C) (\Lambda_{2,j} - \rho_j^C) \right]\end{aligned}$$

where $\Lambda_{1,j} \equiv \Lambda_j$ is the stable root from (D.4), b_j is given by (D.6), and

$$\begin{aligned}\Lambda_{2,j} &\equiv \frac{1}{2} \left[\lambda_j + \frac{1-b_j}{\beta \lambda_j} + \sqrt{\left(\lambda_j + \frac{1-b_j}{\beta \lambda_j} \right)^2 - \frac{4}{\beta}} \right] \quad (\text{unstable root}), \\ a_j &\equiv \left(\sum_i \frac{(1-\beta \lambda_j)(1-\lambda_j)}{(1+\varphi_{ij})} s_{ij} \widehat{Q}_{ijt}^I \right) / \widehat{Q}_{jt}^I \quad \text{with} \quad \widehat{Q}_{jt}^I \equiv \sum_i s_{ij} \widehat{Q}_{ijt}^I, \\ d_j &\equiv \sum_i \frac{(1-\beta \lambda_j)(1-\lambda_j)}{(1+\varphi_{ij})} s_{ij}.\end{aligned}$$

Plugging (D.13) into equation (7), we can solve the optimal reset price as

$$\begin{aligned}\widehat{P}_{ijt}^* &= \frac{1-\beta \lambda_j}{(1-\beta \lambda_j \rho_j^I)(1+\varphi_{ij})} (\widehat{Q}_{ijt}^I - \widehat{Q}_{jt}^I) + \left[\frac{1-\beta \lambda_j}{(1-\beta \lambda_j \rho_j^I)(1+\varphi_{ij})} + \frac{\varphi_{ij}}{1+\varphi_{ij}} \mathcal{H}_j(\rho_j^I) a_j \right] \widehat{Q}_{jt}^I \\ &+ \left[\frac{1-\beta \lambda_j}{(1-\beta \lambda_j \rho_j^C)(1+\varphi_{ij})} + \frac{\varphi_{ij}}{1+\varphi_{ij}} \mathcal{H}_j(\rho_j^C) d_j \right] \widehat{Q}_{jt}^C\end{aligned} \quad (\text{D.14})$$

where $\mathcal{H}_j(\rho)$ is the *two-shock feedback function* defined for any persistence ρ :

$$\mathcal{H}_j(\rho) \equiv \frac{\Lambda_{1,j} \Lambda_{2,j} (1-\beta \lambda_j)}{\lambda_j (1-\Lambda_{1,j} \beta \lambda_j) (\Lambda_{2,j} - \rho) (1-\beta \lambda_j \rho)}. \quad (\text{D.15})$$

Note: $\mathcal{H}_j(\rho)$ differs from the future-reset feedback function $H_j(\tau)$ (defined in Section D.4), which takes a horizon τ as its argument. The function $\mathcal{H}_j(\rho)$ handles two shocks with potentially different

persistence, whereas $H_j(\tau)$ tracks how a single shock propagates across future price resets. When $\rho = \rho_j$ and the shock is common to all firms (so that $a_j = d_j$), the two-shock expression reduces to the single-shock case involving $\varkappa_j$. Therefore, as in the single shock case, the optimal reset price response to the cost shocks can be decomposed into idiosyncratic (the first term of (D.14)) and common (the second and third terms of (D.14)) components. Note that the solution holds for any arbitrary realization of $\{\epsilon_{ijt}\}$ and ϵ_{jt} , and does not require the shocks to be independent.

D.4 Closed-form solutions for future reset prices

This section derives closed-form expressions for future reset prices $\hat{P}_{ijt+\tau}^*$ conditional on the shock history through period t and no shocks after t . We present the two key results—for flexible costs and synchronized sticky costs—as formal propositions with complete proofs. We re-use the definitions of Λ_j , a_j , b_j , and $\varkappa_j$ from Section D.3, and introduce the shorthand D_j and the feedback function $H_j(\tau)$.

Notation. Recall Λ_j , a_j , b_j from (D.4)–(D.6), and $\varkappa_j$ from (D.8). Define the shorthand functions:

$$D_j \equiv \rho_j(1 - b_j) + \lambda_j[\beta\rho_j(\lambda_j - \rho_j) - 1], \quad (\text{D.16})$$

$$\psi_{ij}(\rho_j) \equiv \frac{1}{1 + \varphi_{ij}} \cdot \frac{1 - \beta\lambda_j}{1 - \beta\lambda_j\rho_j},$$

$$H_j(\tau) \equiv (1 - \beta\lambda_j) \varkappa_j \left[\frac{\rho_j^{\tau+1}}{1 - \beta\lambda_j\rho_j} - \frac{\Lambda_j^{\tau+1}}{1 - \beta\lambda_j\Lambda_j} \right], \quad (\text{D.17})$$

$$\tilde{H}_j(\tau) \equiv (1 - \beta\lambda_j\rho_j) \varkappa_j \left[\frac{\rho_j^{\tau+1}}{1 - \beta\lambda_j\rho_j} - \frac{\Lambda_j^{\tau+1}}{1 - \beta\lambda_j\Lambda_j} \right]. \quad (\text{D.18})$$

Note that $\varkappa_j = a_j/D_j$ by definition. We will also use the root identity

$$b_j = \frac{(\Lambda_j - \lambda_j)(1 - \beta\lambda_j\Lambda_j)}{\Lambda_j},$$

which follows from the sectoral price difference equation.

Under Calvo pricing, the expected sector price satisfies

$$\mathbb{E}_t \hat{P}_{jt+\tau} = (1 - \lambda_j) \mathbb{E}_t \hat{P}_{jt+\tau}^* + \lambda_j \mathbb{E}_t \hat{P}_{jt+\tau-1}.$$

Rearranging gives

$$\mathbb{E}_t \hat{P}_{jt+\tau}^* = \frac{\mathbb{E}_t \hat{P}_{jt+\tau} - \lambda_j \mathbb{E}_t \hat{P}_{jt+\tau-1}}{1 - \lambda_j}. \quad (\text{D.19})$$

Recall from (D.3) that

$$\mathbb{E}_t \hat{P}_{jt+\tau} = \frac{\rho_j^{\tau+1} - \Lambda_j^{\tau+1}}{D_j} a_j \hat{Q}_{jt}^* = (\rho_j^{\tau+1} - \Lambda_j^{\tau+1}) \varkappa_j \hat{Q}_{jt}^*.$$

Using $\varkappa_j = a_j/D_j$, $\mathbb{E}_t \hat{P}_{jt+\tau}^*$ can be expressed compactly as

$$\mathbb{E}_t \hat{P}_{jt+\tau}^* = \varkappa_j \left[\frac{\rho_j - \lambda_j}{1 - \lambda_j} \rho_j^\tau - \frac{\Lambda_j - \lambda_j}{1 - \lambda_j} \Lambda_j^\tau \right] \hat{Q}_{jt}^*. \quad (\text{D.20})$$

D.4.1 Future reset price: flexible costs

Proposition D.1 (Future Reset Price: Flexible Costs). *Under flexible costs, where the firm's cost follows an AR(1) process $\mathbb{E}_t \hat{Q}_{ijt+\tau+h}^* = \rho_j^{\tau+h} \hat{Q}_{ijt}^*$ for all $h \geq 0$, the firm-level future reset price at horizon τ is*

$$\hat{P}_{ijt+\tau}^* = \underbrace{\psi_{ij}(\rho_j) \rho_j^\tau (\hat{Q}_{ijt}^* - \hat{Q}_{jt}^*)}_{\text{idiosyncratic component}} + \underbrace{\left\{ \psi_{ij}(\rho_j) \rho_j^\tau + \frac{\varphi_{ij}}{1 + \varphi_{ij}} H_j(\tau) \right\}}_{\substack{\text{direct cost} \\ \text{feedback via } \hat{P}_j}} \hat{Q}_{jt}^*, \quad (\text{D.21})$$

where $\psi_{ij}(\rho_j) \equiv \frac{1}{1 + \varphi_{ij}} \cdot \frac{1 - \beta \lambda_j}{1 - \beta \lambda_j \rho_j}$ and $H_j(\tau)$ is defined in (D.17). Setting $\tau = 0$ recovers Proposition 1.

Proof. At $t + \tau$, the firm's reset price (evaluated at t) satisfies the granular Phillips curve:

$$\hat{P}_{ijt+\tau}^* = \frac{1 - \beta \lambda_j}{1 + \varphi_{ij}} \sum_{h \geq 0} (\beta \lambda_j)^h \left[\mathbb{E}_t \hat{Q}_{ijt+\tau+h}^* + \varphi_{ij} \mathbb{E}_t \hat{P}_{jt+\tau+h} \right]. \quad (\text{D.22})$$

Step 1: Cost decomposition. Write $\mathbb{E}_t \hat{Q}_{ijt+\tau+h}^* = \rho_j^{\tau+h} (\hat{Q}_{ijt}^* - \hat{Q}_{jt}^*) + \rho_j^{\tau+h} \hat{Q}_{jt}^*$.

Step 2: Idiosyncratic term. Evaluate the geometric sum:

$$\frac{1 - \beta \lambda_j}{1 + \varphi_{ij}} \sum_{h \geq 0} (\beta \lambda_j)^h \rho_j^{\tau+h} (\hat{Q}_{ijt}^* - \hat{Q}_{jt}^*) = \psi_{ij}(\rho_j) \rho_j^\tau (\hat{Q}_{ijt}^* - \hat{Q}_{jt}^*).$$

Step 3: Direct cost term. From the $\hat{Q}_{jt}^*$ component:

$$\frac{1 - \beta \lambda_j}{1 + \varphi_{ij}} \sum_{h \geq 0} (\beta \lambda_j)^h \rho_j^{\tau+h} \hat{Q}_{jt}^* = \psi_{ij}(\rho_j) \rho_j^\tau \hat{Q}_{jt}^*.$$

Step 4: Feedback via sector prices. Using (D.3),

$$\sum_{h \geq 0} (\beta \lambda_j)^h \mathbb{E}_t \hat{P}_{jt+\tau+h} = \frac{a_j \hat{Q}_{jt}^*}{D_j} \left[\frac{\rho_j^{\tau+1}}{1 - \beta \lambda_j \rho_j} - \frac{\Lambda_j^{\tau+1}}{1 - \beta \lambda_j \Lambda_j} \right].$$

Multiplying by $\frac{1-\beta\lambda_j}{1+\varphi_{ij}} \varphi_{ij}$ and using $\varkappa_j = a_j/D_j$ gives

$$\frac{1-\beta\lambda_j}{1+\varphi_{ij}} \varphi_{ij} \sum_{h \geq 0} (\beta\lambda_j)^h \mathbb{E}_t \hat{P}_{jt+\tau+h} = \frac{\varphi_{ij}}{1+\varphi_{ij}} H_j(\tau) \hat{Q}_{jt}^*.$$

Step 5: Combine. Adding Steps 2–4 yields (D.21).

Step 6: Verification at $\tau = 0$. The direct-cost term becomes $\psi_{ij}(\rho_j) \hat{Q}_{jt}^*$, and the feedback term equals

$$\frac{\varphi_{ij}}{1+\varphi_{ij}} (1-\beta\lambda_j) \varkappa_j \left[\frac{\rho_j}{1-\beta\lambda_j\rho_j} - \frac{\Lambda_j}{1-\beta\lambda_j\Lambda_j} \right] \hat{Q}_{jt}^* = \frac{\varphi_{ij}}{1+\varphi_{ij}} \frac{1-\beta\lambda_j}{1-\beta\lambda_j\rho_j} \varkappa_j \frac{\rho_j - \Lambda_j}{1-\beta\lambda_j\Lambda_j} \hat{Q}_{jt}^*,$$

using the identity $\frac{\rho_j}{1-\beta\lambda_j\rho_j} - \frac{\Lambda_j}{1-\beta\lambda_j\Lambda_j} = \frac{\rho_j - \Lambda_j}{(1-\beta\lambda_j\rho_j)(1-\beta\lambda_j\Lambda_j)}$. Collecting terms recovers Proposition 1. $\square$

Consistency: sector vs. firm averages. Aggregating (D.21) across i with weights s_{ij} yields

$$\mathbb{E}_t \sum_i s_{ij} \hat{P}_{ijt+\tau}^* = \frac{a_j}{1-\lambda_j} \frac{\rho_j^\tau}{1-\beta\lambda_j\rho_j} \hat{Q}_{jt}^* + \frac{a_j b_j}{(1-\lambda_j) D_j} \left[\frac{\rho_j^{\tau+1}}{1-\beta\lambda_j\rho_j} - \frac{\Lambda_j^{\tau+1}}{1-\beta\lambda_j\Lambda_j} \right] \hat{Q}_{jt}^*. \quad (\text{D.23})$$

Using (i) $D_j + b_j\rho_j = (\rho_j - \lambda_j)(1 - \beta\lambda_j\rho_j)$ and (ii) $\frac{b_j\Lambda_j}{1-\beta\lambda_j\Lambda_j} = \Lambda_j - \lambda_j$ (from $b_j = \frac{(\Lambda_j - \lambda_j)(1 - \beta\lambda_j\Lambda_j)}{\Lambda_j}$), this simplifies to the sector expression in (D.20).

D.4.2 Future reset price: synchronized sticky costs

Proposition D.2 (Future Reset Price: Synchronized Sticky Costs). *When producer and distributor adjustments are synchronized (same Poisson rate λ_j), so that the firm's cost relevant for its price spell remains constant at its reset-time value—i.e., the cost that enters the pricing decision at reset time $t + \tau$ is $\mathbb{E}_t \hat{Q}_{ijt+\tau}^* = \rho_j^\tau \hat{Q}_{ijt}^*$, which the firm expects to remain unchanged throughout that spell—the firm-level future reset price at horizon τ is*

$$\hat{P}_{ijt+\tau}^* = \underbrace{\frac{1}{1+\varphi_{ij}} \rho_j^\tau (\hat{Q}_{ijt}^* - \hat{Q}_{jt}^*)}_{\text{idiosyncratic component}} + \underbrace{\left\{ \underbrace{\frac{1}{1+\varphi_{ij}} \rho_j^\tau}_{\text{direct common}} + \underbrace{\frac{\varphi_{ij}}{1+\varphi_{ij}} \varkappa_j \left[\rho_j^{\tau+1} - \frac{1-\beta\lambda_j\rho_j}{1-\beta\lambda_j\Lambda_j} \Lambda_j^{\tau+1} \right]}_{\text{feedback via } \hat{P}_{j,\cdot}} \right\}}_{\text{feedback via } \hat{P}_{j,\cdot}} \hat{Q}_{jt}^*. \quad (\text{D.24})$$

Setting $\tau = 0$ recovers Proposition 2.

Proof. At future reset time $t + \tau$, the firm's cost (in calendar time) is $\mathbb{E}_t \hat{Q}_{ijt+\tau}^* = \rho_j^\tau \hat{Q}_{ijt}^*$. Under synchronization, this cost stays constant within the price spell starting at $t + \tau$: the firm expects its cost to remain at $\rho_j^\tau \hat{Q}_{ijt}^*$ for all $h \geq 0$ periods while its price is fixed. Meanwhile, the sector's average cost continues to evolve as AR(1) in calendar time: $\mathbb{E}_t \hat{Q}_{jt+\tau+h}^* = \rho_j^{\tau+h} \hat{Q}_{jt}^*$.

Step 1: Cost decomposition. Under synchronization, the firm's cost relevant for pricing is $\mathbb{E}_t \widehat{Q}_{ijt+\tau}^* = \rho_j^\tau \widehat{Q}_{ijt}^*$, which stays constant for all h . Decompose this into sector and firm-level components:

$$\mathbb{E}_t \widehat{Q}_{ijt+\tau}^* = \underbrace{\mathbb{E}_t \widehat{Q}_{jt+\tau+h}^*}_{\text{sector AR(1)}} + \underbrace{(\mathbb{E}_t \widehat{Q}_{ijt+\tau}^* - \mathbb{E}_t \widehat{Q}_{jt+\tau+h}^*)}_{\text{firm-level offset}}.$$

Step 2: Sector component sum. Using $\mathbb{E}_t \widehat{Q}_{jt+\tau+h}^* = \rho_j^{\tau+h} \widehat{Q}_{jt}^*$:

$$\sum_{h \geq 0} (\beta \lambda_j)^h \mathbb{E}_t \widehat{Q}_{jt+\tau+h}^* = \rho_j^\tau \widehat{Q}_{jt}^* \sum_{h \geq 0} (\beta \lambda_j \rho_j)^h = \frac{\rho_j^\tau}{1 - \beta \lambda_j \rho_j} \widehat{Q}_{jt}^*.$$

Step 3: Firm-level offset sum. Using $\mathbb{E}_t \widehat{Q}_{ijt+\tau}^* = \rho_j^\tau \widehat{Q}_{ijt}^*$:

$$\begin{aligned} \sum_{h \geq 0} (\beta \lambda_j)^h (\mathbb{E}_t \widehat{Q}_{ijt+\tau}^* - \mathbb{E}_t \widehat{Q}_{jt+\tau+h}^*) &= \sum_{h \geq 0} (\beta \lambda_j)^h (\rho_j^\tau \widehat{Q}_{ijt}^* - \rho_j^{\tau+h} \widehat{Q}_{jt}^*) \\ &= \frac{\rho_j^\tau}{1 - \beta \lambda_j} \widehat{Q}_{ijt}^* - \frac{\rho_j^\tau}{1 - \beta \lambda_j \rho_j} \widehat{Q}_{jt}^*. \end{aligned}$$

Step 4: Combine cost sums. Adding Steps 2 and 3, the common $\frac{\rho_j^\tau}{1 - \beta \lambda_j \rho_j} \widehat{Q}_{jt}^*$ cancels:

$$\sum_{h \geq 0} (\beta \lambda_j)^h \mathbb{E}_t \widehat{Q}_{ijt+\tau}^* = \frac{\rho_j^\tau}{1 - \beta \lambda_j} \widehat{Q}_{ijt}^*.$$

Multiplying by $\frac{1 - \beta \lambda_j}{1 + \varphi_{ij}}$ gives the direct contribution $\frac{1}{1 + \varphi_{ij}} \rho_j^\tau \widehat{Q}_{ijt}^*$.

Step 5: Split idiosyncratic and common. Add and subtract $\frac{1}{1 + \varphi_{ij}} \rho_j^\tau \widehat{Q}_{jt}^*$:

$$\frac{1}{1 + \varphi_{ij}} \rho_j^\tau \widehat{Q}_{ijt}^* = \underbrace{\frac{1}{1 + \varphi_{ij}} \rho_j^\tau (\widehat{Q}_{ijt}^* - \widehat{Q}_{jt}^*)}_{\text{idiosyncratic}} + \underbrace{\frac{1}{1 + \varphi_{ij}} \rho_j^\tau \widehat{Q}_{jt}^*}_{\text{direct common}}.$$

Step 6: Feedback sum. Under synchronization, the sector price path is given by (D.11):

$$\mathbb{E}_t \widehat{P}_{jt+\tau+h} = (\rho_j^{\tau+h+1} - \Lambda_j^{\tau+h+1}) \varkappa_j \frac{1 - \beta \lambda_j \rho_j}{1 - \beta \lambda_j} \widehat{Q}_{jt}^*.$$

Summing over h :

$$\sum_{h \geq 0} (\beta \lambda_j)^h \mathbb{E}_t \widehat{P}_{jt+\tau+h} = \varkappa_j \frac{1 - \beta \lambda_j \rho_j}{1 - \beta \lambda_j} \left[\frac{\rho_j^{\tau+1}}{1 - \beta \lambda_j \rho_j} - \frac{\Lambda_j^{\tau+1}}{1 - \beta \lambda_j \Lambda_j} \right] \widehat{Q}_{jt}^*.$$

Multiplying by $\frac{1-\beta\lambda_j}{1+\varphi_{ij}} \varphi_{ij}$, the $(1 - \beta\lambda_j)$ factors cancel, yielding:

$$\frac{\varphi_{ij}}{1+\varphi_{ij}} \varkappa_j (1 - \beta\lambda_j \rho_j) \left[\frac{\rho_j^{\tau+1}}{1 - \beta\lambda_j \rho_j} - \frac{\Lambda_j^{\tau+1}}{1 - \beta\lambda_j \Lambda_j} \right] \hat{Q}_{jt}^* = \frac{\varphi_{ij}}{1+\varphi_{ij}} \varkappa_j \left[\rho_j^{\tau+1} - \frac{(1 - \beta\lambda_j \rho_j) \Lambda_j^{\tau+1}}{1 - \beta\lambda_j \Lambda_j} \right] \hat{Q}_{jt}^*.$$

Step 7: Combine. Adding Steps 5 and 6 yields (D.24). For $\tau = 0$, the feedback term simplifies:

$$\rho_j - \frac{(1 - \beta\lambda_j \rho_j) \Lambda_j}{1 - \beta\lambda_j \Lambda_j} = \frac{\rho_j(1 - \beta\lambda_j \Lambda_j) - (1 - \beta\lambda_j \rho_j) \Lambda_j}{1 - \beta\lambda_j \Lambda_j} = \frac{\rho_j - \Lambda_j}{1 - \beta\lambda_j \Lambda_j},$$

recovering Proposition 2. □

Comparison with flexible costs. Both the idiosyncratic and common pass-through coefficients differ between synchronized and flexible costs.

- *Idiosyncratic:* Under synchronization, the coefficient is $\rho_j^\tau / (1 + \varphi_{ij})$, while under flexible costs it is $\psi_{ij}(\rho_j) \rho_j^\tau$. These differ by a factor of $(1 - \beta\lambda_j) / (1 - \beta\lambda_j \rho_j)$, reflecting the fact that under synchronization the firm's cost relevant for pricing remains at its reset-time value throughout the price spell.
- *Common:* The direct common term is $\rho_j^\tau / (1 + \varphi_{ij})$ under synchronization versus $\psi_{ij}(\rho_j) \rho_j^\tau$ under flexible costs—again differing by $(1 - \beta\lambda_j) / (1 - \beta\lambda_j \rho_j)$. The feedback terms also differ: under synchronization the sector price formula (D.11) has an extra factor $(1 - \beta\lambda_j \rho_j) / (1 - \beta\lambda_j)$ relative to the flexible case (D.3). This factor cancels with $(1 - \beta\lambda_j)$ from the granular Phillips curve, producing the simpler feedback expression in (D.24).

Consistency: sector vs. firm averages. The synchronized feedback function $\tilde{H}_j(\tau)$ from (D.18) can be equivalently written as

$$\tilde{H}_j(\tau) = (1 - \beta\lambda_j \rho_j) \varkappa_j \left[\frac{\rho_j^{\tau+1}}{1 - \beta\lambda_j \rho_j} - \frac{\Lambda_j^{\tau+1}}{1 - \beta\lambda_j \Lambda_j} \right].$$

Aggregating (D.24) across i with weights s_{ij} and using the definitions of a_j and b_j yields

$$\begin{aligned} \mathbb{E}_t \sum_i s_{ij} \hat{P}_{ijt+\tau}^* &= \rho_j^\tau \sum_i s_{ij} \frac{\hat{Q}_{ijt}^*}{1 + \varphi_{ij}} + \tilde{H}_j(\tau) \sum_i s_{ij} \frac{\varphi_{ij}}{1 + \varphi_{ij}} \hat{Q}_{jt}^* \\ &= \frac{a_j}{(1 - \beta\lambda_j)(1 - \lambda_j)} \rho_j^\tau \hat{Q}_{jt}^* + \frac{b_j}{(1 - \beta\lambda_j)(1 - \lambda_j)} \tilde{H}_j(\tau) \hat{Q}_{jt}^*. \end{aligned}$$

Substituting the definition of $\tilde{H}_j(\tau)$ and simplifying delivers the sector closed form consistent with the synchronized sector price (D.11).

Remark: Generalization. The results above assume a standard AR(1) shock process where costs decay at rate ρ_j from the moment of impact. Section D.5 extends these formulas to a more

general shock path $\tilde{q}(\tau; x, \rho)$ (defined in Section C.1) that allows the initial drop x to differ from the long-run persistence ρ . Setting $x_j = \rho_j$ in the generalized formulas recovers exactly the propositions above and Propositions 1–2 from the main paper.

D.5 Extension with a more general shock process

This section extends the closed-form solutions of Section D.4 to a case with the generalized shock path $\tilde{q}(\tau; x, \rho)$ introduced in Section C.1. Section C.2 uses these formulas to construct theoretical pass-through predictions and compare them with empirical estimates.

Notation. Let firm i in sector j receive a cost shock $\hat{Q}_{ijt}^*$ at date t . Using the sector-specific notation $\tilde{q}_j(\tau) \equiv \tilde{q}(\tau; x_j, \rho_j)$ from (C.3), the expected cost at horizon τ evolves as

$$\mathbb{E}_t \hat{Q}_{ijt+\tau}^* = \hat{Q}_{ijt}^* \tilde{q}_j(\tau), \quad \text{where } \tilde{q}_j(\tau) = \begin{cases} 1 & \tau = 0 \\ x_j \rho_j^{\tau-1} & \tau \geq 1. \end{cases}$$

The drop parameter $x_j \in (0, 1]$ governs immediate pass-through in period 1, while ρ_j governs subsequent AR(1) decay. When $x_j = \rho_j$, this reduces to the standard AR(1) case $\tilde{q}_j(\tau) = \rho_j^\tau$, and the formulas below recover those in Section D.4.

Define the sector average cost shock as $\hat{Q}_{jt}^* \equiv \sum_k s_{kj} \hat{Q}_{kjt}^*$, so that

$$\mathbb{E}_t \hat{Q}_{jt+\tau}^* = \hat{Q}_{jt}^* \tilde{q}_j(\tau).$$

We retain the definitions from Section D.3: Λ_j from (D.4), a_j and b_j from (D.5)–(D.6), and $\varkappa_j$ from (D.8). We also use D_j from (D.16) in Section D.4. Define the auxiliary quantity

$$E_j \equiv (1 - b_j) + \beta \lambda_j (\lambda_j - \rho_j). \quad (\text{D.25})$$

Note the identity $\rho_j E_j = D_j + \lambda_j$, which we will use in derivations below.

D.5.1 Future reset price with flexible costs

Before deriving firm-level reset prices, we solve for the sector price path $\mathbb{E}_t \hat{P}_{jt+\tau}$ under the generalized shock. The sector price satisfies the second-order difference equation (from (10)):

$$(1 - b_j + \beta \lambda_j^2) \mathbb{E}_t \hat{P}_{jt+\tau} - \lambda_j \mathbb{E}_t \hat{P}_{jt+\tau-1} - \beta \lambda_j \mathbb{E}_t \hat{P}_{jt+\tau+1} = a_j \mathbb{E}_t \hat{Q}_{jt+\tau}^*, \quad (\text{D.26})$$

with forcing $a_j \tilde{q}_j(\tau) \hat{Q}_{jt}^*$ and initial condition $\mathbb{E}_t \hat{P}_{jt-1} = 0$.

Lemma D.1 (Sector Price under Generalized Shock: Flexible Costs). *The sector price path under the generalized shock is*

$$\mathbb{E}_t \hat{P}_{jt+\tau} = \varkappa_j \left[x_j \rho_j^\tau + \Xi_j(x_j) \Lambda_j^{\tau+1} \right] \hat{Q}_{jt}^*, \quad \tau \geq 0, \quad (\text{D.27})$$

where

$$\Xi_j(x_j) \equiv \frac{\rho_j D_j - x_j(D_j + \lambda_j)}{\rho_j \lambda_j} = \frac{D_j - x_j E_j}{\lambda_j}. \quad (\text{D.28})$$

When $x_j = \rho_j$, we have $\Xi_j(\rho_j) = -1$, and (D.27) reduces to $\mathbb{E}_t \hat{P}_{jt+\tau} = \varkappa_j(\rho_j^{\tau+1} - \Lambda_j^{\tau+1}) \hat{Q}_{jt}^*$, recovering (D.3).

Proof. The general solution to (D.26) is the sum of a particular solution and the homogeneous solution $C\Lambda_j^\tau$ (the unstable root is excluded by transversality).

Step 1: Particular solution for $\tau \geq 1$. For $\tau \geq 1$, the forcing is $a_j x_j \rho_j^{\tau-1} \hat{Q}_{jt}^*$. Try $\mathbb{E}_t \hat{P}_{jt+\tau}^{(p)} = c \rho_j^{\tau-1}$. Substituting:

$$(1 - b_j + \beta \lambda_j^2) c \rho_j^{\tau-1} - \lambda_j c \rho_j^{\tau-2} - \beta \lambda_j c \rho_j^\tau = a_j x_j \rho_j^{\tau-1}.$$

Dividing by $\rho_j^{\tau-2}$:

$$\begin{aligned} c[(1 - b_j)\rho_j + \beta \lambda_j^2 \rho_j - \lambda_j - \beta \lambda_j \rho_j^2] &= a_j x_j \rho_j, \\ c[(1 - b_j)\rho_j + \lambda_j(\beta \rho_j(\lambda_j - \rho_j) - 1)] &= a_j x_j \rho_j, \\ c \cdot D_j &= a_j x_j \rho_j. \end{aligned}$$

Since $\varkappa_j = a_j/D_j$, we have $c = x_j \rho_j \varkappa_j$, giving $\mathbb{E}_t \hat{P}_{jt+\tau}^{(p)} = x_j \varkappa_j \rho_j^\tau$ for $\tau \geq 1$.

Step 2: General solution. Write $\mathbb{E}_t \hat{P}_{jt+\tau} = x_j \varkappa_j \rho_j^\tau + A \Lambda_j^\tau$ for $\tau \geq 0$, where A is determined by the $\tau = 0$ equation.

Step 3: Determine A from $\tau = 0$. Substituting into (D.26) at $\tau = 0$ (with $\mathbb{E}_t \hat{P}_{jt-1} = 0$):

$$(1 - b_j + \beta \lambda_j^2)(x_j \varkappa_j + A) - \beta \lambda_j(x_j \varkappa_j \rho_j + A \Lambda_j) = a_j.$$

Using $a_j = D_j \varkappa_j$ and collecting terms:

$$x_j \varkappa_j [(1 - b_j + \beta \lambda_j^2) - \beta \lambda_j \rho_j] + A [(1 - b_j + \beta \lambda_j^2) - \beta \lambda_j \Lambda_j] = D_j \varkappa_j.$$

The coefficient of A simplifies using the characteristic equation: $(1 - b_j + \beta \lambda_j^2) - \beta \lambda_j \Lambda_j = \lambda_j/\Lambda_j$. The coefficient of $x_j \varkappa_j$ equals $(1 - b_j) + \beta \lambda_j(\lambda_j - \rho_j) = E_j$. Thus:

$$x_j E_j \varkappa_j + A \lambda_j / \Lambda_j = D_j \varkappa_j \quad \Rightarrow \quad A = \frac{\Lambda_j \varkappa_j}{\lambda_j} (D_j - x_j E_j) = \Xi_j(x_j) \Lambda_j \varkappa_j.$$

Step 4: Final expression. The sector price is $\mathbb{E}_t \hat{P}_{jt+\tau} = x_j \varkappa_j \rho_j^\tau + \Xi_j(x_j) \Lambda_j \varkappa_j \Lambda_j^\tau = \varkappa_j [x_j \rho_j^\tau + \Xi_j(x_j) \Lambda_j^{\tau+1}]$.

Step 5: Verify baseline case. When $x_j = \rho_j$: $\Xi_j(\rho_j) = (D_j - \rho_j E_j)/\lambda_j = (D_j - D_j - \lambda_j)/\lambda_j = -1$, so $\mathbb{E}_t \hat{P}_{jt+\tau} = \varkappa_j(\rho_j^{\tau+1} - \Lambda_j^{\tau+1})$, matching (D.3). $\square$

Feedback function. Using Lemma D.1, the geometric sum of sector prices is:

$$\sum_{h \geq 0} (\beta \lambda_j)^h \mathbb{E}_t \hat{P}_{jt+\tau+h} = \varkappa_j \left[x_j \frac{\rho_j^\tau}{1 - \beta \lambda_j \rho_j} + \Xi_j(x_j) \frac{\Lambda_j^{\tau+1}}{1 - \beta \lambda_j \Lambda_j} \right] \hat{Q}_{jt}^*. \quad (\text{D.29})$$

Define the generalized feedback function:

$$H_j(\tau; x_j) \equiv (1 - \beta \lambda_j) \varkappa_j \left[x_j \mathcal{G}_{j,\rho}(\tau) + \Xi_j(x_j) \Lambda_j \mathcal{G}_{j,\Lambda}(\tau) \right], \quad (\text{D.30})$$

where $\mathcal{G}_{j,\rho}(\tau) \equiv \rho_j^\tau / (1 - \beta \lambda_j \rho_j)$ and $\mathcal{G}_{j,\Lambda}(\tau) \equiv \Lambda_j^\tau / (1 - \beta \lambda_j \Lambda_j)$. When $x_j = \rho_j$, this reduces to $H_j(\tau)$ from (D.17).

Proposition D.3 (Future Reset Price with Generalized Shock: Flexible Costs). *Under flexible costs with the generalized shock path $\tilde{q}_j(\tau) = \tilde{q}(\tau; x_j, \rho_j)$, the firm-level future reset price at horizon $\tau \geq 0$ is*

$$\hat{P}_{ijt+\tau}^* = \underbrace{\psi_{ij,\tau}^{\text{flex}} (\hat{Q}_{ijt}^* - \hat{Q}_{jt}^*)}_{\text{idiosyncratic response}} + \underbrace{\Psi_{ij,\tau}^{\text{flex}} \hat{Q}_{jt}^*}_{\text{common response}}, \quad (\text{D.31})$$

where the idiosyncratic pass-through coefficient is

$$\psi_{ij,\tau}^{\text{flex}} \equiv \frac{1 - \beta \lambda_j}{1 + \varphi_{ij}} \cdot \begin{cases} \frac{1 - \beta \lambda_j \rho_j + \beta \lambda_j x_j}{1 - \beta \lambda_j \rho_j} & \tau = 0, \\ \frac{\tilde{q}_j(\tau)}{1 - \beta \lambda_j \rho_j} & \tau \geq 1, \end{cases} \quad (\text{D.32})$$

and the common pass-through coefficient is

$$\Psi_{ij,\tau}^{\text{flex}} \equiv \psi_{ij,\tau}^{\text{flex}} + \frac{\varphi_{ij}}{1 + \varphi_{ij}} H_j(\tau; x_j). \quad (\text{D.33})$$

Setting $x_j = \rho_j$ recovers Proposition D.1. Setting $\tau = 0$ and $x_j = \rho_j$ recovers Proposition 1.

Proof. Start from the granular reset-pricing condition:

$$\hat{P}_{ijt+\tau}^* = \frac{1 - \beta \lambda_j}{1 + \varphi_{ij}} \sum_{h \geq 0} (\beta \lambda_j)^h \left[\mathbb{E}_t \hat{Q}_{ijt+\tau+h}^* + \varphi_{ij} \mathbb{E}_t \hat{P}_{jt+\tau+h} \right]. \quad (\text{D.34})$$

Step 1: Cost path. Under flexible costs, firm i 's cost continues to evolve after time t :

$$\mathbb{E}_t \hat{Q}_{ijt+\tau+h}^* = \hat{Q}_{ijt}^* \tilde{q}_j(\tau + h) = \hat{Q}_{ijt}^* x_j \rho_j^{\tau+h-1} \quad \text{for } \tau + h \geq 1.$$

Step 2: Geometric sum of cost path. For $\tau \geq 1$:

$$\sum_{h \geq 0} (\beta \lambda_j)^h \tilde{q}_j(\tau + h) = \sum_{h \geq 0} (\beta \lambda_j)^h x_j \rho_j^{\tau+h-1} = x_j \rho_j^{\tau-1} \sum_{h \geq 0} (\beta \lambda_j \rho_j)^h = \frac{x_j \rho_j^{\tau-1}}{1 - \beta \lambda_j \rho_j} = \frac{\tilde{q}_j(\tau)}{1 - \beta \lambda_j \rho_j}.$$

For $\tau = 0$, the sequence is $\{1, x_j, x_j \rho_j, x_j \rho_j^2, \dots\}$, so:

$$\sum_{h \geq 0} (\beta \lambda_j)^h \tilde{q}_j(h) = 1 + \beta \lambda_j x_j \sum_{k \geq 0} (\beta \lambda_j \rho_j)^k = 1 + \frac{\beta \lambda_j x_j}{1 - \beta \lambda_j \rho_j} = \frac{1 - \beta \lambda_j \rho_j + \beta \lambda_j x_j}{1 - \beta \lambda_j \rho_j}.$$

Step 3: Idiosyncratic term. The idiosyncratic component $(\hat{Q}_{ijt}^* - \hat{Q}_{jt}^*)$ enters only through own costs. Combining with Step 2, the coefficient is $\psi_{ij,\tau}^{\text{flex}}$ as given in (D.32).

Step 4: Direct common term. From the $\hat{Q}_{jt}^*$ component of own costs, the geometric sum from Step 2 gives $\psi_{ij,\tau}^{\text{flex}} \hat{Q}_{jt}^*$.

Step 5: Feedback via sector prices. Using (D.29) and (D.30):

$$\frac{(1 - \beta \lambda_j) \varphi_{ij}}{1 + \varphi_{ij}} \sum_{h \geq 0} (\beta \lambda_j)^h \mathbb{E}_t \hat{P}_{jt+\tau+h} = \frac{\varphi_{ij}}{1 + \varphi_{ij}} H_j(\tau; x_j) \hat{Q}_{jt}^*.$$

Step 6: Combine. Adding Steps 3–5: $\hat{P}_{ijt+\tau}^* = \psi_{ij,\tau}^{\text{flex}} (\hat{Q}_{ijt}^* - \hat{Q}_{jt}^*) + \psi_{ij,\tau}^{\text{flex}} \hat{Q}_{jt}^* + \frac{\varphi_{ij}}{1 + \varphi_{ij}} H_j(\tau; x_j) \hat{Q}_{jt}^*$, yielding (D.31)–(D.33). $\square$

Connection to baseline. When $x_j = \rho_j$ (AR(1) case), we have $\tilde{q}_j(\tau) = \rho_j^\tau$ and $\Xi_j(\rho_j) = -1$. The idiosyncratic coefficient becomes $\psi_{ij,\tau}^{\text{flex}} = \psi_{ij}(\rho_j) \rho_j^\tau$ where $\psi_{ij}(\rho_j) = \frac{1 - \beta \lambda_j}{(1 + \varphi_{ij})(1 - \beta \lambda_j \rho_j)}$, recovering Proposition D.1. Substituting into (D.30):

$$H_j(\tau; \rho_j) = (1 - \beta \lambda_j) \varkappa_j \left[\rho_j \mathcal{G}_{j,\rho}(\tau) - \Lambda_j \mathcal{G}_{j,\Lambda}(\tau) \right] = H_j(\tau),$$

which recovers exactly the baseline feedback function $H_j(\tau)$ from (D.17).

D.5.2 Future reset price with synchronized sticky costs

Under synchronization, the sector difference equation has the same left-hand side as (D.26) but a different forcing term. As derived in Section D.3 (see equations around (D.11)), under synchronization each firm's cost stays constant within its price spell. The sector NKPC forcing involves the term $\hat{Q}_{jt+\tau}^* - \beta \lambda_j \hat{Q}_{jt+\tau+1}^*$:

$$(1 - b_j + \beta \lambda_j^2) \mathbb{E}_t \hat{P}_{jt+\tau}^{\text{sync}} - \lambda_j \mathbb{E}_t \hat{P}_{jt+\tau-1}^{\text{sync}} - \beta \lambda_j \mathbb{E}_t \hat{P}_{jt+\tau+1}^{\text{sync}} = \frac{a_j}{1 - \beta \lambda_j} \left[\mathbb{E}_t \hat{Q}_{jt+\tau}^* - \beta \lambda_j \mathbb{E}_t \hat{Q}_{jt+\tau+1}^* \right]. \quad (\text{D.35})$$

Under the generalized shock $\tilde{q}_j(\tau)$, the forcing simplifies to $\frac{a_j(1 - \beta \lambda_j \rho_j)}{1 - \beta \lambda_j} \tilde{q}_j(\tau) \hat{Q}_{jt}^*$ for $\tau \geq 1$, but at $\tau = 0$ the forcing is $\frac{a_j(1 - \beta \lambda_j x_j)}{1 - \beta \lambda_j} \hat{Q}_{jt}^*$ (since $\tilde{q}_j(1)/\tilde{q}_j(0) = x_j$, not ρ_j).

Lemma D.2 (Sector Price under Generalized Shock: Synchronized Costs). *The synchronized sector price path under the generalized shock is*

$$\mathbb{E}_t \hat{P}_{jt+\tau}^{\text{sync}} = \varkappa_j \left[\frac{1 - \beta \lambda_j \rho_j}{1 - \beta \lambda_j} x_j \rho_j^\tau + \frac{\tilde{\Xi}_j(x_j)}{1 - \beta \lambda_j} \Lambda_j^{\tau+1} \right] \hat{Q}_{jt}^*, \quad \tau \geq 0, \quad (\text{D.36})$$

where

$$\tilde{\Xi}_j(x_j) \equiv \Xi_j(x_j) + \beta\lambda_j x_j = \frac{D_j - x_j E_j + \beta\lambda_j^2 x_j}{\lambda_j}. \quad (\text{D.37})$$

When $x_j = \rho_j$, we have $\tilde{\Xi}_j(\rho_j) = -1 + \beta\lambda_j \rho_j = -(1 - \beta\lambda_j \rho_j)$, and (D.36) reduces to (D.11).

Proof. The proof parallels that of Lemma D.1, but requires careful treatment of the forcing at $\tau = 0$.

Step 1: Forcing term structure. Under the generalized shock, the synchronized forcing involves $\hat{Q}_{jt+\tau}^* - \beta\lambda_j \hat{Q}_{jt+\tau+1}^* = [\tilde{q}_j(\tau) - \beta\lambda_j \tilde{q}_j(\tau+1)]\hat{Q}_{jt}^*$. Since $\tilde{q}_j(\tau+1)/\tilde{q}_j(\tau) = \rho_j$ for $\tau \geq 1$ but $\tilde{q}_j(1)/\tilde{q}_j(0) = x_j$:

- For $\tau \geq 1$: forcing = $\frac{a_j(1-\beta\lambda_j\rho_j)}{1-\beta\lambda_j} \tilde{q}_j(\tau) \hat{Q}_{jt}^*$
- For $\tau = 0$: forcing = $\frac{a_j(1-\beta\lambda_j x_j)}{1-\beta\lambda_j} \hat{Q}_{jt}^*$

Step 2: Particular solution. For $\tau \geq 1$, the forcing is $a_j \frac{1-\beta\lambda_j\rho_j}{1-\beta\lambda_j} x_j \rho_j^{\tau-1} \hat{Q}_{jt}^*$. Following the same algebra as in Lemma D.1, the particular solution is $\mathbb{E}_t \hat{P}_{jt+\tau}^{sync, (p)} = \frac{1-\beta\lambda_j\rho_j}{1-\beta\lambda_j} x_j \varkappa_j \rho_j^\tau \hat{Q}_{jt}^*$.

Step 3: General solution. Write $\mathbb{E}_t \hat{P}_{jt+\tau}^{sync} = \left(\frac{1-\beta\lambda_j\rho_j}{1-\beta\lambda_j} x_j \varkappa_j \rho_j^\tau + A^{sync} \Lambda_j^\tau \right) \hat{Q}_{jt}^*$ for $\tau \geq 0$.

Step 4: Determine A^{sync} from $\tau = 0$. At $\tau = 0$, the forcing is $\frac{a_j(1-\beta\lambda_j x_j)}{1-\beta\lambda_j}$ (not $\frac{a_j(1-\beta\lambda_j\rho_j)}{1-\beta\lambda_j}$). Substituting:

$$(1 - b_j + \beta\lambda_j^2) \left(\frac{1-\beta\lambda_j\rho_j}{1-\beta\lambda_j} x_j \varkappa_j + A^{sync} \right) - \beta\lambda_j \left(\frac{1-\beta\lambda_j\rho_j}{1-\beta\lambda_j} x_j \varkappa_j \rho_j + A^{sync} \Lambda_j \right) = \frac{1-\beta\lambda_j x_j}{1-\beta\lambda_j} a_j.$$

Using $a_j = D_j \varkappa_j$, the characteristic equation $(1 - b_j + \beta\lambda_j^2) - \beta\lambda_j \Lambda_j = \lambda_j / \Lambda_j$, and $(1 - b_j + \beta\lambda_j^2) - \beta\lambda_j \rho_j = E_j$:

$$\frac{1-\beta\lambda_j\rho_j}{1-\beta\lambda_j} x_j \varkappa_j E_j + A^{sync} \frac{\lambda_j}{\Lambda_j} = \frac{1-\beta\lambda_j x_j}{1-\beta\lambda_j} D_j \varkappa_j.$$

Solving for A^{sync} and using the identity $D_j = \rho_j E_j - \lambda_j$:

$$\begin{aligned} A^{sync} &= \frac{\Lambda_j \varkappa_j}{\lambda_j(1-\beta\lambda_j)} \left[(1 - \beta\lambda_j x_j) D_j - (1 - \beta\lambda_j \rho_j) x_j E_j \right] \\ &= \frac{\Lambda_j \varkappa_j}{\lambda_j(1-\beta\lambda_j)} \left[D_j - x_j E_j + \beta\lambda_j x_j (\rho_j E_j - D_j) \right] \\ &= \frac{\Lambda_j \varkappa_j}{\lambda_j(1-\beta\lambda_j)} \left[D_j - x_j E_j + \beta\lambda_j^2 x_j \right] = \frac{\tilde{\Xi}_j(x_j) \varkappa_j \Lambda_j}{1-\beta\lambda_j}. \end{aligned}$$

Step 5: Final expression. The sector price is:

$$\mathbb{E}_t \hat{P}_{jt+\tau}^{sync} = \varkappa_j \left[\frac{1-\beta\lambda_j\rho_j}{1-\beta\lambda_j} x_j \rho_j^\tau + \frac{\tilde{\Xi}_j(x_j)}{1-\beta\lambda_j} \Lambda_j^{\tau+1} \right] \hat{Q}_{jt}^*.$$

Step 6: Verify baseline case. When $x_j = \rho_j$, we have $\tilde{\Xi}_j(\rho_j) = \Xi_j(\rho_j) + \beta\lambda_j \rho_j = -1 + \beta\lambda_j \rho_j = -(1 - \beta\lambda_j \rho_j)$, so

$$\mathbb{E}_t \hat{P}_{jt+\tau}^{sync} = \varkappa_j \left[\frac{1-\beta\lambda_j\rho_j}{1-\beta\lambda_j} \rho_j^{\tau+1} - \frac{1-\beta\lambda_j\rho_j}{1-\beta\lambda_j} \Lambda_j^{\tau+1} \right] \hat{Q}_{jt}^* = \frac{1-\beta\lambda_j\rho_j}{1-\beta\lambda_j} \varkappa_j (\rho_j^{\tau+1} - \Lambda_j^{\tau+1}) \hat{Q}_{jt}^*,$$

matching (D.11). □

Comparison: Flexible vs. Synchronized sector prices. Comparing Lemmas D.1 and D.2:

- *Flexible:* $\mathbb{E}_t \hat{P}_{jt+\tau} = \varkappa_j [x_j \rho_j^\tau + \Xi_j(x_j) \Lambda_j^{\tau+1}] \hat{Q}_{jt}^*$
- *Synchronized:* $\mathbb{E}_t \hat{P}_{jt+\tau}^{sync} = \varkappa_j \left[\frac{1-\beta\lambda_j\rho_j}{1-\beta\lambda_j} x_j \rho_j^\tau + \frac{\tilde{\Xi}_j(x_j)}{1-\beta\lambda_j} \Lambda_j^{\tau+1} \right] \hat{Q}_{jt}^*$

The synchronized and flexible formulas differ in their treatment of the homogeneous term coefficient. Under the generalized shock, the synchronized forcing at $\tau = 0$ involves $(1 - \beta\lambda_j x_j)$ rather than $(1 - \beta\lambda_j \rho_j)$, which leads to the modified coefficient $\tilde{\Xi}_j(x_j) = \Xi_j(x_j) + \beta\lambda_j x_j$. When $x_j = \rho_j$, we have $\tilde{\Xi}_j(\rho_j) = -(1 - \beta\lambda_j \rho_j)$, and the synchronized formula reduces to $(1 - \beta\lambda_j \rho_j)/(1 - \beta\lambda_j)$ times the flexible formula.

Synchronized feedback function. Using (D.36), the geometric sum of synchronized sector prices is:

$$\begin{aligned} \sum_{h \geq 0} (\beta\lambda_j)^h \mathbb{E}_t \hat{P}_{jt+\tau+h}^{sync} &= \varkappa_j \left[\frac{1 - \beta\lambda_j \rho_j}{1 - \beta\lambda_j} \frac{x_j \rho_j^\tau}{1 - \beta\lambda_j \rho_j} + \frac{\tilde{\Xi}_j(x_j)}{1 - \beta\lambda_j} \frac{\Lambda_j^{\tau+1}}{1 - \beta\lambda_j \Lambda_j} \right] \hat{Q}_{jt}^* \\ &= \frac{\varkappa_j}{1 - \beta\lambda_j} \left[x_j \rho_j^\tau + \tilde{\Xi}_j(x_j) \frac{\Lambda_j^{\tau+1}}{1 - \beta\lambda_j \Lambda_j} \right] \hat{Q}_{jt}^*. \end{aligned}$$

Multiplying by $(1 - \beta\lambda_j)$, the factor in the denominator cancels. Define the synchronized feedback function:

$$\tilde{H}_j(\tau; x_j) \equiv \varkappa_j \left[x_j \rho_j^\tau + \tilde{\Xi}_j(x_j) \frac{\Lambda_j^{\tau+1}}{1 - \beta\lambda_j \Lambda_j} \right], \quad (\text{D.38})$$

where (as defined in (D.37))

$$\tilde{\Xi}_j(x_j) \equiv \Xi_j(x_j) + \beta\lambda_j x_j = \frac{D_j - x_j E_j + \beta\lambda_j^2 x_j}{\lambda_j}. \quad (\text{D.39})$$

Then:

$$\frac{(1 - \beta\lambda_j)\varphi_{ij}}{1 + \varphi_{ij}} \sum_{h \geq 0} (\beta\lambda_j)^h \mathbb{E}_t \hat{P}_{jt+\tau+h}^{sync} = \frac{\varphi_{ij}}{1 + \varphi_{ij}} \tilde{H}_j(\tau; x_j) \hat{Q}_{jt}^*. \quad (\text{D.40})$$

When $x_j = \rho_j$, we have $\tilde{\Xi}_j(\rho_j) = \Xi_j(\rho_j) + \beta\lambda_j \rho_j = -1 + \beta\lambda_j \rho_j = -(1 - \beta\lambda_j \rho_j)$, and

$$\begin{aligned} \tilde{H}_j(\tau; \rho_j) &= \varkappa_j \left[\rho_j^{\tau+1} - (1 - \beta\lambda_j \rho_j) \frac{\Lambda_j^{\tau+1}}{1 - \beta\lambda_j \Lambda_j} \right] \\ &= (1 - \beta\lambda_j \rho_j) \varkappa_j \left[\frac{\rho_j^{\tau+1}}{1 - \beta\lambda_j \rho_j} - \frac{\Lambda_j^{\tau+1}}{1 - \beta\lambda_j \Lambda_j} \right], \end{aligned}$$

matching $\tilde{H}_j(\tau)$ from (D.18).

Proposition D.4 (Future Reset Price with Generalized Shock: Synchronized Sticky Costs). *When producer and distributor adjustments are synchronized, so that the firm's cost relevant for pricing at reset time $t + \tau$ —namely $\hat{Q}_{ijt+\tau}^* = \hat{Q}_{ijt}^* \tilde{q}_j(\tau)$ —remains constant throughout that price spell, the firm-level future reset price at horizon τ is*

$$\hat{P}_{ijt+\tau}^* = \underbrace{\psi_{ij,\tau}^{\text{sync}} (\hat{Q}_{ijt}^* - \hat{Q}_{jt}^*)}_{\text{idiosyncratic response}} + \underbrace{\Psi_{ij,\tau}^{\text{sync}} \hat{Q}_{jt}^*}_{\text{common response}}, \quad (\text{D.41})$$

where the idiosyncratic pass-through coefficient is

$$\psi_{ij,\tau}^{\text{sync}} \equiv \frac{1}{1 + \varphi_{ij}} \tilde{q}_j(\tau), \quad (\text{D.42})$$

and the common pass-through coefficient is

$$\Psi_{ij,\tau}^{\text{sync}} \equiv \psi_{ij,\tau}^{\text{sync}} + \frac{\varphi_{ij}}{1 + \varphi_{ij}} \tilde{H}_j(\tau; x_j), \quad (\text{D.43})$$

where $\tilde{H}_j(\tau; x_j)$ is the synchronized feedback function from (D.38). Setting $x_j = \rho_j$ recovers Proposition D.2. Setting $\tau = 0$ recovers Proposition 2.

Proof. Under synchronization, when the firm resets its price at time $t + \tau$, its cost relevant for pricing— $\hat{Q}_{ijt+\tau}^*$ —remains constant throughout that price spell. The granular condition becomes:

$$\hat{P}_{ijt+\tau}^* = \frac{1}{1 + \varphi_{ij}} \hat{Q}_{ijt+\tau}^* + \frac{\varphi_{ij}}{1 + \varphi_{ij}} (1 - \beta \lambda_j) \sum_{h \geq 0} (\beta \lambda_j)^h \mathbb{E}_t \hat{P}_{jt+\tau+h}.$$

Step 1: Constant cost within spell. Under synchronization, the firm's cost stays at its reset-time value $\hat{Q}_{ijt+\tau}^*$ for all $h \geq 0$ periods while its price is fixed. Thus:

$$\frac{1 - \beta \lambda_j}{1 + \varphi_{ij}} \sum_{h \geq 0} (\beta \lambda_j)^h \hat{Q}_{ijt+\tau}^* = \frac{1 - \beta \lambda_j}{1 + \varphi_{ij}} \cdot \frac{\hat{Q}_{ijt+\tau}^*}{1 - \beta \lambda_j} = \frac{1}{1 + \varphi_{ij}} \hat{Q}_{ijt+\tau}^*.$$

Step 2: Direct cost term. Using $\hat{Q}_{ijt+\tau}^* = \hat{Q}_{ijt}^* \tilde{q}_j(\tau)$:

$$\frac{1}{1 + \varphi_{ij}} \hat{Q}_{ijt+\tau}^* = \frac{1}{1 + \varphi_{ij}} \tilde{q}_j(\tau) \hat{Q}_{ijt}^*.$$

Step 3: Split idiosyncratic and common. Add and subtract $\frac{1}{1 + \varphi_{ij}} \tilde{q}_j(\tau) \hat{Q}_{jt}^*$:

$$\frac{1}{1 + \varphi_{ij}} \tilde{q}_j(\tau) \hat{Q}_{ijt}^* = \underbrace{\frac{1}{1 + \varphi_{ij}} \tilde{q}_j(\tau) (\hat{Q}_{ijt}^* - \hat{Q}_{jt}^*)}_{\psi_{ij,\tau}^{\text{sync}} (\hat{Q}_{ijt}^* - \hat{Q}_{jt}^*)} + \underbrace{\frac{1}{1 + \varphi_{ij}} \tilde{q}_j(\tau) \hat{Q}_{jt}^*}_{\psi_{ij,\tau}^{\text{sync}} \hat{Q}_{jt}^*}.$$

Step 4: Feedback term. Under synchronization, the sector price follows (D.36) rather than the

flexible-cost path. By (D.40), the feedback via synchronized sector prices contributes:

$$\frac{(1 - \beta\lambda_j)\varphi_{ij}}{1 + \varphi_{ij}} \sum_{h \geq 0} (\beta\lambda_j)^h \mathbb{E}_t \hat{P}_{jt+\tau+h}^{sync} = \frac{\varphi_{ij}}{1 + \varphi_{ij}} \tilde{H}_j(\tau; x_j) \hat{Q}_{jt}^*.$$

Step 5: Combine. Adding Steps 3 and 4 yields (D.41)–(D.43). $\square$

Comparison with flexible costs. Both the idiosyncratic and feedback components differ between flexible and synchronized cost cases:

- *Idiosyncratic:* Under flexible costs, $\psi_{ij,\tau}^{\text{flex}}$ includes the factor $(1 - \beta\lambda_j)/(1 - \beta\lambda_j\rho_j)$. Under synchronization, $\psi_{ij,\tau}^{\text{sync}} = \frac{1}{1 + \varphi_{ij}} \tilde{q}_j(\tau)$ is simpler because the firm's cost relevant for pricing stays at its reset-time value throughout the price spell.
- *Feedback:* The feedback functions $H_j(\tau; x_j)$ (flexible) and $\tilde{H}_j(\tau; x_j)$ (synchronized) differ because the sector price dynamics differ. Under synchronization, the sector price (D.36) has an extra factor $(1 - \beta\lambda_j\rho_j)/(1 - \beta\lambda_j)$ relative to the flexible case (D.27). This factor cancels with $(1 - \beta\lambda_j)$ from the granular Phillips curve, producing the simpler synchronized feedback (D.38) without the $(1 - \beta\lambda_j)$ prefactor.

Summary of pass-through coefficients. Dropping sector subscripts for notational clarity, the idiosyncratic pass-through coefficients under the generalized shock process $\tilde{q}(\tau; x, \rho)$ are:

$$\text{Synchronized:} \quad \psi_{\tau}^{\text{sync}}(x, \rho) = \frac{1}{1 + \varphi} \tilde{q}(\tau; x, \rho), \quad (\text{D.44})$$

$$\text{Flexible } (\tau \geq 1): \quad \psi_{\tau}^{\text{flex}}(x, \rho) = \frac{1 - \beta\lambda}{(1 + \varphi)(1 - \beta\lambda\rho)} \tilde{q}(\tau; x, \rho), \quad (\text{D.45})$$

$$\text{Flexible } (\tau = 0): \quad \psi_0^{\text{flex}}(x, \rho) = \frac{1 - \beta\lambda}{1 + \varphi} \cdot \frac{1 - \beta\lambda\rho + \beta\lambda x}{1 - \beta\lambda\rho}. \quad (\text{D.46})$$

The common pass-through coefficients are:

$$\text{Synchronized:} \quad \Psi_{\tau}^{\text{sync}}(x, \rho) = \psi_{\tau}^{\text{sync}}(x, \rho) + \frac{\varphi}{1 + \varphi} \tilde{H}(\tau; x, \rho), \quad (\text{D.47})$$

$$\text{Flexible:} \quad \Psi_{\tau}^{\text{flex}}(x, \rho) = \psi_{\tau}^{\text{flex}}(x, \rho) + \frac{\varphi}{1 + \varphi} H(\tau; x, \rho). \quad (\text{D.48})$$

The feedback functions are:

$$\text{Flexible:} \quad H(\tau; x, \rho) = (1 - \beta\lambda) \varkappa \left[\frac{x\rho^{\tau}}{1 - \beta\lambda\rho} + \Xi(x) \frac{\Lambda^{\tau+1}}{1 - \beta\lambda\Lambda} \right], \quad (\text{D.49})$$

$$\text{Synchronized:} \quad \tilde{H}(\tau; x, \rho) = \varkappa \left[x\rho^{\tau} + \tilde{\Xi}(x) \frac{\Lambda^{\tau+1}}{1 - \beta\lambda\Lambda} \right], \quad (\text{D.50})$$

where $\varkappa = a/D$ and

$$\Xi(x) = \frac{D - xE}{\lambda}, \quad \tilde{\Xi}(x) = \Xi(x) + \beta\lambda x = \frac{D - xE + \beta\lambda^2 x}{\lambda}.$$

The auxiliary functions are:

$$\begin{aligned} b(\lambda, \varphi) &= \frac{\varphi(1 - \beta\lambda)(1 - \lambda)}{1 + \varphi}, & a(\lambda, \varphi) &= \frac{(1 - \beta\lambda)(1 - \lambda)}{1 + \varphi}, \\ \Lambda(\lambda, \varphi) &= \frac{1}{2} \left[\lambda + \frac{1 - b}{\beta\lambda} - \sqrt{\left(\lambda + \frac{1 - b}{\beta\lambda} \right)^2 - \frac{4}{\beta}} \right], \\ D(\lambda, \varphi, \rho) &= \rho(1 - b) + \lambda[\beta\rho(\lambda - \rho) - 1], & E(\lambda, \varphi, \rho) &= (1 - b) + \beta\lambda(\lambda - \rho). \end{aligned}$$

Setting $x = \rho$ recovers the baseline AR(1) formulas from Section D.4. When $x = \rho$, we have $\Xi(\rho) = -1$ and $\tilde{\Xi}(\rho) = -(1 - \beta\lambda\rho)$. The empirical analysis in Section C.2 uses the synchronized formulas (D.44) and (D.47).

Figure D1 illustrates how the drop parameter x and persistence ρ shape pass-through dynamics. Panel (a) displays the shock path $\tilde{q}(\tau; x, \rho)$ from (C.3): the standard AR(1) case ($x = \rho = 0.9$, blue) decays smoothly, while $x < \rho$ (green) produces a sharp initial drop followed by slower decay. Panels (b) and (c) plot the synchronized pass-through coefficients ψ_τ^{sync} and Ψ_τ^{sync} from (D.44)–(D.47). Panel (b) shows idiosyncratic pass-through $\psi_\tau^{\text{sync}} = \frac{1}{1+\varphi}\tilde{q}(\tau)$, which is proportional to the shock path since idiosyncratic shocks do not trigger strategic feedback. Panel (c) shows common pass-through Ψ_τ^{sync} , which additionally includes the synchronized feedback term $\frac{\varphi}{1+\varphi}\tilde{H}(\tau; x)$ from competitors' price responses. The key takeaway is that allowing $x \neq \rho$ generates pass-through with decreasing rate of decay, which appears to be the case in the data.

D.6 “Feedback” and “strategic” effects

Our theoretical solutions account for both the “feedback” and “strategic” effects defined by Wang and Werning (2022) (WW). Specifically, WW define the feedback effect as those channels that already exist in static models (channels i and ii mentioned below) and the strategic effect as the additional effect due to the fact that each firm internalizes that, in the words of WW, “its current pricing decision can affect how its rivals will set their prices in the future.” In this appendix, we discuss the difference between these two effects through the lens of our discrete time oligopolistic competition Calvo model.

To fix ideas, it is useful to start by thinking about how equilibrium is reached in a static version of the model (i.e., where firm optimization considers only the current period). Consider a two-period problem: the firms start in their steady state in period t , and there are small shocks to the marginal costs of the firms $\{\widehat{M}_{ijt+1}\}_{i=1}^{N_j}$ in $t + 1$.

In the static setting, the model can be solved fully non-linearly separately for each period (i.e.,

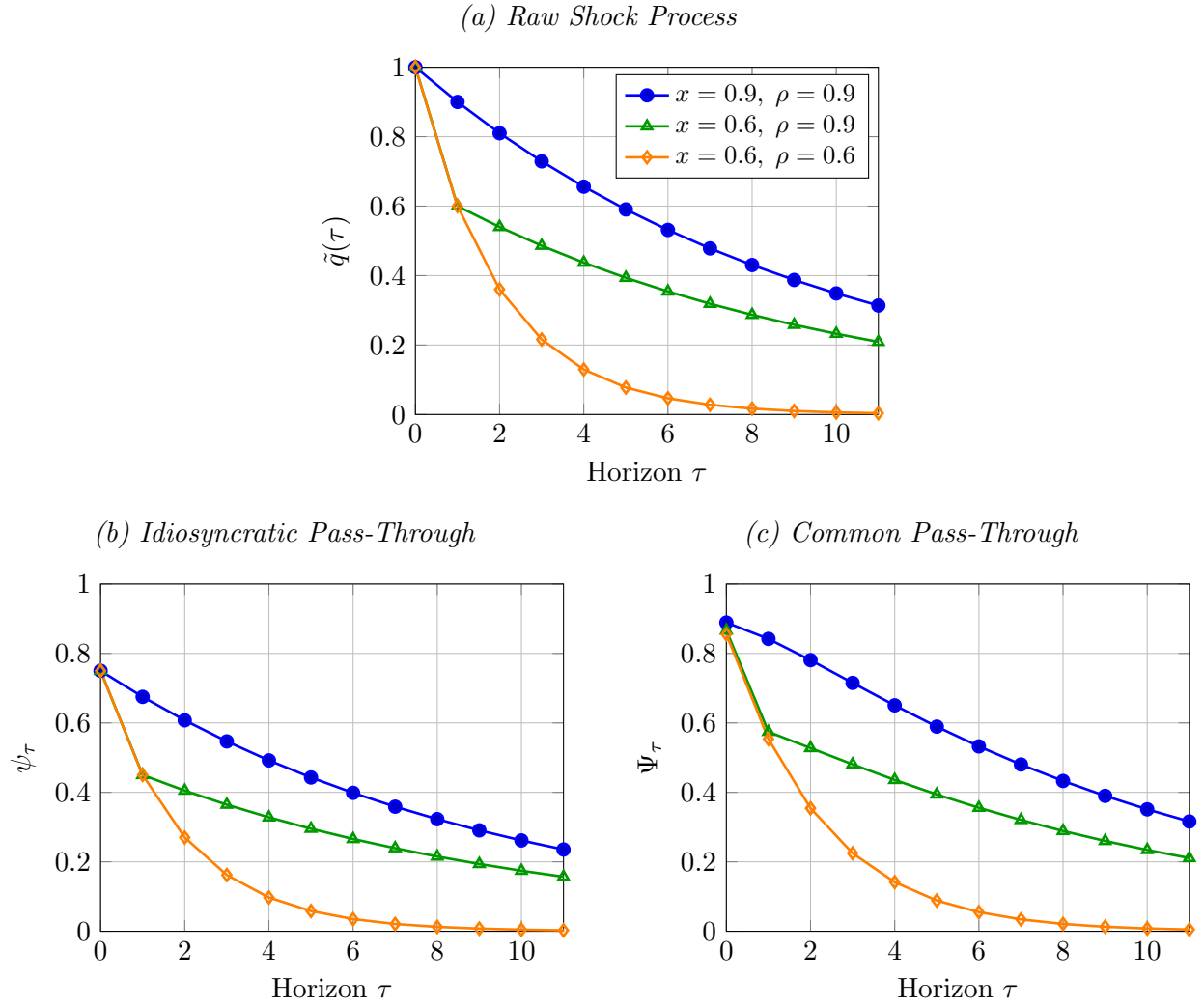


Figure D1: Shock and pass-through dynamics for alternative (x, ρ) combinations.

t and $t + 1$) by solving a fixed point problem that involves a system of first order equations:

$$P_{ijt} = \frac{\vartheta_{ijt}(\{P_{kjt}\}_{k=1}^{N_j})}{\vartheta_{ijt}(\{P_{kjt}\}_{k=1}^{N_j}) - 1} M_{ijt} \quad \forall i = 1, \dots, N_j, \quad (\text{D.51})$$

where P_{ijt} in equation (D.51) is the best-response of firm i for the given marginal cost M_{ijt} and competitors' prices $\{P_{kjt}\}_{k \neq i}$. The Nash equilibrium is formed when no firm wants to deviate given the set of prices $(\{P_{kjt}\}_{k=1}^{N_j})$ and the underlying costs $(\{M_{kjt}\}_{k=1}^{N_j})$ in the industry. It is worth noting that this equilibrium accounts for the fact that

- (i) a firm's pricing decision affects its competitors pricing decisions, i.e., P_{ijt} appears in the best responses of P_{kjt} ; and
- (ii) the firm's competitors' pricing decisions affect its price, i.e., P_{ijt} is a function of $\{P_{kjt}\}_{k \neq i}$ as shown in (D.51).

It is worth noting that taking first order approximations does not affect the properties of the solution. Approximating the solution around the market structure at t , we have the following expression for the firm's optimal markup

$$\hat{\mu}_{ijt+1} \approx -\frac{1}{\vartheta_{ijt} - 1} \hat{\vartheta}_{ijt+1} = -\varphi_{ijt} [\hat{P}_{ijt+1} - \hat{P}_{jt+1}].$$

The log-linearized system of equations in (D.51) is given by

$$\hat{P}_{ijt+1} = \widehat{M}_{ijt+1} - \varphi_{ijt} [\hat{P}_{ijt+1} - \hat{P}_{jt+1}] = \widehat{M}_{ijt+1} - \varphi_{ijt} \left[\hat{P}_{ijt+1} - \sum_k s_{kjt} \hat{P}_{kjt+1} \right] \quad \forall i = 1, \dots, N_j. \quad (\text{D.52})$$

We see (D.52) has the same property of (D.51), i.e., it accounts for both (i) and (ii). The only difference is that, with the log-linear approximation, we can solve the system of equations in closed form.

The “Naïve” model in static setting. WW illustrate the difference between the “feedback” effect and the “strategic effect” by specifying a “Naïve” model where only the “feedback” effect is present. To discuss the key assumptions made by WW, we start with the “Naïve” version of the static model described above. To facilitate the comparison of our results with those of WW, we analyze the case of symmetric firms and common cost shocks (i.e., $\widehat{M}_{kjt+1} = \widehat{M}_{ijt+1} = \widehat{M}_{jt+1} = 1$). Under these assumptions, we can rewrite the first order conditions in (D.52) as

$$\hat{P}_{ijt+1} = \frac{1}{1 + \varphi_{jt}} + \frac{\varphi_{jt}}{1 + \varphi_{jt}} \hat{P}_{jt+1} \quad \forall i = 1, \dots, N_j. \quad (\text{D.53})$$

The key assumption made by WW is that the deviation of the firm's optimal price from its new steady state price is a linear function of its competitors' deviation from their new steady state prices. Applying this assumption in the static model, and integrating the fact that pass through

to a common cost shock is 100%, results in

$$\hat{P}_{ijt+1} - 1 = b \sum_{k \neq i} (\hat{P}_{kjt+1} - 1) \quad \forall i = 1, \dots, N_j, \quad (\text{D.54})$$

where b captures how a firm responds to its competitors' prices.

Summing (D.54) over firms, we have

$$\hat{P}_{jt+1} - 1 = b(N_j - 1) (\hat{P}_{jt+1} - 1).$$

Solving for $\hat{P}_{jt+1}$ gives $\hat{P}_{jt+1} = 1$. Substituting back to (D.53), we have $\hat{P}_{ijt+1} = 1 \forall i$.

The solution also implies a response slope of $B \equiv b(N_j - 1) = \varphi_{jt}/(1 + \varphi_{jt})$ as

$$\hat{P}_{ijt+1} = \frac{1}{1 + \varphi_{jt}} + \frac{\varphi_{jt}}{1 + \varphi_{jt}} \hat{P}_{jt+1} = 1 - B + B \frac{1}{N_j - 1} \sum_{k \neq i} \hat{P}_{kjt+1} = 1 \quad \forall i = 1, \dots, N_j.$$

In this case, we have the same solution as static Nash because the assumption (D.54) does not impose any additional restriction in the static model.

The “Naïve” model in dynamic setting. Does the restriction in (D.54) make a difference in the dynamic version of the model? We proceed to derive price dynamics in the dynamic “Naïve” set up by WW. Our derivation here follows closely from WW, with the key difference being that our model features discrete time while theirs features continuous time.

As in the static case, we start with the first order conditions of the firms, which now depend on the full trajectory of future sector prices:

$$\begin{aligned} \hat{P}_{ijt}^* &= (1 - \beta\lambda) \sum_{\tau=0}^{\infty} (\beta\lambda)^\tau \mathbb{E}_t [\widehat{M}_{t+\tau} - \varphi(\hat{P}_{ijt}^* - \hat{P}_{jt+\tau})] \\ &= \frac{1 - \beta\lambda}{1 + \varphi} \sum_{\tau=0}^{\infty} (\beta\lambda)^\tau \mathbb{E}_t [\widehat{M}_{t+\tau} + \varphi \hat{P}_{jt+\tau}]. \end{aligned} \quad (\text{D.55})$$

We next analyze the expected change in the sector price following the assumption of WW, where a firm's reset price is a function of its competitors' prices. From Calvo, we have

$$\mathbb{E}_t \hat{P}_{kjt+\tau} = (1 - \lambda) \mathbb{E}_t \hat{P}_{kjt+\tau}^* + \lambda \hat{P}_{kjt+\tau-1} \quad (\text{D.56})$$

Imposing WW's restriction that a firm's reset price is a linear function of the sum of the deviations of its competitors' prices from their optimal steady state prices:

$$\mathbb{E}_t (\hat{P}_{kjt+\tau}^* - \hat{P}_{kjT}) = b \sum_{i \neq k} (\mathbb{E}_t \hat{P}_{ijt+\tau} - \hat{P}_{ijT}), \quad (\text{D.57})$$

we can rewrite (D.56) as:

$$\begin{aligned}
\mathbb{E}_t(\hat{P}_{kjt+\tau} - \hat{P}_{kjT}) &= (1 - \lambda)\mathbb{E}_t(\hat{P}_{kjt+\tau}^* - \hat{P}_{kjT}) + \lambda(\hat{P}_{kjt+\tau-1} - \hat{P}_{kjT}) \\
&= (1 - \lambda)b \sum_{i \neq k} (\mathbb{E}_t \hat{P}_{ijt+\tau} - \hat{P}_{ijT}) + \lambda(\hat{P}_{kjt+\tau-1} - \hat{P}_{kjT}) \\
&= (1 - \lambda)b \sum_{i \neq k} (\mathbb{E}_t \hat{P}_{ijt+\tau} - 1) + \lambda(\hat{P}_{kjt+\tau-1} - 1)
\end{aligned} \tag{D.58}$$

where $\hat{P}_{kjT}$ is the (log) change in the price in the new steady state at T (when all prices are fully adjusted) relative to the price in old steady state before the common cost shock at t . The second line imposes the restriction, with $b \equiv B/(N - 1)$ captures how a firm responds to its competitors' prices. The third line uses the fact that the flexible price pass-through of a common cost shock is 100%, i.e., $\hat{P}_{kjT} = 1$.

To obtain the expected sector price dynamics, we sum over the expected individual price dynamics:

$$\frac{1}{N} \sum_k \mathbb{E}_t \hat{P}_{kjt+\tau} - 1 = (1 - \lambda)b \frac{1}{N} \sum_k \sum_{l \neq k} (\mathbb{E}_t \hat{P}_{ljt+\tau} - 1) + \lambda \left(\frac{1}{N} \sum_k \mathbb{E}_t \hat{P}_{kjt+\tau-1} - 1 \right)$$

With the assumption of symmetric firms, $\frac{1}{N} \sum_k \mathbb{E}_t \hat{P}_{kjt+\tau} = \mathbb{E}_t \hat{P}_{jt+\tau}$ and

$$(\mathbb{E}_t \hat{P}_{jt+\tau} - 1) = (1 - \lambda)b(N - 1)(\mathbb{E}_t \hat{P}_{jt+\tau} - 1) + \lambda(\mathbb{E}_t \hat{P}_{jt+\tau-1} - 1).$$

Rearrange and get

$$\mathbb{E}_t \hat{P}_{jt+\tau} = \frac{(1 - \lambda)(1 - B)}{1 - (1 - \lambda)B} + \frac{\lambda}{1 - (1 - \lambda)B} \mathbb{E}_t \hat{P}_{jt+\tau-1}$$

Solving the dynamics gives

$$\mathbb{E}_t \hat{P}_{jt+\tau} = 1 - \left(\frac{\lambda}{1 - (1 - \lambda)B} \right)^{\tau+1} \equiv 1 - (\Lambda^{\text{Naïve}})^{\tau+1}, \tag{D.59}$$

where (D.59) corresponds to the third equation on page A.31 of WW's Online Appendix.

To complete the solution, we now solve for B . First, we substitute (D.59) into the first order condition (D.55). Solving the dynamics gives

$$\begin{aligned}
\hat{P}_{ijt}^* &= \frac{1}{1 + \varphi} + \frac{\varphi}{1 + \varphi} \left(1 - \frac{1 - \beta\lambda}{1 - \beta\lambda\Lambda^{\text{Naïve}}} \Lambda^{\text{Naïve}} \right) \\
&= 1 - \frac{\varphi}{1 + \varphi} \frac{1 - \beta\lambda}{1 - \beta\lambda\Lambda^{\text{Naïve}}} \Lambda^{\text{Naïve}}.
\end{aligned} \tag{D.60}$$

Second, we verify the reset price solution under assumption (D.57):

$$\hat{P}_{ijt}^* - 1 = b \sum_{k \neq i} (\mathbb{E}_t \hat{P}_{kjt} - 1) = B(\hat{P}_{jt} - 1) = -B\Lambda^{\text{Naïve}} \quad (\text{D.61})$$

Together, we get a system of two equations to solve for B and $\Lambda^{\text{Naïve}}$:

$$\begin{aligned} \Lambda^{\text{Naïve}} &= \frac{\lambda}{1 - (1 - \lambda)B} \\ B &= \frac{\varphi}{1 + \varphi} \frac{1 - \beta\lambda}{1 - \beta\lambda\Lambda^{\text{Naïve}}} \end{aligned}$$

Finally, solving B gives

$$B = \frac{1 + (2 - \lambda)\varphi - \beta\lambda(\varphi + \lambda) - \sqrt{(-1 + \beta\lambda^2 - 2\varphi + \lambda\varphi + \beta\lambda\varphi)^2 - 4(1 - \lambda)(1 + \varphi)(1 - \beta\lambda)\varphi}}{2(1 - \lambda)(1 + \varphi)}.$$

Special cases. First, without market power (i.e., $\varphi = 0$), we have $B = 0$ and $\Lambda^{\text{Naïve}} = \lambda$. Second, without price stickiness (i.e., $\lambda = 0$), we have $B = \varphi/(1 + \varphi)$ and $\Lambda^{\text{Naïve}} = 0$.

Comparison. Recall that, under our benchmark solution, the aggregate price dynamics in a model with homogeneous firms and sectors can be written as

$$\hat{P}_{t+\tau} - \hat{P}_{t+\tau-1} = (1 - \Lambda)\Lambda^\tau \quad \text{and} \quad \hat{P}_{t+\tau} = 1 - \Lambda^{\tau+1},$$

where

$$\Lambda \equiv \frac{1}{2} \left[\frac{1 + \lambda\varphi + \beta\lambda(\lambda + \varphi)}{\beta\lambda(1 + \varphi)} - \sqrt{\left(\frac{1 + \lambda\varphi + \beta\lambda(\lambda + \varphi)}{\beta\lambda(1 + \varphi)} \right)^2 - \frac{4}{\beta}} \right].$$

In the “Naïve” model, the price dynamics take the same form, with a different adjustment factor $\Lambda^{\text{Naïve}}$:

$$\hat{P}_{t+\tau} - \hat{P}_{t+\tau-1} = (1 - \Lambda^{\text{Naïve}})(\Lambda^{\text{Naïve}})^\tau \quad \text{and} \quad \hat{P}_{t+\tau} = 1 - (\Lambda^{\text{Naïve}})^{\tau+1}.$$

To compare the aggregate dynamics of the two models, it is sufficient to compare $\Lambda^{\text{Naïve}}$ with Λ . Figure D2 plots the ratio of the two adjustment factors. As in WW, we find the difference is negligible for realistic values of λ and φ .

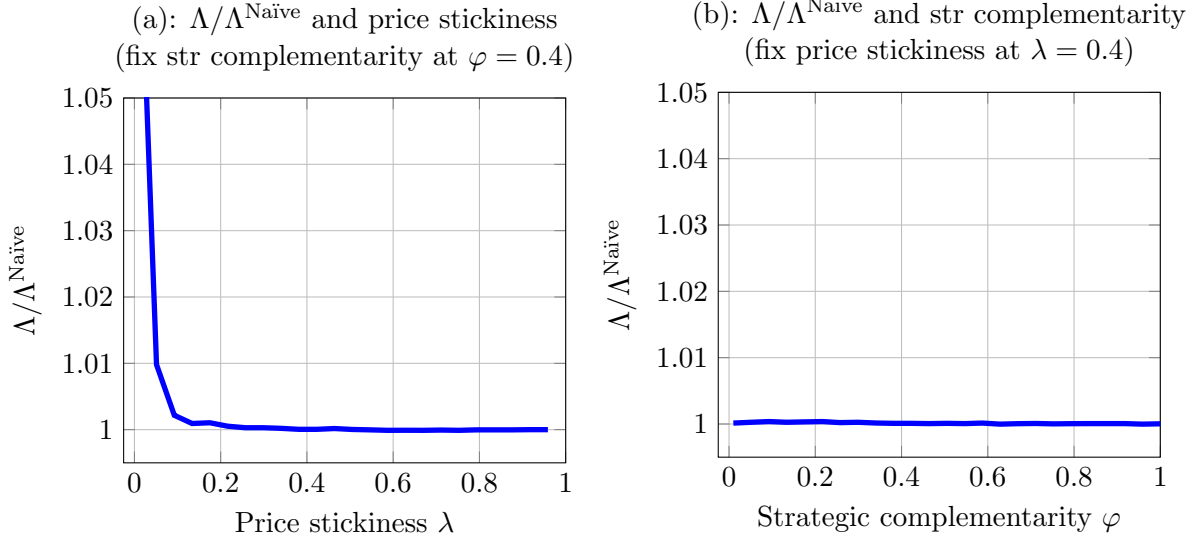


Figure D2: benchmark vs “naïve” dynamics in response to a common cost shock

Discussions. Why does the “strategic effect” not play a big role in the price dynamics of the model? Does this result suggest the firm’s reset price has limited impact on its competitors’ future reset prices?

To answer these questions, we need to understand the restrictions imposed by (D.57). The key difference between the naïve and the benchmark model lies in how firms take expectations about their competitors’ prices. In the naïve model, the expectation is formed based on assumption (D.57) and equation (D.58). In contrast, in our benchmark model, the expectation is formed based on (D.55). The key restriction imposed by (D.57) is that a firm’s reset price in $t + \tau$ only depends on its competitors’ prices in $t + \tau$. However, this does not mean that a firm’s reset price has no impact on its competitors’ future reset prices. To see this, we can iterate expression (D.56) forward to see how firm k ’s reset price at t influences other firms’ reset prices at $t + 1$, $t + 2$, etc.

At $t + 1$:

$$\begin{aligned}\mathbb{E}_t \hat{P}_{kjt+1} &= (1 - \lambda) \mathbb{E}_t \hat{P}_{kjt+1}^* + \lambda \mathbb{E}_t \hat{P}_{kjt} \\ &= (1 - \lambda) \mathbb{E}_t \hat{P}_{kjt+1}^* + \lambda \left[(1 - \lambda) \mathbb{E}_t \hat{P}_{kjt}^* + \lambda \hat{P}_{kjt-1} \right].\end{aligned}\quad (\text{D.62})$$

At $t + 2$:

$$\begin{aligned}\mathbb{E}_t \hat{P}_{kjt+2} &= (1 - \lambda) \mathbb{E}_t \hat{P}_{kjt+2}^* + \lambda \mathbb{E}_t \hat{P}_{kjt+1} \\ &= (1 - \lambda) \mathbb{E}_t \hat{P}_{kjt+2}^* + \lambda \left\{ (1 - \lambda) \mathbb{E}_t \hat{P}_{kjt+1}^* + \lambda \left[(1 - \lambda) \mathbb{E}_t \hat{P}_{kjt}^* + \lambda \hat{P}_{kjt-1} \right] \right\}.\end{aligned}\quad (\text{D.63})$$

Therefore, even if firm i ’s reset price at $t + \tau$ only depends on the expected competitors’ prices at $t + \tau$, the fact that the expected competitors’ prices at $t + \tau$ implicitly depend on their reset prices in earlier periods, i.e., $\mathbb{E}_t \hat{P}_{kjt+\tau}(\hat{P}_{kjt}^*, \hat{P}_{kjt+1}^*, \dots, \hat{P}_{kjt+\tau-1}^*)$, makes the optimal reset price of firm i at

$t + \tau$, $\hat{P}_{ijt+\tau}^*$, an implicit function of $\{\hat{P}_{kjt}^*, \hat{P}_{kjt+1}^*, \dots, \hat{P}_{kjt+\tau-1}^*\}_{k \neq i}$. Consequently, the restriction (D.57) does not preclude the possibility that a firm may implicitly respond to its competitors' reset prices in the earlier periods.

What the restriction (D.57) imposes is that the firm's reset price at $t + \tau$ responds to its competitors' earlier reset prices in a specific way – the importance of firm k 's reset price at t , $\hat{P}_{kjt}^*$, for firm i 's reset price at $t + \tau$, $\hat{P}_{ijt+\tau}^*$, is given by $\lambda^\tau(1 - \lambda)$, as illustrated by (D.62) and (D.63). Figure D2 shows that when the response factor B is solved to ensure consistent expectations (i.e., (D.60) and (D.61) hold), the restricted “Naïve” solution provides a very good proxy for the true dynamic solutions. This implies that the expected future price index of a competitor, $\mathbb{E}_t \hat{P}_{kjt+\tau}$, provides a good summary of all the prices $(\hat{P}_{kjt}^*, \hat{P}_{kjt+1}^*, \dots, \hat{P}_{kjt+\tau}^*)$ that a firm needs to consider when resetting its price.

D.7 Derivation of the sufficient statistic φ under Kimball demand

In this appendix, we show how the key statistic for the degree of strategic complementarity in our model φ_{ij} can be derived under Kimball demand.

Elasticities under a general demand function. The pricing problem of the distributor under a general demand function can be written as

$$\max_{P_{ijt}^*} \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\lambda)^\tau \frac{U_c(c_{t+\tau})/P_{t+\tau}}{U_c(c_t)/P_t} (P_{ijt}^* - Q_{ijt+\tau}^*) c_{ijt+\tau},$$

where $Q_{ijt+\tau}^*$ is the nominal marginal cost of distributor i of sector j in period $t + \tau$.

First order condition w.r.t. the reset price P_{ijt}^* is:

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\lambda)^\tau \frac{U_c(c_{t+\tau})/P_{t+\tau}}{U_c(c_t)/P_t} \left[1 + \left(1 - \frac{Q_{ijt+\tau}^*}{P_{ijt}^*} \right) \vartheta_{ijt+\tau} \right] c_{ijt+\tau} = 0,$$

where $\vartheta_{ijt+\tau}$ is the demand elasticity:

$$\vartheta_{ijt+\tau} \equiv - \frac{\partial c_{ijt+\tau}}{\partial P_{ijt}^*} \frac{P_{ijt}^*}{c_{ijt+\tau}}.$$

With the assumption of log utility, $U_c(c_{t+\tau}) = 1/c_{t+\tau}$, the optimal reset price can be written as

$$P_{ijt}^* = \frac{\mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\lambda_j)^\tau \vartheta_{ijt+\tau} c_{ijt+\tau}}{\mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\lambda_j)^\tau (\vartheta_{ijt+\tau} - 1) c_{ijt+\tau} / Q_{ijt+\tau}^*}.$$

Therefore, we get the same solution as in our benchmark model. Under the assumption of

Kimball demand, we have⁷

$$c_{ijt} \equiv \left[1 - \xi \ln \left(\frac{P_{ijt}}{P_{jt}} \right) \right]^{\frac{\theta}{\xi}} c_{jt},$$

where ξ is the superelasticity that governs the extent to which the firm adjusts its markup to cost shocks. The effective demand elasticity is given by

$$\mathbb{E}_t \vartheta_{ijt+\tau} = \mathbb{E}_t \left[\frac{\theta}{1 - \xi \ln \left(\frac{P_{ijt}^*}{P_{jt+\tau}} \right)} \right]. \quad (\text{D.64})$$

In the more general demand system developed by [Amiti, Itskhoki and Konings \(2019\)](#) that nests the Kimball and [Atkeson and Burstein \(2008\)](#) as special cases, the demand elasticity can be derived as⁸

$$\mathbb{E}_t \vartheta_{ijt+\tau} = \mathbb{E}_t \left[s_{ijt+\tau} + \chi_{ijt+\tau} \left(1 - \frac{s_{ijt+\tau} \chi_{ijt+\tau}}{\sum_i s_{ijt+\tau} \chi_{ijt+\tau}} \right) \right] \quad \text{with} \quad \chi_{ijt+\tau} \equiv \frac{\theta}{1 - \xi \ln \left(\frac{P_{ijt}^*}{P_{jt+\tau}} \right)}. \quad (\text{D.65})$$

Re-deriving theoretical results. It is worthwhile noting that, if we could express the expected markup as a function of the relative prices, then all of our previous steps in deriving the closed-form solutions will carry through. That is, we need to find the expression of φ_{ij} such that the following relationship holds:

$$\mathbb{E}_t \hat{\mu}_{ijt+\tau} \approx -\frac{1}{\vartheta_{ij} - 1} \mathbb{E}_t \hat{\vartheta}_{ijt+\tau} = -\varphi_{ij} \left[\hat{P}_{ijt}^* - \mathbb{E}_t \hat{P}_{jt+\tau} \right].$$

In our benchmark setting, we have

$$\varphi_{ij} \equiv (\theta - 1) \frac{s_{ij}}{1 - s_{ij}} = (\theta - 1) \left(\frac{\theta - 1}{\theta} \mu_{ij} - 1 \right).$$

Under Kimball demand, it can be shown that

$$\varphi_{ij} \equiv \frac{\xi}{\theta} \frac{\vartheta_{ij}}{\vartheta_{ij} - 1}.$$

When firms are *ex ante* homogeneous, $\vartheta_{ij} = \theta$ and

$$\varphi_{ij} = \frac{\xi}{\theta - 1}. \quad (\text{D.66})$$

With the sufficient statistic φ_{ij} , we can follow the same steps to obtain the closed-form solutions of the firms optimal reset prices in Propositions 1 and 2 and calculate the sector and aggregate dynamics according to Proposition 3.

⁷We use the functional form of [Klenow and Willis \(2016\)](#). See Appendix B of [Gopinath and Itskhoki \(2010\)](#) for more details.

⁸See Appendix D of [Amiti, Itskhoki and Konings \(2019\)](#) for more details.

Comparison. Under a first-order approximation, we could obtain the same firm-level, sectoral, and aggregate dynamics in our benchmark sticky-price oligopolistic competition model as in the alternative multi-sector monopolistic competition Kimball model if there exist calibrations of the superelasticity ξ_j such that $\varphi_{ij}^{\text{Benchmark}} = \varphi_{ij}^{\text{Kimball}}$. However, due to the differences in the underlying microfoundations of market power φ_{ij} in the two models, achieving an exact match at the firm level is often difficult.

When firms are symmetric within a sector, it is possible to calibrate the superelasticity ξ_j or θ_j to achieve an exact match, resulting in $\varphi_j^{\text{Benchmark}} = \varphi_j^{\text{Kimball}}$. In this case, the aggregate dynamics in the homogeneous firm sticky-price oligopolistic competition model will be the same as those in the alternative multi-sector monopolistic competition Kimball model.

Finally, if the primary concern is matching the *total* real impact of monetary policy, it is possible to calibrate a one-sector Kimball model to simultaneously match (i) the average degree of price stickiness observed in the data and (ii) the *cumulative* output response to a permanent monetary policy shock produced in our benchmark multi-sector model presented in column 5 of Table 3. Specifically, one can calibrate ξ such that:

$$\frac{\Lambda^{\text{Kimball}}(\xi)}{1 - \Lambda^{\text{Kimball}}(\xi)} = \sum_j \alpha_j \frac{\Lambda_j^{\text{Benchmark}}}{1 - \Lambda_j^{\text{Benchmark}}}.$$

D.8 Mapping to [Amiti, Itskhoki and Konings \(2019\)](#)

D.8.1 Background—The AIK framework

AIK provide a general *static* framework that decomposes a firm's price responses into two components: (1) the reaction to its own cost shocks and (2) the reaction to its competitors' price adjustments. The core theoretical insights of this framework are encapsulated in their Propositions 1 and 2, which are succinctly restated below. These findings are applicable to all industries and, for the sake of brevity, we omit the industry-specific subscript j .

AIK Proposition 1 *For any given invertible demand system and competition structure, there exists a markup function $\mu_{it} = \mu_i(p_{it}, \mathbf{p}_{-it}; \boldsymbol{\xi}_t)$, such that the firm's static profit-maximizing price $\tilde{p}_{it}$ is the solution to the following fixed-point equation for any given price vector of the competitors $\mathbf{p}_{-it}$:*

$$\tilde{p}_{it} = mc_{it} + \mu_i(\tilde{p}_{it}, \mathbf{p}_{-it}; \boldsymbol{\xi}_t), \quad (\text{D.67})$$

where $\boldsymbol{\xi}_t = (\xi_{1t}, \dots, \xi_{Nt})$ is a vector of exogenous demand shifters and N is the number of firms in the industry.

Totally differentiating the best response condition (D.67) around some admissible point $(\tilde{p}_{it}, \mathbf{p}_{-it}; \boldsymbol{\xi}_t)$, e.g., any equilibrium point $(\mathbf{p}_t; \boldsymbol{\xi}_t)$, we obtain the following decomposition for the firm's log price differential:

$$dp_{it} = dmc_{it} + \frac{\partial \mu_i(\mathbf{p}_t; \boldsymbol{\xi}_t)}{\partial p_{it}} dp_{it} + \sum_{k \neq i} \frac{\partial \mu_i(\mathbf{p}_t; \boldsymbol{\xi}_t)}{\partial p_{kt}} dp_{kt} + \sum_{k=1}^N \frac{\partial \mu_i(\mathbf{p}_t; \boldsymbol{\xi}_t)}{\partial \xi_{kt}} d\xi_{kt} \quad (\text{D.68})$$

The markup function $\mu_i(\cdot)$ can be evaluated for an arbitrary price vector $\mathbf{p}_t = (p_{it}, \mathbf{p}_{-it})$, and therefore (D.68) characterizes all possible perturbations to the firm's price in response to shocks to its marginal cost dmc_{it} , the prices of its competitors $\{dp_{kt}\}_{k \neq i}$, and the demand shifters $\{d\xi_{kt}\}_{k=1}^N$. Solving the fixed point for dp_{it} in (D.68) results in:

$$dp_{it} = \frac{1}{1 + \Gamma_{it}} dmc_{it} + \frac{\Gamma_{-it}}{1 + \Gamma_{it}} dp_{-it} + \frac{1}{1 + \Gamma_{it}} \varepsilon_{it}, \quad (\text{D.69})$$

where

$$\begin{aligned} \Gamma_{it} &\equiv -\frac{\partial \mu_i(\mathbf{p}_t; \boldsymbol{\xi}_t)}{\partial p_{it}} \quad \text{and} \quad \Gamma_{-it} \equiv \sum_{k \neq i} \frac{\partial \mu_i(\mathbf{p}_t; \boldsymbol{\xi}_t)}{\partial p_{kt}}, \\ dp_{-it} &\equiv \sum_{k \neq i} \omega_{kt} dp_{kt} \quad \text{with} \quad \omega_{it} \equiv \frac{\partial \mu_i(p_t; \xi_t) / \partial p_{kt}}{\sum_{k \neq i} \partial \mu_i(p_t; \xi_t) / \partial p_{kt}}, \\ \varepsilon_{it} &\equiv \sum_{k=1}^N \frac{\partial \mu_i(\mathbf{p}_t; \xi_t)}{\partial \xi_{kt}} d\xi_{kt}. \end{aligned}$$

AIK Proposition 2 (i) *If the log expenditure function p_t is a sufficient statistic for competitor prices, i.e., if the demand can be written as $q_{it} = q_i(p_{it}, p_t; \xi_t)$, then the weights in the competitor price index are proportional to the competitor revenue market shares s_{kt} , for $k \neq i$, and given by $\omega_{kt} \equiv s_{kt} / (1 - s_{kt})$. Therefore, the index of competitor price changes simplifies to*

$$dp_{-it} \equiv \sum_{k \neq i} \frac{s_{kt}}{1 - s_{kt}} dp_{kt}. \quad (\text{D.70})$$

(ii) *Under the stronger assumption that the perceived demand elasticity is a function of the price of the firm relative to the industry expenditure function, $\sigma_{it} = \sigma_i(p_{it} - p_t; \xi_t)$, the following two markup elasticities are equal:*

$$\Gamma_{-it} \equiv \Gamma_{it}. \quad (\text{D.71})$$

A key implication of AIK Proposition 2 is that, under the relatively mild conditions imposed by AIK, the pass-through to the firm's own cost shock and the firm's competitors' price changes (i.e., the first two coefficients in the price decomposition (D.69)) should sum to one:

$$\frac{1}{1 + \Gamma_{it}} + \frac{\Gamma_{-it}}{1 + \Gamma_{it}} = 1. \quad (\text{D.72})$$

AIK empirically test (D.72) with Belgian data and find strong empirical support of this theoretical relationship.

Remarks. It is worth noting that the decomposition (D.69) cannot be applied directly in empirical estimations. This is because in an oligopolistic competition model the firms' prices are jointly determined. Since a firm's competitors' price changes dp_{-it} are endogenous and depend on the firm's price change dp_{it} , directly estimating (D.69) can result in substantial bias. To address this concern, AIK use proxies of the competitors' cost changes as instruments for the competitors' price

changes.

D.8.2 Response to cost shocks under the AIK framework

In what follows, we derive an alternative decomposition in terms of exogenous shocks, which can be estimated directly using observed cost shocks.

First, note that we can rewrite the price change decomposition (D.69) as follows:

$$\begin{aligned} dp_{it} &= \frac{1}{1 + \Gamma_{it}} dmc_{it} + \frac{\Gamma_{-it}}{1 + \Gamma_{it}} dp_{-it} + \frac{1}{1 + \Gamma_{it}} \varepsilon_{it} \\ \Leftrightarrow dp_{it} &= \frac{1}{1 + \Gamma_{it}} dmc_{it} + \frac{\Gamma_{it}}{1 + \Gamma_{it}} dp_{-it} + \frac{1}{1 + \Gamma_{it}} \varepsilon_{it} \end{aligned} \quad (D.73)$$

$$\Leftrightarrow (1 - s_{it})(1 + \Gamma_{it})dp_{it} = (1 - S_{it}) dmc_{it} + \Gamma_{it} \sum_{k \neq i} s_{kt} dp_{kt} + (1 - s_{it})\varepsilon_{it} \quad (D.74)$$

$$\Leftrightarrow dp_{it} = \frac{1 - s_{it}}{1 - S_{it} + \Gamma_{it}} dmc_{it} + \frac{\Gamma_{it}}{1 - S_{it} + \Gamma_{it}} dp_t + \frac{1 - s_{it}}{1 - S_{it} + \Gamma_{it}} \varepsilon_{it} \quad (D.75)$$

where (D.73) uses relationship (D.71); (D.74) uses relationship (D.70); and (D.75) uses the definition of the sector price index such that $dp_t = \sum_k s_{kt} dp_{kt}$. Note that the only endogenous variable in (D.75) is the changes in the sector price index dp_t .

Next, to solve for dp_t , we aggregate expression (D.75) across all firms:

$$\sum_i s_{it} dp_{it} = \sum_i \frac{s_{it}(1 - s_{it})}{1 - S_{it} + \Gamma_{it}} dmc_{it} + \sum_i \frac{\Gamma_{it}s_{it}}{1 - S_{it} + \Gamma_{it}} dp_t + \sum_i \frac{s_{it}(1 - s_{it})}{1 - S_{it} + \Gamma_{it}} \varepsilon_{it} \quad (D.76)$$

Rearranging (D.76), we can express the changes in the sector price index dp_t as a function of exogenous marginal cost and demand shocks:

$$dp_t = \sum_i \tilde{\varphi}_{it} dmc_{it} + \sum_i \tilde{\varphi}_{it} \varepsilon_{it} \quad (D.77)$$

with

$$\varphi_{it} \equiv \frac{1 - s_{it}}{1 - S_{it} + \Gamma_{it}} \quad \text{and} \quad \tilde{\varphi}_{it} \equiv \frac{\varphi_{it}s_{it}}{\sum_k \varphi_{kt}s_{kt}},$$

where, as we show below in (D.79), $\varphi_{it} > 0$ is the firm's response to idiosyncratic shocks and $\hat{\varphi}_{it} > 0$ is the implicit importance weight of the idiosyncratic shocks with $\sum_i \tilde{\varphi}_{it} = 1$. When $\varepsilon_{it} = 0 \forall i$, expression (D.77) is equivalent to the expression in AIK Proposition 3.

Finally, substitute (D.77) into (D.75) and we get the *solved* version of the price change decomposition:

$$dp_{it} = \underbrace{[\varphi_{it} + (1 - \varphi_{it})\tilde{\varphi}_{it}](mc_{it} + \varepsilon_{it})}_{\text{pass-through to own cost and demand shocks}} + \underbrace{(1 - \varphi_{it}) \left[\sum_{k \neq i} \tilde{\varphi}_{kt} dmc_{kt} + \sum_{k \neq i} \tilde{\varphi}_{kt} \varepsilon_{kt} \right]}_{\text{pass-through to competitors' cost and demand shocks}}$$

It can be shown that, under a common cost or demand shock, the price pass-through of *each firm* is 100% as

$$[\varphi_{it} + (1 - \varphi_{it})\tilde{\varphi}_{it}] + (1 - \varphi_{it}) \sum_{k \neq i} \tilde{\varphi}_{kt} = 1.$$

Defining the common cost and demand shocks as

$$dmc_t \equiv \sum_i \tilde{\varphi}_{it} dmc_{it} \quad \text{and} \quad \varepsilon_t \equiv \sum_i \tilde{\varphi}_{it} \varepsilon_{it}, \quad (\text{D.78})$$

the price change decomposition can be re-expressed in terms of common versus idiosyncratic shocks:

$$dp_{it} = \varphi_{it}(dmc_{it} - dmc_t) + dmc_t + \varphi_{it}(\varepsilon_{it} - \varepsilon_t) + \varepsilon_t. \quad (\text{D.79})$$

The first term of (D.79) shows that the pass-through rate to an idiosyncratic cost shock ($dmc_{it} - dmc_t$) is well defined and given by φ_{it} . The second term shows that, in this static framework, the pass-through rate to a common cost shock is always equal to 100%. Under the structural assumptions of Atkeson and Burstein (2008), φ_{it} is a strictly *decreasing* function of market share s_{it} . Intuitively, this is because large firms with market power absorb part of their cost shocks into markups, while small firms do not adjust markups and thus fully pass-through any idiosyncratic cost shock. It is worth noting that, different from Γ_{it} , which is hump-shaped in market share, φ_{it} is a strictly decreasing function of market share (see Figure D3).⁹

Unlike AIK's original decomposition (D.69), our decomposition (D.79) involves no endogenous variable and thus does not need an instrument. Assuming the demand shocks are exogenous and i.i.d. (as in AIK), equation (D.79) can be directly estimated using OLS provided that a good measure of the common cost shock dmc_t can be constructed.

D.8.3 Cournot vs Bertrand Competition

Consistent with the literature, we have assumed Cournot competition in our benchmark model as it tends to better match the relationship between the estimated pass-through rates and empirical market share distributions (see Atkeson and Burstein 2008 and Amiti, Itskhoki and Konings 2019). Figure D4 contrasts the sufficient statistic $\varphi(\theta, s)$ in our model under Bertrand and Cournot competition. We observe that Cournot competition results in larger strategic complementarity (measured by φ) for a given market share, s , and is more sensitive to the assumed elasticity of substitution, θ .

⁹From the definition of $\varphi_{it} = 1/[1 + \Gamma(s_{it})/(1 - s_{it})]$, we see there are two market share effects as a firm becomes larger: (1) a direct market share effect captured by $(1 - s_{it})$ and (2) an indirect effect through markup adjustment captured by $\Gamma(s_{it})$. The two effects go in the same direction and reduce price pass-through to an idiosyncratic shock until the firm becomes extremely large (e.g., when it accounts for over 80% of the market share). For those extremely large firms, the two effects go in opposite directions: as a firm becomes extremely large it cares less about the cost shocks of other smaller firms (captured by effect 1) and makes smaller markup adjustments (captured by effect 2). It turns out that the direct effect (1) dominates as a firm becomes extremely large, and, therefore, the price pass-through to an idiosyncratic shock is strictly decreasing in the firm's market share.

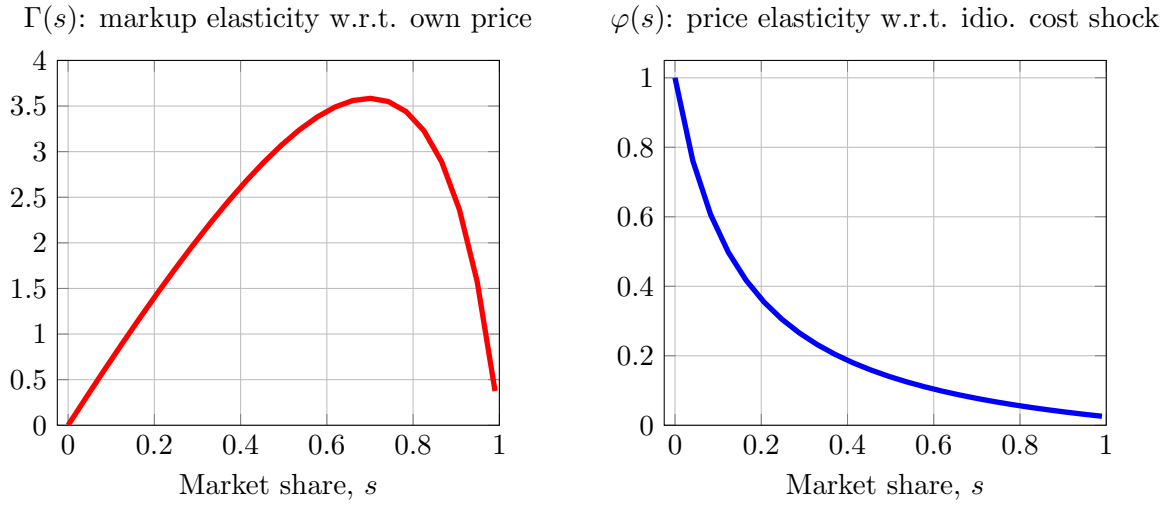


Figure D3: Market share and key elasticities

Note: The two figures plot $\Gamma(s)$ and $\varphi(s)$ under the nested-CES demand preference of [Atkeson and Burstein \(2008\)](#), where the within-industry elasticity of substitution is set to 10 and the cross-industry elasticity of substitution is set to 1.2.

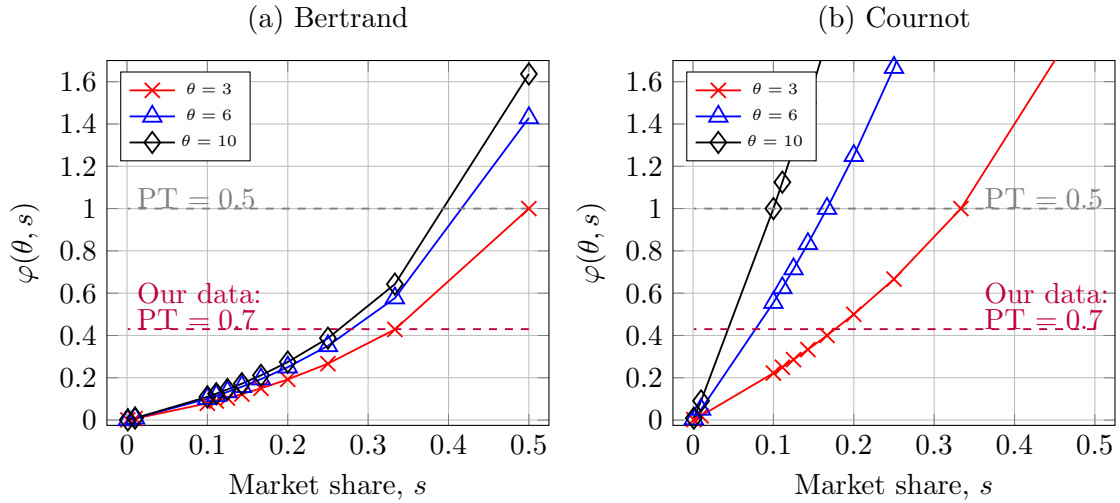


Figure D4: Sufficient statistic $\varphi(\theta, s)$ under Bertrand vs Cournot competition

D.9 Nonlinear duopoly model with Calvo price adjustments

To what extent do our closed-form solutions provide a good approximation for theoretical responses? In particular, two concerns may arise: (1) theoretical responses are derived under the assumption of small shocks, but in practice, the idiosyncratic shocks can be large; and (2) by taking first-order approximations, we might miss important channels revealed in a fully nonlinear dynamic model.

To address these concerns, we fully solve a dynamic oligopolistic Calvo model without taking any approximations and compare the model solutions to our theoretical counterparts. Due to computational constraints, we solve a model with duopoly distributors—a market structure in which strategic interactions among firms are the largest. If we indeed missed any important channels in our approximated model, it would likely be reflected in this setting. We also remove the restriction of perfect synchronization so that our setting is more comparable to [Wang and Werning \(2022\)](#) and [Mongey \(2021\)](#).

Model setting. In the beginning of each period, the firm observes its own and competitors' past prices $\{P_{ij,t-1}, P_{-ij,t-1}\}$, and its own and competitors' current costs $\{Q_{ij,t}, Q_{-ij,t}\}$. These are the four state variables in each sector $S_{j,t} \equiv \{P_{ij,t-1}, P_{-ij,t-1}, Q_{ij,t}, Q_{-ij,t}\}$. Within the period, firms recognize that each firm faces an exogenous probability $1 - \lambda_j$ that it can reset its price. As in [Mongey \(2021\)](#), we assume that within a period, all moves are simultaneous, meaning that firms do not respond to each other's new prices.¹⁰

Formally, the problem can be written as

$$V(S_{j,t}) = (1 - \lambda_j)V^{adj}(S_{j,t}) + \lambda_j V^{stay}(S_{j,t})$$

with

$$\begin{aligned} V^{adj}(S_{j,t}) = \max_{P_{ij,t}^*} (1 - \lambda_j) [\pi(P_{ij,t}^*, P_{-ij}(S_{j,t}), Q_{ij,t}) + \beta \mathbb{E}V(P_{ij,t}^*, P_{-ij}(S_{j,t}), Q_{ij,t+1}, Q_{-ij,t+1})] \\ + \lambda_j [\pi(P_{ij,t}^*, P_{-ij,t-1}, Q_{ij,t}) + \beta V(P_{ij,t}^*, P_{-ij,t-1}, Q_{ij,t+1}, Q_{-ij,t+1})]; \end{aligned} \quad (\text{D.80})$$

and

$$\begin{aligned} V^{stay}(S_{j,t}) = (1 - \lambda_j) [\pi(P_{ij,t-1}, P_{-ij}(S_{j,t}), Q_{ij,t}) + \beta \mathbb{E}V(P_{ij,t-1}, P_{-ij}(S_{j,t}), Q_{ij,t+1}, Q_{-ij,t+1})] \\ + \lambda_j [\pi(P_{ij,t-1}, P_{-ij,t-1}, Q_{ij,t}) + \beta \mathbb{E}V(P_{ij,t-1}, P_{-ij,t-1}, Q_{ij,t+1}, Q_{-ij,t+1})]; \end{aligned} \quad (\text{D.81})$$

where $P_{-ij}(S_{j,t})$ is the firm's competitor's reset price for state $S_{j,t}$ and the expectation $\mathbb{E}$ is taken

¹⁰The key difference in the model we set up here and that of [Mongey \(2021\)](#) is that the probability of price adjustment is exogenous in our model. When the probability of price adjustment is endogenous and state dependent (due to a menu cost), obtaining analytical solutions is very hard (if not impossible). A key benefit of relying on a Calvo framework lies in its analytical tractability, where we are able to solve the model for an arbitrary market structure of a sector (rather than restricting the solution to duopoly markets).

over the cost processes $Q_{ij,t+1}, Q_{-ij,t+1}$. The first line of (D.80) and (D.81) gives the profit and expected future value when the firm's competitor has the opportunity to reset its price to $P_{-ij}(S_{j,t})$, whereas the second line gives the corresponding values when its competitor does not have the opportunity to reset its price (and thus stick to $P_{-ij,t-1}$). As in Wang and Werning (2022) and Mongey (2021), we consider symmetric policy solutions. That is, if the two firms' market conditions are swapped, they would have chosen the same strategy as their competitor has chosen, i.e., $P_{-ij}(P_{ij,t-1}, P_{-ij,t-1}, Q_{ij,t}, Q_{-ij,t}) = P_{ij}(P_{-ij,t-1}, P_{ij,t-1}, Q_{-ij,t}, Q_{ij,t})$.

Simulation setting. The firm's profit function is given by

$$\pi(P_{i,t}, P_{-ij,t}, Q_{ij,t}) = (P_{i,t} - Q_{ij,t})c_{ij,t}$$

where

$$\begin{aligned} \text{Demand for distributor } i\text{'s good: } c_{ij,t} &\equiv (P_{ij,t}/P_{j,t})^{-\theta} P_{j,t} \\ \text{Sector price index: } P_{j,t} &\equiv \left[(P_{ij,t})^{1-\theta} + (P_{-ij,t})^{1-\theta} \right]^{\frac{1}{1-\theta}} \\ \text{Distributor } i\text{'s cost: } \ln(Q_{ij,t}) &= \rho \ln(Q_{ij,t-1}) + \epsilon_{ij,t} \end{aligned} \quad (\text{D.82})$$

The within-sector elasticity of substitution is set to $\theta = 3$, the household discount factor is set to the standard value $\beta = 0.98^{1/12}$, and the idiosyncratic shock is drawn from a normal distribution $N(0, \sigma_\epsilon^2)$ with $\sigma_\epsilon = 0.25$. We simulate a partial equilibrium model with heterogeneous duopoly firms, incorporating AR(1) cost shocks as described by (D.82).¹¹ We simulate 10*2 different models, varying the underlying price stickiness λ_j and the persistence of the cost shock ρ . In each model, we simulate 1,000 industries for 100 periods. We estimate our empirical specification on the simulated data from the fully non-linear model.

Figure D5 compares the estimated and theoretical relationships. The dashed lines show the theoretical pass-through coefficients under our first order approximated solution in Proposition 1, evaluated at the mean market share ($s = 0.5$) across simulations periods. The dots represent the pass-through estimates from the simulated data. As seen in Figure D5, the estimated coefficients align well with theoretical predictions.

Remarks. The empirical and theoretical coefficients may not be exactly the same for two reasons. First, our theoretical prediction is evaluated at the average market share of a firm across all periods. With large idiosyncratic shocks, a firm's market share can substantially deviate from this average in some periods. As a result, the estimated coefficient may differ from theoretical prediction because the latter is evaluated with the "wrong" parameters. For example, in the full nonlinear model, the competitor's price adjustment may be slightly different when accounting for the fact that the market structure, upon receiving an idiosyncratic shock or a price adjustment in the last period,

¹¹Note that, under the assumptions of (1) log consumption and linear labor and (2) Cobb-Douglas aggregated consumption across sectors, the price dynamics can be solved separately in each sector (and the aggregate price change can be obtained by aggregating the sector prices). Therefore, verifying our solution in the partial equilibrium model is sufficient for this purpose.

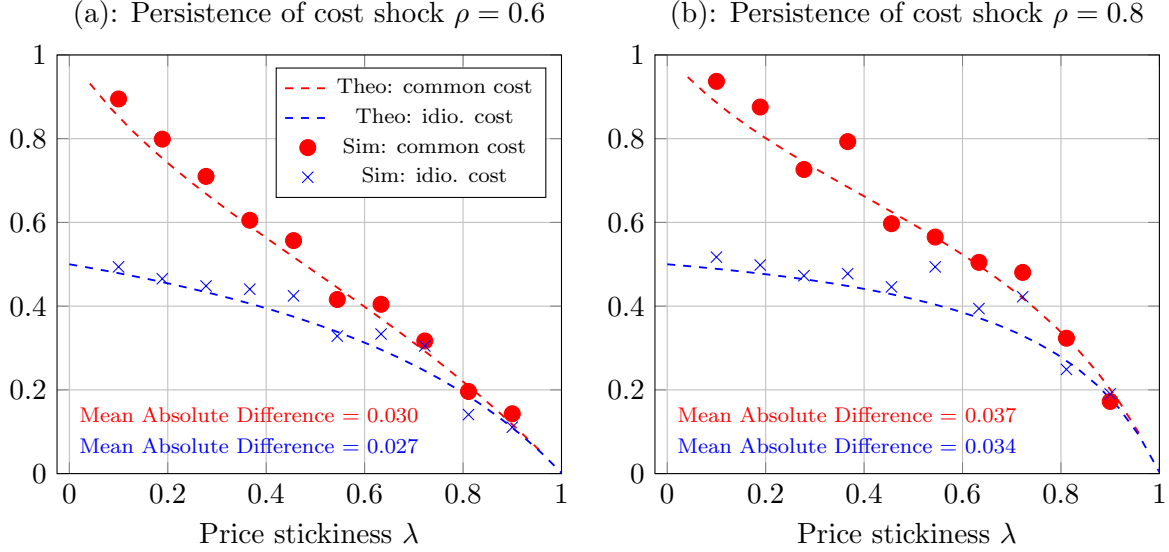


Figure D5: Comparing theoretical vs estimated responses

differs from the steady-state market structure where our theoretical predictions are evaluated.

As seen in Figure D5, this discrepancy does not seem to be significant because the positive biases largely offset the negative biases. This offsetting effect occurs both across firms and over time. Within each period, the presence of a larger-than-average firm simultaneously implies the existence of a smaller-than-average firm. Thus, evaluating the solutions at the average market share results in an upward bias and a downward bias, which largely cancel each other out, thereby having limited impact on the pass-through estimates. Similarly, over time, a firm has an equal probability of receiving positive or negative idiosyncratic cost shocks. Therefore, the periods where the pass-through is higher than theoretical prediction evaluated at the average (steady-state) market share largely offset the periods where the pass-through is lower than theoretical prediction evaluated at the average, resulting in a very small overall bias in the estimated pass-through coefficients using our empirical strategy.

Second, the numerical solution is slightly sensitive to the grid points used to solve the model. For example, alternative numerical settings could bring the fourth red point in panel (b) closer to our theoretical prediction, though this may cause other points to be slightly off. We could achieve a slightly better match between theoretical and estimated responses by taking the median or mean over several numerical solutions. However, we deliberately choose to show the non-averaged version of responses in Figure D5 to illustrate that fully nonlinear numerical solutions may also suffer from accuracy issues and do not always provide a more informative picture.

E Aggregate Dynamics and Implications

E.1 An example of aggregate price evolution with homogeneous duopoly sectors

In this subsection, we describe a simplified version of the model with homogenous duopoly sectors. We do this to illustrate that, while having firms with market power means that aggregate price dynamics are more complex than in a model with monopolistic competitive firms, these dynamics can still be succinctly represented in a simple Calvo form. Figure E1 illustrates the evolution of sector and aggregate prices in this version of the model. Starting from the steady state at $t = 0$, we characterize the exact price dynamics in the model. As shown in the figure, at $t = 1$ there are four types of sectors based on the price adjustment patterns: (1) sectors where both firms adjust their prices, denoted as $[A, A]$; (2) sectors where only the first firm adjusts its price, denoted as $[A, N]$; (3) sectors where only the second firm adjusts its price, denoted as $[N, A]$; and (4) sectors where neither firm adjusts their prices, denoted as $[N, N]$. The proportions of these sectors are given by $(1 - \lambda)^2$, $(1 - \lambda)\lambda$, $(1 - \lambda)\lambda$, and λ^2 , respectively. It is evident that the *realization* of sector prices no longer follows standard Calvo due to the limited number of firms and the discrete realization of the Calvo process in each sector. However, with a large enough number of similar sectors, the evolution of the aggregate price can still be expressed in Calvo form as

$$\hat{P}_1 = (1 - \lambda)\hat{P}_{1A} + \lambda\hat{P}_{1N} = (1 - \lambda) \left[(1 - \lambda)\hat{P}_{[A,A]} + \lambda\hat{P}_{[A,N]} \right] + \lambda \left[(1 - \lambda)\hat{P}_{[N,A]} + \lambda\hat{P}_{[N,N]} \right], \quad (\text{E.1})$$

with

$$\begin{aligned} \hat{P}_{[A,A]} &= s_1\hat{P}_{A_1|[A,A]} + s_2\hat{P}_{A_2|[A,A]} \\ \hat{P}_{[A,N]} &= s_1\hat{P}_{A_1|[A,N]} + s_2\hat{P}_0 \\ \hat{P}_{[N,A]} &= s_1\hat{P}_0 + s_2\hat{P}_{A_2|[N,A]} \\ \hat{P}_{[N,N]} &= \hat{P}_0 \end{aligned}$$

where s_1 is the (within-sector) market share of firm 1 and s_2 is the (within-sector) market share of firm 2; $\hat{P}_{A_1|[A,A]}$ is the price change of firm 1 in the sector where both firms adjusted their prices, etc. It is worth noting that, since the firm does not observe its competitor's price in t when making its price decision, we have

$$\hat{P}_{A_1|[A,A]} = \hat{P}_{A_1|[A,N]} \equiv \hat{P}_{A_1|[A,.]}, \quad \text{and} \quad \hat{P}_{A_2|[A,A]} = \hat{P}_{A_2|[N,A]} \equiv \hat{P}_{A_2|[.,A]}.$$

Define

$$\hat{P}_{1,1} \equiv s_1 \left(\hat{P}_{A_1|[A,A]} + \hat{P}_{A_1|[A,N]} \right) + s_2 \left(\hat{P}_{A_2|[A,A]} + \hat{P}_{A_2|[N,A]} \right) = s_1\hat{P}_{A_1|[A,.] } + s_2\hat{P}_{A_2|[.,A]}.$$

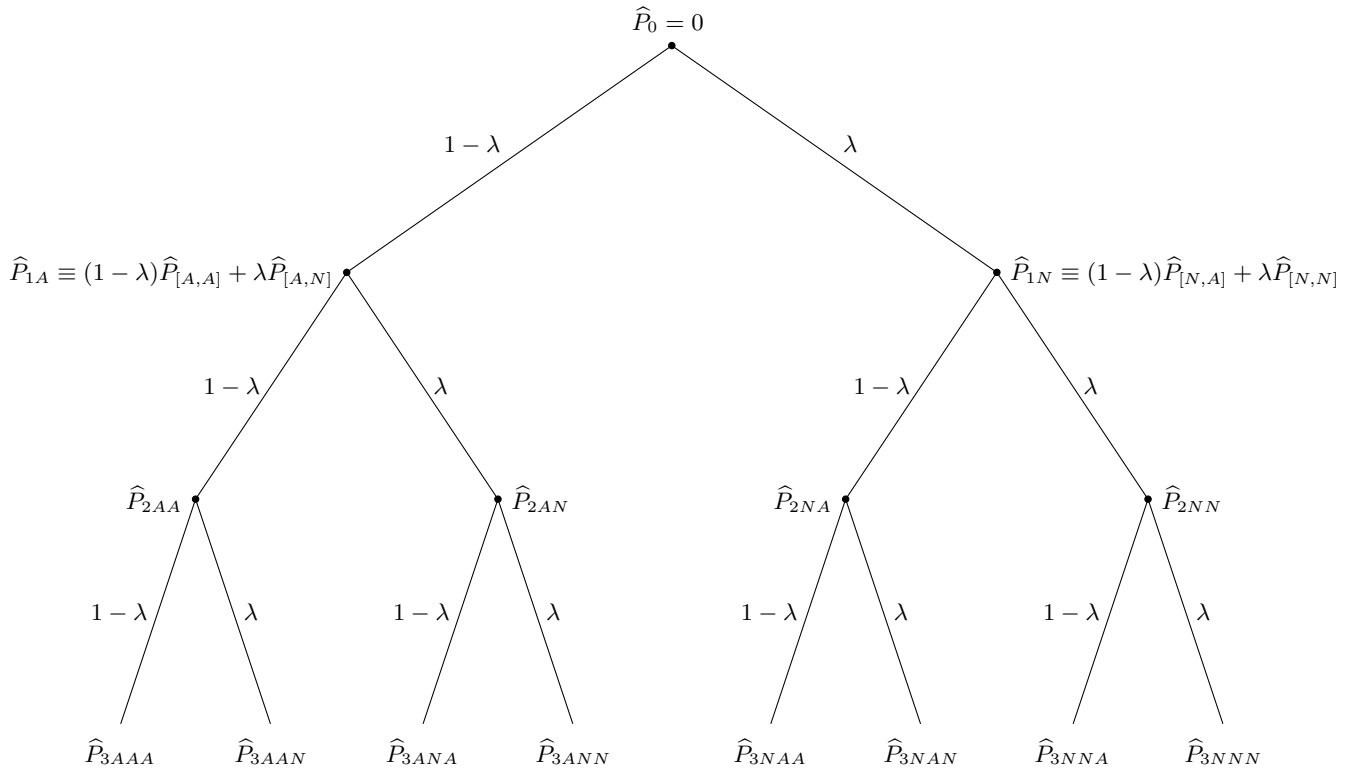


Figure E1: Illustrating the realization of sector and aggregate prices

Notes: This figure illustrates the discrete realization of the sector prices in an economy with ex-ante symmetric firms and homogeneous duopoly sectors.

We can rewrite the aggregate price as

$$\hat{P}_1 = (1 - \lambda)\hat{P}_{1,1} + \lambda\hat{P}_0.$$

Similarly to expression (E.1), for price adjustments in period 2 we have

$$\hat{P}_2 = (1 - \lambda)^2\hat{P}_{2AA} + (1 - \lambda)\lambda\hat{P}_{2AN} + \lambda(1 - \lambda)\hat{P}_{2NA} + \lambda^2\hat{P}_{2NN},$$

where

$$\begin{aligned}\hat{P}_{2AA} &= (1 - \lambda)^2\hat{P}_{[AA,AA]} + (1 - \lambda)\lambda\hat{P}_{[AA,AN]} + \lambda(1 - \lambda)\hat{P}_{[AA,NA]} + \lambda^2\hat{P}_{[AA,NN]}; \\ \hat{P}_{2AN} &= (1 - \lambda)^2\hat{P}_{[AN,AA]} + (1 - \lambda)\lambda\hat{P}_{[AN,AN]} + \lambda(1 - \lambda)\hat{P}_{[AN,NA]} + \lambda^2\hat{P}_{[AN,NN]}; \\ \hat{P}_{2NA} &= (1 - \lambda)^2\hat{P}_{[NA,AA]} + (1 - \lambda)\lambda\hat{P}_{[NA,AN]} + \lambda(1 - \lambda)\hat{P}_{[NA,NA]} + \lambda^2\hat{P}_{[NA,NN]}; \\ \hat{P}_{2NN} &= (1 - \lambda)^2\hat{P}_{[NN,AA]} + (1 - \lambda)\lambda\hat{P}_{[NN,AN]} + \lambda(1 - \lambda)\hat{P}_{[NN,NA]} + \lambda^2\hat{P}_{[NN,NN]}.\end{aligned}$$

With some rearrangements, it can be shown that

$$\hat{P}_2 = (1 - \lambda)\hat{P}_{2,2} + \lambda\hat{P}_1.$$

where

$$\begin{aligned}\hat{P}_{2,2} &\equiv s_1(1 - \lambda) \left[(1 - \lambda)\hat{P}_{A_1|[AA,A]} + \lambda\hat{P}_{A_1|[AA,N]} \right] + s_1\lambda \left[(1 - \lambda)\hat{P}_{A_1|[AN,A]} + \lambda\hat{P}_{A_1|[AN,N]} \right] \\ &\quad + s_1\lambda \left[(1 - \lambda)\hat{P}_{A_1|[NA,A]} + \lambda\hat{P}_{A_1|[NA,N]} \right] \\ &\quad + s_2(1 - \lambda) \left[(1 - \lambda)\hat{P}_{A_2|[A,AA]} + \lambda\hat{P}_{A_2|[N,AA]} \right] + s_2\lambda \left[(1 - \lambda)\hat{P}_{A_2|[A,AN]} + \lambda\hat{P}_{A_2|[N,AN]} \right] \\ &\quad + s_2\lambda \left[(1 - \lambda)\hat{P}_{A_2|[A,NA]} + \lambda\hat{P}_{A_2|[N,NA]} \right].\end{aligned}$$

Further iteration provides expressions for $\hat{P}_t$ when $t \geq 3$. The key takeaway is as follows: although the exact price dynamics are complex, the aggregate price dynamics can be succinctly represented in simple Calvo forms. As discussed in Section 2, three assumptions are crucial for arriving at this result: (1) the frequency of price adjustment is fixed and independent of the firms' pricing behaviour; (2) there is a sufficiently large number of similar sectors, allowing the law of large numbers to be applicable; and (3) the shocks are small, ensuring that a first-order approximation remains accurate.

E.2 Aggregate dynamics in the symmetric benchmark and proof of Corollary 1

In this subsection, we prove Corollary 1 and discuss the aggregate price and output dynamics of our model using a simplified version of the model with symmetric firms and homogeneous sectors.

We start by aggregating the sector prices. From (9), we know the expected sector price follows

$$\mathbb{E}_t \hat{P}_{jt+\tau} \approx (1 - \lambda) \mathbb{E}_t \hat{P}_{jt+\tau}^* + \lambda \mathbb{E}_t \hat{P}_{jt+\tau-1}.$$

As illustrated in Appendix E.1, the realization of the sector prices depends on the realization of the Calvo process, and in general,

$$\hat{P}_{jt+\tau} \neq (1 - \lambda) \hat{P}_{jt+\tau}^* + \lambda \hat{P}_{jt+\tau-1}.$$

However, if there is a large number of ex-ante identical sectors, the law of large numbers implies that the aggregate price will still follow a Calvo process:

$$\hat{P}_{t+\tau} = \frac{1}{J} \sum_j \hat{P}_{jt+\tau} \approx \frac{1}{J} \sum_j (1 - \lambda) \hat{P}_{jt+\tau}^* + \frac{1}{J} \lambda \sum_j \hat{P}_{jt+\tau-1} = (1 - \lambda) \hat{P}_{t+\tau}^* + \lambda \hat{P}_{t+\tau-1},$$

where J is the number of ex-ante homogeneous sectors.

Similarly, we can aggregate the sector NKPC (10) and re-express it as a second-order difference equation of aggregate price levels:

$$\hat{P}_{t+\tau} - \lambda \hat{P}_{t+\tau-1} - \beta \lambda (\hat{P}_{t+\tau+1} - \lambda \hat{P}_{t+\tau}) = \frac{(1 - \beta \lambda)(1 - \lambda)}{(1 + \varphi)} (\mathbb{E}_t \hat{Q}_{t+\tau} + \varphi \hat{P}_{t+\tau}) \quad \forall \tau \geq 0. \quad (\text{E.2})$$

Under a permanent monetary supply shock at t (i.e., $\widehat{M}_{t+\tau} = 1 \quad \forall \tau \geq 0$), the desired producer price $\hat{Q}_t^*$ moves one-to-one with the shock:

$$\mathbb{E}_t \hat{Q}_{t+\tau} = \hat{Q}_{t+\tau}^* = \widehat{M}_{t+\tau} = 1 \quad \forall \tau \geq 0. \quad (\text{E.3})$$

Substituting (E.3) into (E.2), the aggregate price dynamics can be solved as

$$\hat{P}_{t+\tau} - \hat{P}_{t+\tau-1} = (1 - \Lambda) \Lambda^\tau \quad \text{and} \quad \hat{P}_{t+\tau} = 1 - \Lambda^{\tau+1},$$

where

$$\Lambda \equiv \frac{1}{2} \left[\frac{1 + \lambda \varphi + \beta \lambda (\lambda + \varphi)}{\beta \lambda (1 + \varphi)} - \sqrt{\left(\frac{1 + \lambda \varphi + \beta \lambda (\lambda + \varphi)}{\beta \lambda (1 + \varphi)} \right)^2 - \frac{4}{\beta}} \right].$$

Since $P_t C_t = M_t$, the total cumulative change in consumption in response to a permanent monetary supply shock at t can be calculated as

$$\sum_{\tau=0}^{\infty} \hat{C}_{t+\tau} = \sum_{\tau=0}^{\infty} (1 - \hat{P}_{t+\tau}) = \sum_{\tau=0}^{\infty} \Lambda^{\tau+1} = \frac{\Lambda}{1 - \Lambda}.$$

As $\varphi \rightarrow 0$, the dynamics of the model converge to a standard Calvo model with $\Lambda \rightarrow \lambda$. The output amplification relative to a standard Calvo model is given by $\frac{\Lambda(1-\lambda)}{\lambda(1-\Lambda)}$ and the additional price stickiness relative to a standard Calvo model is given by Λ/λ . Figure E2 illustrates the effect of

strategic complementarity on price and output dynamics in this homogeneous sector model. The figure also reports the effect of strategic complementarity on NKPC slope factor $1/(1 + \varphi)$, as described in Corollary 1. ■

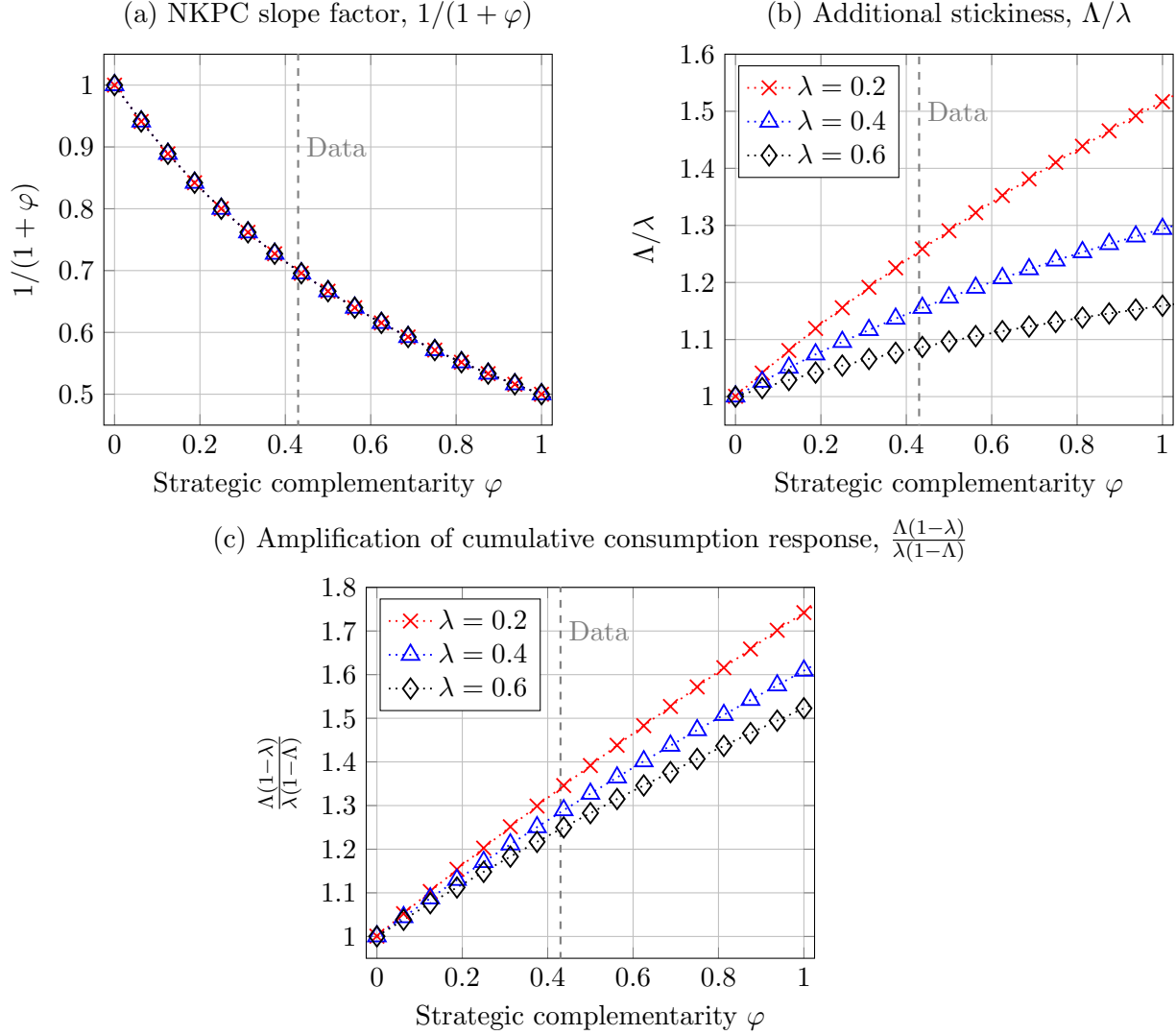


Figure E2: Effect of strategic complementarity on price and output dynamics in the homogeneous sector model (relative to monopolistic competition)

E.2.1 Synchronized two-layer models versus standard one-layer models

In our benchmark model, we assume (i) oligopolistic distributors and monopolistically competitive producers and (ii) both producers and distributors face nominal rigidity, with the timing of price adjustments determined by an identical Poisson process. Alternative models in the literature, such as [Mongey \(2021\)](#) and [Wang and Werning \(2022\)](#), have a different market structure, featuring oligopolistically competitive producers that face nominal rigidity but no distributors. Our assump-

tion (ii) is primarily to match the key feature in the data that distributor prices tend to adjust simultaneously with cost adjustments.¹² In what follows, we show that our benchmark model yields the same aggregate dynamics in response to a permanent monetary policy shock as an alternative one-layer model with oligopolistically competitive sticky price producers.

First, we note that adding an additional layer of *flexible price* monopolistically competitive producers or distributors does not change the aggregate dynamics, as the price pass-through to cost shocks is 100% in this additional layer. For example, adding flexible price monopolistically competitive distributors to the model of Wang and Werning (2022) does not change the aggregate dynamics. Similarly, if the prices of the producers in our benchmark model were fully flexible, the corresponding aggregate price dynamics would be the same as in an alternative model with only oligopolistically competitive sticky price producers and no distributors.

Second, when the additional layer of monopolistically competitive producers or distributors also faces nominal rigidity but the timing of the adjustment is completely *random* and not synchronized with the other layer, the aggregate price sluggishness is amplified, resulting in a larger output response. This is because cost shocks are transmitted more slowly due to the additional layer of nominal rigidity. For example, if we remove the perfect synchronization assumption in our model, the expected change in the distributor's cost becomes

$$\mathbb{E}_t \widehat{Q}_{t+\tau} = [1 - (\lambda^Q)^{\tau+1}] \widehat{Q}_{t+\tau}^* = [1 - (\lambda^Q)^{\tau+1}] \widehat{M}_{t+\tau} = 1 - (\lambda^Q)^{\tau+1} \quad \forall \tau \geq 0,$$

where λ^Q is the degree of price stickiness in the monopolistically competitive producer industry.

Finally, when the timing of price adjustments is perfectly synchronized between monopolistically competitive producers and oligopolistic distributors, the aggregate price and output dynamics in response to a permanent monetary policy shock are identical to those in the model with sticky price oligopolistically competitive distributors and flexible price monopolistically competitive producers. This occurs because, when the distributor adjusts its price, its cost is also fully adjusted. More formally, Propositions 1 and 2 show that the pass-through rate to a permanent common cost shock ($\rho = 1$) is the same in both models. Together with (E.3) and Proposition 3, we can infer that the aggregate dynamics in response to a permanent monetary policy shock should be exactly the same in these two models.

In this context, our assumption on the synchronization between the timing of cost and price adjustments not only mirrors the observed data characteristics of wholesalers but also facilitates the comparison of our theoretical results with recent models featuring oligopolistic competitors and no distributors (e.g., Mongey 2021, Wang and Werning 2022).

E.2.2 Multi-country version

The takeaways of the above results carry over in a multi-country version of the model. The key difference in the multi-country version of the model is that a monetary policy or an exchange rate

¹² Apart from the evidence in Section 3, a similar synchronization pattern has been documented noted in the retail sector by Eichenbaum, Jaimovich and Rebelo (2011) and Goldberg and Hellerstein (2012).

shock can no longer be considered as a “common” shock. For example, a monetary shock may not directly affect the costs of distributors that source their products from abroad, while exchange rate movements may not directly influence the costs of distributors sourcing domestically.

Our theoretical result on sector prices (D.3) suggests that the impact of an unanticipated permanent monetary shock depends on how it affects the average cost index of the industry $\hat{Q}_{jt}^*$:

$$\mathbb{E}_t \hat{P}_{jt+\tau} = \frac{\rho^{\tau+1} - \Lambda^{\tau+1}}{\rho(1-b) + \lambda[\beta\rho(\lambda - \rho) - 1]} a \hat{Q}_{jt}^* = (1 - \Lambda^{\tau+1}) \hat{Q}_{jt}^* \quad \forall \tau \geq 0$$

In a simple setting where the change in marginal cost $\widehat{MC}_{ijt} = \widehat{M}_t = 1$ for domestically-sourced firms and $\widehat{MC}_{ijt} = 0$ for foreign-sourced firms, the change in the average cost index of the industry $\hat{Q}_{jt}^*$ depends on the total market share of domestically-sourced firms s_{jD} :

$$\hat{Q}_{jt}^* = s_{jD} \widehat{M}_t = s_{jD}.$$

The aggregate price dynamics can be expressed as a function of the mean market share of domestically-sourced firms $s_D \equiv \sum_j \alpha_j s_{jD}$,

$$\hat{P}_{t+\tau} = (1 - \Lambda^{\tau+1}) s_D,$$

and when $\varphi \rightarrow 0$, we have $\hat{P}_{t+\tau} = (1 - \lambda^{\tau+1}) s_D$ in a standard Calvo model. Therefore, although the magnitude of price change is attenuated by the fact that not all firms are directly affected by the monetary shock, the amplification effect of strategic complementarity relative to Calvo remains the same.

E.3 Aggregate dynamics in the full model and proof of Proposition 3

In this subsection, we prove Proposition 3 and show how introducing heterogeneous sectors into the model changes the aggregate price and output responses to a monetary shock.

Aggregate price dynamics. Let $\alpha_z = \sum_{j \in z} \alpha_j$ denote the total market share of sectors with market structure z . Assuming a sufficiently large number of sectors of each market structure z , the price index for type z sectors can be expressed as

$$\hat{P}_{zt+\tau} = \frac{1}{n_z} \sum_{j \in z} \hat{P}_{jt+\tau} = \frac{1}{n_z} \sum_{j \in z} (1 - \lambda_j) \hat{P}_{jt+\tau}^* + \frac{1}{n_z} \sum_{j \in z} \lambda_j \hat{P}_{jt+\tau-1} = (1 - \lambda_z) \hat{P}_{zt+\tau}^* + \lambda_z \hat{P}_{zt+\tau-1}$$

where the law of large numbers is applied to derive the second equality. However, due to sector heterogeneity, the aggregate price no longer follows the standard Calvo form:

$$\hat{P}_t = \sum_z \alpha_z \hat{P}_{zt} = \sum_z \alpha_z (1 - \lambda_z) \hat{P}_{zt}^* + \sum_z \alpha_z \lambda_z \hat{P}_{zt-1} \neq (1 - \lambda) \hat{P}_t^* + \lambda \hat{P}_{t-1}, \quad (\text{E.4})$$

where $\lambda \equiv \sum_z \alpha_z \lambda_z$. Note that the last inequality holds because $\hat{P}_{zt}$ is correlated with λ_z :

$$E[\lambda_z \hat{P}_{zt}] = E[\lambda_z]E[\hat{P}_{zt}] + Cov(\lambda_z, \hat{P}_{zt})$$

where the expectation and covariance are taken over sectors z . To be more concrete, note that (E.4) can be rewritten as

$$\begin{aligned} \hat{P}_t &= (1 - \lambda) \hat{P}_t^* + \lambda \hat{P}_{t-1} + Cov(\lambda_z, \hat{P}_{zt-1}) - Cov(\lambda_z, \hat{P}_{zt}^*) \\ &= (1 - \lambda) \hat{P}_t^* + \lambda \hat{P}_{t-1} + Cov(\lambda_z, \hat{P}_{zt-1}) - Cov \left[\lambda_z, \frac{1}{1 - \lambda_z} \left(\hat{P}_{zt} - \lambda_z \hat{P}_{zt-1} \right) \right] \\ &= (1 - \lambda) \hat{P}_t^* + \lambda \hat{P}_{t-1} - Cov \left[\lambda_z, \frac{1}{1 - \lambda_z} \left(\hat{P}_{zt} - \lambda_z \hat{P}_{zt-1} \right) \right] \end{aligned} \quad (E.5)$$

where $\lambda \equiv \sum_z \alpha_z \lambda_z$ and $Cov \left[\lambda_z, \frac{1}{1 - \lambda_z} \left(\hat{P}_{zt} - \lambda_z \hat{P}_{zt-1} \right) \right]$ represent the additional price stickiness due to sector heterogeneity.

Price and output responses to monetary shock. Under a permanent monetary shock at t , the expected future price of sector j can be solved as

$$\mathbb{E}_t \hat{P}_{jt+\tau} = 1 - \Lambda_j^{\tau+1} \quad \forall \tau \geq 0,$$

where

$$\begin{aligned} \Lambda_j &\equiv \frac{1}{2} \left[\lambda_j + \frac{1 - b_j}{\beta \lambda_j} - \sqrt{\left(\lambda_j + \frac{1 - b_j}{\beta \lambda_j} \right)^2 - \frac{4}{\beta}} \right], \\ b_j &\equiv (1 - \beta \lambda_j)(1 - \lambda_j) \sum_i s_{ij} \frac{\varphi_{ij}}{(1 + \varphi_{ij})}. \end{aligned}$$

Summing over similar sectors in each structure type z , we have

$$\hat{P}_{zt+\tau} \equiv \frac{1}{n_z} \sum_{j \in z} \hat{P}_{jt+\tau} = 1 - \Lambda_z^{\tau+1} \quad \forall \tau \geq 0, \quad (E.6)$$

Using (E.6), the inflation and output dynamics can be derived as

$$\hat{\pi}_{zt+\tau} = \hat{P}_{zt+\tau} - \hat{P}_{zt+\tau-1} = (1 - \Lambda_z) \Lambda_z^\tau \quad \text{and} \quad \hat{c}_{zt+\tau} = 1 - \hat{P}_{zt+\tau} = \Lambda_z^{\tau+1} \quad \forall \tau \geq 0.$$

Substituting (E.6) into (E.5), we have

$$\hat{P}_t = (1 - \lambda) \hat{P}_t^* + \lambda \hat{P}_{t-1} - Cov \left[\lambda_z, \frac{1 - \Lambda_z}{1 - \lambda_z} \Lambda_z^\tau \right].$$

Cumulative output response. The cumulative output response in a heterogeneous sector monopolistic competition Calvo model is given by

$$E_z \left[\frac{\lambda_z}{1 - \lambda_z} \right]. \quad (\text{E.7})$$

It is well known that the cumulative output response evaluated at the average frequency of price adjustments of the heterogeneous sector economy, $\lambda \equiv E_z[\lambda_z]$, is downward biased. Taking a second-order approximation of the function (E.7), we have

$$E_z \left[\frac{\lambda_z}{1 - \lambda_z} \right] \approx \frac{\lambda}{1 - \lambda} + \frac{1}{(1 - \lambda)^3} \sigma_{\lambda_z}^2 \geq \frac{\lambda}{1 - \lambda}.$$

But, as shown in [Carvalho \(2006\)](#), the cumulative output response evaluated at the average duration of price adjustment $x_z \equiv 1/(1 - \lambda_z)$ gives the correct impact. To see this, note

$$E_z \left[\frac{\lambda_z}{1 - \lambda_z} \right] = E_z [x_z - 1] = x - 1,$$

where $x \equiv E_z[1/(1 - \lambda_z)]$. Under monopolistic competition, the cumulative real impact of a monetary shock can be replicated by simply targeting the observed average duration of price adjustments.

With market power, the cumulative output response becomes

$$E_z \left[\frac{\Lambda_z}{1 - \Lambda_z} \right].$$

By the same token, the cumulative output response can be obtained by evaluating the average *market power adjusted* duration of price adjustments: $X_z \equiv 1/(1 - \Lambda_z)$

$$E_z \left[\frac{\Lambda_z}{1 - \Lambda_z} \right] = E_z [X_z - 1] = X - 1,$$

where $X \equiv E_z[1/(1 - \Lambda_z)]$.

Note that, when firms have market power, the conventional average duration of price adjustment x_z is no longer sufficient to capture cumulative output amplification. In this case, accounting for the impact of real rigidity, by targeting the duration implied by Λ_z rather than λ_z , is important. To see this, we can decompose the cumulative output response into two components

$$\begin{aligned} E_z \left[\frac{\Lambda_z}{1 - \Lambda_z} \right] &= E_z \left[\frac{\lambda_z}{1 - \lambda_z} \frac{\Lambda_z(1 - \lambda_z)}{(1 - \Lambda_z)\lambda_z} \right] = E_z \left[\frac{\lambda_z}{1 - \lambda_z} \right] E_z \left[\frac{\Lambda_z(1 - \lambda_z)}{(1 - \Lambda_z)\lambda_z} \right] + Cov_z \left[\frac{\lambda_z}{1 - \lambda_z}, \frac{\Lambda_z(1 - \lambda_z)}{(1 - \Lambda_z)\lambda_z} \right] \\ &= (x - 1)E_z \left(\frac{X_z - 1}{x_z - 1} \right) + Cov_z \left(x_z - 1, \frac{X_z - 1}{x_z - 1} \right). \end{aligned}$$

This completes the proof of Proposition 3. ■

Table 3 summarizes the key quantitative impacts in different versions of the model (weighted). Figure E3 shows the correlation between market power and price stickiness across industries and

products.

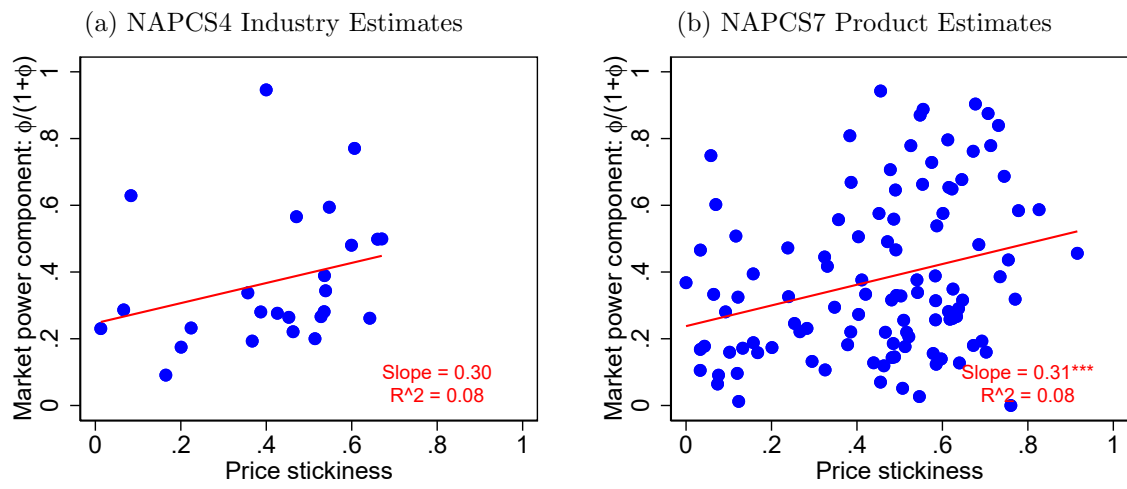


Figure E3: Correlation between market power and price stickiness

E.4 Secular changes in market power and markups

In this section, we calculate the slope implied by secular changes in market power and price stickiness. We combine time series on the post-war evolution of markups and price stickiness with numerical simulations from the model to construct time series for the implied slope of the NKPC. We assume that time variation in Canadian sector-level market power and price stickiness is the same as the variation in aggregate U.S. market power and price stickiness. We obtain time series' for average U.S. markups from [De Loecker, Eeckhout and Unger \(2020\)](#) and the average frequency of the U.S. consumer price changes from [Nakamura et al. \(2018\)](#).¹³ Figure E4 provides the two aggregate time series.

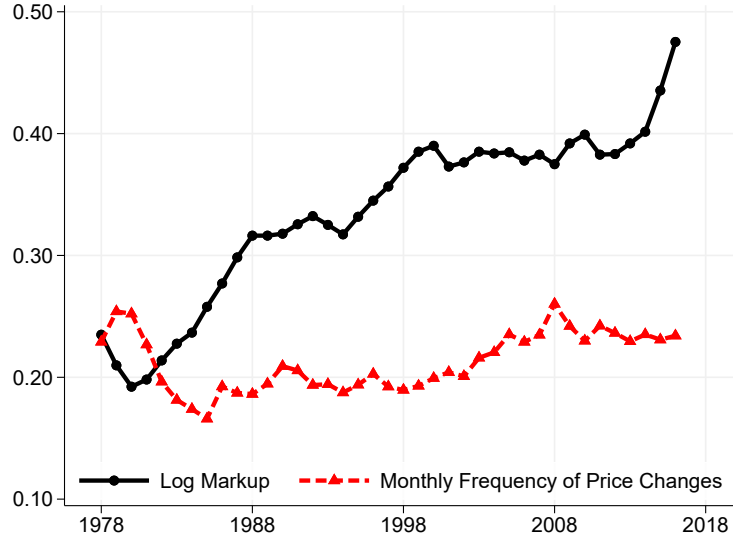


Figure E4: Log Markups and the Frequency of Consumer Price Changes in the United States

We construct the implied NKPC slope using the following steps:

1. Calculate sectoral strategic complementarity φ_j from estimated idiosyncratic cost pass-through coefficients using Canadian data: $\varphi_j = 1/\psi_j - 1$, where ψ_j denotes the idiosyncratic pass-through in sector j .
2. Back out the sectoral elasticity of substitution θ_j implied by φ_j and the sectoral markup μ_j in

$$\varphi_j = \left(\frac{\theta_j - 1}{\theta_j} \mu_j - 1 \right) (\theta_j - 1).$$

i.e., using the quadratic formula:

$$\theta_j = \frac{2\mu_j + \varphi_j - 1 + \sqrt{\varphi_j^2 + (4\mu_j - 2)\varphi_j + 1}}{2(\mu_j - 1)}, \quad \forall \mu_j > 1, \theta_j > 1.$$

¹³We thank Daniel Villar for providing us with the series for consumer price changes.

3. Construct time-varying Canadian markups by assuming they follow the same trend as U.S. aggregate markups: $\mu_{jt} = \mu_j \cdot \mu_t^{US} / \mu_{2016}^{US}$.
4. Impose the constraint $\mu_{jt} \geq \theta_j / (\theta_j - 1)$ to ensure non-negative φ_{jt} . This constraint binds for approximately half of the sector-year observations.
5. Calculate the time-varying strategic complementarity $\varphi_{jt} = \left(\frac{\theta_j - 1}{\theta_j} \mu_{jt} - 1 \right) (\theta_j - 1)$.
6. Calculate time-varying price stickiness using an additive adjustment: $\lambda_{jt} = \lambda_j + \lambda_t^{US} - \lambda_{2016}^{US}$, where $\lambda_t^{US} = 1 - \text{freq}_t^{US}$ and freq_t^{US} is the frequency of U.S. consumer price changes. We restrict $\lambda_{jt} \in [0, 0.99]$.
7. Compute the implied slope κ_t :

$$\kappa_t = \frac{(1 - \beta \Lambda_t)(1 - \Lambda_t)}{\Lambda_t},$$

where Λ_t is chosen so that the cumulative output impulse response in homogeneous-sector model (i.e., $S_t = \Lambda_t / (1 - \Lambda_t)$) equals to the cumulative output response in heterogeneous sector model (i.e., $\bar{S}_t = \frac{1}{J} \sum_{j=1}^J \Lambda_{jt} / (1 - \Lambda_{jt})$) with

$$\Lambda_{jt} = \frac{1}{2} \left(\lambda_{jt} + \frac{1 - b_{jt}}{\beta \lambda_{jt}} - \sqrt{\left(\lambda_{jt} + \frac{1 - b_{jt}}{\beta \lambda_{jt}} \right)^2 - \frac{4}{\beta}} \right),$$

and $b_{jt} = \frac{\varphi_{jt}}{1 + \varphi_{jt}} (1 - \beta \lambda_{jt}) (1 - \lambda_{jt})$. Throughout, we set the monthly discount factor to $\beta = 0.97^{1/12} \approx 0.9975$. Solving $S_t = \bar{S}_t$ gives $\Lambda_t = \frac{\bar{S}_t}{1 + \bar{S}_t}$.

We compare three scenarios: (i) homogeneous price rigidity with no market power, where $\varphi_{jt} = 0$ for all j and t , and $\lambda_{jt} = \bar{\lambda}_t$ (the cross-sectional mean); (ii) homogeneous price rigidity with heterogeneous market power, where φ_{jt} varies across sectors but $\lambda_{jt} = \bar{\lambda}_t$; and (iii) heterogeneous price rigidity with heterogeneous market power, where both φ_{jt} and λ_{jt} vary across sectors.

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