Supplemental Appendix

Uncertainty and Change: Survey Evidence of Firms' Subjective Beliefs

Rüdiger Bachmann, Kai Carstensen, Stefan Lautenbacher,
Manuel Menkhoff, and Martin Schneider

Appendix A Representativeness of the sample

In this appendix, we investigate whether participation in our quantitative uncertainty module is driven by selection, conditional on participation in the main manufacturing survey. We base our analysis on all 69,163 complete firm-quarter responses available in the main survey for the months the uncertainty module was executed. We then ask whether firm size, time dummies, industry dummies, and interacted time-industry dummies are able to predict participation in the uncertainty module. To this end, we run a probit regression of a participation dummy that is 1 for the 14,150 observations of the quantitative uncertainty module and zero otherwise, on these predictors. Since ifo placed the quantitative uncertainty module more prominently after 2017:Q1, the number of respondents increased considerably, which prompted us to split the sample around that date for the purpose of this analysis. We report the estimated coefficients in column (1) of Tables A1 and A2. We find that there is no significant selection with respect to quarter/survey wave and industry suggesting that the uncertainty sample does not misrepresent specific quarters or industries. While firm size turns out to be significantly negative indicating that large firms are slightly underrepresented in the uncertainty module compared to the main manufacturing survey, the pseudo R-squared of 0.016 shows that this selection is quantitatively irrelevant. This is also reflected by an ROC curve which differs only slightly from the diagonal that indicates no discriminatory power, see the left panels of Figures A1 and A2.[1]

We repeat the analysis starting from the subset of 43,211 complete firm-quarter responses available from the online part of the main survey. We thus account for the fact that some firms reply to the main survey by fax and thus, essentially mechanically, do not participate in the uncertainty module, which is solely implemented online. The results of an analogous probit regression are reported in column (2) of Tables A1 and A2. Again, the very low pseudo R-squared suggests that selectivity is not a relevant issue for the uncertainty module. This conclusion is supported by a largely unaltered ROC curve near the non-discriminatory diagonal, see the right panels of Figures A1 and A2.

[1]The receiver operating characteristic (ROC) visualizes the discriminatory power of a binary classifier as follows: By varying the classification threshold—here: the probability above which an observation is predicted to participate in the uncertainty module—the classifier can produce any true positive rate (type II error). The ROC curve plots the true positive rate so obtained against its corresponding false positive rate (type I error). In the case of no discriminatory power, true and false positive rates are always the same, the ROC curve equals the diagonal, and the area under the ROC curve (AUC) is 0.5. A good classifier has a ROC curve well above the diagonal and an AUC that is near the maximum of 1.0.

Table A1: Probit regression of a dummy indicating special survey participation on ifo variables until 2017 Q1

	(1) probit vs. ifo all	(2) probit vs. ifo online
Log of number of employees	-0.0754***	-0.132***
	(0.0127)	(0.0138)
Dummy survey wave 1	-0.0765	0.0215
	(0.276)	(0.295)
Dummy survey wave 2	0.0323	0.185
	(0.275)	(0.294)
Dummy survey wave 3	0.215	0.403
	(0.232)	(0.245)
Dummy survey wave 4	0.243	0.390
	(0.236)	(0.251)
Dummy survey wave 5	0.0933	0.204
	(0.243)	(0.255)
Dummy survey wave 6	0.284	0.441**
	(0.214)	(0.219)
Dummy survey wave 7	0.191	0.338
	(0.222)	(0.231)
Dummy survey wave 8	-0.277	-0.198
	(0.269)	(0.283)
Dummy survey wave 9	0.0777	0.178
	(0.213)	(0.220)
Dummy survey wave 10	0.192	0.312
	(0.237)	(0.251)
Dummy survey wave 11	-0.0657	0.0238
	(0.216)	(0.226)
Dummy survey wave 12	0.122	0.226
	(0.205)	(0.211)
Dummy survey wave 13	0.289*	0.285
	(0.166)	(0.175)
Dummy survey wave 14	-0.0324	-0.0494
	(0.209)	(0.219)
Dummy survey wave 15	0.0954	0.0615
	(0.150)	(0.163)
Dummy supersector 1	0.0375	0.394
	(0.258)	(0.279)
Dummy supersector 2	-0.432	-0.334
	(0.292)	(0.310)
Dummy supersector 3	-0.146	0.0878
	(0.243)	(0.260)
Dummy supersector 4	-0.0135	0.0562
	(0.266)	(0.284)
Dummy supersector 5	0.0316	0.210
	(0.253)	(0.268)
Dummy supersector 6	-0.265	-0.188
	(0.260)	(0.275)
Dummy supersector 7	-0.239	-0.00426
	(0.270)	(0.290)
Dummy supersector 8	0.00565	0.228
	(0.274)	(0.294)
Dummy supersector 9	-0.387	-0.233
	(0.247)	(0.263)
Dummy supersector 10	0.0800	0.230
	(0.269)	(0.288)
Dummy supersector 11	-0.337	-0.211
	(0.261)	(0.277)
Dummy supersector 12	-0.203	-0.110
	(0.238)	(0.251)
Dummy supersector 13	0.0698	0.270
	(0.276)	(0.295)
Constant	-0.572**	-0.216
	(0.228)	(0.242)
Additional wave-supersector dummies	YES	YES
No. of observations	38958	26520
No. of firms	3481	2440
No. of parameters (excl. intercept)	224	224
Pseudo R-squared	0.016	0.031

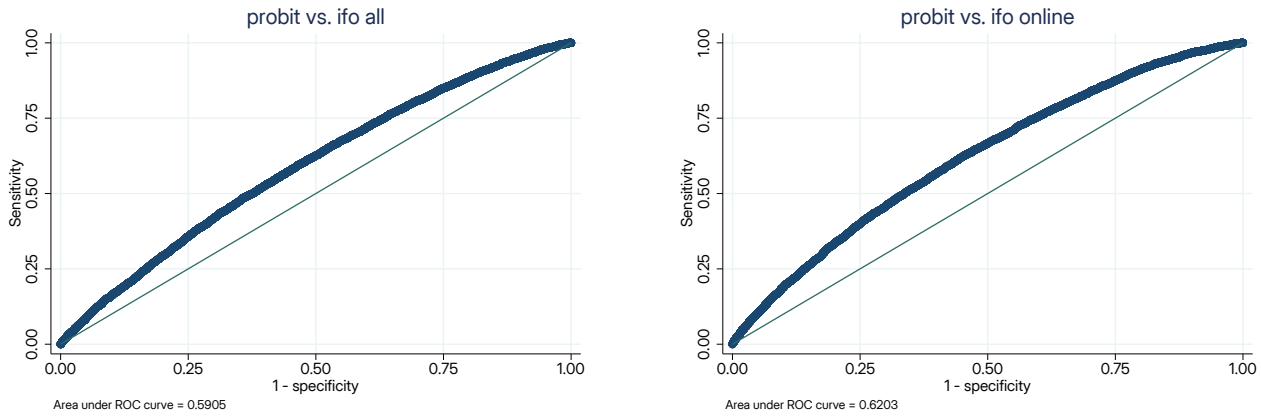
Standard errors in parentheses, clustered by firm; * p < 0.10, ** p < 0.05, *** p < 0.01.

Table A2: Probit regression of a dummy indicating special survey participation on ifo variables since 2017 Q2

	(1) probit vs. ifo all	(2) probit vs. ifo online
Log of number of employees	0.0319***	-0.0491***
	(0.0118)	(0.0136)
Dummy survey wave 1	0.166	-0.174
	(0.189)	(0.214)
Dummy survey wave 2	0.368*	0.00766
	(0.208)	(0.233)
Dummy survey wave 3	0.440**	0.106
	(0.175)	(0.213)
Dummy survey wave 4	0.647***	0.305
	(0.234)	(0.266)
Dummy survey wave 5	0.198	0.307
	(0.191)	(0.248)
Dummy survey wave 6	0.179	0.223
	(0.207)	(0.273)
Dummy survey wave 7	0.130	0.235
	(0.150)	(0.217)
Dummy survey wave 8	0.340**	0.510**
	(0.173)	(0.244)
Dummy survey wave 9	0.214	0.186
	(0.186)	(0.243)
Dummy survey wave 10	0.315*	0.183
	(0.171)	(0.226)
Dummy supersector 1	-0.240	0.224
	(0.201)	(0.253)
Dummy supersector 2	-0.116	0.0410
	(0.222)	(0.273)
Dummy supersector 3	-0.217	-0.114
	(0.194)	(0.234)
Dummy supersector 4	-0.358	-0.332
	(0.225)	(0.273)
Dummy supersector 5	-0.0230	0.0236
	(0.202)	(0.241)
Dummy supersector 6	0.138	0.236
	(0.196)	(0.238)
Dummy supersector 7	0.0315	0.243
	(0.202)	(0.248)
Dummy supersector 8	-0.150	0.100
	(0.208)	(0.255)
Dummy supersector 9	0.0213	0.388*
	(0.185)	(0.226)
Dummy supersector 10	0.125	0.368
	(0.199)	(0.245)
Dummy supersector 11	0.0909	0.278
	(0.198)	(0.240)
Dummy supersector 12	-0.178	0.0462
	(0.184)	(0.222)
Dummy supersector 13	-0.179	-0.0345
	(0.223)	(0.268)
Constant	-0.664***	0.174
	(0.184)	(0.219)
Additional wave-supersector dummies	YES	YES
No. of observations	30205	16691
No. of firms	4271	2442
No. of parameters (excl. intercept)	154	154
Pseudo R-squared	0.016	0.016

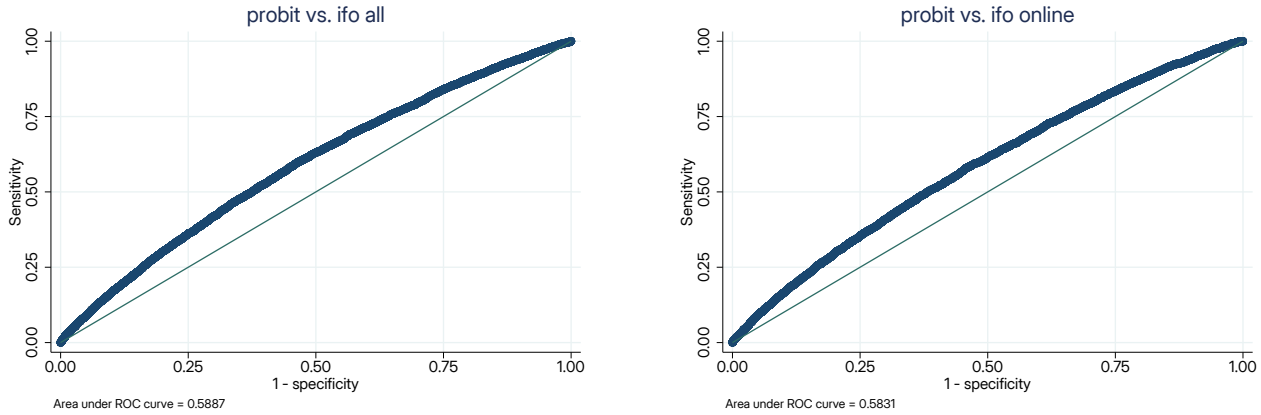
Standard errors in parentheses, clustered by firm; * p < 0.10, ** p < 0.05, *** p < 0.01.

Figure A1: ROC curves for probit estimations until 2017Q1



Notes: The two plots depict ROC curves that correspond to the probit estimations in columns (1) and (2) of Table A1, respectively.

Figure A2: ROC curves for probit estimations since 2017Q1



Notes: The two plots depict ROC curves that correspond to the probit estimations in columns (1) and (2) of Table A2, respectively.

Appendix B Seasonality

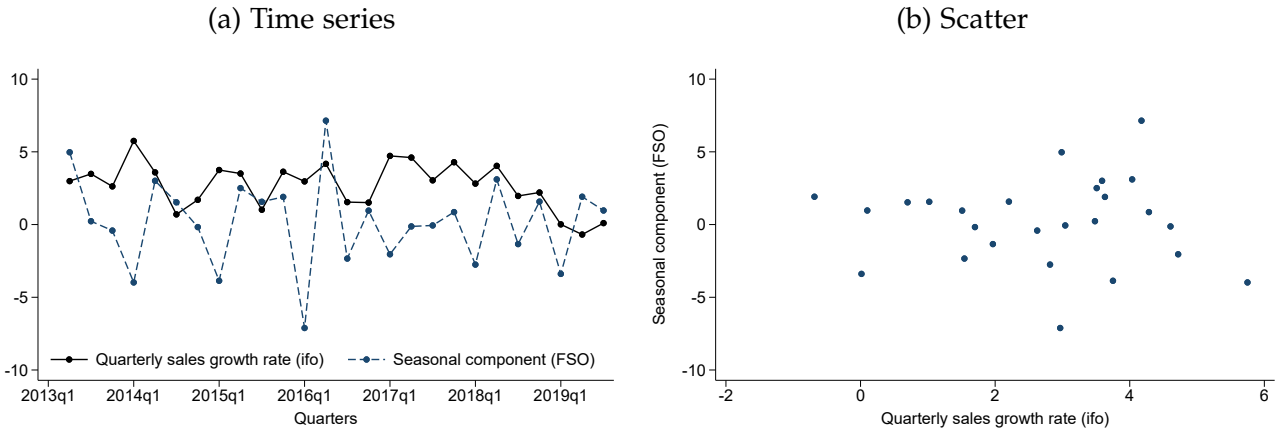
The main ifo Business Survey requests that firms ignore seasonal fluctuations in their answers. For every survey wave it is emphasized that firm should exclude seasonal fluctuations in their answers. Indeed, the survey instructions point to seasonal patterns due to changes in demand, holidays, and vacations. We also note that firms in our sample have participated for on average of 8.8 waves (more than two years) and the median firm participated 8 times in our quantitative online module. Most of these firms have participated in the general ifo survey for much longer. It is therefore reasonable to assume that the firms are well aware of this instruction. In fact, we show next that this assumption is borne out in the data.

We begin by comparing the aggregated quarterly sales growth rate from the ifo survey with the seasonal component of aggregate quarterly sales growth for the German manufacturing sector from the national accounting data (FSO: Federal Statistical Office). Figure B1 in the left panel shows the time series of these two variables and the right panel shows them as a scatterplot. It is clearly visible that they are uncorrelated ($\rho = -0.02$), meaning that the ifo sales growth rate—at least in the aggregate—has no seasonal pattern.

In a second step, we look at the micro level and we regress firm level quarterly sales growth on seasonal dummies (and firm fixed effects as well as, in some specifications, time fixed effects). Table B1 shows the results: all seasonal dummies are statistically insignificant. Columns (1) and (2) do this for all firms in our sample, while columns (3) and (4) restrict the sample to firms that are observed at least 8 times.

To sum up, both the macro and the micro level analysis corroborate that firms follow the survey's instruction to deseasonalize their answers.

Figure B1: Sales growth—survey vs seasonal component from national accounting data



Notes: The figure shows the aggregated quarterly sales growth rate from the ifo survey (from our final sample) and the seasonal component of aggregate quarterly sales growth for the German manufacturing sector from the national accounting data as published by the Federal Statistical Office (FSO). Source: FSO table 42152-0001, retrieved 05/27/2024.

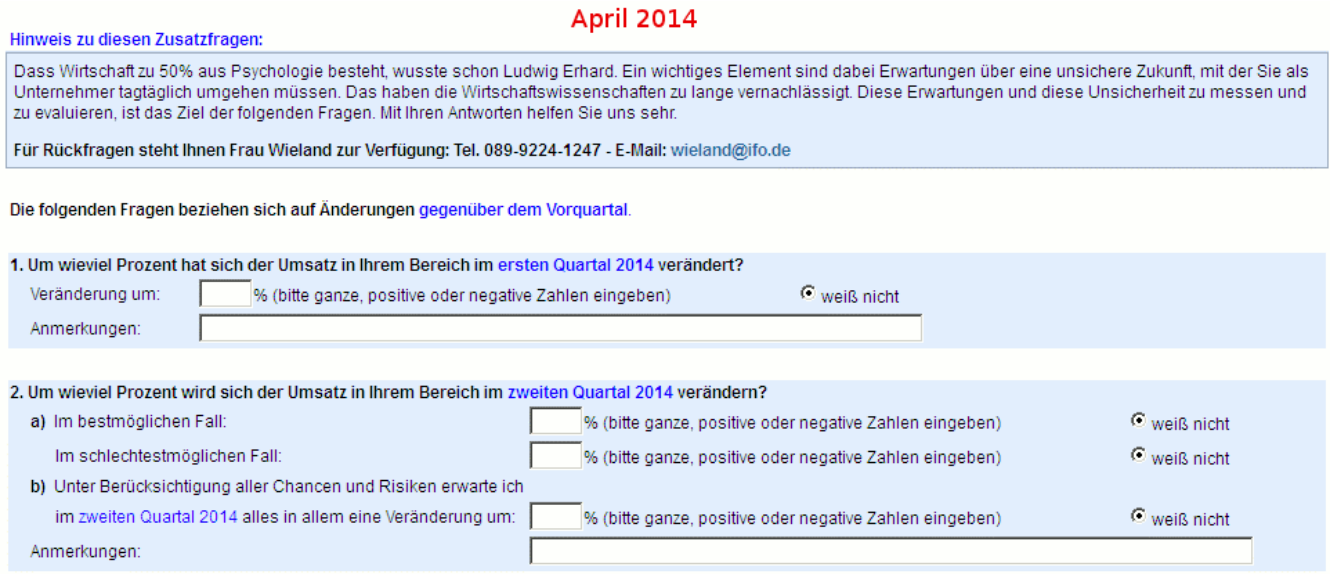
Table B1: Regression of survey sales growth on quarterly dummies

	(1)	(2)	(3)	(4)
Quarter 2	0.732	0.561	1.118	0.933
	(0.612)	(0.619)	(0.775)	(0.783)
Quarter 3	0.942	0.697	0.764	0.532
	(0.577)	(0.590)	(0.738)	(0.754)
Quarter 4	-0.313	-0.578	-0.185	-0.396
	(0.497)	(0.507)	(0.619)	(0.631)
Constant	1.820***	2.000***	1.724***	1.889***
	(0.375)	(0.384)	(0.468)	(0.479)
Observations	8882	8882	5660	5660
Firm FE	Yes	Yes	Yes	Yes
Year FE	No	Yes	No	Yes

Notes: The table shows the results of regressing firm level quarterly sales growth on seasonal dummies. All specifications include firm fixed effects. Columns (2) and (4) additionally include time fixed effects. Columns (1) and (2) use all firms in our sample, while columns (3) and (4) restrict the sample to firms that are observed at least 8 times. Standard errors in parentheses, clustered by firm; * p < 0.10, ** p < 0.05, *** p < 0.01.

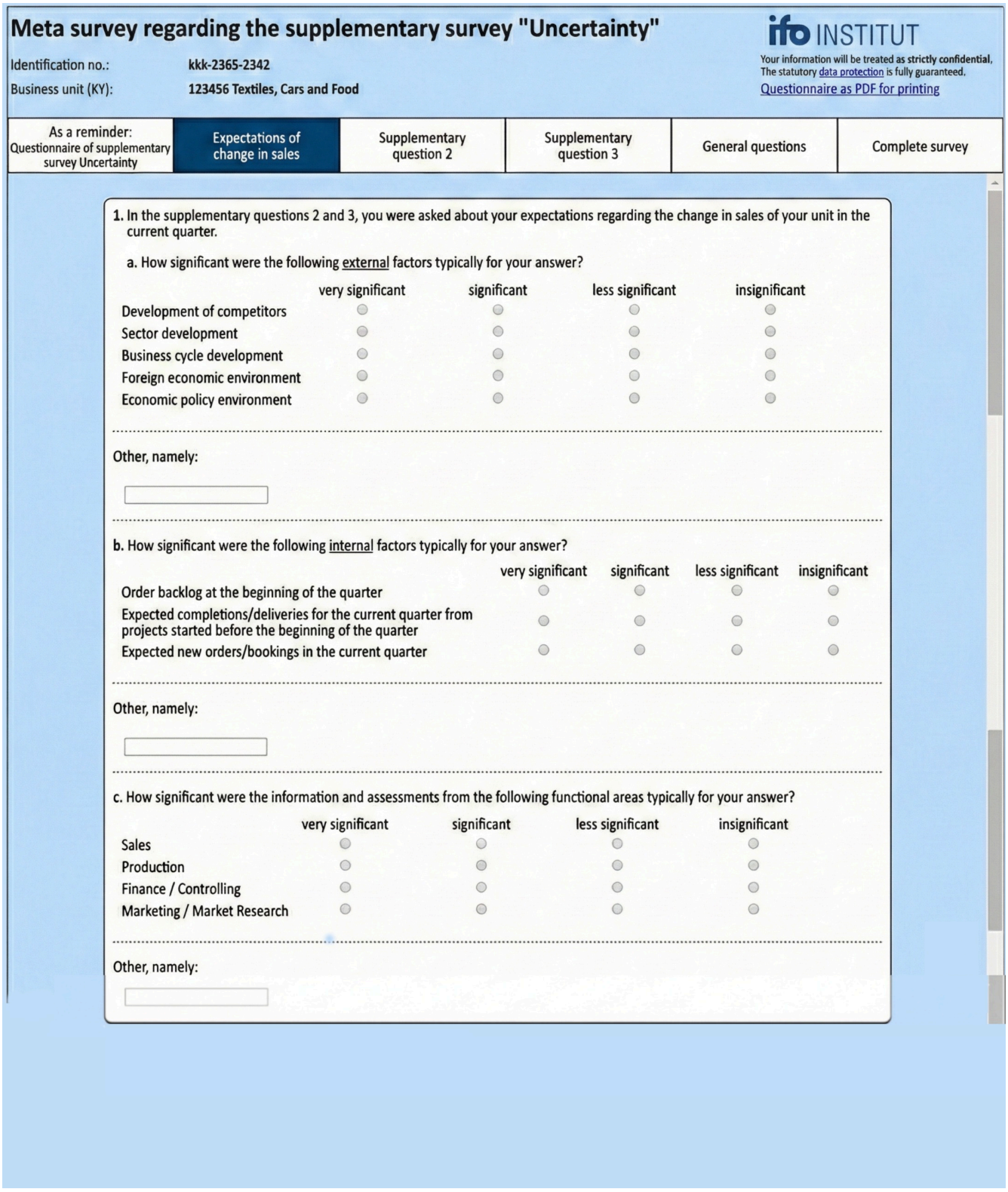
Appendix C Questionnaire screenshots

Figure C1: Original survey questionnaire in German



Notes: Original questionnaire from ifo's online module on subjective uncertainty in German; screenshot from April 2014.

Figure C2: Survey questionnaire translated to English



Meta survey regarding the supplementary survey "Uncertainty"

ifo INSTITUT

Identification no.: kkk-2365-2342

Business unit (KY): 123456 Textiles, Cars and Food

Your information will be treated as strictly confidential, The statutory data protection is fully guaranteed.

Questionnaire as PDF for printing

As a reminder: Questionnaire of supplementary survey Uncertainty | Expectations of change in sales | Supplementary question 2 | Supplementary question 3 | General questions | Complete survey

1. In the supplementary questions 2 and 3, you were asked about your expectations regarding the change in sales of your unit in the current quarter.

a. How significant were the following external factors typically for your answer?

	very significant	significant	less significant	insignificant
Development of competitors	○	○	○	○
Sector development	○	○	○	○
Business cycle development	○	○	○	○
Foreign economic environment	○	○	○	○
Economic policy environment	○	○	○	○

Other, namely:

b. How significant were the following internal factors typically for your answer?

	very significant	significant	less significant	insignificant
Order backlog at the beginning of the quarter	○	○	○	○
Expected completions/deliveries for the current quarter from projects started before the beginning of the quarter	○	○	○	○
Expected new orders/bookings in the current quarter	○	○	○	○

Other, namely:

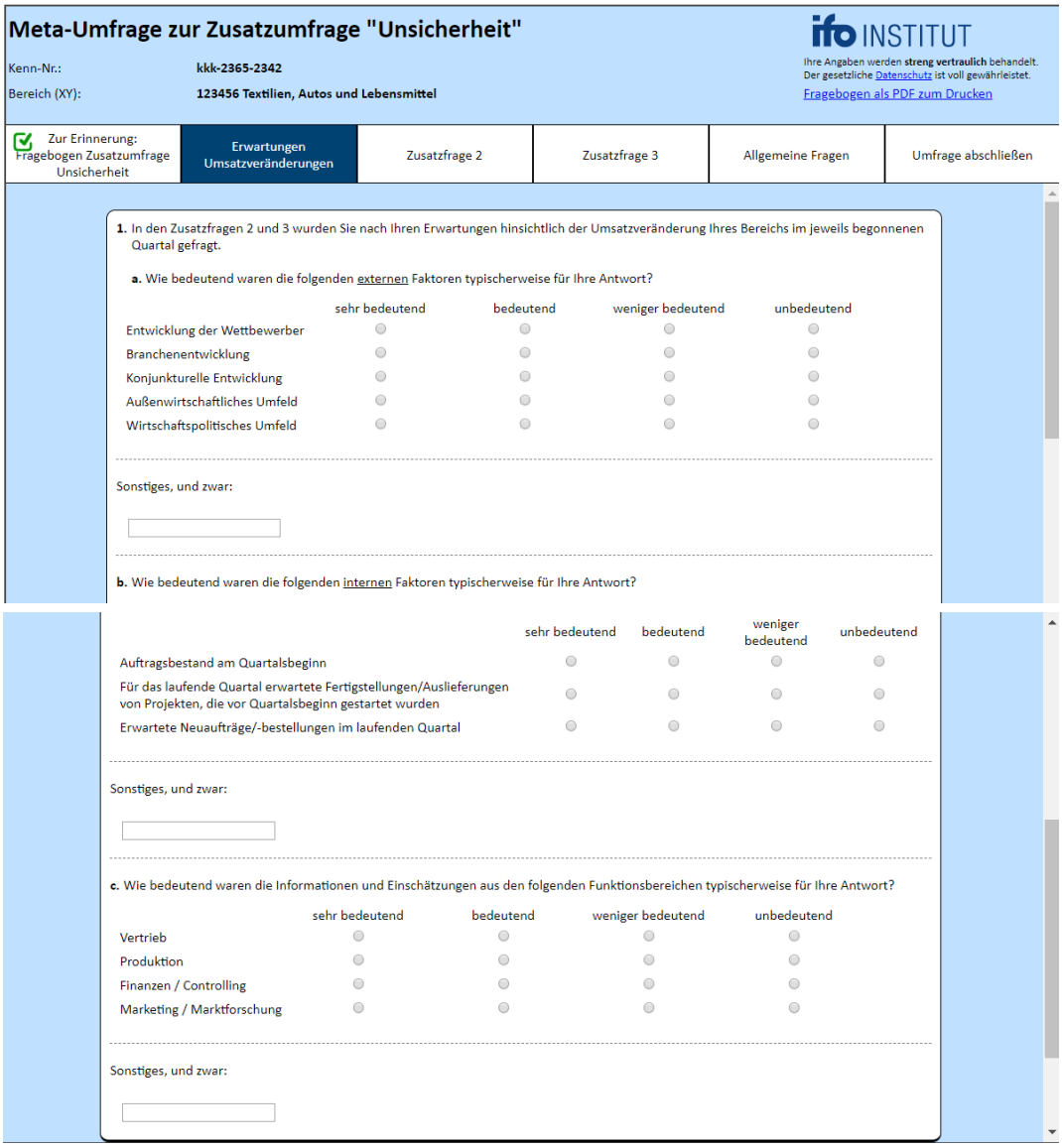
c. How significant were the information and assessments from the following functional areas typically for your answer?

	very significant	significant	less significant	insignificant
Sales	○	○	○	○
Production	○	○	○	○
Finance / Controlling	○	○	○	○
Marketing / Market Research	○	○	○	○

Other, namely:

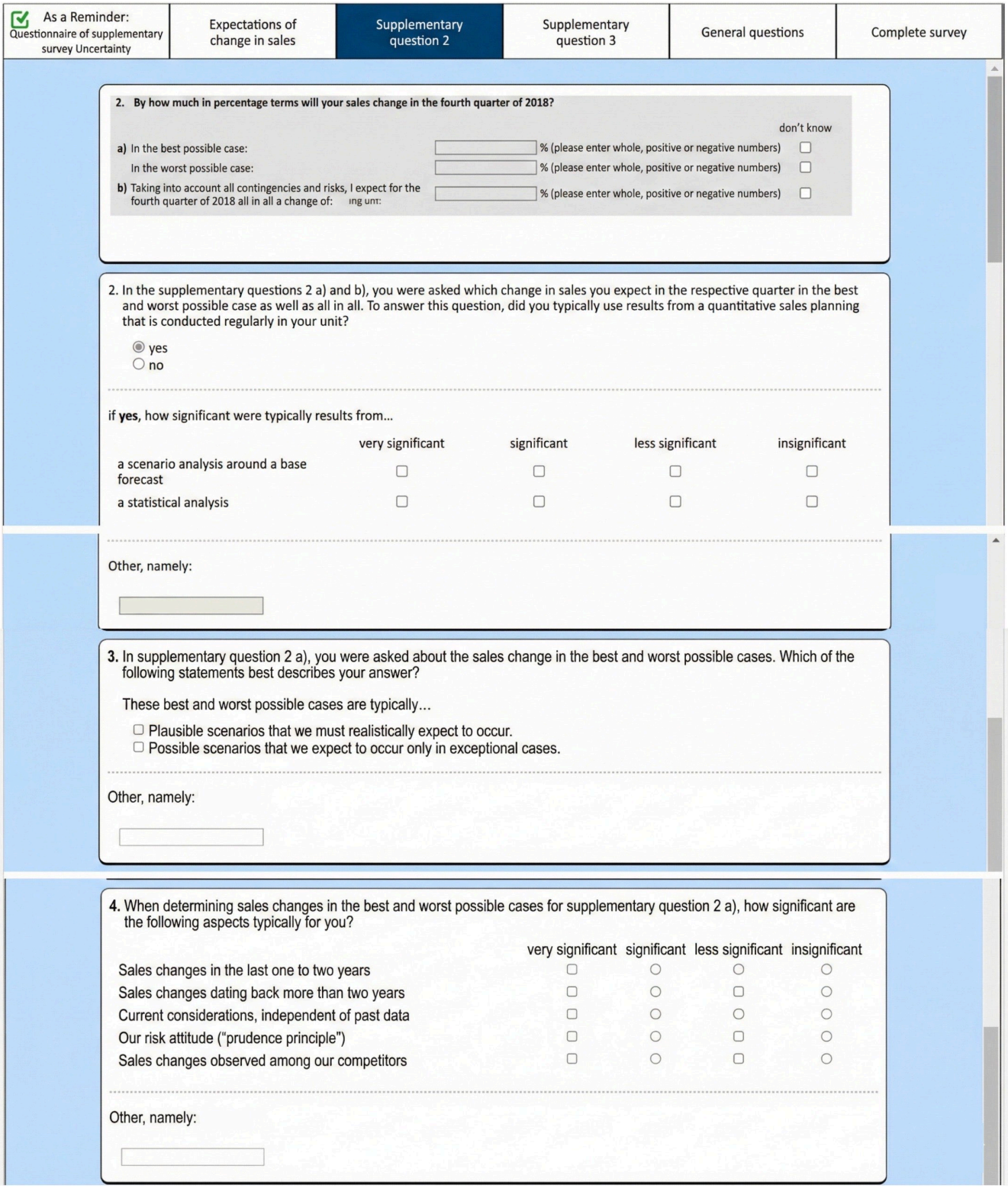
Notes: Questionnaire from ifo's one-time online special survey on its "uncertainty module"; translated to English from a screenshot from fall 2018. (1/3)

Figure C3: Original survey questionnaire in German



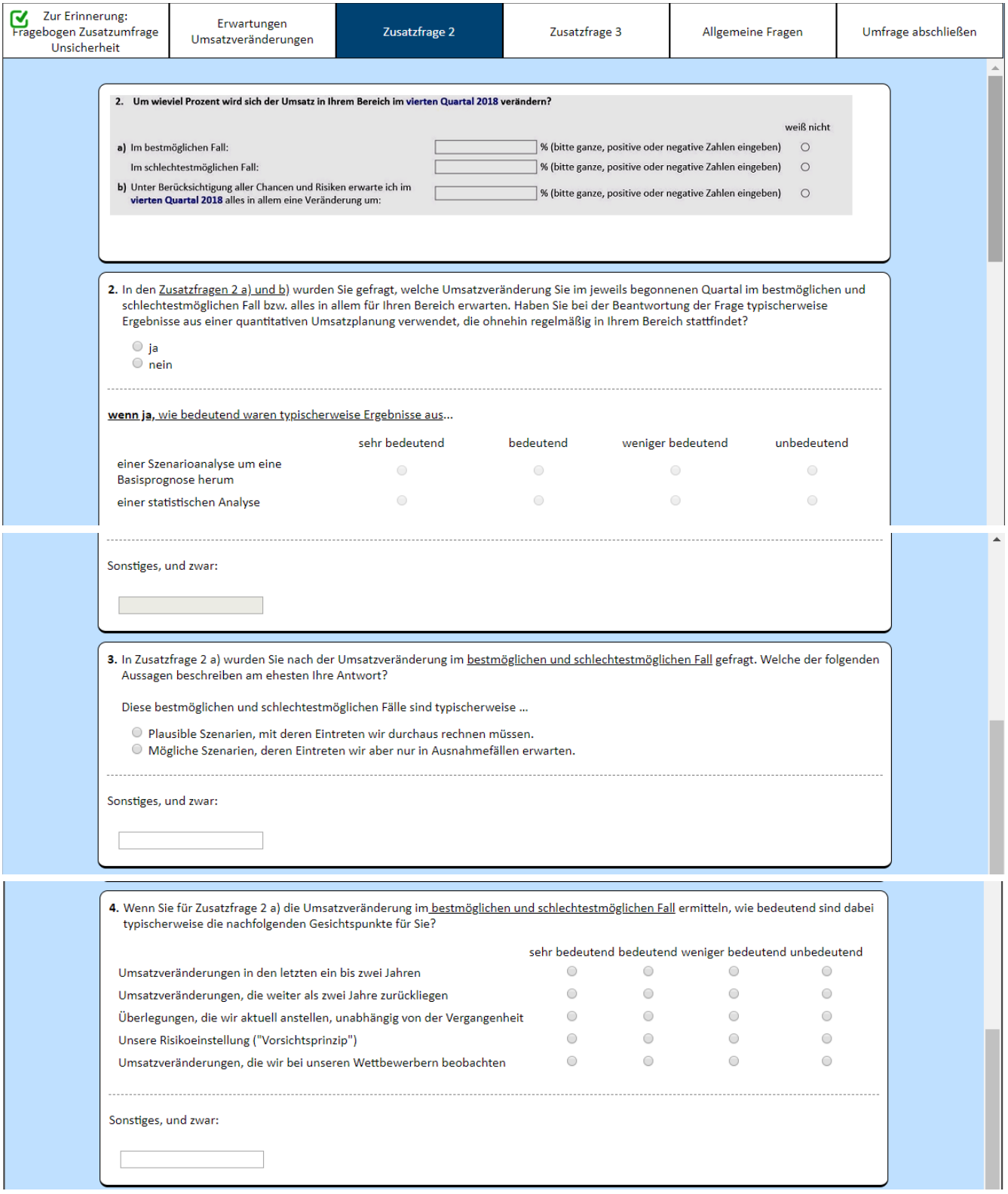
Notes: Original questionnaire from ifo's one-time online special survey on its "uncertainty module" in German, from fall 2018. (1/3)

Figure C4: Survey questionnaire translated to English



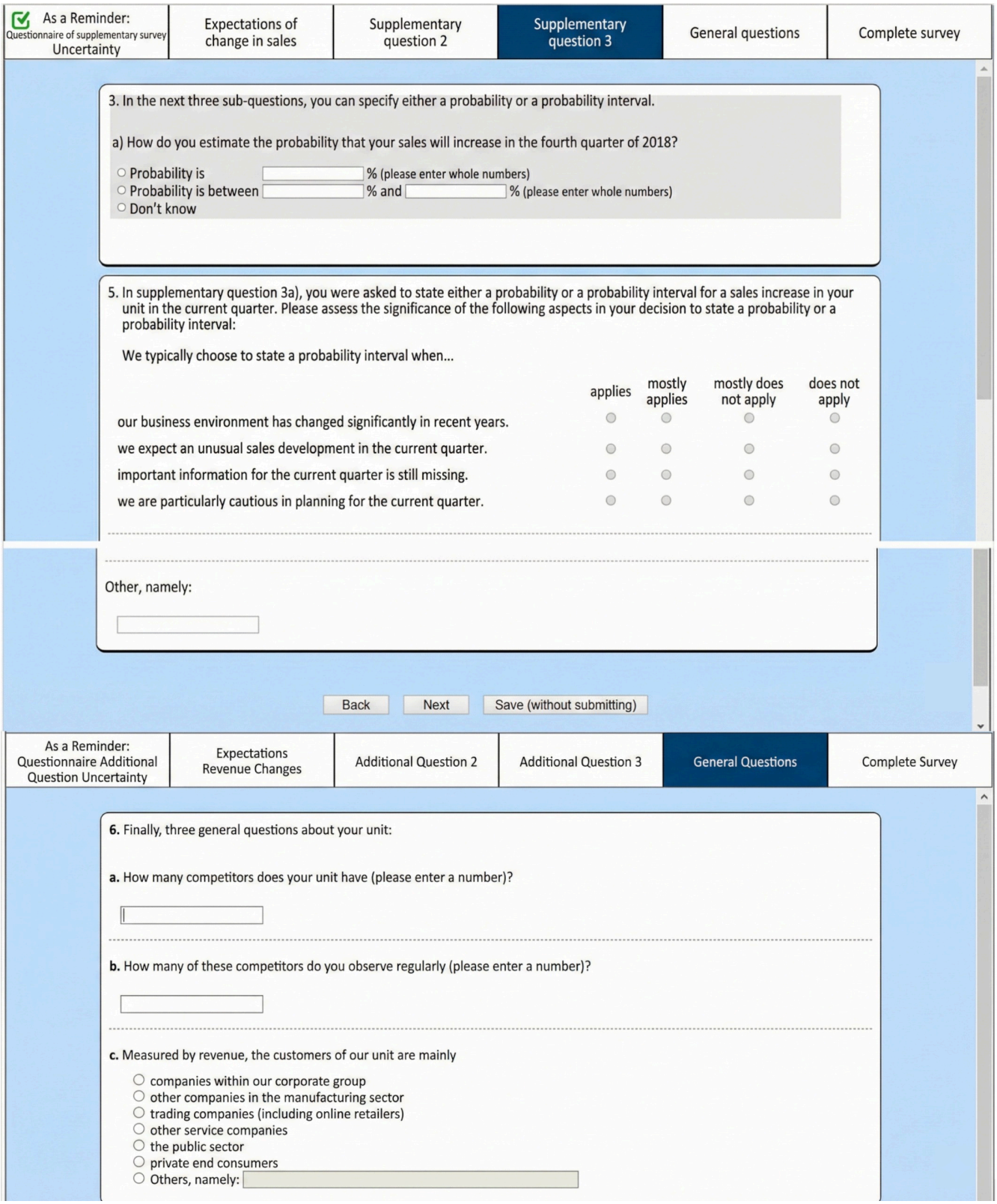
Notes: Questionnaire from ifo's one-time online special survey on its "uncertainty module"; translated to English from a screenshot from fall 2018. (2/3)

Figure C5: Original survey questionnaire in German



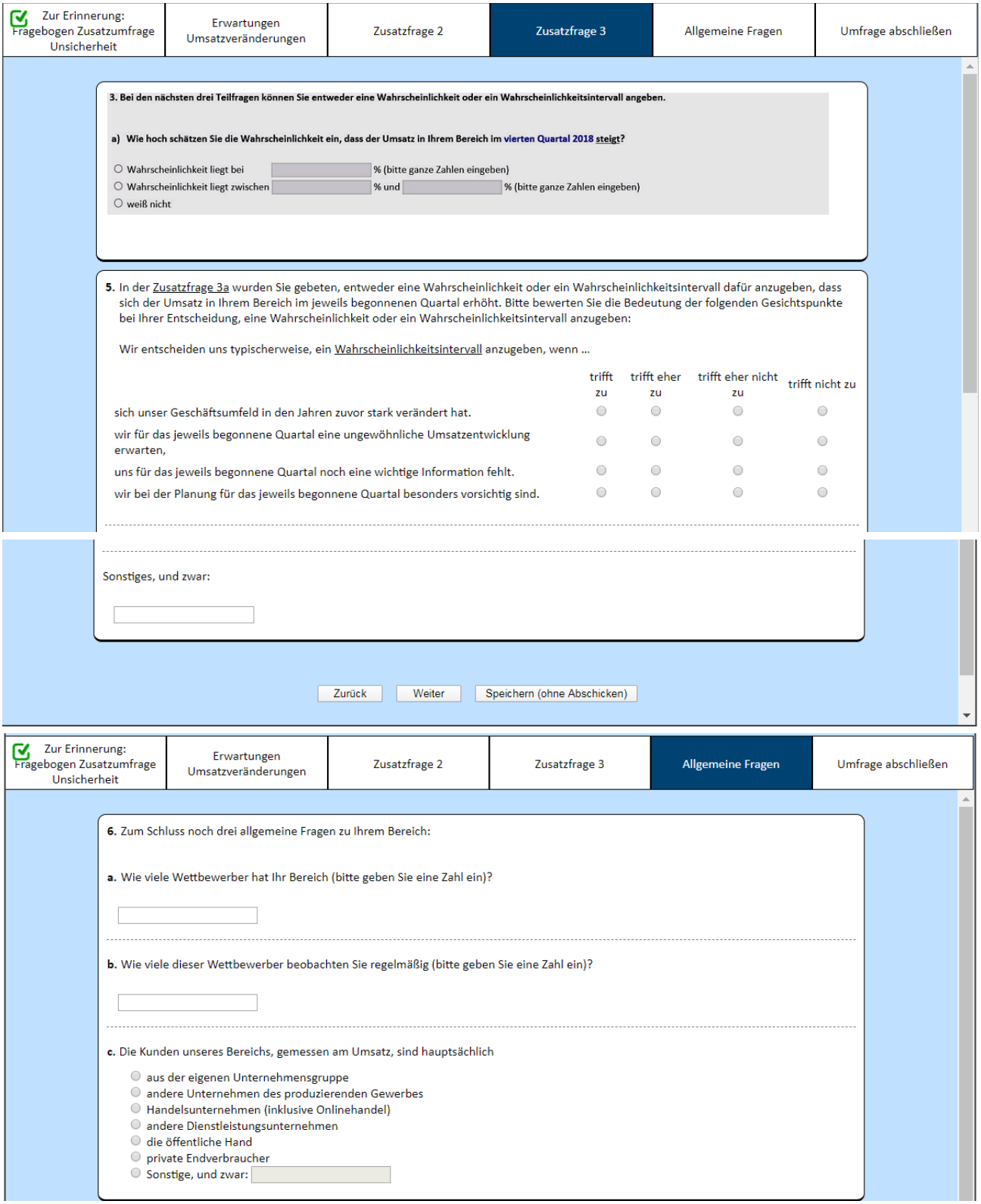
Notes: Original questionnaire from ifo's one-time online special survey on its "uncertainty module" in German, from fall 2018. (2/3)

Figure C6: Survey questionnaire translated to English



Notes: Questionnaire from ifo's one-time online special survey on its "uncertainty module"; translated to English from a screenshot from fall 2018. (3/3)

Figure C7: Original survey questionnaire in German



Notes: Original questionnaire from ifo's one-time online special survey on its "uncertainty module" in German, from fall 2018.

Appendix D Sample creation

In this appendix, we describe the construction of our baseline sample and also explain the number of observations from specific regressions in this paper. The observation numbers that remain after each step are listed in Table D1. Our starting sample from 27 survey waves consists of 14,150 firm-quarter observations. A firm-quarter observation is in the starting sample if the firm at that point in time answered at least one question of our online survey module and pressed the send button to return it.

We start by excluding 1,228 firm-quarter observations that were lacking an answer to question 1, realized sales growth. Then we carefully read the free text comments respondents can give below each of the questions (see Figure 1 in the main text for the questionnaire). We exclude 166 observations for which a comment indicates that the respondent was not able to calculate sales growth rates on a quarterly basis. For example, some firms in some quarters stated that they responded with annual growth rates. Moreover, we drop 60 observations for which the comment raises doubts about the validity and quality of the answer. For example, some firms in some quarters were not able to state realized past growth rates and used estimates instead. Overall, we exclude 1,454 firm-quarter observations based on missing or low-quality answers to question 1, leaving us with 12,696 firm-quarter observations.

Next, we exclude outliers which we define for question 1 (sales growth rate realizations) as lying outside the interval [-100%, 100%]. 59 firm-quarter observations are thus excluded. We have also experimented with a [-15%, 15%] cutoff, that is the bottom and top deciles of the final baseline sample, and found very similar results. We set the upper bounds quite high because large (two-digit) growth rates typically appear to be deliberate responses as many text comments reveal. Firms give explanations such as "Many projects were moved into this quarter" and "Invoice of a major project." This leaves us with 12,637 firm-quarter observations. After these cleaning steps, we require for the firm-quarter observations of a firm to remain in the sample that it have at least five clean firm-quarter observations on question 1, leaving us with 10,059 firm-quarter of 1,014 firm observations. It is this sample that we base the calculation of the trend and turbulence dummies on.

Subsequently, we exclude, respectively, outliers and inconsistencies related to question 2. Question 2-outliers were excluded according to the following two criteria:

1. The best case and worst case sales growth rates elicited in question 2.a lie outside the intervals [-100%; 300%] and [-100%; 100%], respectively.

2. The forecast growth rate elicited in question 2.b lies outside the interval [-100%; 100%].

Then we check whether respondents order the numbers in question 2 consistently, that is, as worst case < forecast < best case. We exclude firm-quarter observations with the orderings worst case $\geq$ forecast $\leq$ best case or worst case $\leq$ forecast $\geq$ best case because it is unclear what the respondents had in mind with these answers. However, we keep those firm-quarter observations with the inverse ordering worst case $\geq$ forecast $\geq$ best case and simply swap the worst case and best case numbers. Most likely inverse orderings were not intended by

Table D1: Sample creation

	firm-quarter obs. in sample	firm-quarter obs. excluded	firms in sample	firms excluded
Original sample	14,150		2,361	
Require response to question 1				
Response to Q1 exists	12,922	1,228	2,266	95
Text comment				
Wrong reference time excluded	12,756	166	2,257	9
Uncertain data quality excluded	12,696	60	2,252	5
Outliers to question 1				
Outliers in Q1 responses excluded	12,637	59	2,248	4
Number of observations by firm				
At least 5 clean responses to Q1	10,093	2,544	1,014	1,234
Outliers and inconsistencies to question 2				
Inconsistent & outlier responses to question 2 excluded	9,701	392	1,014	0
Require responses to question 2.a and 2.b				
Baseline sample: Response to Q2a and Q2b exists	8,889	247	1,005	6
Forecast error exists alongside span observations	5,379	3,510	961	44
Response to Q2a and Q2b and one forecast error	8,659	230	950	55

the respondent and rather a simple clerical error. Altogether, we eliminate 392 firm-quarter observations in this step.

In a final step, we eliminate those 247 firm-quarter observations which do not have answers to question 2.a or 2.b, the best or worst case scenarios for sales growth and the forecast, leaving us with our baseline sample of 8,889 of firm-quarter observations for 1,005 firms (*baseline sample*).

Starting from the baseline sample, some further exercises require also observations of forecast errors. In one subsample, the forecast errors exist in the same quarterly survey waves as the span observations, which leaves us with 5,379 firm-quarter observations. For other, purely cross-sectional exercises, we require at least one forecast error in the sample. This subsample contains 950 firm observations.

Appendix E Definition of manufacturing industries

Table E1 presents the definition of the 14 industries we use. They are based on the original 24 two-digit manufacturing industries, which are defined by the WZ08 (WZ stands for the German *Wirtschaftszweig*) code of the German Statistical Office. Since not all these industries have a large number of observations in the survey, we aggregate, for the purposes of this paper, some of them, resulting in 14 manufacturing industries. The column with the number of observations by industry refers to our baseline sample of 8,889 observations.

Table E1: Definition of industries

Industry	Industry WZ08	Industry WZ08 name	No. of obs.
1	10, 11, 12	Food products; Beverages; Tobacco products	439
2	13, 14, 15	Textiles; Wearing apparel; Leather and related products	247
3	16, 17, 31	Wood, products of wood and cork except furniture, articles of straw and plaiting materials; Paper and paper products; Furniture	871
4	18	Printing and reproduction of recorded media	497
5	19, 20, 21	Coke and refined petroleum products; Chemicals and chemical support; Basic pharmaceutical products and pharmaceutical preparations	754
6	22	Rubber and plastic products	730
7	23	Other non-metallic mineral products	473
8	24	Basic metals	391
9	25	Fabricated metal products, except machinery and equipment	1,073
10	26	Computer, electronic and optical products	493
11	27	Electrical equipment	822
12	28	Machinery and equipment n.e.c.	1,475
13	29, 30	Motor vehicles, trailers and semi-trailers; Other transport equipment	340
14	32, 33	Other manufacturing; Repair and installation of machinery and equipment	284
All			8,889

Appendix F Proof of Proposition 1

This appendix provides the proof of the Proposition 1, restated here:

Proposition. Suppose g can be written as $g = \mu + \sigma\varepsilon$ where ε has a continuous density with mean zero and unit variance: Then the optimal span is a linear function of σ, that is, $\hat{g}_n - \hat{g}_1 = \sigma(\hat{s}_n - \hat{s}_1)$, where $(\hat{s}_n - \hat{s}_1)$ is the optimal span for $\sigma = 1$.

Proof. Since $g = \mu + \sigma\varepsilon$ and ε has density $\mathfrak{g}$, the density of g is $\mathfrak{g}\left(\left(g-\mu\right)/\sigma\right)\sigma^{-1}$. A scenario $\hat{g}_i$ contributes to cost only when g realizes in an interval where $\hat{g}_i$ is the closest scenario. For an interior scenario other than the best or worst case, that interval is bounded by $\frac{1}{2}\left(\hat{g}_{i-1} + \hat{g}_i\right)$ and $\frac{1}{2}\left(\hat{g}_i + \hat{g}_{i+1}\right)$. We can therefore write the firm's expected cost as:

$$\int_{-\infty}^{(\hat{g}_1+\hat{g}_2)/2} (g-\hat{g}_1)^2\,\mathfrak{g}\left(\frac{g-\mu}{\sigma}\right)\sigma^{-1}dg + \sum_{i=2}^{n-1}\int_{(\hat{g}_{i-1}+\hat{g}_i)/2}^{(\hat{g}_i+\hat{g}_{i+1})/2} (g-\hat{g}_i)^2\,\mathfrak{g}\left(\frac{g-\mu}{\sigma}\right)\sigma^{-1}dg + \int_{(\hat{g}_{n-1}+\hat{g}_n)/2}^{\infty} (g-\hat{g}_n)^2\,\mathfrak{g}\left(\frac{g-\mu}{\sigma}\right)\sigma^{-1}dg.$$

The first-order conditions with respect to $\hat{g}_1, ..., \hat{g}_n$, and using Leibniz' rule, are:

$$\int_{(\hat{g}_{i-1}+\hat{g}_i)/2}^{(\hat{g}_i+\hat{g}_{i+1})/2} (g-\hat{g}_i)\,\mathfrak{g}\left(\frac{g-\mu}{\sigma}\right)\sigma^{-1}dg = 0\,; \qquad i = 2, ..., n-1$$

$$\int_{(\hat{g}_{n-1}+\hat{g}_n)/2}^{\infty} (g-\hat{g}_n)\,\mathfrak{g}\left(\frac{g-\mu}{\sigma}\right)\sigma^{-1}dg = \int_{-\infty}^{(\hat{g}_1+\hat{g}_2)/2} (g-\hat{g}_1)\,\mathfrak{g}\left(\frac{g-\mu}{\sigma}\right)\sigma^{-1}dg = 0$$

We note that increasing any given $\hat{g}_i$ increases the upper bound of the interval containing $\hat{g}_i$ as well as the lower bound of the next higher interval. Since the distribution is continuous, these effects cancel, so that the marginal benefit of increasing $\hat{g}_i$ is proportional to expected marginal cost over the interval containing $\hat{g}_i$.

Now consider the change of variable $s = \left(g-\mu\right)/\sigma$:

$$\int_{(\hat{g}_{i-1}+\hat{g}_i-2\mu)/2\sigma}^{(\hat{g}_i+\hat{g}_{i+1}-2\mu)/2\sigma} \left(s - \frac{\hat{g}_i-\mu}{\sigma}\right)\mathfrak{g}(s)\,ds = 0\,; \qquad i = 2, ..., n-1$$

$$\int_{(\hat{g}_{n-1}+\hat{g}_n-2\mu)/2\sigma}^{\infty} \left(s - \frac{\hat{g}_n-\mu}{\sigma}\right)\mathfrak{g}(s)\,ds = \int_{-\infty}^{(\hat{g}_1+\hat{g}_2-2\mu)/2\sigma} \left(s - \frac{\hat{g}_1-\mu}{\sigma}\right)\mathfrak{g}(s)\,ds = 0$$

Define the vector $\hat{s}$ of "standardized scenarios" that solves this system of equations for $\mu = 0$ and $\sigma = 1$. The first order conditions for given μ and σ imply that $\hat{g}_i = \mu + \sigma\hat{s}_i$. It follows in particular that span is $\hat{g}_n - \hat{g}_1 = \sigma(\hat{s}_n - \hat{s}_1)$. ■

Appendix G Modeling learning and uncertainty in unfamiliar territory

G.1 Proofs of the propositions for Example R

Under rational expectations, the firm belief coincides with the true DGP

$$g_{t+1} = f + \sigma(s_t)\varepsilon_t, \tag{8}$$

where ε_t has moments $\mathrm{E}(\varepsilon_t) = 0$, $\mathrm{E}(|\varepsilon_t|) = c$, and $\mathrm{Var}(\varepsilon_t) = 1$, which are all three unrelated to firm characteristics and independent of information known by the firm when it forms beliefs concerning the next period. In particular, it is independent of the signal s_t the firm receives. The firm forecasts next period's growth as f and perceives uncertainty $\sigma(s_t)$, where both f and the function $\sigma(\cdot)$, that maps the signal to the conditional standard deviation of growth, may depend of firm characteristics.

In the following, we restate and prove propositions R1 and R2.

Proposition R1 (Uncertainty looks like volatility in the cross section). Consider two firms A and B with different fixed characteristics, both described by Example R. Firm A has a higher firm-level mean subjective uncertainty than firm B if and only if firm A has higher firm-level mean absolute forecast error than firm B.

Proof. The forecast error of a firm is $fe_t = g_{t+1} - f = \sigma(s_t)\varepsilon_t$. Its mean absolute forecast error is

$$\mathrm{E}(|fe_t|) = \mathrm{E}(\sigma(s_t)|\varepsilon_t|) = \mathrm{E}(\sigma(s_t))\,\mathrm{E}(|\varepsilon_t|), \tag{9}$$

where the last step follows from independence of ε_t from s_t. The result shows that the mean absolute forecast error, $\mathrm{E}(|fe_t|)$, equals the mean uncertainty a firm perceives, $\mathrm{E}(\sigma(s_t))$, rescaled by the factor $c > 0$.[2] Two firms A and B may differ in their fixed characteristics reflected in different mean growth rates f^A and f^B as well as different conditional standard deviations of growth, $\sigma^A(s_t^A)$ and $\sigma^B(s_t^B)$. Using (9), we obtain the the relation

$$\frac{\mathrm{E}(|fe_t^A|)}{\mathrm{E}(|fe_t^B|)} = \frac{\mathrm{E}(\sigma^A(s_t^A))c}{\mathrm{E}(\sigma^B(s_t^B))c} = \frac{\mathrm{E}(\sigma^A(s_t^A))}{\mathrm{E}(\sigma^B(s_t^B))}.$$

If firm A has a higher mean subjective uncertainty than firm B, $\mathrm{E}(\sigma^A(s_t^A))/\,\mathrm{E}(\sigma^B(s_t^B) > 1$, it thus also has a higher mean absolute forecast error than firm B, $\mathrm{E}(|fe_t^A|)/\,\mathrm{E}(|fe_t^B|) > 1$, and vice versa. ■

Proposition R2 (Time-series uncertainty V is volatility V). Consider the time series of lagged growth rates $s_t = g_t$, forecasts and subjective uncertainty for a firm described by Example R. A piecewise linear regression of demeaned uncertainty on demeaned growth with a break at zero shows a V-shaped pattern if and only if a regression of the demeaned subjective absolute forecast error on demeaned growth shows the same V-shaped pattern.

[2] This factor depends on the shape of the distribution of ε_t. For example, under normality, it is $\sqrt{2/\pi}$.

Proof. Let us consider the general case of some regressor vector x_t with invertible variance matrix $\text{Var}(x_t)$. The regressors we are interested in in this paper are $x_t = [g_t^+, g_t^-]'$, where g_t^+ is g_t if $g_t \geq f$ and zero otherwise, and g_t^- is g_t if $g_t < f$ and zero otherwise. Next, note that a time-series regression of mean-adjusted subjective uncertainty on an intercept and some regressors, x_t, yields the same slope parameters as a time-series regression of undajusted subjective uncertainty, $\sigma(s_t)$, on an intercept and the same regressors. Hence, let us consider the latter. Under standard conditions, the slope estimates converge in probability towards the slope parameters of a linear projection of subjective uncertainty on the regressors which can be shown to be

$$\beta = \text{Var}(x_t)^{-1} \text{Cov}(x_t, \sigma(s_t)).$$

By the same argument, the slope parameters of a regression of the deviation of the subjective absolute forecast error from its firm-level mean on an intercept and the regressors x_t converge towards

$$\tilde{\beta} = \text{Var}(x_t)^{-1} \text{Cov}(x_t, |fe_t|).$$

Now,

$$\text{Cov}(x_t, |fe_t|) = \text{Cov}(x_t, \sigma(s_t)|\varepsilon_t|) = \text{Cov}(x_t, \sigma(s_t))c,$$

where the latter equality is a general result that follows from the independence of ε_t from information known in period t.[3] Substituting into the expression for $\tilde{\beta}$ yields

$$\tilde{\beta} = \text{Var}(x_t)^{-1} \text{Cov}(x_t, \sigma(s_t)) \, \text{E}(|\varepsilon_t|) = \beta c.$$

Since $c > 0$ is a positive constant, the two regressions yield, on average, the same shape of the regression line, adjusted by a scale factor. In particular, if the slopes for $x_t = [g_t^+, g_t^-]'$ have a V-shape in the first regression, they also have a V-shape in the second. ■

G.2 Proofs of the propositions for Example L

G.2.1 Preliminary results

For later use, we state and prove two lemmas that will be useful for proving the propositions for Model L.

Lemma 1. Consider the population regression of the random variable Y on an intercept, Z^+, and Z^-, where Z is a normally distributed random variable with zero mean and variance σ^2, Z^+ is Z if $Z \geq 0$ and zero otherwise, and Z^- is Z if $Z < 0$ and zero otherwise. Then the regression coefficients, β^+ and β^-, of Z^+ and Z^-, respectively, are given by

$$\begin{pmatrix} \beta^+ \\ \beta^- \end{pmatrix} = \frac{2/\sigma^2}{(\pi - 2)} \begin{bmatrix} (\pi - 1)\,\text{Cov}(y, Z^+) - \text{Cov}(y, Z^-) \\ (\pi - 1)\,\text{Cov}(y, Z^-) - \text{Cov}(y, Z^+) \end{bmatrix}. \tag{10}$$

[3] Consider random variables X, Y, and Z with means μ_x, μ_y, and μ_z, respectively. Assume Z is independent of X and Y such that, e.g., $\text{E}(YZ) = \mu_y\mu_z$. Then $\text{Cov}(X, YZ) = \text{E}[(X - \mu_x)(YZ - \mu_y\mu_z)] = \text{E}[(X - \mu_x)YZ] - \text{E}[(X - \mu_x)]\mu_y\mu_z = \text{E}[(X - \mu_x)Y]\mu_z = \text{Cov}(X, Y)\mu_z$, where the last equality follows from the fact that $\text{Cov}(X, Y) = \text{E}[(X - \mu_x)(Y - \mu_z)] = \text{E}[(X - \mu_x)Y] - \text{E}[(X - \mu_x)]\mu_z = \text{E}[(X - \mu_x)Y]$ because $\text{E}[(X - \mu_x)] = 0$.

If, in addition, $\operatorname{Cov}(y, Z^+) = -\operatorname{Cov}(y, Z^-) > 0$*, the regression line has a symmetric V-shape.*

Proof. The regression parameters are, in general, given by

$$\begin{pmatrix}\beta^+\\ \beta^-\end{pmatrix} = \begin{pmatrix}\operatorname{Var}(Z^+) & \operatorname{Cov}(Z^+, Z^-)\\ \operatorname{Cov}(Z^+, Z^-) & \operatorname{Var}(Z^+)\end{pmatrix}^{-1}\begin{pmatrix}\operatorname{Cov}(Z^+, Y)\\ \operatorname{Cov}(Z^-, Y)\end{pmatrix}$$

Using the properties of the half-normal distribution, we obtain

$$\begin{aligned}\operatorname{Var}(Z^+) &= \operatorname{E}[(Z^+)^2] - \operatorname{E}(Z^+)^2 = E(Z^2|Z \geq 0)P(Z \geq 0) - \operatorname{E}(Z|Z \geq 0)^2 P(Z \geq 0)^2\\ &= 0.5\sigma^2 - 0.25\frac{2\sigma^2}{\pi} = \frac{\pi - 1}{2\pi}\sigma^2.\end{aligned}$$

By symmetry, we also obtain

$$\operatorname{Var}(Z^-) = \frac{\pi - 1}{2\pi}\sigma^2.$$

Noting that $\operatorname{E}[(Z^+)(Z^-)] = 0$ and using $\operatorname{E}(Z^+) = -\operatorname{E}(Z^-) = \sqrt{2/\pi}\sigma$, we obtain the covariance as

$$\operatorname{Cov}(Z^+, Z^-) = -\operatorname{E}(Z^+)\operatorname{E}(Z^-) = \operatorname{E}(Z|Z \geq 0)^2 P(Z \geq 0)^2 = 0.25\left(\frac{\sqrt{2}\sigma}{\sqrt{\pi}}\right)^2 = \frac{1}{2\pi}\sigma^2.$$

Substituting these results into the regressor variance matrix yields

$$\begin{pmatrix}\operatorname{Var}(Z^+) & \operatorname{Cov}(Z^+, Z^-)\\ \operatorname{Cov}(Z^+, Z^-) & \operatorname{Var}(Z^+)\end{pmatrix}^{-1} = \frac{\sigma^2}{2\pi}\begin{pmatrix}\pi - 1 & 1\\ 1 & \pi - 1\end{pmatrix}^{-1} = \frac{2/\sigma^2}{(\pi - 2)}\begin{pmatrix}\pi - 1 & -1\\ -1 & \pi - 1\end{pmatrix}.$$

Substituting yields the regression parameters

$$\begin{pmatrix}\beta^+\\ \beta^-\end{pmatrix} = \frac{2/\sigma^2}{\pi - 2}\begin{pmatrix}\pi - 1 & -1\\ -1 & \pi - 1\end{pmatrix}\begin{pmatrix}\operatorname{Cov}(Z^+, Y)\\ \operatorname{Cov}(Z^-, Y)\end{pmatrix} = \frac{2/\sigma^2}{\pi - 2}\begin{bmatrix}(\pi - 1)\operatorname{Cov}(y, Z^+) - \operatorname{Cov}(y, Z^-)\\ (\pi - 1)\operatorname{Cov}(y, Z^-) - \operatorname{Cov}(y, Z^+)\end{bmatrix}.$$

If, in addition, $\operatorname{Cov}(y, Z^+) = -\operatorname{Cov}(y, Z^-) > 0$, the parameters simplify to

$$\begin{pmatrix}\beta^+\\ \beta^-\end{pmatrix} = \frac{2\pi/\sigma^2}{(\pi - 2)}\begin{pmatrix}\operatorname{Cov}(y, Z^+)\\ -\operatorname{Cov}(y, Z^+)\end{pmatrix}.$$

From the assumption $\operatorname{Cov}(y, Z^+) > 0$ we conclude that $\beta^+ > 0$, $\beta^- < 0$, and $\beta^+ = -\beta^-$. Hence, the regression line is upward sloping in the positive support and downward sloping with the same slope in the negative support. The branches meet at zero and thus show a symmetric V-shape. ■

Lemma 2. Consider the population regression of the random variable $Q = c_0 + c_1(Z + \mu) + c_2(Z + \mu)^2$ *on an intercept,* Z^+*, and* Z^-*, where* Z *is a normally distributed random variable with zero mean and variance* σ^2*,* Z^+ *is* Z *if* $Z \geq 0$ *and zero otherwise, and* Z^- *is* Z *if* $Z < 0$ *and zero otherwise. Then the*

regression coefficients, β^+ and β^-, of Z^+ and Z^-, respectively, are given by

$$\begin{pmatrix}\beta^+\\ \beta^-\end{pmatrix} = \begin{pmatrix}c_1 + 2c_2\mu + \frac{\sqrt{2\pi}}{\pi-2}c_2\sigma\\ c_1 + 2c_2\mu - \frac{\sqrt{2\pi}}{\pi-2}c_2\sigma\end{pmatrix}. \tag{11}$$

Proof. To apply the results of Lemma 1, we need to compute $\mathrm{Cov}(Q, Z^+)$ and $\mathrm{Cov}(Q, Z^-)$. To this end, we re-write $Q = a_0 + a_1 Z + c_2 Z^2$, where $a_0 = c_0 + c_1\mu + c_2\mu^2$ and $a_1 = c_1 + 2c_2\mu$. Then we compute $\mathrm{E}(Q) = a_0 + c_2\sigma^2$ and $\mathrm{E}(QZ|Z \geq 0) = a_0\,\mathrm{E}(Z|Z \geq 0) + a_1\,\mathrm{E}(Z^2|Z \geq 0) + c_2\,\mathrm{E}(Z^3|Z \geq 0) = \sqrt{2/\pi}a_0\sigma + a_1\sigma^2 + 2\sqrt{2/\pi}c_2\sigma^3$, where in the last step we used $\mathrm{E}(Z|Z \geq 0) = -\mathrm{E}(Z|Z < 0) = \sqrt{2/\pi}\sigma$, $\mathrm{E}(Z^2|Z \geq 0) = \mathrm{E}(Z^2|Z < 0) = \sigma^2$, and $\mathrm{E}(Z^3|Z \geq 0) = -\mathrm{E}(Z^3|Z < 0) = 2\sqrt{2/\pi}c_2\sigma^3$, which follow from the properties of the half-normal distribution. Similarly, we obtain $\mathrm{E}(QZ|Z < 0) = -\sqrt{2/\pi}a_0\sigma + a_1\sigma^2 - 2\sqrt{2/\pi}c_2\sigma^3$. Using these results, it is straightforward to compute

$$\mathrm{Cov}(Q, Z^+) = \frac{1}{2}E(QZ|Z \geq 0) - \frac{1}{2}E(Z|Z \geq 0)E(Q) = \frac{1}{2}a_1\sigma^2 + \frac{1}{2}\sqrt{2/\pi}c_2\sigma^3$$

and

$$\mathrm{Cov}(Q, Z^-) = \frac{1}{2}E(QZ|Z < 0) - \frac{1}{2}E(Z|Z < 0)E(Q) = \frac{1}{2}a_1\sigma^2 - \frac{1}{2}\sqrt{2/\pi}c_2\sigma^3.$$

Substituting these covariances into the framework derived in Lemma 1 yields

$$\begin{pmatrix}\beta^+\\ \beta^-\end{pmatrix} = \frac{2/\sigma^2}{\pi-2}\begin{bmatrix}(\pi-1)\,\mathrm{Cov}(y, Z^+) - \mathrm{Cov}(y, Z^-)\\ (\pi-1)\,\mathrm{Cov}(y, Z^-) - \mathrm{Cov}(y, Z^+)\end{bmatrix} = \begin{pmatrix}c_1 + 2c_2\mu + \frac{\sqrt{2\pi}}{\pi-2}c_2\sigma\\ c_1 + 2c_2\mu - \frac{\sqrt{2\pi}}{\pi-2}c_2\sigma\end{pmatrix}.$$

Note that a sufficient condition for a symmetric V is that the two shift parameters of the quadratic function Q are zero, i.e., $c_1 = 0$ and $\mu = 0$. ■

G.2.2 Example L: the Bayesian learning model

Example L is a basic Bayesian mean-variance learning approach as discussed in standard textbooks such as Koop (2003). Importantly, under Bayesian learning, objective and subjective expectations do not necessarily coincide. In this section, we therefore use $\mathrm{E}^*(\cdot)$, $\mathrm{Var}^*(\cdot)$, and $\mathrm{Cov}^*(\cdot)$ to denote moments under the objective probability measure and $\mathrm{E}(\cdot)$, $\mathrm{Var}(\cdot)$, and $\mathrm{Cov}(\cdot)$ to denote moments under the firm's subjective probability measure.

Next-quarter growth, g_{t+1}, is iid normal with mean g^* and variance σ_g^{*2}. Firms observe a noisy signal $s_t = g_{t+1} + n_t$ in period t, where noise n_t is independent of growth and normally distributed with mean zero and variance σ_n^{*2} (and thus precision σ_n^{*-2}). Hence, $s_t \sim \mathcal{N}(g^*, \sigma_g^{*2} + \sigma_n^{*2})$. We define

$$\gamma^* = \frac{\mathrm{Var}^*(g_{t+1})}{\mathrm{Var}^*(s_t)} = \frac{\mathrm{Var}^*(g_{t+1})}{\mathrm{Var}^*(g_{t+1}) + \mathrm{Var}^*(n_t)} = \frac{\sigma_g^{*2}}{\sigma_g^{*2} + \sigma_n^{*2}} = \frac{SNR^*}{1 + SNR^*} \in (0, 1) \tag{12}$$

as a measure of the information content of the signal with $SNR^* = \sigma_g^{*2}/\sigma_n^{*2}$ being the signal-to-noise ratio.

Under the true DGP, the conditional distribution of g_{t+1} given the signal s_t is, by the rules of the normal distribution, conditional normal with moments

$$\mathrm{E}^*(g_{t+1}|s_t) = g^* + \frac{\mathrm{Cov}^*(g_{t+1}, s_t)}{\mathrm{Var}^*(s_t)}(s_t - g^*) = g^* + \gamma^*(s_t - g^*) = (1-\gamma^*)g^* + \gamma^* s_t \tag{13}$$

and

$$\mathrm{Var}^*(g_{t+1}|s_t) = \mathrm{Var}^*(g_{t+1}) - \frac{\mathrm{Cov}^*(g_{t+1}, s_t)^2}{\mathrm{Var}^*(s_t)} = (1-\gamma^*)\sigma_g^{*2} = \gamma^*\sigma_n^{*2}. \tag{14}$$

We can write this conditional distribution in regression form as

$$g_{t+1} = (1-\gamma^*)g^* + \gamma^* s_t + \sqrt{(1-\gamma^*)\sigma_g^{*2}}\,\varepsilon_{t+1}^*,$$

where the innovation ε_{t+1}^* is standard normal and independent of s_t.

Firms update beliefs concerning g_{t+1} and precision σ_n^{*-2} in a Bayesian fashion using the normal-gamma prior $\mathcal{NG}(0, \frac{\gamma}{1-\gamma}, \Phi^{-2}, \nu)$ where $\gamma \in (0,1)$, $\Phi^2 > 0$, and $\nu > 2$.[4] Concerning the noise precision, this implies an a priory belief of $\mathrm{E}(\sigma_n^{*-2}) = \Phi^{-2}$ and $\mathrm{Var}(\sigma_n^{*-2}) = 2\Phi^{-4}\nu^{-1}$. Hence, Φ^{-2} defines the a priori expectation, while ν governs the tightness of this belief with larger ν signaling higher certainty.[5]

Concerning future growth, the prior implies an a priori expectation of zero. To interpret γ, it is straightforward to show that γ is the prior belief concerning the information content of the signal, i.e.,

$$\gamma = \frac{\mathrm{Var}(g_{t+1})}{\mathrm{Var}(g_{t+1}) + \mathrm{E}(\sigma_n^2)}$$

under the probability measure of the subjective prior belief. Furthermore, the implied prior variance of next-quarter growth is $\mathrm{Var}(g_{t+1}) = \frac{\nu}{\nu-2}\frac{\gamma}{1-\gamma}\Phi^2$. In the following, we denote this prior variance as σ_g^2 and use it instead of Φ^2 to characterize the parameterization of the learning model.

[4] We use the parameterization of the gamma and normal-gamma distributions as defined in Koop (2003). Specifically, a gamma distributed random variable z with parameters Φ^{-2} and ν, $z \sim \mathcal{G}(\Phi^{-2}, \nu)$, has moments $\mathrm{E}(z) = \Phi^{-2}$, $\mathrm{Var}(z) = 2\Phi^{-4}\nu^{-1}$, and $\mathrm{E}(z^{-1}) = \frac{\nu}{\nu-2}\Phi^2$. The joint distribution of $y|z \sim \mathcal{N}(\mu, \lambda^{-1}z^{-1})$ and z is normal-gamma, $(y,z) \sim \mathcal{NG}(\mu, \lambda^{-1}, \Phi^{-2}, \nu)$. It implies that the marginal distribution of y is a location-scale t distribution, $y \sim t(\mu, \Phi^2/\lambda, \nu)$, with moments $\mathrm{E}(y) = \mu$ and $\mathrm{Var}(y) = \frac{\nu\Phi^2}{(\nu-2)\lambda}$ if $\nu > 2$.

[5] Taking the expressions for the prior moments $\mathrm{E}(\sigma_n^{*-2})$ and $\mathrm{Var}(\sigma_n^{*-2})$ together yields $\nu/2 = \left(\frac{\mathrm{Var}(\sigma_n^{*-2})}{\mathrm{E}(\sigma_n^{*-2})^2}\right)^{-1} = (SCV)^{-1}$, where SCV is the prior squared coefficient of variation of σ_n^{*-2}. Hence, a higher coefficient of variation in prior beliefs and thus a larger uncertainty with respect to the inverse noise variance is reflected in a smaller prior parameter ν.

After observing the signal s_t, firms' posterior beliefs are normal-gamma, $(g_{t+1}, \sigma_n^{*-2})|s_t \sim \mathcal{NG}(\gamma s_t, \gamma, \Phi_1^{-2}, \nu+1)$, where

$$\Phi_1^2 = \frac{\nu}{1+\nu}\Phi^2 + \frac{1-\gamma}{1+\nu}s_t^2 = \frac{(\nu-2)(1-\gamma)}{(1+\nu)\gamma}\sigma_g^2 + \frac{1-\gamma}{1+\nu}s_t^2.$$

By the rules of the normal-gamma distribution, the marginal posterior of g_{t+1} is a location-scale t distribution with $\nu+1$ degrees of freedom and moments

$$\mathrm{E}(g_{t+1}|s_t) = \gamma s_t$$

and

$$\mathrm{Var}(g_{t+1}|s_t) = \frac{\nu+1}{\nu-1}\Phi_1^2\gamma = \frac{(\nu-2)(1-\gamma)}{\nu-1}\sigma_g^2 + \frac{\gamma(1-\gamma)}{\nu-1}s_t^2.$$

We can write this subjective conditional distribution in regression form as

$$g_{t+1} = \gamma s_t + \sqrt{\frac{(\nu-2)(1-\gamma)}{\nu+1}\sigma_g^2 + \frac{\gamma(1-\gamma)}{\nu+1}s_t^2}\,\varepsilon_{t+1},$$

where the innovation ε_{t+1} follows, under the probability measure of the subjective posterior belief, Student's t distribution with $\nu+1$ degrees of freedom and is independent of s_t.

To forecast g_{t+1}, firms use the posterior mean of their subjective belief,

$$f_t \equiv \mathrm{E}(g_{t+1}|s_t) = \gamma s_t. \tag{15}$$

Hence, their forecast error is $fe_t = g_{t+1} - \gamma s_t$. To express their uncertainty with respect to next-quarter growth, firms use the square root of their posterior variance, i.e.,

$$\omega_t \equiv \omega(s_t) \equiv \sqrt{a + bs_t^2}. \tag{16}$$

where $a \equiv \frac{(\nu-2)(1-\gamma)}{\nu-1}\sigma_g^2$ and $b \equiv \frac{\gamma(1-\gamma)}{\nu-1}$. Note that $a > 0$ and $b > 0$ except in borderline cases such as $\sigma_g^2 = 0$ or $\gamma = 0$.

An econometrician observes sample moments that relate to population moments under the objective probability measure. In the following, we characterize some of these moments. The forecast error is a linear function of growth and the signal, both of which are normally distributed. Given the signal, it is thus normally distributed with conditional expectation

$$\mathrm{E}^*(fe_t|s_t) = \mathrm{E}^*(g_{t+1}|s_t) - \gamma s_t = (1-\gamma^*)g^* + \gamma^* s_t - \gamma s_t = (1-\gamma^*)g^* + (\gamma^* - \gamma)s_t$$

and conditional variance

$$\mathrm{Var}^*(fe_t|s_t) = \mathrm{Var}^*(g_{t+1}|s_t) = \gamma^*\sigma_n^{*2}.$$

Unconditionally, the forecast error is normally distributed with moments

$$\mathrm{E}^*(fe_t) = (1-\gamma^*)g^* + (\gamma^* - \gamma)g^* = (1-\gamma)g^* \tag{17}$$

and

$$\begin{aligned}\mathrm{Var}^*(fe_t) &= \mathrm{Var}^*(g_{t+1}) + \gamma^2 \mathrm{Var}^*(s_t) - 2\gamma \mathrm{Cov}^*(g_{t+1}, s_t) \\ &= \left(1 - 2\gamma + \gamma^2\right)\sigma_g^{*2} + \gamma^2 \sigma_n^{*2} \qquad (18) \\ &= \left[(\gamma - \gamma^*)^2 + \gamma^*(1-\gamma^*)\right]\sigma_s^{*2}, \qquad (19)\end{aligned}$$

where we used $\gamma^* = \sigma_g^{*2}/(\sigma_g^{*2} + \sigma_n^{*2})$ and $\sigma_s^{*2} \equiv \sigma_g^{*2} + \sigma_n^{*2} = \sigma_g^{*2}/\gamma^*$ in the last step.

Squared subjective uncertainty has, under the true DGP, expectation

$$\mathrm{E}^*\left[\omega_t^2\right] = a + b\left(\sigma_g^{*2} + \sigma_n^{*2} + g^{*2}\right), \tag{20}$$

while the expectation of its square root does not have a simple closed-form solution.

G.2.3 Proof of Proposition L0

In the following, we restate and prove Proposition L0.

Proposition L0 (Uncertainty moves with volatility in the cross section). Consider firms described by Model L. Other things equal, if a firm experiences more volatile growth σ_g^*, then both its firm-level mean subjective uncertainty and the firm-level mean absolute deviation (MAD) of its forecast error are higher.

Proof. While it is obvious from (20) that expected squared uncertainty, $\mathrm{E}^*(\omega_t^2)$, is an increasing function of σ_g^*, a few steps are needed to show it for $\mathrm{E}^*(\omega_t)$. Let us start by noting that $\omega_t \equiv \omega(s_t)$ is a convex function of s_t as its second derivative w.r.t. s_t is $\omega''(s_t) = ab/(a + bs_t^2)^{3/2}$ which is positive because $a > 0$ and $b > 0$. Hence, Jensen's inequality, $\mathrm{E}^*[\omega(s_t)] \geq \omega(\mathrm{E}^*[s_t])$, holds. Now consider two signals, $s_{1t} \sim \mathcal{N}(g^*, \sigma_{1s}^{*2})$ and $s_{2t} \sim \mathcal{N}(g^*, \sigma_{2s}^{*2})$, where $\sigma_{2g}^* > \sigma_{1g}^*$ and thus $\sigma_{2s}^{*2} = \sigma_{2g}^{*2} + \sigma_n^{*2} > \sigma_{1s}^{*2} = \sigma_{1g}^{*2} + \sigma_n^{*2}$. Write $s_{2t} = s_{1t} + z_t$, where $z_t \sim \mathcal{N}(0, \sigma_{2g}^{*2} - \sigma_{1g}^{*2})$ is independent of s_{1t}. Hence, $\mathrm{E}^*[s_{2t}|s_{1t}] = s_{1t} + \mathrm{E}^*[z_t|s_{1t}] = s_{1t}$. Now use Jensen's inequality for the conditional moments, $\mathrm{E}^*[\omega(s_{2t})|s_{1t}] \geq \omega(\mathrm{E}^*[s_{2t}|s_{1t}]) = \omega(s_{1t})$. By the law of iterated expectations, we finally have $\mathrm{E}^*[\omega(s_{2t})] = \mathrm{E}^*\left(\mathrm{E}^*[\omega(s_{2t})|s_{1t}]\right) \geq \mathrm{E}^*[\omega(s_{1t})]$. Hence, if the firm experiences the more volatile signal s_{2t} it exhibits higher average uncertainty $\mathrm{E}^*[\omega(s_{2t})]$.

The mean absolute deviation of the forecast error is defined as

$$\mathrm{MAD}^*(fe_t) = \mathrm{E}^*\left|fe_t - \mathrm{E}^*(fe_t)\right|.$$

The mean-adjusted forecast error, $fe_t - \mathrm{E}^*(fe_t)$, is normally distributed with a mean of zero and the variance $\mathrm{Var}^*(fe_t)$ given in (18). The expectation of its absolute value is, by the rules

of the half-normal distribution,

$$\text{MAD}^*(fe_t) = \sqrt{2/\pi}\left[\text{Var}^*(fe_t)\right]^{\frac{1}{2}} = \sqrt{2/\pi}\left[\left(1-2\gamma+\gamma^2\right)\sigma_g^{*2}+\gamma^2\sigma_n^{*2}\right]^{\frac{1}{2}}.$$

Since $1-2\gamma+\gamma^2 = (1-\gamma)^2 > 0$, the mean absolute deviation of the forecast error is an increasing function of σ_g^*. ■

G.2.4 Proof of Proposition L1

In the following, we restate and prove Proposition L1.

Proposition L1 (Uncertainty is more than volatility in the cross section). Consider firms described by Model L. Other things equal,
(a) as the volatility of growth σ_g^* converges to zero, firm-level mean subjective uncertainty converges to a positive constant.
(b) if a firm's mean growth rate is more positive (negative), then its firm-level mean subjective uncertainty is higher, but the MAD of its forecast error is the same. Moreover, the firm's average forecast error is more positive (negative) and hence further away from zero.
(c) if a firm is more confident about its environment in the sense of lower prior uncertainty about growth σ_g, then its firm-level mean subjective uncertainty is lower, but the firm-level MAD of its forecast error is the same.

Proof of part (a). By (20), as σ_g^* converges to zero, mean squared subjective uncertainty as observed by an econometrician converges to $a + b\left(\sigma_n^{*2} + g^{*2}\right)$ which is strictly positive because $a > 0$, $b > 0$, $\sigma_n^{*2} > 0$, and $g^{*2} \geq 0$. While there is no simple closed-form expression for mean subjective uncertainty, $\text{E}^*(\omega_t)$, note that ω_t is a strictly positive random variable even if $\sigma_g^* = 0$. Hence, its expectation is strictly positive as well. ■

Proof of part (b). Let us consider the mean absolute deviation of the forecast error, $\text{MAD}^*(fe_t) = \text{E}^*\left|fe_t - \text{E}^*(fe_t)_t\right|$ first. The mean-adjusted forecast error, $\widetilde{fe}_t = fe_t - \text{E}^*(fe_t)$, is a normal random variable with a mean of zero and the same variance as fe_t, which does not depend on g^*, see (19). Hence, the distribution of $\widetilde{fe}_t$ does not depend on g^* which is why the expectation of its absolute value, and thus the MAD of fe_t, is not affected by a change in g^*.

Second, consider the firm's average forecast error, $\text{E}^*(fe_t) = (1-\gamma)g^*$. As long as $\gamma < 1$, it is further away from zero if a firm's mean growth rate is further away from zero.

Finally, consider the firm's average uncertainty. Again, it is straightforward to see that a mean growth rate, g^*, further away from zero leads to higher average squared uncertainty because its expectation is a linear function of $(g^*)^2$ with slope $b = \gamma(1-\gamma)/(\nu-1) > 0$, see (20). To show this for average uncertainty, let us define the mean-adjusted signal $\tilde{s}_t = s_t - g^*$. Hence,

$$\text{E}^*[\omega(s_t)] = \text{E}^*[\omega(\tilde{s}_t + g^*)] = \int_{-\infty}^{\infty} \omega(\tilde{s}_t + g^*)\frac{1}{\sigma_s^*}\phi\left(\frac{\tilde{s}_t}{\sigma_s^*}\right)d\tilde{s}_t,$$

where $\phi(\cdot)$ is the pdf of the standard normal distribution which does not depend on g^*. Hence, the first derivative w.r.t. g^* is

$$\frac{\partial \mathrm{E}^*[\omega(s_t)]}{\partial g^*} = \int_{-\infty}^{\infty} \frac{\partial \omega(\tilde{s}_t + g^*)}{\partial g^*} \frac{1}{\sigma_s^*} \phi\left(\frac{\tilde{s}_t}{\sigma_s^*}\right) d\tilde{s}_t = \mathrm{E}^*\left[\omega'(s_t)\right],$$

where $\omega'(s_t) = bs_t/\sqrt{a + bs_t^2}$. Due to $\omega'(-s_t) = \omega'(-s_t)$, we can split the integral into the positive and negative support which yields

$$\begin{aligned}
\frac{\partial \mathrm{E}^*[\omega(s_t)]}{\partial g^*} &= \int_{-\infty}^{0} \omega'(s_t) \frac{1}{\sigma_s^*} \phi\left(\frac{s_t - g^*}{\sigma_s^*}\right) ds_t + \int_{0}^{\infty} \omega'(s_t) \frac{1}{\sigma_s^*} \phi\left(\frac{s_t - g^*}{\sigma_s^*}\right) ds_t \\
&= -\int_{0}^{\infty} \omega'(s_t) \frac{1}{\sigma_s^*} \phi\left(\frac{-s_t - g^*}{\sigma_s^*}\right) ds_t + \int_{0}^{\infty} \omega'(s_t) \frac{1}{\sigma_s^*} \phi\left(\frac{s_t - g^*}{\sigma_s^*}\right) ds_t \\
&= \int_{0}^{\infty} \omega'(s_t) \frac{1}{\sigma_s^*} \left[\phi\left(\frac{s_t - g^*}{\sigma_s^*}\right) - \phi\left(\frac{-s_t - g^*}{\sigma_s^*}\right)\right] ds_t.
\end{aligned}$$

Now, if $g^* > 0$ then $\phi\left(\frac{s_t - g^*}{\sigma_s^*}\right) - \phi\left(\frac{-s_t - g^*}{\sigma_s^*}\right) > 0$, so expected uncertainty increases as g^* increases away from zero. Conversely, if $g^* < 0$ then $\phi\left(\frac{s_t - g^*}{\sigma_s^*}\right) - \phi\left(\frac{-s_t - g^*}{\sigma_s^*}\right) < 0$, so expected uncertainty increases as g^* decreases away from zero. ■

Proof of part (c). Let us first consider the firm's average uncertainty. Again, it is straightforward to see that average squared uncertainty is a linear function of σ_g^2 with slope $(\nu - 2)(1 - \gamma)/(\nu - 1) > 0$, see (20). Hence, lower prior uncertainty about growth reduces average squared uncertainty. For average uncertainty, a simple closed-form solution is not feasible. Nevertheless, uncertainty $\omega_t = \sqrt{a + bs_t^2}$ is, due to $a = \sigma_g^2(\nu - 2)(1 - \gamma)/(\nu - 1)$, shifted downwards at every point s_t as σ_g decreases. Consequently, its expectation $E^*[\omega_t]$ also goes down as as σ_g decreases.

The forecast error $fe_t = g_{t+1} - \gamma s_t$ is normally distributed with mean and variance stated in (17) and (19), respectively. Both moments are unrelated to the subjective prior uncertaint σ_g which is why fe_t and all functions of it, such as its mean absolute deviation, are unaffected by a change in σ_g. ■

G.2.5 Proof of Proposition L2

In the following, we restate and prove Proposition L2.

Proposition L2 (Time-series uncertainty V without volatility V). Consider time series of lagged growth rates s_t, forecasts and subjective uncertainty for a firm described by Example L. Suppose that $|g^*| < \sigma_g^*/\sqrt{\gamma^*}$. Up to a second order Taylor expansion, a piecewise linear regression of demeaned subjective uncertainty on demeaned lagged growth with a break at zero generates a V-shaped pattern. In contrast, a regression of the demeaned debiased absolute forecast error on demeaned lagged growth shows no relationship.

Proof of the first part. Let us first consider a regression of the deviation of squared subjective uncertainty from its firm-level mean on the deviation of past growth from its firm-level mean, run separately for positive and negative growth deviations, because this yields a simple closed-form solution. Note that this regression is equivalent to the regression

$$\omega_t^2 = \beta_0 + \beta_1^+ \tilde{s}_t^+ + \beta_1^- \tilde{s}_t^- + u_t,$$

where $\tilde{s}_t^+ = \tilde{s}_t$ if $\tilde{s}_t \geq 0$ and zero otherwise, $\tilde{s}_t^- = \tilde{s}_t$ if $\tilde{s}_t < 0$ and zero otherwise, and $\tilde{s}_t \equiv s_t - \mathrm{E}^*(s_t)$ is the deviation of the signal (which we identify with past growth in the empirical part of this paper) from its firm-level mean.

Using the definition $\omega_t^2 = a + bs_t^2$ and applying Lemma 2, we obtain

$$\beta_1^+ = 2bg^* + \frac{\sqrt{2\pi}}{\pi - 2} b\sigma_s^*$$

and

$$\beta_1^+ = 2bg^* - \frac{\sqrt{2\pi}}{\pi - 2} b\sigma_s^*.$$

We obtain an asymmetric V if the slope of the right (left) branch is positive (negative). This requires $2g^* + \frac{\sqrt{2\pi}}{\pi-2}\sigma_s^* > 0$ and $2g^* - \frac{\sqrt{2\pi}}{\pi-2}\sigma_s^* < 0$. Noting that $\frac{\sqrt{2\pi}}{\pi-2} \approx 2$, it approximately requires $-\sigma_s^* < g^* < \sigma_s^*$ and thus $-\sigma_g^*/\sqrt{\gamma^*} < g^* < \sigma_g^*/\sqrt{\gamma^*}$. A symmetric V around zero additionally requires g^*.

An equivalent regression of $\omega_t = \sqrt{a + bs_t^2}$ on the same regressors does not allow a simple closed-form solution. Since ω_t looks approximately quadratic in the neighborhood of $s_t = 0$, we apply a second-order Taylor expansion around zero and use it instead of ω_t. Noting that $\omega'(s_t) = bs_t/\sqrt{a + bs_t^2}$ and $\omega''(s_t) = ab/(a + bs_t^2)^{3/2}$, the Taylor polynomial is

$$T_2(s_t) = \omega(0) + \omega'(0)s_t + \frac{1}{2}\omega''(0)s_t^2 = \sqrt{a} + \frac{b}{2\sqrt{a}} s_t^2.$$

The coefficients of the regression

$$T_2(s_t) = \beta_0 + \beta_1^+ \tilde{s}_t^+ + \beta_1^- \tilde{s}_t^- + u_t$$

can again be found by applying Lemma 2. This yields

$$\beta_1^+ = \left[2bg^* + \frac{\sqrt{2\pi}}{\pi - 2} b\sigma_s^* \right] / (2\sqrt{a})$$

and

$$\beta_1^+ = \left[2bg^* - \frac{\sqrt{2\pi}}{\pi - 2} b\sigma_s^* \right] / (2\sqrt{a}).$$

Hence, compared to the piecewise regression fitted to ω_t^2, the slopes of both branches are simply multiplied by the factor $1/(2\sqrt{a}) > 0$.

This implies that we expect an asymmetric V under the same condition $-\sigma_g^*/\sqrt{\gamma^*} < g^* < \sigma_g^*/\sqrt{\gamma^*}$*. It means that long-run growth g^* must not be too big compared to the standard deviation of the signal which is the square root of $\sigma_g^{*2} + \sigma_n^{*2}$. Hence, we obtain an asymmetric V as long as either the volatility of growth or the standard deviation of the noise are not small. A perfectly symmetric V around zero additionally requires g^* but for $|g^*| << \sigma_s^*$, the V will look approximately symmetric. ■

Proof of the second part. Now consider a regression with the same regressors as in part (a) but with the absolute deviation of the debiased forecast error from its firm-level mean on the left-hand side. Debiasing here means that we use the residual of a regression of the forecast error on an intercept, $\tilde{s}_t^+$, and $\tilde{s}_t^-$. Since the forecast error is, by construction, a linear function of $\tilde{s}_t$, this effectively takes out the conditional mean, $\mathrm{E}(fe_t|\tilde{s}_t)$, from the forecast error. Consequently, the debiased forecast error is uncorrelated with the signal. Due to normality, it is also independent from it. Hence, any regression of some function of the debiased forecast error, such as the absolute deviation, on functions of $\tilde{s}_t$ will yield a slope of zero. ■

G.2.6 Proof of proposition L3

In the following, we restate and prove proposition L3.

Proposition L3 (Uncertainty and overreaction). Consider time series of lagged growth rates s_t, forecasts and subjective uncertainty for a firm described by Example L.
(a) If the firm overreacts to signals, $\gamma > \gamma^*$, the covariance between its forecast and forecast error is negative.
(b) Other things equal, a decline in the true information content γ^* implies more overreaction to signals and a lower (more negative) covariance between forecast and forecast error. If the manager is not too uncertain about information quality (ν large enough), then it also implies lower excess uncertainty.

Proof of part (a). While a rational firms reacts to signals with weight γ^*, see (13), a learning firms uses the weight γ, see (15). Hence, $\gamma > \gamma^*$ characterizes a firm that overreacts compared to a rational firm. The covariance between forecast and forecast error is, in general,

$$\mathrm{Cov}^*(f_t, fe_t) = \mathrm{Cov}^*(\gamma s_t, g_{t+1} - \gamma s_t) = \gamma\sigma_g^2 - \gamma^2(\sigma_g^2 + \sigma_n^2). \tag{21}$$

Using the definition (12) of γ^* to substitute $\sigma_g^2 = \gamma^*(\sigma_g^2 + \sigma_n^2)$ yields

$$\mathrm{Cov}^*(f_t, fe_t) = (\gamma\gamma^* - \gamma^2)(\sigma_g^2 + \sigma_n^2) = \gamma(\gamma^* - \gamma)(\sigma_g^2 + \sigma_n^2). \tag{22}$$

Since $0 < \gamma, \gamma^* < 1$ and $\sigma_g^2, \sigma_n^2 > 0$, the covariance is negative if and only if $\gamma > \gamma^*$. ■

Proof of part (b). The true information content of the signal is measured by $\gamma^* = \sigma_g^2/(\sigma_g^2 + \sigma_n^2)$. Reducing γ^*, keeping the variance of growth, σ_g^2, unchanged, implies an increase in

the noise variance σ_n^2. Formally, we are interested in $d\gamma^* < 0$ which is equivalent to $d\sigma_n^2 = -\frac{(\sigma_g^2+\sigma_n^2)^2}{\sigma_g^2} d\gamma^* > 0$.

For a given prior belief, i.e., invariant γ, the firm overreacts more if γ^* is reduced, so $\gamma - \gamma^*$ is higher.

Concerning the covariance between forecast and forecast error, we use (21) and obtain

$$\frac{\partial Cov^*(f_t, fe_t)}{\partial \sigma_n^2} = -\gamma^2 < 0.$$

Hence, if a firm overreacts more to the signal, then the covariance between forecast and forecast error becomes more negative.

Concerning the firm-level mean absolute deviation of the forecast error, first note that, by (18), the variance of the forecast error increases:

$$\frac{\partial \operatorname{Var}^*(fe_t)}{\partial \sigma_n^2} = \gamma^2 > 0.$$

Now, by the rules of the half-normal distribution, the mean absolute deviation of the forecast error is an increasing function of the variance,

$$\operatorname{MAD}^*(fe_t) = \operatorname{E}^* |fe_t - \operatorname{E}^*(fe_t)| = \sqrt{\frac{2}{\pi}} \left[\operatorname{Var}^*(fe_t)\right]^{1/2}.$$

Hence, it also increases as the firm overreacts more. The derivative is

$$\frac{\partial \operatorname{MAD}^*(fe_t)}{\partial \sigma_n^2} = \frac{\gamma^2}{\sqrt{2\pi}} \left[\operatorname{Var}^*(fe_t)\right]^{-1/2} = \frac{\gamma^2/\sigma_s^*}{\sqrt{2\pi}} \left[(\gamma - \gamma^*)^2 + \gamma^* (1 - \gamma^*)\right]^{-1/2}.$$

Concerning firm-level uncertainty, let us again consider squared uncertainty first. By (20), its expectation increases as the firm overreacts more:

$$\frac{\partial \operatorname{E}^*[\omega_t^2]}{\partial \sigma_n^2} = b = \frac{\gamma(1-\gamma)}{\nu - 1} > 0.$$

For uncertainty, ω_t, there is no simple closed-form solution, hence we resort to the second-order Taylor approximation, $T_2(s_t) = \sqrt{a} + \frac{b}{2\sqrt{a}} s_t^2$, introduced for the proof of Proposition L2 above. Its expectation is $\operatorname{E}^*[T_2(s_t)] = \sqrt{a} + \frac{b}{2\sqrt{a}}(\sigma_g^{*2} + \sigma_n^{*2} + g^{*2})$. Thus, its derivative is

$$\frac{\partial \operatorname{E}^*[\omega_t]}{\partial \sigma_n^2} \approx \frac{\partial \operatorname{E}^*[T_2(s_t)]}{\partial \sigma_n^2} = \frac{b}{2\sqrt{a}} = \frac{\gamma/\sigma_g}{2} \frac{\sqrt{1-\gamma}}{\sqrt{(\nu-1)(\nu-2)}}.$$

What increases more, the volatility of forecast errors or subjective uncertainty? For the squared measures, $\mathrm{Var}^*(fe_t)$ and $\mathrm{E}^*[\omega_t^2]$, the answer is straightforward. The relative increase,

$$\frac{\partial \mathrm{Var}^*(fe_t)/\partial\sigma_n^2}{\partial \mathrm{E}^*[\omega_t^2]/\partial\sigma_n^2} = (\nu-1)\frac{\gamma}{1-\gamma},$$

is larger than 1, i.e., the variance of the forecast error increases more than average squared uncertainty, if ν is large. This condition implies that firms are relatively confident regarding the precision of the signal.

For our preferred measures, $\mathrm{MAD}^*(fe_t)$ and $\mathrm{E}^*[\omega_t]$, the relative increase is

$$\frac{\partial \mathrm{MAD}^*(fe_t)/\partial\sigma_n^2}{\partial \mathrm{E}^*[\omega_t]/\partial\sigma_n^2} = \sqrt{\frac{2}{\pi}}\,\frac{\gamma\sqrt{(\nu-1)(\nu-2)}}{\sqrt{(1-\gamma)\left[(\gamma-\gamma^*)^2+\gamma^*(1-\gamma^*)\right]}}\,\frac{\sigma_g}{\sigma_s^*}.$$

Again, it is larger than 1 if ν is large, i.e., firms are relatively confident regarding the precision of the signal. ■

G.3 Hybrid learning model

The hybrid learning model combines elements of both Example R and Example L without adding new learning mechanisms. We simply assume that next-quarter growth, now denoted by h_{t+1}, consists of two mutually independent shocks, a learnable shock $g_{t+1} \sim \mathcal{N}(g^*, \sigma_g^2)$ and an unlearnable shock $z_{t+1} \sim \mathcal{N}(0, \sigma_z^2)$, i.e.,

$$h_{t+1} = g_{t+1} + z_{t+1}.$$

In the same fashion as in Example L, firms receive a signal concerning the learnable shock, $s_t = g_{t+1} + n_t$. In contrast, as in Example R, they have no a priori knowledge concerning the realization of z_{t+1} but know the mechanism that generates z_{t+1} and come up with a rational forecast of it. For concreteness, reflecting the results of our empirical exercise, let us assume that firms are aware that, under the true DGP, σ_z^2 is a deterministic function of current growth:

$$\mathrm{Var}^*(z_{t+1}|g_t) = \sigma_z^2(g_t).$$

This information structure reflects that firms typically have a good understanding of their own business environment which enables them, unlike the econometrician, to distinguish between learnable and unlearnable components of future sales growth. For example, they might observe signals concerning their stance in their home market but not in new or external markets with which their are not familiar.

Under the true DGP, the conditional distribution of h_{t+1} given the signal s_t is conditionally normal. Due to independence of its components and unpredictability of z_{t+1}, we obtain a

simple extension of the models in Example L and R:

$$\mathrm{E}^*(h_{t+1}|s_t) = \mathrm{E}^*(g_{t+1}|s_t) + \mathrm{E}^*(z_{t+1}) = (1-\gamma^*)g^* + \gamma^* s_t$$

and

$$\mathrm{Var}^*(h_{t+1}|s_t) = \mathrm{Var}^*(g_{t+1}|s_t) + \mathrm{Var}^*(z_{t+1}) = \gamma^* \sigma_n^2 + \sigma_z^2(g_t).$$

Learning of g_{t+1} works as in the Example L, while z_{t+1} is unlearnable which is why firms form rational expectations as in Example R. Since firms know that the two shocks are mutually independent, their beliefs after observing s_t are characterized by the location-scale t distribution of g_{t+1}, discussed in the Example L, and the true DGP of z_{t+1}. Hence, the subjective posterior of growth, h_{t+1}, is characterized by the sum of a t and a normal distribution. From this distribution, firms derive the forecast

$$f_t \equiv \mathrm{E}(h_{t+1}|s_t) = \mathrm{E}(g_{t+1}|s_t) + \mathrm{E}^*(z_{t+1}) = \gamma s_t$$

and uncertainty

$$\sigma^2(s_t) \equiv \mathrm{Var}(h_{t+1}|s_t) = \mathrm{Var}(g_{t+1}|s_t) + \mathrm{Var}^*(z_{t+1}) = \frac{(\nu-2)(1-\gamma)}{\nu-1}\sigma_g^2 + \frac{\gamma(1-\gamma)}{\nu-1}s_t^2 + \sigma_z^2(g_t).$$

Now suppose the unlearnable shock, z_{t+1}, shows the specific form of heteroskedasticity documented in the empirical part of this paper:

$$\sigma_z^2(g_t) = \begin{cases} b_0 + b_1 g_t^2 & \text{if } g_t < 0 \\ b_0 & \text{if } g_t \geq 0 \end{cases}$$

where $b_0, b_1 > 0$. Under the true DGP, growth will inherit this shape of heteroskedasticity because the learnable shock is homoskedastic. Hence, the variance of growth g_{t+1} increases with last period's absolute growth rate, g_t, if that was negative, while it is homoskedastic if g_t was positive. Moreover, the forecast error, $fe_t = g_{t+1} - \gamma s_t$, is a linear combination of growth and the homoskedastic signal which is why its variance exhibits the same basic heteroskedastic shape as growth.

Let us additionally assume that the signal used by firms to learn g_{t+1} is last period's growth rate, i.e., $s_t \equiv g_t$. Then subjective uncertainty becomes the square root of

$$\omega^2(g_t) = \frac{(\nu-2)(1-\gamma)}{\nu-1}\sigma_g^2 + b_0 + \begin{cases} (a_1+b_1)\, g_t^2 & \text{if } g_t < 0 \\ a_1 g_t^2 & \text{if } g_t \geq 0 \end{cases}$$

where $a_1 \equiv \frac{\gamma(1-\gamma)}{\nu-1} > 0$. Hence, firms experience uncertainty approximately as an asymmetric V with a steeper left branch.

Appendix H Detailed summary statistics

In this appendix, we report summary statistics for the answers to the questions in our survey module. Table H1 pools all firm-quarter observations and reports mean, standard deviation, and key quantiles for this pooled sample. The numbers here reflect variation both in the time series and in the cross section of firms. For Table H2, we compute, for each individual firm, the time-series mean and standard deviations. The panel reports mean, standard deviation and quantiles of the cross-sectional distributions of firm-level statistics. The number of observations for (functions of) forecast errors naturally drops because, in order to compute firm-level forecast errors, we need to observe the expected sales growth rate and the realized sales growth rate of a firm in two consecutive quarters; for details see Appendix D. In addition, we present in this appendix the same two summary statistics tables split by our three firm characteristics: firm size (Tables H3 to H10), trend (Tables H11 to H16), and turbulence (Tables H17 to H24).

Table H1: Summary statistics of survey answers, span, and forecast errors; pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	8889	2.18	14.62	-14	-5	3	10	16
Forecast sales growth rate for the current quarter	8889	2.65	11.61	-10	0	2	5	12
Worst case sales growth rate for the current quarter	8889	-4.52	12.17	-20	-10	-2	0	5
Best case sales growth rate for the current quarter	8889	7.79	12.99	-1	2	5	10	20
Span between worst and best case forecast	8889	12.31	10.08	4	5	10	15	25
Forecast error	5379	-.85	13.38	-15	-6	0	5	11
Forecast error from random walk model	5379	.56	17.94	-15	-5	0	5	17
Forecast error from iid process	5379	.7	11.19	-9.53	-3.56	.43	4.56	10.8
Absolute forecast error	5379	8.49	10.37	0	2	5	10	20

Notes: The summary statistics for the first five rows in the table above are based on the *baseline sample* with 8,889 firm-quarter observations. The last four rows are based on the *baseline sample* with a forecast error, leading to 5,379 firm-quarter observations (see Table D1 in Appendix D). For comparison, we also compute the forecast error from a random walk model and from an *iid* process, respectively, based on the same sample as for the firms' forecast errors. P10 to P90 denote the corresponding percentiles of the distribution.

Table H2: Summary statistics of survey answers, span, and forecast errors; by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Time series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	1005	2.23	7.05	-5	-.83	2.12	5.5	9.8
Mean by Firm: Forecast case sales growth rate for the current quarter	1005	2.91	6.66	-3	0	2.5	5	9.08
Mean by Firm: Worst sales growth rate for the current quarter	1005	-4.3	6.94	-12.22	-7.56	-3.33	-.25	2.36
Mean by Firm: Best case sales growth rate for the current quarter	1005	8.06	8	1.43	3.88	6.7	10.69	15.83
Mean by Firm: Span between worst and best case forecast	1005	12.36	7.38	5.18	7.33	10.73	15.08	21.67
Mean by Firm: Forecast error	950	-.96	9.54	-9.9	-4	-.67	2.5	6.25
Mean by Firm: Forecast error from random walk model	950	.99	10.45	-7.11	-3.25	0	3.75	9.96
Mean by Firm: Forecast error from iid process	950	1.1	7.58	-6.01	-2.14	.46	3.33	8.58
Mean by Firm: Absolute forecast error	950	8.98	8.41	2.5	4.14	6.71	11	17.5
Time series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	998	11.57	8.34	3.92	6.06	9.13	14.72	21.73
Std. Dev. by Firm: Forecast case sales growth rate for the current quarter	998	8.16	7.36	2.03	3.51	6.21	10.16	16.44
Std. Dev. by Firm: Worst sales growth rate for the current quarter	998	8.82	7.25	2.48	3.95	6.97	11.14	17.67
Std. Dev. by Firm: Best case sales growth rate for the current quarter	998	8.91	8.07	2.45	4.04	6.63	10.8	17.45
Std. Dev. by Firm: Span between worst and best case forecast	998	6.29	5.17	2.24	3.2	5.01	7.6	11.29
Std. Dev. by Firm: Forecast error	835	10.3	8.61	2.75	4.8	8.1	13.06	20.41
Std. Dev. by Firm: Forecast error from random walk model	835	12.83	12.73	2.12	4.51	8.96	16.67	28.21
Std. Dev. by Firm: Forecast error from iid process	835	7.56	7.53	1.52	2.89	5.45	9.79	15.6
Std. Dev. by Firm: Absolute forecast error	835	6.98	6.38	1.71	3.06	5.29	8.62	13.93

Notes: The upper panel shows summary statistics of the cross-sectional distribution of firm-specific time series averages. To this end, we first compute the mean of the respective variables for each firm and then calculate moments of this cross-sectional distribution. The lower panel shows summary statistics of the cross-sectional distribution of firm-specific time series standard deviations. To this end, we first compute the standard deviation of the respective variables for each firm and then calculate moments of this cross-sectional distribution. The summary statistics for the first five rows in the upper panel are based on the *baseline sample* with 1005 firms. The summary statistics for the last four rows in the upper panel are based on the *baseline sample* with a forecast error, leading to 950 firms (see Table D1 in Appendix D). For the summary statistics in the lower panel, we use the same observations. However, not all firms have a sufficient number to compute a standard deviation, which is why the number of firms shrink to 998 and 835, respectively. For comparison, we also compute the forecast error from a random walk model and from an *iid* process, respectively, based on the same sample as for the firms' forecast errors. P10 to P90 denote the corresponding percentiles of the distribution.

Table H3: Summary statistics of survey answers and derived variables from tiny firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	289	-1.65	18.18	-20	-10	0	10	20
Expected case sales growth rate for the current quarter	289	.73	14.34	-15	-5	1	5	15
Worst sales growth rate for the current quarter	289	-10.34	16.15	-30	-20	-10	0	5
Best case sales growth rate for the current quarter	289	9.18	16.43	-10	0	10	15	30
Span between worst and best case forecast	289	19.52	15.32	4	10	15	30	40
Forecast error	187	-4.6	18.51	-30	-12	-2	5	15
Forecast error from random walk model	187	3.24	22.13	-20	-5	0	10	30
Forecast error from iid process	187	3.39	13.52	-10.79	-3	2.09	8.57	18.5
Absolute forecast error	187	12.85	14.06	0	3	10	17	30

Table H4: Summary statistics of survey answers and derived variables from tiny firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Time series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	31	.86	8.91	-11.5	-4.21	.5	7.22	11.82
Mean by Firm: Expected sales growth rate for the current quarter	31	3.1	6.08	-6.1	-.62	4.11	7.8	9.88
Mean by Firm: Worst sales growth rate for the current quarter	31	-6.86	7.74	-16.32	-12.5	-6.93	0	1.5
Mean by Firm: Best case sales growth rate for the current quarter	31	11.29	7.49	2	7.11	11.07	15	23.44
Mean by Firm: Span between worst and best case forecast	31	18.15	8.36	9.57	11	16.5	24.17	30.63
Mean by Firm: Forecast error	30	-4.73	13.42	-20.2	-13.57	-2.25	3	6.5
Mean by Firm: Forecast error from random walk model	30	3.48	11.49	-5.37	-3.5	.76	9.18	15.83
Mean by Firm: Forecast error from iid process	30	4.39	9.68	-7.43	-.21	3.2	9.23	16.83
Mean by Firm: Absolute forecast error	30	14.21	11.08	4.1	5.5	9.24	20.55	31.8
Time series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	30	15.61	8.52	6.28	8.49	13.26	23.18	26.96
Std. Dev. by Firm: Expected sales growth rate for the current quarter	30	11.79	7.2	4.37	6.88	10.24	13.87	25.83
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	30	11.98	7.36	3.37	6.89	11.24	15.05	23.23
Std. Dev. by Firm: Best case sales growth rate for the current quarter	30	12.58	8.52	3.52	6.65	11.4	13.8	24.65
Std. Dev. by Firm: Span between worst and best case forecast	30	9.64	5.01	4.69	6.98	8.51	11.81	16.63
Std. Dev. by Firm: Forecast error	28	15.73	10.85	4.55	7.71	11.95	22.42	31.91
Std. Dev. by Firm: Forecast error from random walk model	28	15.54	14.64	2.36	5.83	10.03	22.47	35.49
Std. Dev. by Firm: Forecast error from iid process	28	9.51	6.24	2.89	4.86	8.59	13.26	17.69
Std. Dev. by Firm: Absolute forecast error	28	11.14	7.9	3.25	4.94	8.69	19.19	21.21

Table H5: Summary statistics of survey answers and derived variables from small firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	2019	2.88	16.48	-15	-5	3	10	20
Expected case sales growth rate for the current quarter	2019	3.7	11.56	-5	0	3	8	15
Worst sales growth rate for the current quarter	2019	-5.04	12.2	-20	-10	-3	0	5
Best case sales growth rate for the current quarter	2019	9.87	13.4	0	3	10	15	20
Span between worst and best case forecast	2019	14.91	11.62	5	7	10	20	30
Forecast error	1222	-.59	16.02	-18	-8	0	5	15
Forecast error from random walk model	1222	.94	18.27	-15	-5	0	7	20
Forecast error from iid process	1222	.76	12.22	-10.33	-4	.52	5.29	12.2
Absolute forecast error	1222	10.41	12.19	0	3	6	15	25

Table H6: Summary statistics of survey answers and derived variables from small firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Time series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	243	2.72	7.96	-5.62	-1.67	2.27	6.41	11.6
Mean by Firm: Expected sales growth rate for the current quarter	243	3.8	7.38	-3.2	-.27	2.8	6.42	12
Mean by Firm: Worst sales growth rate for the current quarter	243	-5.2	7.75	-14.4	-9.5	-4.17	-.83	2.5
Mean by Firm: Best case sales growth rate for the current quarter	243	10.15	9.37	2	4.63	8.1	13.29	20
Mean by Firm: Span between worst and best case forecast	243	15.35	8.59	6.78	9.29	13.4	19.38	26.67
Mean by Firm: Forecast error	236	-.51	13.39	-15	-5	-.5	3.55	8.43
Mean by Firm: Forecast error from random walk model	236	1.5	12.56	-7.67	-4.84	0	4.93	13.91
Mean by Firm: Forecast error from iid process	236	.97	8.92	-7.5	-2.93	.17	4.12	10
Mean by Firm: Absolute forecast error	236	11.25	11.1	2.73	5	8.5	15	21.5
Time series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	241	13.13	8.9	4.71	7.07	10.59	17.09	24.2
Std. Dev. by Firm: Expected sales growth rate for the current quarter	241	8.27	6.46	2.12	4.04	6.99	10.37	15.65
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	241	9.15	6.56	2.74	4.35	8.16	11.14	19.92
Std. Dev. by Firm: Best case sales growth rate for the current quarter	241	9.32	7.29	2.92	4.7	7.49	11.05	16.15
Std. Dev. by Firm: Span between worst and best case forecast	241	7.81	6.28	2.74	4.35	6.38	9.58	14.14
Std. Dev. by Firm: Forecast error	201	12.64	8.58	3.86	7.07	10.07	17.03	22.63
Std. Dev. by Firm: Forecast error from random walk model	201	13.23	11.59	2.89	5	10.02	17.64	27.18
Std. Dev. by Firm: Forecast error from iid process	201	7.83	7.59	1.53	3.45	6	9.7	15.56
Std. Dev. by Firm: Absolute forecast error	201	8.66	7.05	2.12	4.17	7.07	10.67	17.31

Table H7: Summary statistics of survey answers and derived variables from medium sized firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	3961	2.15	14.6	-14	-5	2	10	16
Expected case sales growth rate for the current quarter	3961	2.51	11.86	-10	0	2	6	12
Worst sales growth rate for the current quarter	3961	-4.66	12.41	-20	-10	-2	0	5
Best case sales growth rate for the current quarter	3961	7.62	12.92	-2	2	5	10	20
Span between worst and best case forecast	3961	12.28	9.18	4	6	10	15	21
Forecast error	2481	-.92	13.03	-15	-6	0	5	11
Forecast error from random walk model	2481	.49	17.67	-15	-5	0	5	17
Forecast error from iid process	2481	.62	10.73	-8.87	-3.44	.61	4.31	10.24
Absolute forecast error	2481	8.42	9.99	0	2	5	10	20

Table H8: Summary statistics of survey answers and derived variables from medium sized firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Time series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	431	2.1	7.66	-5	-1	2.18	5.5	9.43
Mean by Firm: Expected sales growth rate for the current quarter	431	2.54	6.57	-3.2	-.43	2.36	5.06	8.75
Mean by Firm: Worst sales growth rate for the current quarter	431	-4.63	7.26	-12.5	-8.12	-3.75	-.6	2.25
Mean by Firm: Best case sales growth rate for the current quarter	431	7.65	7.62	1.29	3.57	6.7	10.67	15
Mean by Firm: Span between worst and best case forecast	431	12.28	6.44	5.6	7.8	11.14	15	21.33
Mean by Firm: Forecast error	409	-1.25	8.38	-9.5	-4.22	-1	1.86	6.8
Mean by Firm: Forecast error from random walk model	409	.69	9.98	-7.5	-2.86	.25	3.5	8.5
Mean by Firm: Forecast error from iid process	409	1.12	7.49	-5.78	-1.9	.69	3.25	9
Mean by Firm: Absolute forecast error	409	9.01	7.75	2.58	4.46	6.71	10.86	18
Time series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	429	11.63	8.24	3.75	6.06	9.32	14.75	22.5
Std. Dev. by Firm: Expected sales growth rate for the current quarter	429	8.51	7.85	2.12	3.7	6.24	10.38	17.72
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	429	9.14	7.69	2.51	4.01	6.83	12.1	18.07
Std. Dev. by Firm: Best case sales growth rate for the current quarter	429	9.25	8.4	2.44	4.08	6.69	11.58	18.6
Std. Dev. by Firm: Span between worst and best case forecast	429	6.07	4.59	2.42	3.52	5	7.47	10.46
Std. Dev. by Firm: Forecast error	368	10.55	9.38	2.83	4.97	8.04	13.14	20.03
Std. Dev. by Firm: Forecast error from random walk model	368	13.27	12.69	2.28	4.84	9.43	16.43	30.61
Std. Dev. by Firm: Forecast error from iid process	368	7.82	7.67	1.53	3.02	5.5	9.82	16.82
Std. Dev. by Firm: Absolute forecast error	368	7.09	6.51	1.83	3.23	5.62	8.55	14.04

Table H9: Summary statistics of survey answers and derived variables from large firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	2620	2.12	12.46	-10	-4	3	8	15
Expected case sales growth rate for the current quarter	2620	2.25	10.84	-8	0	2	5	10
Worst sales growth rate for the current quarter	2620	-3.27	10.99	-15	-8	-2	1	5
Best case sales growth rate for the current quarter	2620	6.29	12.08	-3	1	5	10	15
Span between worst and best case forecast	2620	9.55	8.32	3	5	7	10	20
Forecast error	1643	-.42	11.08	-11	-5	0	4	10
Forecast error from random walk model	1643	.15	17.74	-15	-5	0	5	14
Forecast error from iid process	1643	.46	10.85	-9.73	-3.47	-.17	3.88	10
Absolute forecast error	1643	6.86	8.72	0	2	5	9	15

Table H10: Summary statistics of survey answers and derived variables from large firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Time series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	300	2.17	4.75	-3.41	0	2	4.72	7.46
Mean by Firm: Expected sales growth rate for the current quarter	300	2.7	6.18	-2.07	.39	2.37	4.32	6.87
Mean by Firm: Worst sales growth rate for the current quarter	300	-2.84	5.26	-9.05	-5.67	-2.29	.25	2.38
Mean by Firm: Best case sales growth rate for the current quarter	300	6.63	6.91	1.44	3.61	5.69	8.15	12.01
Mean by Firm: Span between worst and best case forecast	300	9.47	6.1	4	5.65	8.32	11.4	16.1
Mean by Firm: Forecast error	286	-.76	5.96	-7	-3.5	-.17	2.1	5
Mean by Firm: Forecast error from random walk model	286	1.17	10.65	-6.33	-2.67	0	3.2	8
Mean by Firm: Forecast error from iid process	286	.87	6.19	-4.83	-1.9	0	2.69	6.7
Mean by Firm: Absolute forecast error	286	6.87	5.26	2	3.43	5.83	8.67	12.5
Time series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	298	9.83	7.62	3.16	5.42	7.89	11.52	18.32
Std. Dev. by Firm: Expected sales growth rate for the current quarter	298	7.2	7.21	1.75	3.01	5.16	8.95	14.27
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	298	7.78	7	2.19	3.71	5.94	9.68	14.33
Std. Dev. by Firm: Best case sales growth rate for the current quarter	298	7.73	7.99	2.16	3.32	5.35	9.16	14.35
Std. Dev. by Firm: Span between worst and best case forecast	298	5.06	4.52	1.7	2.57	4.02	6.21	8.86
Std. Dev. by Firm: Forecast error	257	7.96	6.57	2.12	3.58	6.36	10.41	14.68
Std. Dev. by Firm: Forecast error from random walk model	257	11.73	13.07	1.85	3.79	7.81	15.28	25.2
Std. Dev. by Firm: Forecast error from iid process	257	6.82	7.25	1.41	2.52	4.49	9.27	14.14
Std. Dev. by Firm: Absolute forecast error	257	5.3	4.51	1.5	2.38	4.1	6.56	10.55

Table H11: Summary statistics of survey answers and derived variables from 'bad' trend firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	2129	-4.49	14.67	-20	-10	-5	4	10
Expected case sales growth rate for the current quarter	2129	-.12	13.12	-12	-5	0	5	10
Worst sales growth rate for the current quarter	2129	-8.7	14.12	-25	-15	-6	0	3
Best case sales growth rate for the current quarter	2129	5.96	14.53	-5	0	5	10	20
Span between worst and best case forecast	2129	14.66	11.83	5	7	10	20	30
Forecast error	1329	-4.29	14.58	-20	-10	-3	3	10
Forecast error from random walk model	1329	3.93	18.45	-12	-4	2	10	22
Forecast error from iid process	1329	4.25	11.77	-7.78	-.6	3.31	8.32	16.18
Absolute forecast error	1329	10.11	11.34	0	3	7	14	23

Table H12: Summary statistics of survey answers and derived variables from 'bad' trend firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Time series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	250	-4.82	6.72	-10.75	-6.6	-3.56	-1.5	-.54
Mean by Firm: Expected sales growth rate for the current quarter	250	.3	8.51	-6.39	-3	0	2.5	5.9
Mean by Firm: Worst sales growth rate for the current quarter	250	-8.32	8.66	-17.33	-11.8	-7.25	-3.18	-.31
Mean by Firm: Best case sales growth rate for the current quarter	250	6.34	9.6	-1	2	5	8.75	15
Mean by Firm: Span between worst and best case forecast	250	14.66	8.4	6.06	9.24	12.5	17.63	26.44
Mean by Firm: Forecast error	242	-5.42	8.95	-16	-8.96	-3.68	-.5	3
Mean by Firm: Forecast error from random walk model	242	5.73	14.27	-4.89	-.25	2.55	8.5	16.33
Mean by Firm: Forecast error from iid process	242	5.4	8.63	-2.43	1.1	3.35	8	14.95
Mean by Firm: Absolute forecast error	242	10.72	8.13	3.38	5.25	8.33	14.31	19.75
Time series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	247	12.14	7.74	4.65	7.07	10.05	15.56	22.63
Std. Dev. by Firm: Expected sales growth rate for the current quarter	247	9.25	8.39	2.4	4.47	7.16	10.84	17.96
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	247	10.3	8.43	2.95	5	8.55	12.7	17.92
Std. Dev. by Firm: Best case sales growth rate for the current quarter	247	10	9.16	2.64	4.35	7.44	11.93	19.66
Std. Dev. by Firm: Span between worst and best case forecast	247	7.52	6.36	2.79	3.98	5.8	8.65	13.13
Std. Dev. by Firm: Forecast error	211	11.9	9.8	3.54	5.8	9.08	15.19	21.79
Std. Dev. by Firm: Forecast error from random walk model	211	12.89	11.96	3.54	5.5	9.07	16.28	27.46
Std. Dev. by Firm: Forecast error from iid process	211	8.15	7.82	2	3.46	6.12	9.79	15.56
Std. Dev. by Firm: Absolute forecast error	211	8.16	7.14	2.12	3.54	6.5	10.05	16.49

Table H13: Summary statistics of survey answers and derived variables from 'normal' trend firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	4695	2.04	11.43	-10	-4	2	7	14
Expected case sales growth rate for the current quarter	4695	2.31	9.31	-6	0	2	5	10
Worst sales growth rate for the current quarter	4695	-4.01	9.87	-15	-10	-2	0	5
Best case sales growth rate for the current quarter	4695	6.84	10.55	0	2	5	10	20
Span between worst and best case forecast	4695	10.85	8.51	3	5	10	15	20
Forecast error	2973	-.68	10.72	-12	-5	0	4	10
Forecast error from random walk model	2973	.21	15.06	-14	-5	0	5	15
Forecast error from iid process	2973	.46	8.89	-7.78	-2.43	.27	3.5	9
Absolute forecast error	2973	6.95	8.19	0	2	5	10	16

Table H14: Summary statistics of survey answers and derived variables from 'normal' trend firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Time series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	503	1.99	2.88	-.33	.67	2	3.55	4.83
Mean by Firm: Expected sales growth rate for the current quarter	503	2.49	4.74	-1.46	.33	2.29	4.18	6.73
Mean by Firm: Worst sales growth rate for the current quarter	503	-3.81	5.1	-10	-6.12	-3.2	-.6	1.2
Mean by Firm: Best case sales growth rate for the current quarter	503	7.03	6.3	2	3.71	6	8.75	13
Mean by Firm: Span between worst and best case forecast	503	10.83	6.33	4.71	6.14	9.17	13.92	19.22
Mean by Firm: Forecast error	482	-1.01	5.78	-7	-3.25	-.59	1.6	5
Mean by Firm: Forecast error from random walk model	482	.52	7.61	-6	-2.6	0	3	7
Mean by Firm: Forecast error from iid process	482	.78	5.04	-4.4	-1.27	.33	2.79	5.7
Mean by Firm: Absolute forecast error	482	7.05	5.49	1.89	3.5	5.94	8.73	13.67
Time series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	501	9.99	6.66	3.39	5.62	8.48	12.61	17.85
Std. Dev. by Firm: Expected sales growth rate for the current quarter	501	6.81	5.74	1.75	3.09	5.3	8.87	13.33
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	501	7.48	5.61	2.12	3.71	6.12	9.64	13.82
Std. Dev. by Firm: Best case sales growth rate for the current quarter	501	7.36	6.36	2.12	3.51	5.53	9.35	14.75
Std. Dev. by Firm: Span between worst and best case forecast	501	5.38	3.6	1.89	2.84	4.63	7.21	9.54
Std. Dev. by Firm: Forecast error	433	8.7	6.78	2.12	4.11	7.21	10.8	16.01
Std. Dev. by Firm: Forecast error from random walk model	433	11.28	9.47	1.85	4.32	8.8	15.28	25.01
Std. Dev. by Firm: Forecast error from iid process	433	6.29	5.29	1.41	2.56	4.78	8.87	12.89
Std. Dev. by Firm: Absolute forecast error	433	5.79	4.75	1.6	2.78	4.82	7.36	10.74

Table H15: Summary statistics of survey answers and derived variables from 'good' trend firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	2065	9.39	17.42	-8	2	8	15	25
Expected case sales growth rate for the current quarter	2065	6.26	13.58	-5	2	5	10	17
Worst sales growth rate for the current quarter	2065	-1.36	13.47	-15	-5	0	5	10
Best case sales growth rate for the current quarter	2065	11.82	15.28	0	5	10	15	25
Span between worst and best case forecast	2065	13.19	10.84	4	6	10	15	25
Forecast error	1231	2.58	16.84	-13	-5	1	9	20
Forecast error from random walk model	1231	-2.15	22.84	-20	-8	-2	3	15
Forecast error from iid process	1231	-2.58	14.19	-14.11	-6.9	-2.87	1.64	8.33
Absolute forecast error	1231	10.68	13.27	0	3	6	14	25

Table H16: Summary statistics of survey answers and derived variables from 'good' trend firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Time series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	252	9.72	5.58	5.5	6.32	8.2	11.41	17.13
Mean by Firm: Expected sales growth rate for the current quarter	252	6.34	6.46	.13	3.3	5.21	8.48	13.14
Mean by Firm: Worst sales growth rate for the current quarter	252	-1.3	6.36	-9	-4.31	-.87	2.12	5.4
Mean by Firm: Best case sales growth rate for the current quarter	252	11.83	8.1	4.94	7.13	10	14.82	22
Mean by Firm: Span between worst and best case forecast	252	13.13	7.57	6	8.21	11.39	15.33	22.44
Mean by Firm: Forecast error	237	3.41	13.33	-6	-1.43	2	6.33	14.5
Mean by Firm: Forecast error from random walk model	237	-2.37	11.08	-13.43	-5.37	-2.58	.5	7.25
Mean by Firm: Forecast error from iid process	237	-2.59	8.64	-12.08	-6.07	-2.29	0	3.7
Mean by Firm: Absolute forecast error	237	11.54	11.95	3	4.92	7.5	13.5	24
Time series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	250	14.2	10.88	4.03	6.3	10.2	19.15	30.83
Std. Dev. by Firm: Expected sales growth rate for the current quarter	250	9.81	8.58	2.67	3.97	6.99	12.55	21.78
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	250	10.05	8.38	2.69	4.16	7.59	12.89	21.3
Std. Dev. by Firm: Best case sales growth rate for the current quarter	250	10.94	9.3	3.19	4.73	8.38	13.37	24.78
Std. Dev. by Firm: Span between worst and best case forecast	250	6.92	6.11	2.49	3.43	5.2	8.19	12.93
Std. Dev. by Firm: Forecast error	210	12.55	10.22	3.34	5.84	9.2	16.85	28.13
Std. Dev. by Firm: Forecast error from random walk model	210	16.14	17.49	2.44	4.24	10.19	20.51	36.48
Std. Dev. by Firm: Forecast error from iid process	210	9.66	10.1	1.63	3.3	6.25	12.58	23.01
Std. Dev. by Firm: Absolute forecast error	210	8.53	7.67	1.63	3.77	6.16	10.61	17.58

Table H17: Summary statistics of survey answers and derived variables from low turbulence firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	2243	2.52	5.98	-5	0	3	5	10
Expected case sales growth rate for the current quarter	2243	2.57	4.82	-2	0	2	5	8
Worst sales growth rate for the current quarter	2243	-1.85	5.66	-10	-5	0	1	5
Best case sales growth rate for the current quarter	2243	5.73	5.98	0	3	5	10	10
Span between worst and best case forecast	2243	7.57	6.11	2	4	6	10	15
Forecast error	1405	-.33	5.91	-7	-3	0	3	5
Forecast error from random walk model	1405	.13	6.12	-6	-2	0	2	7
Forecast error from iid process	1405	.31	4.85	-4.33	-1.82	.18	2.25	5.24
Absolute forecast error	1405	3.99	4.37	0	1	3	5	9

Table H18: Summary statistics of survey answers and derived variables from low turbulence firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Time series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	253	2.5	4.3	-2	.29	2.33	4.56	7.63
Mean by Firm: Expected sales growth rate for the current quarter	253	2.7	3.02	-.4	.94	2.36	4.33	6.33
Mean by Firm: Worst sales growth rate for the current quarter	253	-1.73	3.55	-6.67	-3.54	-1.37	.33	2.4
Mean by Firm: Best case sales growth rate for the current quarter	253	5.81	3.71	2.22	3.36	5	8	10.73
Mean by Firm: Span between worst and best case forecast	253	7.54	4.06	3.56	4.91	6.56	9.33	12.22
Mean by Firm: Forecast error	244	-.59	4.51	-4.83	-2.21	0	1.5	3
Mean by Firm: Forecast error from random walk model	244	.38	4.93	-3.33	-1.8	-.1	2	4
Mean by Firm: Forecast error from iid process	244	.56	4.35	-3.1	-1.32	.03	1.88	3.7
Mean by Firm: Absolute forecast error	244	4.22	3.48	1.33	2.5	3.63	5	7
Time series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	253	4.39	1.55	2.18	3.24	4.47	5.68	6.22
Std. Dev. by Firm: Expected sales growth rate for the current quarter	253	3.48	2.24	1.26	1.89	2.97	4.55	6.23
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	253	4.12	2.42	1.41	2.32	3.74	5.29	7.31
Std. Dev. by Firm: Best case sales growth rate for the current quarter	253	4.17	3.1	1.64	2.31	3.61	5.03	6.88
Std. Dev. by Firm: Span between worst and best case forecast	253	3.88	3.41	1.55	2.14	3.06	4.58	7.14
Std. Dev. by Firm: Forecast error	223	4.5	2.74	1.41	2.53	4.04	6.36	7.83
Std. Dev. by Firm: Forecast error from random walk model	223	4.38	3.03	.84	2.12	3.68	5.79	8.8
Std. Dev. by Firm: Forecast error from iid process	223	3.15	2.34	.82	1.53	2.74	3.86	6.36
Std. Dev. by Firm: Absolute forecast error	223	3.11	2.17	.71	1.62	2.65	4.13	5.57

Table H19: Summary statistics of survey answers and derived variables from medium low turbulence firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	2289	1.53	8.9	-10	-5	2	7	12
Expected case sales growth rate for the current quarter	2289	1.75	7.08	-5	0	2	5	10
Worst sales growth rate for the current quarter	2289	-4.21	8.26	-15	-10	-3	0	5
Best case sales growth rate for the current quarter	2289	5.94	7.7	0	2	5	10	15
Span between worst and best case forecast	2289	10.15	7.05	4	5	10	13	20
Forecast error	1398	-.72	8.54	-11	-5	0	5	10
Forecast error from random walk model	1398	.61	9.97	-10	-5	0	5	12
Forecast error from iid process	1398	.64	6.58	-6.42	-2.67	.58	3.88	7.73
Absolute forecast error	1398	6.27	5.84	0	2	5	9	14

Table H20: Summary statistics of survey answers and derived variables from medium low turbulence firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Time series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	252	1.32	5.24	-5.4	-1.13	1.3	4.15	7
Mean by Firm: Expected sales growth rate for the current quarter	252	1.62	4.4	-3.5	-.7	2.29	4.18	5.86
Mean by Firm: Worst sales growth rate for the current quarter	252	-4.45	5.65	-10.53	-6.65	-3.77	-.89	1.43
Mean by Firm: Best case sales growth rate for the current quarter	252	5.86	4.63	.73	2.94	5.88	8.04	11.22
Mean by Firm: Span between worst and best case forecast	252	10.31	5.08	5.67	6.92	9.13	12.62	16
Mean by Firm: Forecast error	238	-.79	4.9	-6.71	-3.12	-1	1.5	4.57
Mean by Firm: Forecast error from random walk model	238	.89	5.09	-5	-2.25	.71	3.71	7.33
Mean by Firm: Forecast error from iid process	238	.77	4.08	-4.57	-1.08	.93	2.73	5.83
Mean by Firm: Absolute forecast error	238	6.33	3.15	2.67	4.35	6	7.75	10.75
Time series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	251	7.85	1.48	6.4	7.17	8.05	8.82	9.36
Std. Dev. by Firm: Expected sales growth rate for the current quarter	251	5.49	2.99	2.43	3.54	4.97	6.88	9.46
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	251	6.31	3.27	2.89	3.93	5.48	7.99	10.92
Std. Dev. by Firm: Best case sales growth rate for the current quarter	251	6.04	3.2	2.58	3.85	5.48	7.31	10.14
Std. Dev. by Firm: Span between worst and best case forecast	251	5.08	2.97	2.47	3.17	4.46	6.03	8.54
Std. Dev. by Firm: Forecast error	213	7.55	3.39	3.54	5.26	7.64	9.19	11.47
Std. Dev. by Firm: Forecast error from random walk model	213	8.65	5.11	3.06	5.51	7.57	11.68	14.67
Std. Dev. by Firm: Forecast error from iid process	213	5.22	3.71	1.73	2.89	4.47	6.65	9.25
Std. Dev. by Firm: Absolute forecast error	213	5.02	2.63	2.07	3.37	5	6.24	7.64

Table H21: Summary statistics of survey answers and derived variables from medium high turbulence firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	2255	1.89	12.41	-15	-7	3	10	17
Expected case sales growth rate for the current quarter	2255	2.12	9.97	-10	-3	3	7	12
Worst sales growth rate for the current quarter	2255	-6.11	11.17	-20	-10	-5	0	5
Best case sales growth rate for the current quarter	2255	7.8	10.46	-3	2	7	10	20
Span between worst and best case forecast	2255	13.91	8.82	5	8	10	20	25
Forecast error	1424	-.63	12.49	-15	-8	0	5	15
Forecast error from random walk model	1424	.16	15.39	-18	-7	0	8	20
Forecast error from iid process	1424	.37	10.05	-11.25	-4.79	.71	5.55	11.33
Absolute forecast error	1424	9.19	8.48	0	3	7	13	20

Table H22: Summary statistics of survey answers and derived variables from medium high turbulence firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Time series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	250	2.14	5.38	-4.11	-1.4	2.1	5	8.42
Mean by Firm: Expected sales growth rate for the current quarter	250	2.38	4.94	-3.6	-.5	2.32	5	8.67
Mean by Firm: Worst sales growth rate for the current quarter	250	-5.67	6.08	-14.2	-9.55	-4.95	-1.75	1.5
Mean by Firm: Best case sales growth rate for the current quarter	250	7.81	5.62	1.32	4.5	7.38	11.06	15
Mean by Firm: Span between worst and best case forecast	250	13.48	5.58	7.64	9.75	12.8	15.67	21.62
Mean by Firm: Forecast error	236	-.49	7.21	-9	-5	-.55	4.09	7.38
Mean by Firm: Forecast error from random walk model	236	.08	7.98	-7.67	-5	-.1	4.42	10
Mean by Firm: Forecast error from iid process	236	.27	5.88	-6.55	-3.45	.39	3.38	8.04
Mean by Firm: Absolute forecast error	236	9.35	4.79	4.89	6.07	8.65	11.6	15
Time series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	249	11.68	2.66	9.07	10.23	11.52	13.24	14.75
Std. Dev. by Firm: Expected sales growth rate for the current quarter	249	8.33	4.31	3.4	5.42	8.04	10.37	12.99
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	249	9.11	4.87	3.31	6.07	8.77	11.7	14.43
Std. Dev. by Firm: Best case sales growth rate for the current quarter	249	8.66	4.31	3.77	5.77	8.26	10.79	14.14
Std. Dev. by Firm: Span between worst and best case forecast	249	6.49	3.22	2.75	4.47	6.26	8.24	10.04
Std. Dev. by Firm: Forecast error	205	11.18	5.57	4.97	7.64	10.51	13.91	19.09
Std. Dev. by Firm: Forecast error from random walk model	205	13.66	7.64	4.99	8.49	12.75	17.68	22.63
Std. Dev. by Firm: Forecast error from iid process	205	7.94	4.86	2.83	4.79	7.07	10.31	13.6
Std. Dev. by Firm: Absolute forecast error	205	7.26	3.84	2.89	4.55	7.07	9.07	11.93

Table H23: Summary statistics of survey answers and derived variables from high turbulence firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	2102	2.84	24.76	-25	-13	3	15	30
Expected case sales growth rate for the current quarter	2102	4.26	19.5	-15	-5	3	10	25
Worst sales growth rate for the current quarter	2102	-5.99	19.27	-30	-15	-5	3	10
Best case sales growth rate for the current quarter	2102	11.99	21.62	-10	0	10	20	30
Span between worst and best case forecast	2102	17.99	13.74	5	10	15	20	33
Forecast error	1306	-1.68	21.95	-25	-12	0	8	21
Forecast error from random walk model	1306	1.48	31.1	-33	-14	0	15	37
Forecast error from iid process	1306	1.52	18.73	-17.67	-7.57	.54	10	21.67
Absolute forecast error	1306	15.18	15.95	1	5	10	20	35

Table H24: Summary statistics of survey answers and derived variables from high turbulence firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Time series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	250	2.99	11.1	-8	-1.89	2.93	8.83	15
Mean by Firm: Expected sales growth rate for the current quarter	250	4.94	10.93	-4.78	-.75	3.58	9.17	16.11
Mean by Firm: Worst sales growth rate for the current quarter	250	-5.39	10.12	-15.69	-10.67	-5.32	.2	5.19
Mean by Firm: Best case sales growth rate for the current quarter	250	12.8	12.58	1.49	5.69	10.83	17.5	27.08
Mean by Firm: Span between worst and best case forecast	250	18.19	9.08	8.75	11.77	16.19	22.33	30.25
Mean by Firm: Forecast error	243	-2.24	16.17	-20	-10	-2.3	4	13
Mean by Firm: Forecast error from random walk model	243	3.09	18.83	-14.75	-6.25	.4	9.69	25
Mean by Firm: Forecast error from iid process	243	2.82	12.52	-11.01	-5	1.25	9	18.88
Mean by Firm: Absolute forecast error	243	16.42	12.33	5.25	8.2	13.67	20.77	30.5
Time series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	245	22.71	8.96	15.35	17.07	20	26.78	35.25
Std. Dev. by Firm: Expected sales growth rate for the current quarter	245	15.57	10.15	5.77	8.57	13.06	20.19	29.24
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	245	15.97	9.65	6.34	9.66	13.72	20.69	27.46
Std. Dev. by Firm: Best case sales growth rate for the current quarter	245	17	11.36	5.73	9.31	14.29	21.73	32.47
Std. Dev. by Firm: Span between worst and best case forecast	245	9.83	7.58	4.16	5.43	7.84	11.73	17.15
Std. Dev. by Firm: Forecast error	213	18.82	11.77	6.29	10.61	16.07	24.11	35.39
Std. Dev. by Firm: Forecast error from random walk model	213	25.21	17.39	7.07	13.4	22.41	33.23	43.91
Std. Dev. by Firm: Forecast error from iid process	213	14.21	10.64	3.54	7.27	11.58	18.26	27.14
Std. Dev. by Firm: Absolute forecast error	213	13.01	8.89	4.24	6.66	10.61	16.49	26.89

Appendix I Additional tables

Table I1: Within-firm regressions of sales growth on past sales growth

	Sales growth in $t+1$	
	(1)	(2)
Sales growth in t	-0.0253	
	(0.0286)	
Sales growth in t		
$\times$ positive		0.0297
		(0.0482)
$\times$ negative		-0.0775
		(0.0526)
Observations	4445	4445
R^2	0.000	0.001

Notes: The table shows within-firm regressions of realized sales growth in $t+1$ on realized sales growth in the previous quarter t. Sales growth is within-transformed and in column (2) interacted with a dummy variable for whether this within-transformed sales growth rate is positive or negative. We restrict the sample to p5–p95 of within-transformed past sales growth. Standard errors in parentheses, clustered by firm; * p < 0.10, ** p < 0.05, *** p < 0.01.

Table I2: Variance decomposition of sales growth and span

	(1)	(2)	(3)
	Sales growth		
Constant	2.182***	2.182***	2.182***
	(0.199)	(0.202)	(0.200)
R^2	0.006	0.014	0.071
	Span		
Constant	12.31***	12.31***	12.31***
	(0.239)	(0.243)	(0.239)
R^2	0.016	0.006	0.056
Observations	8889	8889	8889
Industry FE	Yes	No	No
Time FE	No	Yes	No
Industry $\times$ Time FE	No	No	Yes

Notes: The table shows the results of fixed effects regressions of sales growth and span: industry fixed effects in column (1), time fixed effects in column (2), and industry $\times$ time fixed effects in column (3). Standard errors in parentheses, clustered by firm; * p < 0.10, ** p < 0.05, *** p < 0.01.

Table I3: Properties of span

	Shares	N
Skewness of span		
Symmetric	28.63 %	1540
Negative skew	50.94 %	2740
Positive skew	20.43 %	1099
Realized growth outside of span		
Above	22.66 %	1219
Below	21.94 %	1180

Notes: The upper panel of the table shows the shares of observations, where (i) the forecast of sales growth is at the midpoint between the best and worst case scenario (symmetric), (ii) the worst case scenario of sales growth is further away from the forecast than the best case scenario (negative skew), and (iii) the best case scenario of sales growth is further away from the forecast than the worst case scenario (positive skew). The lower panel shows the shares of observations, where realized growth fell outside of span.

Table I4: Within-firm regressions of subjective uncertainty and absolute forecast errors on past sales growth (robustness)

	(1) Span	(2) Span	(3) DownSpan	(4) UpSpan	(5) Abs. debiased FE	(6) Abs. debiased FE	(7) Abs. stat. FE	(8) Abs. stat. FE
Past sales growth	-0.0511***				-0.0311**		-0.0377**	
	(0.0123)				(0.0137)		(0.0157)	
Past sales growth								
× positive		0.124***	0.0804***	0.0435***		-0.00408		0.0125
		(0.0214)	(0.0143)	(0.0111)		(0.0210)		(0.0252)
× negative		-0.216***	-0.121***	-0.0942***		-0.0561**		-0.0843***
		(0.0211)	(0.0141)	(0.0127)		(0.0247)		(0.0275)
Constant	12.00***	10.97***	6.386***	4.586***	7.449***	7.292***	8.453***	8.160***
	(0.0364)	(0.105)	(0.0709)	(0.0563)	(0.0374)	(0.109)	(0.0380)	(0.122)
Observations	7355	7355	7355	7355	4573	4573	4518	4518
R^2	0.004	0.023	0.016	0.013	0.002	0.002	0.002	0.004

Notes: The table shows within-firm regressions of different measures of uncertainty and volatility on sales growth rates in the previous quarter. The right-hand side for all regressions is the within-transformed firm-level sales growth rate, in the even columns and column (3) interacted with dummy variables for whether this within-transformed sales growth rate is positive or negative. In columns (1) and (2), the left-hand side variable is span. In column (3), the left-hand side variable is downside span, i.e., the absolute distance from the worst case scenario to the forecast. In column (4), the left-hand side variable is upside span, i.e., the absolute distance from the best case scenario to the forecast. In columns (5) and (6), the left-hand side variable is the absolute debiased subjective forecast error. To be specific, we demean the firms' forecast error and regress it on the same right-hand side variables defined above for column (2). This is to ensure that the results we find are not driven by possible bias in the subjective forecast errors. We then take the absolute value of the residuals of this regression. In columns (7) and (8), the left-hand side variable is the absolute statistical forecast error defined as the absolute difference between realized growth and its firm-specific mean. In all cases, the left-hand side variables are within-transformed (and the overall mean added). In addition, we restrict the sample to p5–p95 of within-transformed past sales growth and require at least 5 span observations for each firm. Standard errors in parentheses, clustered by firm; * p < 0.10, ** p < 0.05, *** p < 0.01.

Table I5: Subjective uncertainty and firms' employment plans

	(1) Emp down	(2) Emp down	(3) Emp down	(4) Emp up	(5) Emp up	(6) Emp up
Forecast	-0.503***		-0.530***	0.379***		0.390***
	(0.0526)		(0.0524)	(0.0455)		(0.0459)
Span		0.334***	0.391***		-0.116**	-0.158***
		(0.0566)	(0.0575)		(0.0540)	(0.0529)
Constant	12.61***	7.396***	8.008***	13.55***	15.86***	15.41***
	(0.179)	(0.693)	(0.690)	(0.143)	(0.647)	(0.630)
Observations	7943	7943	7943	7943	7943	7943

Notes: The table shows the coefficients of linear probability regressions of an indicator for a planned decrease of employment in the next three months (Columns (1) to (3)) and an indicator for a planned increase of employment in the next three months (Columns (4) to (6)) on firms' sales growth forecast and subjective uncertainty (span). We control for firm & time fixed effects. In addition, we restrict the sample to p5–p95 of within-transformed past sales growth. Standard errors in parentheses, clustered by firm; * p < 0.10, ** p < 0.05, *** p < 0.01.

Table I6: Subjective uncertainty and firms' employment and price plans (robustness)

	(1) Emp down	(2) Emp down	(3) Emp down	(4) P down	(5) P down	(6) P down	(7) P up	(8) P up	(9) P up
Forecast	-0.562***		-0.582***	-0.223***		-0.233***	0.255***		0.262***
	(0.0616)		(0.0611)	(0.0425)		(0.0429)	(0.0599)		(0.0594)
Span		0.293***	0.344***		0.156***	0.175***		-0.0956	-0.117*
		(0.0596)	(0.0613)		(0.0452)	(0.0453)		(0.0700)	(0.0687)
Constant	12.83***	7.902***	8.762***	7.126***	4.693***	5.050***	15.68***	17.48***	17.07***
	(0.201)	(0.736)	(0.748)	(0.136)	(0.541)	(0.519)	(0.192)	(0.849)	(0.857)
Observations	7304	7304	7304	7309	7309	7309	7309	7309	7309

Notes: The table shows the coefficients of linear probability regressions of an indicator for a planned decrease of employment in the next three months (Columns (1) to (3)), an indicator for a planned decrease of prices in the next three months (Columns (4) to (6)), and an indicator for a planned increase of prices in the next three months (Columns (7) to (9)) on firms' sales growth forecast and subjective uncertainty (span). We control for firm & time fixed effects. In addition, we restrict the sample to p5–p95 of within-transformed past sales growth and require at least 5 span observations for each firm. Standard errors in parentheses, clustered by firm; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.