

Market opacity and fragility: Why liquidity evaporates when it is most needed

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IESE

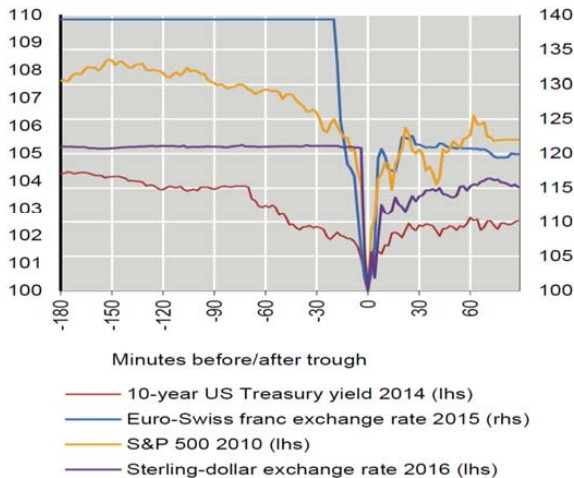
June 2025

Flash events

Selected historic intraday moves

Index: 100 = intraday trough

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BIS (2017)

Introduction

Flash event: sudden evaporation of liquidity *in conjunction* with strong liquidity demand and extreme price changes over short time intervals, *followed by quick recovery*—all this *in the absence of any fundamentals news*.

- ▶ Concern for crashes has recently revived, in the wake of the sizeable number of “flash events” (equities, FX, bonds. . .).
- ▶ Unifying feature: jamming of the rationing function of market illiquidity—as the market becomes **more** illiquid, traders demand **more** liquidity. . .
- ▶ This makes liquidity “fragile.”

Rationing from high illiquidity?

Liquidity demand driven crashes:

- ▶ May 2010 “flash-crash”: Waddell & Reed persists demanding liquidity while the crash is ongoing. “[The Flash Crash is explained by] unsettled market conditions [...] *combined* with the size and execution strategy of the Waddell & Reed trades.” (Aldrich et al (2017))
- ▶ In 40 “mini” flash events in FTSE350 (UK): “HFTs lean against the wind initially [...] They subsequently tend to follow and exacerbate the price change [...] their liquidity is consumed through trades more quickly than it is replenished with new orders.” (Aquilina et al. (2018))

Why do traders fail to internalize higher trading costs?
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Change in market structure

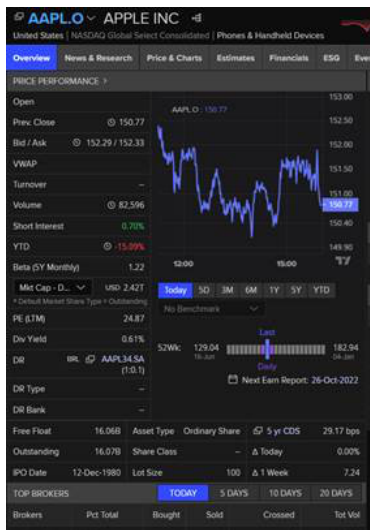


(a) Trading floor: liquidity supplied by professional agents.

Change in market structure



(a) Trading floor: liquidity supplied by professional agents.



(b) LOB: all-to-all trading

The structural change in market organization

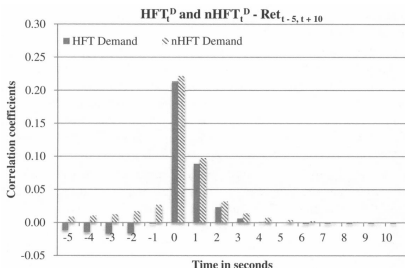
Over the past 20 years exchanges have undergone a substantial transformation

- ▶ Electronification $\implies$ Markets have moved from floor trading to “all to all” structure (Foucault (2022)).
- ▶ “All to all” structure $\implies$ Informational friction: only some traders access market information (Ding et al. (2014)).

Market information has become vital to facilitate liquidity provision.

All to all trading and liquidity supply (1/2)

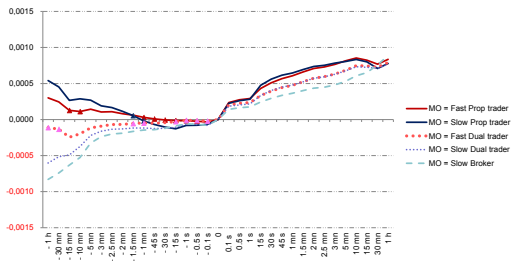
“HFTs trade (buy or sell) in the direction of permanent price changes and in the opposite direction of transitory pricing errors. This is done through their liquidity demanding (marketable) orders. . . ” (Brogaard et al., RFS, 2014)



*Correlation of returns with HFT and non-HFT liquidity demand: HFTs marketable orders (HFT^D) are negatively correlated with lagged returns (5 to 2 seconds lags), liquidity “demanding” HFTs orders *de-facto* supply liquidity.*

All to all trading and liquidity supply (2/2)

“[p]roprietary traders, be they fast or slow... place marketable buy orders after price declines, and marketable sell orders after price increases... While proprietary traders’ contrarian strategies rely on marketable orders, they supply liquidity to the market.” (Biais et al., WP, 2017)



Proprietary (Slow/Fast) traders post buy marketable orders after midquote declines: liquidity is supplied by both *fast* and *slow* traders via contrarian marketable orders.

This paper

Dynamic (3-period) model of liquidity provision with symmetric information, competitive CARA hedgers and dealers:

- ▶ Imperfect observability of market information (market opacity) prevents the participation of non-standard liquidity providers.
- ▶ This may cause liquidity demand to become **increasing in illiquidity**.
- ▶ *Strategic complementarity*: $\downarrow$ liquidity $\implies \uparrow$ liquidity demand $\implies \downarrow$ liquidity, generating: (i) liquidity dry-ups, (ii) flash-events.
- ▶ Our model rationalizes momentum and the assumption that noise trading follows an AR(1) process.

Related literature

- ▶ Liquidity fragility: Brunnermeier and Pedersen (2009), Gromb and Vayanos (2002), Cespa and Foucault (2014), Cespa and Vives (2015), Menkveld and Yueshen (2019). *We propose a liquidity demand theory of liquidity fragility.*
- ▶ Liquidity provision via contrarian marketable orders: Brogaard et al (2014), Biais et al (2017), Anand et al (2013, 2021). *We show how liquidity supply via market(able) orders arises in equilibrium.*
- ▶ Early literature on price crashes: Gennotte and Leland (1990), Jacklin et al. (1992), Madrigal and Scheinkman (1997). *Differently from these papers, in our setup all traders are rational and the crash occurs because of the self-sustaining loop triggered by traders' liquidity demand.*
- ▶ Pros of market fragmentation: Chen and Duffie (2021), Malamud and Rostek (2017), Manzano and Vives (2021). *All advantages of fragmentation occur w/traders having a price impact, while in our setting, traders are competitive.*

The model

Two rounds of trade $t \in \{1, 2\}$, liquidation at $t = 3$ of a single risky asset with liquidation value $v \sim N(0, \tau_v^{-1})$, and a risk-less asset with unit return. Two classes of traders:

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- ▶ Two continua of competitive CARA dealers with risk-tolerance coefficient γ :
 1. A continuum of mass $\mu \in (0, 1]$, who submit price-contingent orders x_t^D , to maximize the expected utility (EU) of final wealth $W^D = (v - p_2)x_2^D + (p_2 - p_1)x_1^D$.

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- ▶ CARA liquidity traders with risk-tolerance γ_H who hedge an endowment shock u_t in a non-tradable security that they receive at $t = 3$:
 - ▶ At $t = 1$, a unit mass enters, learns u_1 , and posts market orders x_{t1} , $t \in \{1, 2\}$ to maximize EU of $u_1 v + (v - p_2)x_{21} + (p_2 - p_1)x_{11}$.

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 - ▶ At $t = 2$, a new unit enters, learns u_2 , observes $s_{u_1} = u_1 + \eta$, and posts a market order x_2 to max EU of $u_2 v + (v - p_2)x_2$.

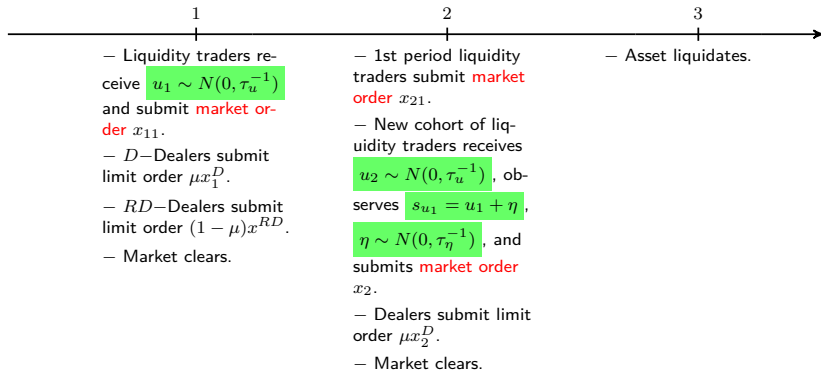
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- ▶ $u_t \sim N(0, \tau_{u_t}^{-1})$, $\eta \sim N(0, \tau_\eta^{-1})$ and $u_t \perp v$, $u_t \perp \eta$, $u_1 \perp u_2$.

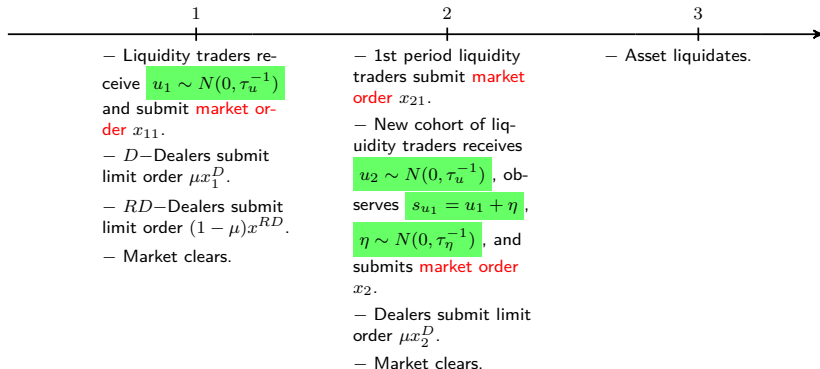
The model

Timeline:



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Alternative: 1 and 2 are separate venues trading the same asset.

Information and equilibrium

- ▶ Information sets:
 - ▶ $\{p_1, p_2\}$ for D in period 2, $\{p_1\}$ for RD and for D in period 1.
 - ▶ $\Omega_1 = \{u_1\}$ for first period hedgers.
 - ▶ $\Omega_2 = \{s_{u_1}, u_2\}$ for second period hedgers.
- ▶ We restrict attention to linear equilibria.

Equilibrium (1/2)

When $\mu \in (0, 1]$ and $\tau_\eta \in (0, \infty)$, a linear equilibrium is given by:

► Dealers' strategies:

$$X_1^D(p_1) = \gamma \left(\frac{1}{\gamma_H \Lambda_1} - \tau_v \right) p_1, \quad X^{RD}(p_1) = -\gamma \tau_v p_1$$
$$X_2^D(p_1, p_2) = -\gamma \tau_v p_2$$

► Hedgers' strategies:

$$X_{11}(u_1) = a_1 u_1, \quad X_{21}(u_1) = a_{21} u_1$$
$$X_2(u_2, s_{u_1}) = a_2 u_2 + b s_{u_1}$$

► Prices:

$$p_1 = -\Lambda_1 u_1$$
$$p_2 = -\Lambda_2 u_2 - \Lambda_{21} u_1 - \Lambda_{22} \eta$$

Equilibrium (2/2)

Price coefficients	Strategy coefficients	Variances
$\Lambda_2 = -\frac{a_2}{\mu\gamma\tau_v}$	$a_2 = \frac{\gamma_H\tau_v\Lambda_2 - 1}{\tau_v\text{Var}_2[v - p_2]}$	$\text{Var}_2[v - p_2] = \frac{1}{\tau_v} + \frac{(\Lambda_{21} - \Lambda_{22})^2}{\tau_\eta + \tau_u}$
$\Lambda_{21} = -\frac{b + a_{21} + (1 - \mu)\gamma\tau_v\Lambda_1}{\mu\gamma\tau_v}$	$b = \gamma_H \frac{\Lambda_{21}\tau_\eta + \Lambda_{22}\tau_u}{(\tau_\eta + \tau_u)\text{Var}_2[v - p_2]}$	$\text{Var}_1[v - p_2] = \frac{1}{\tau_v} + \frac{\Lambda_2^2}{\tau_u} + \frac{\Lambda_{22}^2}{\tau_\eta}$
$\Lambda_{22} = -\frac{b}{\mu\gamma\tau_v}$	$a_{21} = \frac{\gamma_H\Lambda_{21}\tau_v - 1}{\tau_v\text{Var}_1[v - p_2]}$	$\text{Var}_1[p_2] = \frac{\Lambda_2^2}{\tau_u} + \frac{\Lambda_{22}^2}{\tau_\eta}$
$\Lambda_1 = -\frac{\mu\gamma + \gamma_H}{\gamma\gamma_H\tau_v}a_1$	$a_1 = -\gamma_H \frac{\Lambda_{21} - \Lambda_1}{\text{Var}_1[p_2]}$	

Table: Equilibrium price and strategy coefficients and variances.

A measure of illiquidity at $t = 1$

At a linear equilibrium:

$$p_1 = -\Lambda_1 u_1,$$

and the natural measure of illiquidity is

$$\left| \frac{\partial p_1}{\partial u_1} \right| = \Lambda_1,$$

i.e., the price impact of a marginal increase in first period traders' endowment shock.

A measure of illiquidity at $t = 2$: WAPI

At the second round:

$$p_2 = - \overbrace{\Lambda_2 u_2 - \Lambda_{21} u_1 - \Lambda_{22} \eta}^{\text{Different price impacts}},$$

and none of the endowment shock coefficients per-se captures illiquidity:

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$$WAPI = \frac{(|a_{21} + b| \Lambda_{21} + |a_2| \Lambda_2) \sigma_u + \Lambda_{22} b \sigma_\eta}{(|a_{21} + b| + |a_2|) \sigma_u + b \sigma_\eta},$$

- ▶ The “Weighted Average Price Impact.”
- ▶ Weights correspond to the volume associated with the responses to endowment shocks and possible noisy information.
- ▶ Related to the weighted average spread.

Equilibrium

Two polar benchmarks with $\mu = 1$:

- (a) “Fully” transparent: $\tau_\eta \rightarrow \infty$, where second period traders can perfectly anticipate the impact of u_1 on p_2 .
- (b) “Fully” opaque: $\tau_\eta \rightarrow 0$, where second period traders cannot anticipate *at all* the impact of u_1 on p_2 .

Fully transparent benchmark

There exists a unique equilibrium in linear strategies where

$$\begin{aligned} p_2 &= - \underbrace{\Lambda_2}_{>0} u_2 - \underbrace{\Lambda_{21}}_{>0} u_1 \\ &= -\Lambda_2 \theta_2, \end{aligned}$$

with $\theta_2 \equiv u_2 + \beta u_1$ (AR(1) process for noise with endogenous persistence parameter $\beta \equiv \Lambda_{21}/\Lambda_2$),

$$p_1 = - \underbrace{\Lambda_1}_{>0} u_1,$$

price impact coefficients reflect limited risk bearing capacity and

$$0 < \Lambda_1 < \Lambda_{21} < \Lambda_2 \implies \beta < 1.$$

The unique equilibrium is obtained in a partially opaque market where second period hedgers receive a noisy signal with precision $\tau_\eta \in (0, \infty)$, as $\tau_\eta \rightarrow \infty$.

Fully transparent benchmark

Hedgers' strategies:

$$x_{11} = a_1 u_1, \quad x_{21} = a_{21} u_1, \quad x_2 = a_2 u_2 + b u_1$$

where

$$-1 < a_{21} < a_1 < 0, \quad -1 < a_2 < 0, \quad b > 0.$$

- ▶ At the first round, liquidity traders hedge their risk exposure (x_{11}) anticipating that they will increase their position at the second round (x_{21}), when second period liquidity traders are in the market, leading to $\Lambda_1 < \Lambda_{21}$.
- ▶ Second period traders hedge and provide liquidity with a contrarian market(able) order with aggressiveness $b > 0$.

Fully transparent benchmark

Dealers' strategies:

$$X_2^D(p_1, p_2) = -\gamma\tau_v p_2, \quad X_1^D(p_1) = \frac{\gamma}{\gamma_H \Lambda_1} p_1 - \gamma\tau_v p_1.$$

- ▶ At the second round, dealers take the other side of the market.
- ▶ At the first round, they also trade with liquidity traders (with short-term speculation in the direction of the “permanent” price change).

Liquidity supply and demand

- ▶ We measure dealers' *supply* of liquidity with the price impact of the endowment shocks Λ_1 , Λ_{21} and Λ_2 (the risk-weighted compensation that dealers demand to absorb first and second period hedgers' aggregate demand).
- ▶ We measure hedgers' *demand* for liquidity via their hedging aggressiveness $|a_1|$, $|a_{21}|$ and $|a_2|$.

With full transparency:

- ▶ Second period traders take a contrarian position against first period liquidity traders' second period order.
- ▶ They de-facto provide liquidity via a contrarian market order.

Fully transparent benchmark

► Prices:

$$p_2 = -\Lambda_2 u_2 - \Lambda_{21} u_1 = -\Lambda_2 \underbrace{\theta_2}_{=u_2+\beta u_1}, \quad \beta \equiv \Lambda_{21}/\Lambda_2,$$

$$p_1 = -\Lambda_1 u_1, \quad 0 < \Lambda_1 < \Lambda_{21} < \Lambda_2.$$

price impact coefficients reflect limited risk bearing capacity, our measure of **liquidity supply**.

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$$x_1 = a_1 u_1, \quad x_{21} = a_{21} u_1, \quad x_2 = a_2 u_2 + b u_1$$

with $-1 < a_{21} < a_1 < 0$, $a_2 \in (-1, 0)$, $b > 0$, $|a_t|$ our measure of **liquidity demand**.

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Liquidity supply

$\Lambda_2(a_2)$

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Liquidity supply

Contrarian order

$$b > 0$$

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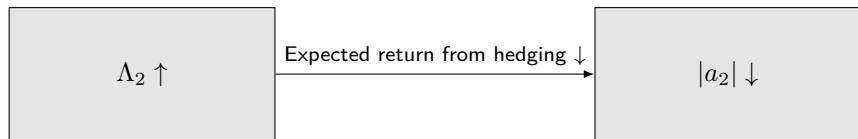
$$X_2^D(p_1, p_2) = -\gamma \tau_v p_2, \quad X_1^D(p_1) = \frac{\gamma}{\gamma_H \Lambda_1} p_1 - \gamma \tau_v p_1.$$

$$|a_2(\Lambda_2)|$$

Downward sloping demand for liquidity

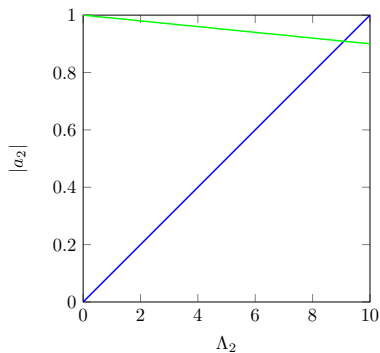
Hedgers' demand for liquidity is a decreasing function of the price impact it induces:

- ▶ *Higher illiquidity discourages liquidity demand.*
- ▶ Illiquidity works as a *rationing* device.



Fully transparent benchmark

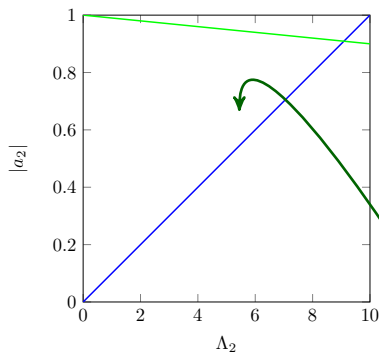
$$\tau_u = \tau_v = 0.1, \tau_\eta \rightarrow \infty, \gamma = 1, \gamma_H = 0.1$$



$$\frac{\partial |a_2|}{\partial \Lambda_2} < 0 \text{ and } \frac{\partial \Lambda_2}{\partial |a_2|} > 0$$

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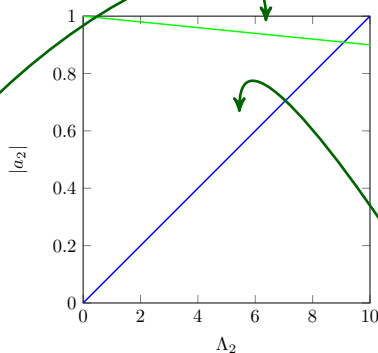


Increasing in liquidity demand.

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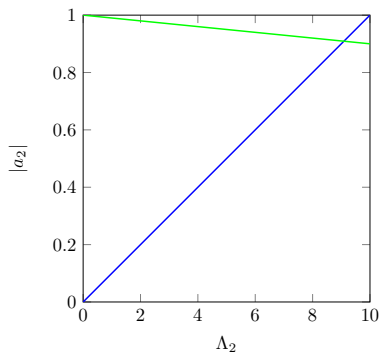
Decreasing in illiquidity.

Increasing in liquidity demand.

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$$\frac{\partial |a_2|}{\partial \Lambda_2} < 0 \text{ and } \frac{\partial \Lambda_2}{\partial |a_2|} > 0$$

Illiquidity works as a rationing device.

Fully opaque benchmark

When the market is fully opaque, we have $p_1 = -\Lambda_1 u_1$,
 $p_2 = -\Lambda_2 u_2 - \Lambda_{21} u_1$, with $\Lambda_{21} = -(a_{21}/\gamma\tau_v) > \Lambda_1 > 0$ and $\Lambda_2 > 0$. At equilibrium in the second period:

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(a) Hedgers stop posting contrarian orders ($b = 0$):

$x_2 = \frac{\gamma_H \Lambda_2 \tau_v - 1}{\underbrace{\text{Var}_2[v - p_2]}_{a_2}} u_2$ and face execution risk that increases in the price impact Λ_{21} of first period hedgers.

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(b) First period hedgers trade $x_{21} = \underbrace{\frac{\gamma_H \Lambda_{21} \tau_v - 1}{\text{Var}_1[v - p_2]}}_{a_{21}} u_1$ and face execution risk that increases in the price impact Λ_2 of second period hedgers.

- $|a_2|$ and $|a_{21}|$ decrease with execution risk and increase in the return from hedging the risky endowment.

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- (a) Hedgers stop posting contrarian orders ($b = 0$):

$x_2 = \underbrace{\frac{\gamma_H \Lambda_2 \tau_v - 1}{\text{Var}_2[v - p_2]}}_{a_2} u_2$ and face execution risk that increases in the price impact Λ_{21} of first period hedgers.

- (b) First period hedgers trade $x_{21} = \underbrace{\frac{\gamma_H \Lambda_{21} \tau_v - 1}{\text{Var}_1[v - p_2]}}_{a_{21}} u_1$ and face execution risk that increases in the price impact Λ_2 of second period hedgers.

- ▶ $|a_2|$ and $|a_{21}|$ decrease with execution risk and increase in the return from hedging the risky endowment.
- ▶ Induced changes in execution risk may dominate those of expected return from hedging.

Liquidity demand and supply with opacity (1/2)

Recall $x_2 = a_2 u_2$, $a_2 \in (-1, 0)$:

$$|a_2| = \frac{\overbrace{\gamma_H \tau_v \Lambda_2 - 1}^{(a) \text{ Expected return from hedging } \downarrow}}{\underbrace{\tau_v \left(\tau_v^{-1} + \left(\frac{\tau_u}{((\gamma + \gamma_H) \tau_u + \gamma \tau_v \Lambda_2^2) \tau_v} \right)^2 \tau_u^{-1} \right)}_{(b) \text{ Execution risk } \downarrow}}.$$

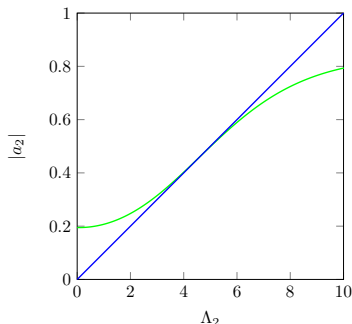
An increase in Λ_2 has two contrasting effects:

$$\frac{d|a_2(\Lambda_2, \Lambda_{21}(\Lambda_2))|}{d\Lambda_2} = \underbrace{\frac{\partial |a_2(\Lambda_2, \Lambda_{21})|}{\partial \Lambda_2}}_{(-)} + \underbrace{\frac{\partial |a_2(\Lambda_2, \Lambda_{21}(\Lambda_2))|}{\partial \Lambda_{21}}}_{(-)} \underbrace{\frac{d\Lambda_{21}}{d\Lambda_2}}_{(-)}.$$

- (a) A direct effect that reduces liquidity demand.
- (b) An indirect effect that boosts liquidity demand and that may dominate.

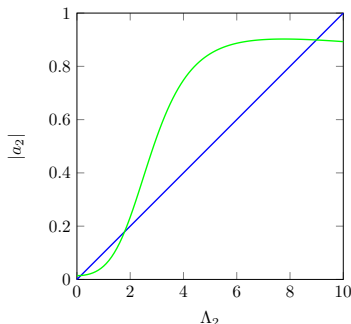
Liquidity demand and supply with opacity (2/2)

$$\tau_u = 2, \tau_v = 0.1, \tau_\eta = 0, \gamma = 1, \gamma_L = 0.1$$



(a) Unique equilibrium

$$\tau_u = \tau_v = 0.1, \tau_\eta = 0, \gamma = 1, \gamma_L = 0.1$$



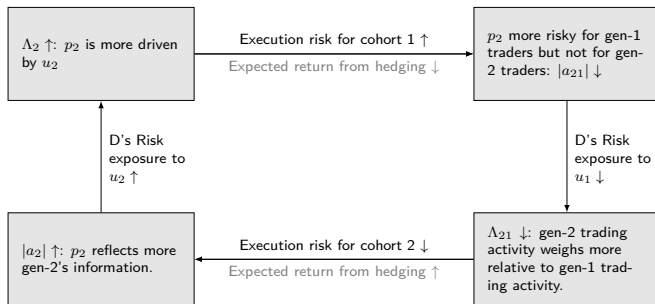
(b) Multiple equilibria

$$\frac{\partial |a_2|}{\partial \Lambda_2} \leq 0 \text{ and } \frac{\partial \Lambda_2}{\partial |a_2|} > 0$$

Fully opaque benchmark: second period

Λ_{21} and Λ_2 are strategic substitutes (heterogeneous shocks to price in period 2) and this leads an increase in the cost of trading in period 2 associated to u_2 (Λ_2) to become self-sustaining (symmetrically for Λ_{21} and u_1) and that may lead to multiplicity:

$$p_2 = -\Lambda_2 u_2 - \Lambda_{21} u_1$$



Equilibrium properties with opacity (1/2)

► If

$$0 < \tau_u \tau_v < \gamma / (4(\gamma + \gamma_H)^3),$$

three equilibria arise. Otherwise a unique equilibrium obtains with $\Lambda_2 = \Lambda_{21}$.

► Multiplicity more likely to obtain when:

- (a) More dispersed endowment shock (τ_u smaller)
- (b) The volatility of the security's payoff is larger (τ_v smaller)
- (c) Traders are more risk averse (γ_H lower)

► ... that is, when traders need to hedge the most (higher unpredictability of endowment shock and risk aversion), and dealers are less willing to supply liquidity (higher volatility of the security's payoff).

Equilibrium properties with opacity (2/2)

- With ME, at a stable equilibrium we have either that p_2 reacts more to u_2 than to u_1 , or the opposite. In the former (latter) case the first period market is more (less) liquid:

$$\Lambda_1^{***} < \Lambda_{21}^{***} < \Lambda_2^{***}, \text{ or } \Lambda_1^* < \Lambda_2^* < \Lambda_{21}^*.$$

- Equilibria can be ranked in terms of the price sensitivity to first and second period endowment shocks and liquidity consumption:

$$\Lambda_2^* < \Lambda_2^{**} < \Lambda_2^{***}, \quad \Lambda_{21}^{***} < \Lambda_{21}^{**} < \Lambda_{21}^*, \quad \Lambda_1^{***} < \Lambda_1^{**} < \Lambda_1^*.$$

$$-1 < a_2^{***} < a_2^{**} < a_2^* < 0,$$

$$-1 < a_{21}^* < a_{21}^{**} < a_{21}^{***} < 0,$$

$$-1 < a_1^* < a_1^{**} < a_1^{***} < 0.$$

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$$-1 < a_1^* < a_1^{**} < a_1^{***} < 0.$$

Traders consume **more** liquidity when it costs the **most**.

Comparative Statics with opacity

- An increase in the volatility of the endowment shock or a decline in dealers' risk-bearing capacity heightens strategic complementarity at the intermediate equilibrium:

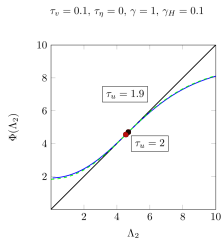
$$\left. \frac{\partial}{\partial \tau_u} \left(\frac{\partial \Phi(\Lambda_2)}{\partial \Lambda_2} \right) \right|_{\Lambda_2 = \Lambda_2^{**}} < 0, \quad \left. \frac{\partial}{\partial \gamma} \left(\frac{\partial \Phi(\Lambda_2)}{\partial \Lambda_2} \right) \right|_{\Lambda_2 = \Lambda_2^{**}} < 0.$$

Small shock: $\uparrow$ execution risk $\implies \downarrow |a_2| (\downarrow |a_{21}|)$ and $\downarrow \Lambda_2 (\downarrow \Lambda_{21})$.

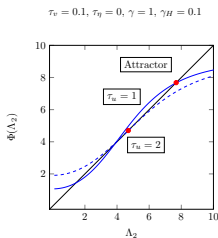
- When the market is fully opaque and a unique equilibrium is obtained:
 1. A shock increasing liquidity traders' endowment volatility large enough to induce multiple equilibria (ME), leads the market towards the extremal equilibrium with high Λ_2 (and low Λ_{21}) at $t = 2$.
 2. A shock increasing dealers' risk aversion large enough to induce ME, leads the market towards the extremal equilibrium with low Λ_2 (and high Λ_{21}) at $t = 2$.
 3. In both cases, the total illiquidity ($WAPI$) and $\text{Var}[p_2]$ are higher.

A liquidity “dry-up”

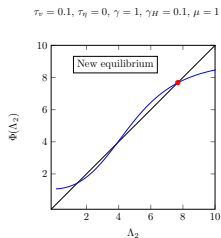
Larger shock: stronger strategic complementarities. . . Suppose that unexpectedly $\downarrow \tau_u$:



(a) Small shock



(b) Large Shock

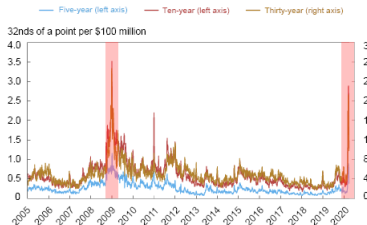


(c) Low liquidity equilibrium

A more dispersed endowment shock

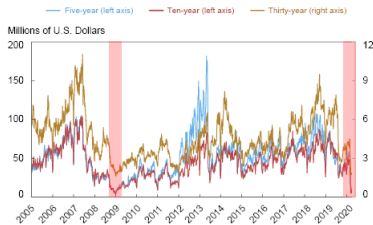
Impact of Covid pandemic on USGov Bonds' liquidity, March 2020 (BrokerTec segment).

Price Impact at its Highest since 2007-09 Financial Crisis



(a) Price impact

Order Book Depth Comparable with Nadir of 2007-09 Financial Crisis

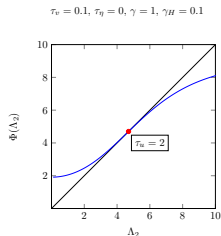


(b) Market depth

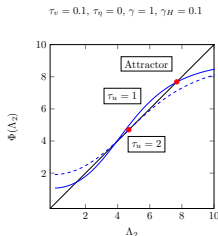
(Source: Federal Reserve Bank of NY)

A “flash-crash”

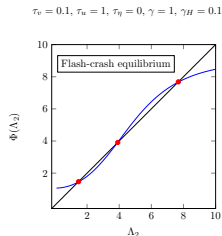
Suppose that temporarily $\tau_u \downarrow$:



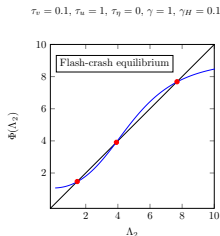
(a) Unique equilibrium



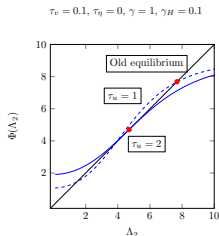
(b) Shock



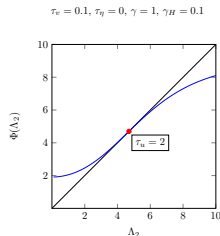
(c) Low liquidity eq.



(d) Low liquidity eq.



(e) Shock

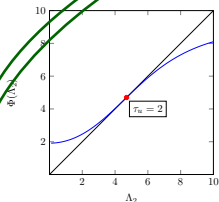


(f) Unique equilibrium

A “flash-crash”

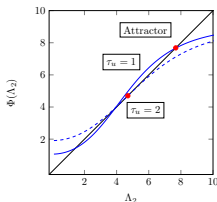
Suppose that temporarily $\tau_u \downarrow$:

$$\tau_v = 0.1, \tau_\eta = 0, \gamma = 1, \gamma_H = 0.1$$



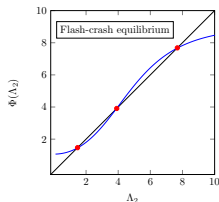
$\Delta WAPI(\uparrow) 44\%$ Unique equilibrium

$$\tau_v = 0.1, \tau_\eta = 0, \gamma = 1, \gamma_H = 0.1$$



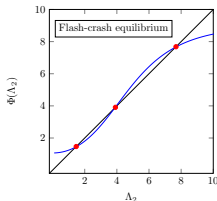
(b) Shock

$$\tau_v = 0.1, \tau_u = 1, \tau_\eta = 0, \gamma = 1, \gamma_H = 0.1$$



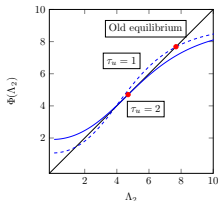
(c) Low liquidity eq.

$$\tau_v = 0.1, \tau_u = 1, \tau_\eta = 0, \gamma = 1, \gamma_H = 0.1$$



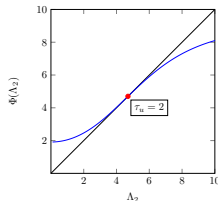
(d) Low liquidity eq.

$$\tau_v = 0.1, \tau_\eta = 0, \gamma = 1, \gamma_H = 0.1$$



(e) Shock

$$\tau_v = 0.1, \tau_\eta = 0, \gamma = 1, \gamma_H = 0.1$$

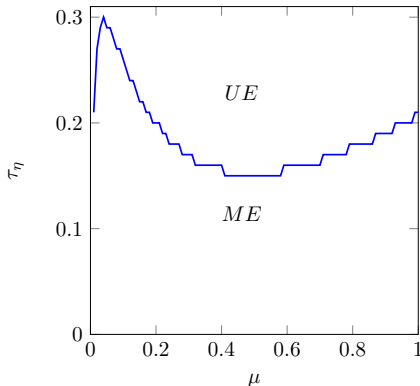


(f) Unique equilibrium

The effect of restricted dealers

Back to the general model (μ, τ_η) : nonlinear effects.

$$\tau_u = \tau_v = 0.1, \gamma = 1, \gamma_L = 0.1$$



For low τ_η :

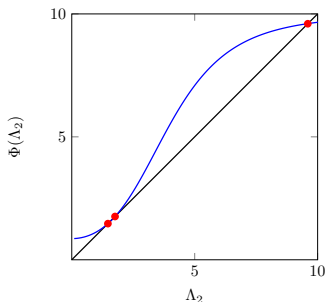
- (a) An increase in μ has a non-monotonic effect on SC.
- (b) A decrease in μ may induce a liquidity crash for μ large.

Shock to dealers' participation

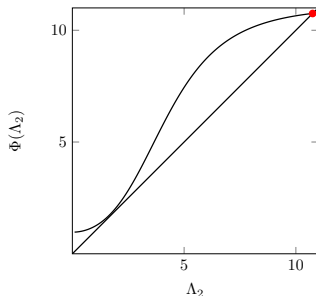
A 0.11% decrease in μ (from $\mu = 0.9$ to $\mu = 0.8$) yields a switch from multiple equilibria to uniqueness, potentially inducing a liquidity crash for: from $\Lambda_2 = 1.47$ at the low illiquidity equilibrium to $\Lambda_2 = 9.6$ (a 653% illiquidity increase).

$$\tau_v = 0.1, \tau_u = 0.5, \tau_\eta = 0.2, \gamma = 1, \gamma_L = 0.1, \mu = 0.9$$

$$\tau_v = 0.1, \tau_u = 0.5, \tau_\eta = 0.2, \gamma = 1, \gamma_L = 0.1, \mu = 0.8$$



(a) $\mu = 0.9$



(b) $\mu = 0.8$

Welfare analysis (general model)

Total welfare:

$$TW(\mu; \tau_\eta) = \mu CE^D + (1 - \mu)CE^{RD} + CE_1^H + CE_2^H.$$

Numerical Result

When a unique equilibrium obtains, $TW(\mu; \tau_\eta)$ is increasing in μ and τ_η . The total welfare improvement is driven by the increase of CE_t^H , $t = 1, 2$, with μ and τ_η . CE^{RD} decreases with τ_η as well as CE^D when τ_η is not too small and CE^D decreases in μ .

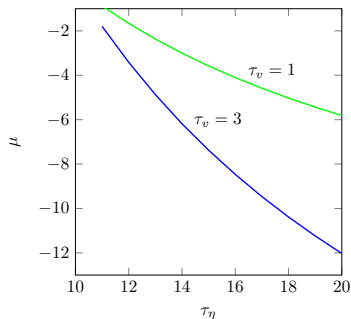
Welfare analysis

An increase in transparency:

- ▶ Spurs second period hedgers' speculative activity, reducing the rewards to liquidity provision enjoyed by dealers.
- ▶ May come at the expense of a reduced dealer participation to the market since the entry/exit of dealers will be affected by their prospective profits.
- ▶ What is the reduction in μ that is needed to keep total welfare constant when transparency increases in regimes with low and high payoff volatility?
- ▶ Result: Moderate increases in transparency may induce a reduction in the mass μ of dealers always present in the market which may hurt welfare, in particular for riskier securities.

Welfare analysis

$$\tau_u = 2, \tau_\eta = 10, \gamma = 1, \gamma_H = 1, \mu = 0.5$$



(Percentage change in μ that keeps $TW(\mu; \tau_\eta)$ constant against an increase in τ_η)

Consistent with Bessembinder and Maxwell (2008)'s evidence on TRACE.

Opacity vs. transparency (1/2)

Transparency improves TW but has mixed effects on total illiquidity $WAPI$.

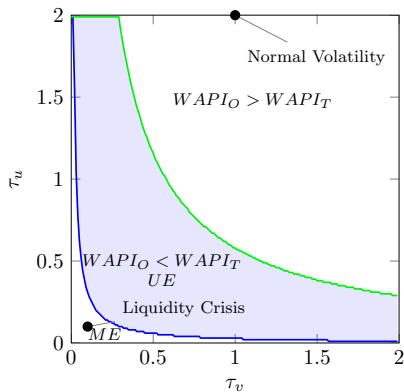
Two scenarios:

1. “Normal” volatility: calibrated to yield $\sqrt{\text{Var}[p_2 - p_1]} \approx 30\%$. We find that $WAPI$ is larger with opacity.
2. “Liquidity crisis” volatility: calibrated to yield much higher volatility ($\sqrt{\text{Var}[p_2 - p_1]} \in \{1448\%, 691\%, 188\%\}$). We find that $WAPI$ is larger with transparency.

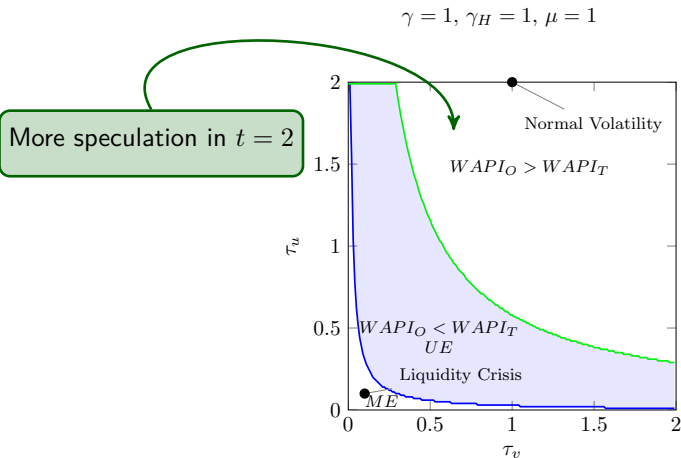
Furthermore, $WAPI$ is U-shaped in transparency. (As τ_η increases second period hedgers demand more liquidity and speculate more aggressively against u_1 . These two effects have an opposite impact on $WAPI$.)

Opacity vs. transparency (2/2)

$$\gamma = 1, \gamma_H = 1, \mu = 1$$

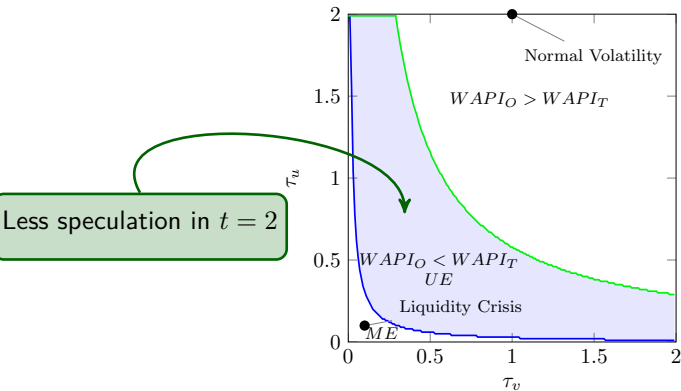


Opacity vs. transparency (2/2)



Opacity vs. transparency (2/2)

$$\gamma = 1, \gamma_H = 1, \mu = 1$$



Conclusions

Technological developments and regulatory changes have favored an “all-to-all” market structure:

- ▶ Opacity can make markets fragile and impair the rationing function of illiquidity.
- ▶ Trading costs are heterogeneous for different investors when the market is fragile and investors consuming more liquidity are those paying more for it.
- ▶ Channel: “non-standard” liquidity providers supply liquidity via contrarian marketable orders.
- ▶ Policy implication: provision of public information about the trading process has a stabilizing impact on the market (consolidated tape in the EU, equal access to price information in the US Treasury market).

Thanks!