

Supplemental Appendix for

‘General Equilibrium Effects in Space:
Theory and Measurement’

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A Online Appendix: Proofs and Additional Results

A.1 Proofs for Section 3

A.1.1 Proof of Equation (14)

The definitions of γ_{ij} in (14) and $D_i(\mathbf{w}|\boldsymbol{\tau})$ in (12) immediately imply that

$$\gamma_{ij} = \mathbb{I}_{i=j}(\phi - (\phi - 1)n_i) - (\phi - 1)(1 - n_i)\omega_j^0 - \frac{1}{R_i^0} \frac{\partial R_i(\mathbf{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_j}$$

where

$$\frac{1}{R_i^0} \frac{\partial R_i(\mathbf{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_j} = \sum_s \sum_d \ell_{i,s}^0 r_{id,s}^0 [\mathbb{I}_{d=j}(\phi - (\phi - 1)n_d) - (\phi - 1)(1 - n_d)\omega_j^0 - (\sigma - 1)(1 - \psi\phi)(\mathbb{I}_{i=j} - x_{jd,s}^0)].$$

Thus, using the definitions of $\kappa \equiv (\sigma - 1)(1 - \psi\phi)$, $\phi_i^0 \equiv \phi - (\phi - 1)n_i$ and $r_{ij}^0 \equiv \sum_s \ell_{i,s}^0 r_{ij,s}^0$, we can re-write this expression as

$$\begin{aligned} \gamma_{ij} &= \mathbb{I}_{i=j}\phi_i^0 + (1 - \phi_i^0)\omega_j^0 - \sum_s \sum_d \ell_{i,s}^0 r_{id,s}^0 [\mathbb{I}_{d=j}\phi_d^0 + (1 - \phi_d^0)\omega_j^0 - \kappa(\mathbb{I}_{i=j} - x_{jd,s}^0)] \\ &= \mathbb{I}_{i=j}(\phi_i^0 + \kappa) - \sum_s \ell_{i,s}^0 r_{ij,s}^0 \phi_j^0 - \kappa \sum_s \sum_d \ell_{i,s}^0 r_{id,s}^0 x_{jd,s}^0 - \omega_j^0 (\phi_i^0 - \sum_s \sum_d \ell_{i,s}^0 r_{id,s}^0 \phi_d^0) \\ &= \mathbb{I}_{i=j}(\phi_i^0 + \kappa) - r_{ij}^0 \phi_j^0 - \kappa \sum_s \sum_d \ell_{i,s}^0 r_{id,s}^0 x_{jd,s}^0 - \omega_j^0 (\phi_i^0 - \sum_d r_{id}^0 \phi_d^0) \end{aligned}$$

which is equivalent to (19) since $\sum_d r_{id}^0 = 1$.

A.1.2 Proof of Theorem 1

We re-define the system in (15) to set the change in the wage of market m to zero. Consider the matrix $\bar{\mathbf{M}}$ obtained by deleting the m -th row from the identity matrix with dimension equal to the number of markets. If $\bar{\mathbf{M}}\bar{\boldsymbol{\gamma}}\bar{\mathbf{M}}'$ is non-singular, then we can write

$$\bar{\mathbf{M}}\hat{\mathbf{w}} = (\bar{\mathbf{M}}\bar{\boldsymbol{\gamma}}\bar{\mathbf{M}}')^{-1} \bar{\mathbf{M}}\hat{\boldsymbol{\eta}},$$

which yields the representation in (21) when we define $\bar{\boldsymbol{\beta}} \equiv \bar{\mathbf{M}}'(\bar{\mathbf{M}}\bar{\boldsymbol{\gamma}}\bar{\mathbf{M}}')^{-1} \bar{\mathbf{M}}$.

In the rest of the proof, we first show that $\bar{\mathbf{M}}\bar{\boldsymbol{\gamma}}\bar{\mathbf{M}}'$ is non-singular and then establish that $\bar{\boldsymbol{\beta}}$ admits the series representation in (21). To simplify exposition, we abuse notation by defining

$$\bar{\boldsymbol{\gamma}} \equiv \bar{\mathbf{M}}\bar{\boldsymbol{\gamma}}\bar{\mathbf{M}}', \quad \hat{\mathbf{w}} \equiv \bar{\mathbf{M}}\hat{\mathbf{w}} \quad \text{and} \quad \hat{\boldsymbol{\eta}} \equiv \bar{\mathbf{M}}\hat{\boldsymbol{\eta}}.$$

This modified system does not include the row associated with the market clearing condition of market m and imposes that $\hat{w}_m = 0$. To obtain a characterization for the solution of this system, let $\bar{\boldsymbol{\lambda}}$ be the diagonal matrix defined by the vector of $\lambda_i^0 \equiv \kappa + \phi_i$, and $\tilde{\boldsymbol{\gamma}}$ be the matrix with entries $\tilde{\gamma}_{ij} \equiv \rho_{ij}/(\kappa + \phi_i)$, so that

$$\bar{\boldsymbol{\gamma}} = \bar{\boldsymbol{\lambda}}(\bar{\mathbf{I}} - \tilde{\boldsymbol{\gamma}}).$$

Consider the vector $\{h_i\}_{i \neq m} \gg 0$ that guarantees the diagonal dominance of $\bar{\boldsymbol{\gamma}}$ in the initial equilibrium. Let

$\bar{\mathbf{h}}$ be the diagonal matrix such that h_i is the diagonal entry in row i . Thus, the system in (15) is equivalent to

$$\begin{aligned}\bar{\lambda}(\bar{\mathbf{I}} - \bar{\tilde{\gamma}})(\bar{\mathbf{h}}\bar{\mathbf{h}}^{-1})\hat{\mathbf{w}} &= \hat{\boldsymbol{\eta}} \\ \bar{\lambda}(\bar{\mathbf{h}} - \bar{\tilde{\gamma}}\bar{\mathbf{h}})\bar{\mathbf{h}}^{-1}\hat{\mathbf{w}} &= \hat{\boldsymbol{\eta}} \\ (\bar{\lambda}\bar{\mathbf{h}})(\bar{\mathbf{I}} - (\bar{\mathbf{h}}^{-1}\bar{\tilde{\gamma}}\bar{\mathbf{h}}))\bar{\mathbf{h}}^{-1}\hat{\mathbf{w}} &= \hat{\boldsymbol{\eta}}\end{aligned}$$

which implies that

$$\hat{\mathbf{w}} = \bar{\mathbf{h}}(\bar{\mathbf{I}} - \bar{\tilde{\gamma}})^{-1}(\bar{\lambda}\bar{\mathbf{h}})^{-1}\hat{\boldsymbol{\eta}}, \quad \bar{\tilde{\gamma}} \equiv \bar{\mathbf{h}}^{-1}\bar{\tilde{\gamma}}\bar{\mathbf{h}}. \quad (1)$$

Notice that, for all i , $\tilde{\gamma}_{ij} = \tilde{\gamma}_{ij}h_j/h_i = \rho_{ij}h_j/(\kappa + \phi_i)h_i$.

First, we show that $(\bar{\mathbf{I}} - \bar{\tilde{\gamma}})$ is non-singular, so that we can write the expression in (1). We proceed by contradiction. Suppose that $(\bar{\mathbf{I}} - \bar{\tilde{\gamma}})$ is singular, so $\mu = 0$ is an eigenvalue of $(\bar{\mathbf{I}} - \bar{\tilde{\gamma}})$. Take the eigenvector $\mathbf{x}$ associated with the zero eigenvalue and normalize it such that $x_i = 1$ and $|x_j| \leq 1$. Notice that $(\bar{\mathbf{I}} - \bar{\tilde{\gamma}})\mathbf{x} = 0$, so that the i -row of this system is

$$1 - \sum_{j \neq m} \tilde{\gamma}_{ij}x_j = 0 \implies 1 - \frac{\rho_{ii}}{\kappa + \phi_i} - \sum_{j \neq i, m} \frac{\rho_{ij}}{\kappa + \phi_i} \frac{h_j}{h_i} x_j = 0$$

Thus, because $|x_j| \leq 1$ for all j , we can write

$$(\kappa + \phi_i - \rho_{ii})h_i = \sum_{j \neq i, m} \rho_{ij}h_jx_j \leq \sum_{j \neq i, m} |\rho_{ij}||h_j||x_j| \leq \sum_{j \neq i, m} |\rho_{ij}||h_j|,$$

which contradicts the second part of condition (20).

Second, we show that $(\bar{\mathbf{I}} - \bar{\tilde{\gamma}})^{-1}$ admits the series representation in (21). This is true whenever the largest eigenvalue of $\bar{\tilde{\gamma}}$ is below one. To show this, we proceed by contradiction. Assume that the largest eigenvalue μ is weakly greater than one. Take the eigenvector $\mathbf{x}$ associated with the largest eigenvalue and normalize it such that $x_i = 1$ and $|x_j| \leq 1$. Notice that $\mu\mathbf{x} = \bar{\tilde{\gamma}}\mathbf{x}$ so that the i -row of this system is

$$1 \leq \mu = \sum_{j \neq m} \frac{\rho_{ij}}{\kappa + \phi_i} \frac{h_j}{h_i} x_j.$$

Since $\kappa + \phi_i$ and h_i are positive by (20), the same steps used above imply that

$$(\kappa + \phi_i - \rho_{ii})h_i \leq \sum_{j \neq i, m} |\rho_{ij}||h_j|,$$

which contradicts the assumption of diagonal dominance. Thus, the largest eigenvalue of $\bar{\tilde{\gamma}}$ is below one, allowing us to write $(\bar{\mathbf{I}} - \bar{\tilde{\gamma}})^{-1} = \sum_{d=0}^{\infty} (\bar{\tilde{\gamma}})^d$. Substituting this series expansion into (1) yields

$$\hat{\mathbf{w}} = \sum_{d=0}^{\infty} \left(\bar{\mathbf{h}}(\bar{\tilde{\gamma}})^d \bar{\mathbf{h}}^{-1} \right) \bar{\lambda}^{-1} \hat{\boldsymbol{\eta}}.$$

Finally, to establish the result, we now show that $\bar{\mathbf{h}}(\bar{\tilde{\gamma}})^d \bar{\mathbf{h}}^{-1} = (\bar{\tilde{\gamma}})^d$. We proceed by induction. For $d=1$,

it is trivial to see that $\bar{h}(\bar{\gamma})\bar{h}^{-1} = \bar{\gamma}$. Assume that it holds for d , we now show that it also holds for $d+1$:

$$\bar{h}(\bar{\gamma})^{d+1}\bar{h}^{-1} = \bar{h}(\bar{\gamma})^d\bar{\gamma}\bar{h}^{-1} = \bar{h}(\bar{\gamma})^d(\bar{h}^{-1}\bar{\gamma}\bar{h})\bar{h}^{-1} = \left(\bar{h}(\bar{\gamma})^d\bar{h}^{-1}\right)\bar{\gamma} = (\bar{\gamma})^{d+1}.$$

Thus,

$$\hat{w} = \sum_{d=0}^{\infty} (\bar{\gamma})^d \bar{\lambda}^{-1} \hat{\eta},$$

which immediately implies the result.

A.1.3 Proof of Corollary 1

The series expansion representation of β_{ij} indicates that $\beta_{ij} > 0$ if $\tilde{\gamma}_{ij} > 0$ for all i and j . We now show that $\tilde{\gamma}_{ij} > 0$ whenever $\max_{o,d} |n_o - n_d|$ is low enough. Since $\sum_d r_{id}^0 (\phi_i^0 - \phi_d^0) = (\phi - 1) \sum_d r_{id}^0 (n_d^0 - n_i^0) > -(\phi - 1) \max_{o,d} |n_o - n_d|$,

$$\tilde{\gamma}_{ij} > \frac{r_{ij}^0 \phi_j^0 + \kappa \sum_s \sum_d \ell_{i,s}^0 r_{id,s}^0 x_{jd,s}^0 - \omega_j (\phi - 1) \max_{o,d} |n_o - n_d|}{\phi_i^0 + \kappa}$$

and, thus,

$$\max_{o,d} |n_o - n_d| < \frac{r_{ij}^0 \phi_j^0 + \kappa \sum_s \sum_d \ell_{i,s}^0 r_{id,s}^0 x_{jd,s}^0}{\omega_j (\phi - 1)} \Rightarrow \tilde{\gamma}_{ij} > 0.$$

Since the numerator is positive in our model, there exists $\max_{o,d} |n_o - n_d| \geq 0$ such that the condition above holds for all i and j .

A.1.4 Proof of Corollary 2

Recall that the special case of a foreign shock in (18) implies that $\hat{\eta}_i = -\sum_s \ell_{i,s}^0 \hat{\zeta}_{F,s}$. For each region i , we consider the covariance between the direct effect, $DE_i \equiv \beta_{ii} \hat{\eta}_i = -\beta_{ii} \sum_s \ell_{i,s}^0 \hat{\zeta}_{F,s}$, and the indirect effect, $IE_i \equiv \sum_{j \neq i} \beta_{ij} \hat{\eta}_j = -\sum_s (\sum_{j \neq i} \beta_{ij} \ell_{j,s}^0) \hat{\zeta}_{F,s}$, across realizations of the shock $\hat{\zeta}_F$. Conditional on all initial conditions in $\mathbf{W}^0$,

$$\begin{aligned} C_i &= Cov(DE_i, IE_i | \mathbf{W}^0) \\ &= E[\beta_{ii}(\hat{\eta}_i - E[\hat{\eta}_i | \mathbf{W}^0]) \sum_{j \neq i} \beta_{ij} \hat{\eta}_j | \mathbf{W}^0] \\ &= E[\sum_s (\beta_{ii} \ell_{i,s}^0) \hat{\zeta}_{F,s} \sum_k (\sum_{j \neq i} \beta_{ij} \ell_{j,k}^0) \hat{\zeta}_{F,k} | \mathbf{W}^0] \\ &= \sum_s \sum_k (\beta_{ii} \ell_{i,s}^0) (\sum_{j \neq i} \beta_{ij} \ell_{j,k}^0) E[\hat{\zeta}_{F,s} \hat{\zeta}_{F,k} | \mathbf{W}^0] \\ &= \sum_s \sum_k \Gamma_{i,sk} \Sigma_{sk} \end{aligned}$$

A.1.5 Proof of Corollary 3

We establish this result in two steps.

Step 1. First we show that, if $\bar{\gamma} \equiv \lambda^0 (\bar{\mathbf{I}} - \mathbf{1} \boldsymbol{\rho}')$ where $\mathbf{1}$ is a column vector of ones and $\boldsymbol{\rho} \equiv \{\rho_j\}_{j \neq m}$ is column vector, then $\bar{\gamma}^{-1} = (\lambda^0)^{-1} (\bar{\mathbf{I}} + \rho_m^{-1} \mathbf{1} \boldsymbol{\rho}')$.

$$\begin{aligned} \bar{\gamma}^{-1} \bar{\gamma} &= \bar{\mathbf{I}} + \rho_m^{-1} \mathbf{1} \boldsymbol{\rho}' - \mathbf{1} \boldsymbol{\rho}' - \rho_m^{-1} \mathbf{1} \boldsymbol{\rho}' \mathbf{1} \boldsymbol{\rho}' \\ &= \bar{\mathbf{I}} + \rho_m^{-1} \mathbf{1} \boldsymbol{\rho}' - \mathbf{1} \boldsymbol{\rho}' - \left(\rho_m^{-1} \sum_{j \neq m} \rho_j \right) \mathbf{1} \boldsymbol{\rho}' \\ &= \bar{\mathbf{I}} + \left(\rho_m^{-1} (1 - \sum_{j \neq m} \rho_j) - 1 \right) \mathbf{1} \boldsymbol{\rho}' \\ &= \bar{\mathbf{I}} \end{aligned}$$

where the second equality follows from $\boldsymbol{\rho}' \mathbf{1} = \sum_{j \neq m} \rho_j$, and the fourth from $\sum_j \rho_j = 1$ (since $\sum_j \gamma_{ij} = 0$ for all i).

Step 2. We now establish conditions that allow us to write $\gamma_{ij} = \lambda^0(\mathbb{I}_{i=j} - \rho_j)$. Assume that $n_i^0 = n^0$, so $\phi_i^0 = \phi + (1 - \phi)n^0$ and we can define $\lambda^0 \equiv \kappa + \phi^0$. We now use the fact that $x_{ij,s} = x_{i,s}$ and $\xi_{j,s} = \xi_s$ so that

$$r_{ij}^0 = \frac{\sum_s x_{ij,s}^0 \xi_{j,s} E_j^0}{\sum_j \sum_s x_{ij,s}^0 \xi_{j,s} E_j^0} = \frac{\sum_s x_{i,s}^0 \xi_s E_j^0}{\sum_j \sum_s x_{i,s}^0 \xi_s E_j^0} = \frac{E_j^0}{\sum_j E_j^0} = e_{Wj}^0.$$

In addition,

$$\kappa \sum_s \sum_d \ell_{i,s}^0 r_{id,s}^0 x_{jd,s}^0 = \kappa \sum_s \sum_d \frac{x_{i,s}^0 \xi_s E_d^0}{\sum_j \sum_s x_{i,s}^0 \xi_s E_j^0} x_{j,s}^0 = \kappa \frac{\sum_s \xi_s x_{i,s}^0 x_{j,s}^0}{\sum_s \xi_s x_{i,s}^0} = \kappa \chi_j$$

Since $n_i^0 = n^0$, then $\sum_d r_{id}^0 (\phi_i^0 - \phi_d^0) = 0$. This implies that $\rho_{ij}^0 = e_{Wj}^0 \phi^0 + \kappa \chi_j$, which we can use to define $\rho_j \equiv (e_{Wj}^0 \phi^0 + \kappa \chi_j) / (\kappa + \phi^0)$ and $\beta_j \equiv \mathbb{I}_{j \neq m} \rho_j / \rho_m$.

A.1.6 Proof of alternative version of Corollary 3

We consider an alternative version of Corollary 3. Specifically, we assume that markets have the same labor supply elasticity ($\phi_j^0 = \phi^0$) and spending shares ($\xi_{j,s} = \xi_s$). We further restrict markets to have the same size $E_j^0 = E$, but we allow for a richer structure of trade costs. We assume that there are S^{NT} non-tradable sectors $s \in S^{NT}$ with $x_{ii,s} = 1$ and $x_{ij,s} = 0$ for $i \neq j$ and S^T tradable sectors $s \in S^T$ with home bias and symmetric trade costs, such that $x_{ij,s} = g \mathbb{I}_{[i=j]} + x_{i,s}(1 - g)$ with $\frac{\sum_s \xi_s x_{i,s}^0 x_{j,s}^0}{\sum_s \xi_s x_{i,s}^0} = \chi_j$. In this setting, we establish the following result.

Corollary 1. Assume that $E_j^0 = E$, $\phi_j^0 = \phi^0$, $\xi_{j,s} = \xi_s$ and $\frac{\sum_s \xi_s x_{i,s}^0 x_{j,s}^0}{\sum_s \xi_s x_{i,s}^0} = \chi_j$ for all i and j . Further assume that $x_{ij,s} = g \mathbb{I}_{[i=j]} + x_{i,s}(1 - g)$ for $s \in S^T$, and $x_{ij,s} = \mathbb{I}_{[i=j]}$ for $s \in S^{NT}$. Then,

$$\hat{w}_i = \frac{1}{\lambda^0} \hat{\eta}_i(\hat{\tau}) + \bar{\eta} \quad \text{such that} \quad \bar{\eta} \equiv \sum_j \frac{\beta_j}{\lambda^0} \hat{\eta}_j(\hat{\tau}), \quad (2)$$

with $\lambda^0 = \phi^0 + \kappa - (\phi^0(\xi_{NT} + g(1 - \xi_{NT})) + \kappa(\xi_{NT} + S^T g^2))$.

Proof. Following the same argument as in Corollary 3, we only need to establish that we can write $\gamma_{ij} = \lambda^0(\mathbb{I}_{i=j} - \rho_j)$. We start by establishing three relationships that hold in equilibrium. First, note that the assumption of $E_j^0 = E$ implies that

$$E_i^0 = \sum_s \sum_j x_{ij,s}^0 \xi_{j,s} E_j^0 \implies \sum_s \sum_j x_{ij,s}^0 \xi_s = 1 \quad \forall i. \quad (3)$$

Second, we plug the spending shares into this expression to obtain:

$$\xi_{NT} + \sum_{s \in S^T} \sum_j (g \mathbb{I}_{[i=j]} + x_{i,s}(1 - g)) \xi_s = 1 \implies \sum_{s \in S^T} x_{i,s} \xi_s = (1 - \xi_{NT}) \quad (4)$$

with $\xi_{NT} \equiv \sum_{s \in S^{NT}} \xi_s$.

Finally, the assumption of $\phi_j^0 = \phi^0$ implies that

$$\gamma_{ij}^0 = (\phi^0 + \kappa) \mathbb{I}_{[i=j]} - \rho_{ij}^0$$

with

$$\rho_{ij}^0 = \phi^0 r_{ij}^0 + \kappa \sum_{s,d} \ell_{i,s}^0 r_{id,s}^0 x_{jd,s}^0. \quad (5)$$

We now derive an expression for ρ_{ij}^0 using the relationships above:

$$\begin{aligned}
&= \phi^0 \frac{\sum_s x_{ij,s}^0 \xi_{j,s} E_j^0}{\sum_s \sum_d x_{id,s}^0 \xi_{d,s} E_d^0} + \kappa \sum_{s,d} \frac{x_{id,s}^0 \xi_{d,s} E_d^0}{\sum_o \sum_s x_{io,s}^0 \xi_{o,s} E_o^0} x_{jd,s}^0 \\
&= \phi^0 \sum_s x_{ij,s}^0 \xi_s + \kappa \sum_{s,d} x_{id,s}^0 x_{jd,s}^0 \xi_s \\
&= \phi^0 (\xi_{NT} \mathbb{I}_{[i=j]} + \sum_{s \in \mathcal{S}^T} (g \mathbb{I}_{[i=j]} + x_{i,s}(1-g)) \xi_s) \\
&+ \kappa (\xi_{NT} \mathbb{I}_{[i=j]} + \sum_{s \in \mathcal{S}^T} \sum_d (g \mathbb{I}_{[i=d]} + x_{i,s}(1-g)) (g \mathbb{I}_{[j=d]} + x_{j,s}(1-g)) \xi_s) \\
&= \phi^0 ((\xi_{NT} + g(1-\xi_{NT})) \mathbb{I}_{[i=j]} + (1-g)(1-\xi_{NT})) \\
&+ \kappa (\xi_{NT} \mathbb{I}_{[i=j]} + (S^T g^2 \mathbb{I}_{[i=j]} + 2(1-\xi_{NT})(1-g)g + N(1-g)^2 \sum_{s \in \mathcal{S}^T} x_{i,s} x_{j,s} \xi_s)) \\
&= \phi^0 ((\xi_{NT} + g(1-\xi_{NT})) \mathbb{I}_{[i=j]} + (1-g)(1-\xi_{NT})) \\
&+ \kappa ((\xi_{NT} + S^T g^2) \mathbb{I}_{[i=j]} + (1-\xi_{NT})(2(1-g)g + N(1-g)^2 \chi_j))
\end{aligned}$$

where the first equality uses the definition of r_{ij}^0 and $\ell_{i,s}^0$, the second equality uses $E_j^0 = E$ and (3), the third equality uses the trade share assumption, the fourth equality uses (4), and the fifth equality uses $\sum_{s \in \mathcal{S}^T} x_{i,s} x_{j,s} \xi_s = \sum_s x_{i,s} x_{j,s} \xi_s = \chi_j \sum_s \xi_s x_{i,s}^0 = \chi_j (1-\xi_{NT})$. Thus,

$$\rho_{ij}^0 = (\phi^0 (\xi_{NT} + g(1-\xi_{NT})) + \kappa (\xi_{NT} + S^T g^2)) \mathbb{I}_{[i=j]} + (1-g)(1-\xi_{NT}) (\phi^0 + \kappa (2g + N(1-g) \chi_j)).$$

Together with (5), we can then write

$$\gamma_{ij}^0 = \lambda^0 (\mathbb{I}_{[i=j]} - \rho_j)$$

where

$$\begin{aligned}
\lambda^0 &= \phi^0 + \kappa - (\phi^0 (\xi_{NT} + g(1-\xi_{NT})) + \kappa (\xi_{NT} + S^T g^2)) \\
\rho_j &\equiv (1-g)(1-\xi_{NT}) (\phi^0 + \kappa (2g + N(1-g) \chi_j)) / \lambda^0.
\end{aligned}$$

□

A.1.7 Proof of Corollary 4

By Shepard's lemma, the price index expression in (7) implies that

$$\hat{P}_i = \sum_{s,o} \xi_{i,s} x_{oi,s}^0 (\hat{\tau}_{oi,s} + \hat{p}_o) = \hat{\eta}_i^C(\hat{\tau}) + \sum_o x_{oi}^0 \hat{p}_o$$

where $x_{oi}^0 \equiv \sum_s \xi_{i,s} x_{oi,s}^0$ is the share of o in the total spending of i . Thus,

$$\begin{aligned}
\hat{P}_i &= \hat{\eta}_i^C(\hat{\tau}) + \sum_o x_{oi}^0 \left((1-\psi\phi) \hat{w}_o + \psi\phi \hat{\Omega} \right) \\
&= \hat{\eta}_i^C(\hat{\tau}) + \sum_o (x_{oi}^0 (1-\psi\phi) + \psi\phi \omega_o^0) \hat{w}_o, \\
&= \hat{\eta}_i^C(\hat{\tau}) + \sum_o \left(x_{oi}^0 \frac{\kappa}{\sigma-1} + \left(1 - \frac{\kappa}{\sigma-1} \right) \omega_o^0 \right) \hat{w}_o
\end{aligned}$$

where the first expression follows from $\hat{p}_o$ in (10), the second from $\hat{b}_i = \hat{\Omega} = \sum_o \omega_o^0 \hat{w}_o$, and the last from the definition of $\kappa = (\sigma-1)(1-\psi\phi)$. The combination of this expression and the expression for $\hat{w}_o$ in (21) immediately implies (23).

A.1.8 Proof of Equation (25)

The labor supply equation in (8) with $\hat{b}_i = \sum_d \omega_d^0 \hat{w}_d$ implies that $\hat{n}_i = \phi(1-n_i^0)(\hat{w}_i - \sum_d \omega_d^0 \hat{w}_d)$. Using (21), we get that

$$\hat{n}_i = \phi(1 - n_i^0) \sum_j \left(\beta_{ij} - \sum_d \omega_d^0 \beta_{dj} \right) \hat{\eta}_j, \quad (6)$$

which immediately implies the second expression in (25).

The combination of (9), (21), and (6) implies that

$$\Delta \overline{\ln w_i} = \sum_j \beta_{ij} \hat{\eta}_j - (1 - n_i^0) \sum_j \left(\beta_{ij} - \sum_d \omega_d^0 \beta_{dj} \right) \hat{\eta}_j,$$

and, thus,

$$\Delta \overline{\ln w_i} = \sum_j \left[n_i^0 \beta_{ij} + (1 - n_i^0) \sum_d \omega_d^0 \beta_{dj} \right] \hat{\eta}_j.$$

This immediately implies the first expression in (25).

A.1.9 Proof of Equation (28)

To establish condition (28) first notice that, by definition,

$$\begin{aligned} \nu_i^w &= (1 - \sigma) \sum_j \beta_{ij}^w(\boldsymbol{\theta} | \mathbf{W}^0) \sum_s \ell_{j,s}^0 \left(\sum_d r_{jd,s}^0 \hat{\tau}_{jd,s}^{\text{unbs}} - \sum_o \sum_d r_{jd,s}^0 x_{od,s}^0 \hat{\tau}_{od,s}^{\text{unbs}} \right) - \alpha^w \\ &= \sum_{s,d,o} \left[\sum_j (1 - \sigma) \beta_{ij}^w(\boldsymbol{\theta} | \mathbf{W}^0) \ell_{j,s}^0 r_{jd,s}^0 \left(\mathbb{I}_{o=j} - x_{od,s}^0 \right) \right] \hat{\tau}_{od,s}^{\text{unbs}} - \alpha^w. \end{aligned}$$

Using the definition $Z_j \equiv \sum_s \ell_{j,s}^0 \hat{\zeta}_{j,s}^{\text{obs}}$, we have that

$$\begin{aligned} Z_j &= (1 - \sigma) \sum_s \ell_{j,s}^0 \left(\sum_d r_{jd,s}^0 \hat{\zeta}_{id,s} \mathbb{I}_{(i,d,s) \in \mathcal{T}^{\text{obs}}} - \sum_o \sum_d r_{jd,s}^0 x_{od,s}^0 \hat{\zeta}_{od,s} \mathbb{I}_{(o,d,s) \in \mathcal{T}^{\text{obs}}} \right) \\ &= \sum_{(s,d,o) \in \mathcal{T}^{\text{obs}}} \left[(1 - \sigma) \ell_{j,s}^0 r_{jd,s}^0 \left(\mathbb{I}_{o=j} - x_{od,s}^0 \right) \right] \hat{\zeta}_{od,s}. \end{aligned}$$

For arbitrary i and j , the two expressions above and Assumption 1 imply that

$$\begin{aligned} &E \left[\nu_i^w Z_j | \mathbf{W}^0, \mathbf{V}^0, \hat{\boldsymbol{\tau}}^{\text{unbs}} \right] \\ &= E \left[\left(\sum_{(s,d,o)} \left[\sum_h \beta_{ih}^w(\boldsymbol{\theta} | \mathbf{W}^0) \ell_{h,s}^0 r_{hd,s}^0 \left(\mathbb{I}_{o=h} - x_{od,s}^0 \right) \right] \hat{\tau}_{od,s}^{\text{unbs}} - \alpha^w \right) \left(\sum_{(s',d',o') \in \mathcal{T}^{\text{obs}}} \ell_{j,s'}^0 r_{jd',s'}^0 \left(\mathbb{I}_{o'=j} - x_{o'd',s'}^0 \right) \hat{\zeta}_{o'd',s'} \right) | \mathbf{W}^0, \mathbf{V}^0, \hat{\boldsymbol{\tau}}^{\text{unbs}} \right] \\ &= \left(\sum_{(s,d,o)} \left[\sum_h \beta_{ih}^w(\boldsymbol{\theta} | \mathbf{W}^0) \ell_{h,s}^0 r_{hd,s}^0 \left(\mathbb{I}_{o=h} - x_{od,s}^0 \right) \right] \hat{\tau}_{od,s}^{\text{unbs}} - \alpha^w \right) \left(\sum_{(s',d',o') \in \mathcal{T}^{\text{obs}}} \ell_{j,s'}^0 r_{jd',s'}^0 \left(\mathbb{I}_{o'=j} - x_{o'd',s'}^0 \right) E \left[\hat{\zeta}_{o'd',s'} | \mathbf{W}^0, \mathbf{V}^0, \hat{\boldsymbol{\tau}}^{\text{unbs}} \right] \right) \\ &= \left(\sum_{(s,d,o)} \left[\sum_h \beta_{ih}^w(\boldsymbol{\theta} | \mathbf{W}^0) \ell_{h,s}^0 r_{hd,s}^0 \left(\mathbb{I}_{o=h} - x_{od,s}^0 \right) \right] \hat{\tau}_{od,s}^{\text{unbs}} - \alpha^w \right) \left(\sum_{(s',d',o') \in \mathcal{T}^{\text{obs}}} \ell_{j,s'}^0 r_{jd',s'}^0 \left(\mathbb{I}_{o'=j} - x_{o'd',s'}^0 \right) 0 \right) \\ &= 0 \end{aligned}$$

Thus, $E[\nu_i^w Z_j] = E \left[E[\nu_i^w Z_j | \mathbf{W}^0, \mathbf{V}^0, \hat{\boldsymbol{\tau}}^{\text{unbs}}] \right] = 0$ for any i and j . This immediately establishes that, for any real matrix h_{ij}^w , $E \left[\nu_i^w \sum_j h_{ij}^w Z_j \right] = \sum_j h_{ij}^w E[\nu_i^w Z_j] = 0$. We can follow the same steps to show that $E \left[\nu_i^n \sum_j h_{ij}^n Z_j \right] = 0$.

A.1.10 Model-Implied Optimal Moment Condition

To simplify exposition without loss of generality, we assume that all variables are demeaned, so that (25) and (28) can be written in the following vector form:

$$v_i(\boldsymbol{\theta}) = Y_i - (\beta_i(\boldsymbol{\theta} | \mathbf{W}))' \mathbf{Z} \quad \text{such that} \quad E \left[v_i(\boldsymbol{\theta}) \sum_j h_{ij} Z_j \right] = 0. \quad (7)$$

Define $\mathbf{H}_i = \left[(\mathbf{h}_i^k \mathbf{Z})' \right]_{k=1}^{dim(\boldsymbol{\theta})}$ where $\mathbf{h}_i^k \mathbf{Z}$ has dimension $dim(v_i) \times 1$. Thus, for any $\mathbf{h}_i^k$, the condition above is equivalent to

$$E[\mathbf{H}_i v_i(\boldsymbol{\theta})] = 0,$$

which yields the following class of GMM estimators of $\boldsymbol{\theta}$,

$$\hat{\boldsymbol{\theta}}_H \equiv \argmin_{\boldsymbol{\theta} \in \Theta} \left[\sum_i \mathbf{H}_i v_i(\boldsymbol{\theta}) \right]' \left[\sum_i \mathbf{H}_i v_i(\boldsymbol{\theta}) \right].$$

Optimal Moment Conditions with Independent Residuals. We follow [Chamberlain \(1987\)](#) to derive the optimal moment conditions. We start with the assumption that v_i are independent across (clusters of) markets. We show below how the formula changes when we instead assume that the observed shocks are independent, while allowing residuals to have an arbitrary correlation.

When v_i is independent across markets and independent from initial conditions in $\mathbf{W}$, the usual optimal IV formula in [Chamberlain \(1987\)](#) holds, so that the optimal moment condition is

$$\mathbf{H}_i^* \equiv \left(E[v_i(\boldsymbol{\theta}) v_i(\boldsymbol{\theta})' | \mathbf{Z}] \right)^{-1} \nabla_{\boldsymbol{\theta}} v_i(\boldsymbol{\theta})$$

where, given (7),

$$\nabla_{\boldsymbol{\theta}} v_i(\boldsymbol{\theta}) = -\nabla_{\boldsymbol{\theta}} \beta_i(\boldsymbol{\theta} | \mathbf{W}) \mathbf{Z}.$$

The term $(E[v_i(\boldsymbol{\theta}) v_i(\boldsymbol{\theta})' | \mathbf{Z}])^{-1}$ adjusts the weight of each observation to the inverse of the variance of its residuals. It is the usual adjustment that arises in generalized least squares under heteroskedasticity. This term is irrelevant under homoskedasticity. For each observation, $\nabla_{\boldsymbol{\theta}} v_i(\boldsymbol{\theta})$ attributes a higher weight to the exposure of the markets whose bilateral reduced-form elasticities are more sensitive to changes in each parameter.

Intuition for Efficiency Gains. We now illustrate the source of the efficiency gains from the use of the optimal moment conditions in the context of our model. To fix ideas, we assume that the goal is to estimate ϕ when the labor demand elasticity κ is already known. In this case, the efficiency gains arise from the fact that the reduced-form expression for wage changes in (25) is the model-consistent first-stage specification for the estimation of ϕ .

To see this, notice that the labor supply structure in (8)–(9) implies that

$$\frac{n_i^0}{1 - n_i^0} \Delta \ln n_i = \hat{\Omega} + \phi \Delta \overline{\ln w_i} + v_i^n \quad (8)$$

where $v_i^n \equiv -\Delta \ln \bar{b}_i$ and $\hat{\Omega}$ is the change in the numeraire of transfers.

Given that κ is assumed to be known, the reduced-form expression for wage changes in (25) yields the model-consistent first-stage expression:

$$\Delta \overline{\ln w_i} = \alpha^w + \sum_j \beta_{ij}^w(\phi) \hat{\eta}_j + v_i^w. \quad (9)$$

Notice that if we substitute (9) into (8), then we recover the reduced-form expression for employment in (25). Thus, equations (8)–(9) imply the same relationship between market-level shock exposure (as measured by $\hat{\eta}_j$) and employment and wage changes across markets.

Using the general equilibrium predictions of the model in (8)–(9), the optimal moment condition for the estimation of ϕ is

$$(E[v_i(\phi)v_i(\phi)'|\mathbf{Z}])^{-1}\nabla_{\phi}v_i(\phi)=\begin{bmatrix}\sigma_n^2 & \sigma_{wn} \\ \sigma_{wn} & \sigma_w^2\end{bmatrix}^{-1}\sum_j\begin{bmatrix}\beta_{ij}^w(\phi)+\phi\nabla_{\phi}\beta_{ij}^w(\phi) \\ \nabla_{\phi}\beta_{ij}^w(\phi)\end{bmatrix}\hat{\eta}_j, \quad (10)$$

where σ_n^2 and σ_w^2 denote the variance of v_i^n and v_i^w respectively and σ_{wn} denotes the covariance between v_i^w and v_i^n .

We now compare the optimal moment condition implied by the general equilibrium predictions of the model to the optimal moment condition that a researcher would obtain if she estimates ϕ with the equilibrium relationship in (8) but imposes an *arbitrary* first-stage relationship between the average log-wage in market i (i.e., the endogenous variable) and that market's exogenous shock exposure (i.e., the instrumental variable):

$$\Delta\overline{\ln w}_i = \alpha + \beta\hat{\eta}_i + u_i. \quad (11)$$

This approach yields a consistent estimator of ϕ under the same orthogonality conditions introduced in Section 3.3. However, it yields a less efficient estimator. To see this, we now apply the optimal moment condition to (8) and (11) (which is equivalent to the 2SLS estimator of ϕ when $\hat{\eta}_i$ is the IV):

$$(E[v_i(\phi)v_i(\phi)'|\mathbf{Z}])^{-1}\nabla_{\phi}v_i(\phi)=\begin{bmatrix}\sigma_n^2 & 0 \\ 0 & \sigma_u^2\end{bmatrix}^{-1}\begin{bmatrix}\beta\hat{\eta}_i \\ 0\end{bmatrix}=\begin{bmatrix}\frac{\beta}{\sigma_n^2}\hat{\eta}_i \\ 0\end{bmatrix}\propto\begin{bmatrix}\hat{\eta}_i \\ 0\end{bmatrix}, \quad (12)$$

where we assume that the researcher also imposed that the covariance between u_i and v_i is zero – as typically done in practice.

Under the assumption that the general equilibrium model is well specified, the fact that (10) and (12) are different indicates that the arbitrary first-stage assumption leads to a less efficient estimator. The difference is that the derivation of (12) does not take into account the general equilibrium structure of the model, and, thus, ignores two channels through which shocks affect endogenous outcomes in general equilibrium: it ignores (i) information on shocks in other regions that may affect wages and employment in i (i.e., the arbitrary first-stage ignores spatial indirect effects), and (ii) the impact that the parameter ϕ has on wage responses (i.e., the arbitrary first-stage ignores that wage responses depend on ϕ).

Optimal Moment Conditions with Correlated Residuals. We now also derive the optimal moment conditions using the results in [Borusyak and Hull \(2023\)](#) that allow for arbitrary cross-market correlation in v_i but assume that the observed shocks are independent from each other. In this case, they show that

$$\mathbf{H}^* \equiv (E[\mathbf{v}(\boldsymbol{\theta})\mathbf{v}(\boldsymbol{\theta})'|\mathbf{W}])^{-1}\nabla_{\boldsymbol{\theta}}\mathbf{v}(\boldsymbol{\theta}).$$

We then apply this formula to our reduced-form representation of the predictions of general equilibrium spatial models: using (7),

$$\nabla_{\boldsymbol{\theta}}v_i(\boldsymbol{\theta}) = -\nabla_{\boldsymbol{\theta}}\beta_i(\boldsymbol{\theta}|\mathbf{W})\mathbf{Z}.$$

This formula is similar to the one above. The term $\nabla_{\boldsymbol{\theta}}v_i(\boldsymbol{\theta})$ is identical in the two formulas. The only difference is the first term, which now accounts for the potential covariance between the residuals of different markets. In our model, such a correlation may arise if markets are exposed to similar unobserved shocks in economic fundamentals – a point recently raised by [Adão et al. \(2019b\)](#) in the context of shift-share specifications. To see this, assume that $\hat{\tau}_{od,s}^{\text{unbs}}$ are independently drawn from an arbitrary distribution with mean zero and variance of σ_{τ}^2 . As shown in Appendix A.1.9, the residual can be written in a general form, $v_i(\boldsymbol{\theta}) = \sum_{s,d,o}\beta_{i,ods}^v(\boldsymbol{\theta}|\mathbf{W})\hat{\tau}_{od,s}^{\text{unbs}}$ where $\beta_{i,ods}^v = \sum_j[\beta_{ij}^w(\boldsymbol{\theta}|\mathbf{W}), \beta_{ij}^n(\boldsymbol{\theta}|\mathbf{W})]'\ell_{j,s}y_{jd,s}(\mathbb{I}_{o=j} - x_{od,s})$. Thus,

$$E[v_i(\boldsymbol{\theta})v_j(\boldsymbol{\theta})'|\mathbf{W}] = \sigma_{\tau}^2 \sum_{s,d,o} (\beta_{i,ods}^v(\boldsymbol{\theta}|\mathbf{W}))(\beta_{j,ods}^v(\boldsymbol{\theta}|\mathbf{W}))'.$$

This expression shows that the correlation between the residuals of markets i and j is higher if they have higher exposure to the same shocks. This is the case if the two markets are similar in terms of employment shares across sectors and/or within-sector revenue shares across destinations.

Implementation Comments. We conclude with three comments on the implementation of our methodology. First, estimation can be implemented within the compact set of parameters that are feasible in the theory. Second, it is hard to compute the variance adjustment term in the optimal moment conditions, since it requires knowledge of the unobserved residuals. Third, to simplify the estimator's implementation, one can adopt an asymptotically equivalent two-step GMM estimator of θ where, in the first-stage, we obtain a consistent estimator $\hat{\theta}_1$ using $\nabla_{\theta} v_i(\theta_0)$ computed with an arbitrary guess θ_0 and, in the second-stage, we estimate $\hat{\theta}_2$ using $\nabla_{\theta} v_i(\hat{\theta}_1)$ computed with the first-stage estimate $\hat{\theta}_1$. In our application, we rely on a consistent estimator of θ that imposes parameter restrictions that align with the theory, but ignores the other issues due to computational costs.

A.1.11 Reduced-form Responses in Sectoral Employment Outcomes

To derive the change in sectoral employment, recall that, in our model, the share of employment in sector s is equal to the share of revenue in that sector, so that $\hat{\ell}_{i,s} = \hat{R}_{i,s} - \sum_k \ell_{i,k}^0 \hat{R}_{i,k}$. The definitions $R_{i,s} \equiv \sum_j x_{ij,s} \xi_{j,s} E_j$, $r_{ij,s} \equiv x_{ij,s} \xi_{j,s} E_j / R_{i,s}$ and $\mu_{i,s}(\hat{\tau}) \equiv \sum_j r_{ij,s}^0 (\hat{\tau}_{ij,s} - \sum_o x_{oj,s}^0 \hat{\tau}_{oj,s})$ imply

$$\hat{R}_{i,s} = (1-\sigma)\mu_{i,s}(\hat{\tau}) + (1-\sigma)\hat{p}_i + (\sigma-1) \sum_o \left(\sum_j r_{ij,s}^0 x_{oj,s}^0 \right) \hat{p}_o + \sum_j r_{ij,s}^0 \hat{E}_j.$$

Thus, since $\hat{\eta}_i(\hat{\tau}) \equiv (1-\sigma) \sum_k \ell_{i,k} \mu_{i,k}(\hat{\tau})$,

$$\hat{\ell}_{i,s} = (1-\sigma)\mu_{i,s}(\hat{\tau}) - \hat{\eta}_i(\hat{\tau}) + (\sigma-1) \sum_o \left(\sum_j r_{ij,s}^0 x_{oj,s}^0 - \sum_k \ell_{i,k}^0 \sum_j r_{ij,k}^0 x_{oj,k}^0 \right) \hat{p}_o + \sum_j (r_{ij,s}^0 - r_{ij}^0) \hat{E}_j.$$

Using the fact that $\hat{p}_o \equiv (1-\psi\phi)\hat{w}_o + \psi\phi\hat{\Omega}$, this expression is equivalent to

$$\hat{\ell}_{i,s} = (1-\sigma)\mu_{i,s}(\hat{\tau}) - \hat{\eta}_i(\hat{\tau}) + \kappa \sum_o \left(\sum_j r_{ij,s}^0 x_{oj,s}^0 - \sum_k \ell_{i,k}^0 \sum_j r_{ij,k}^0 x_{oj,k}^0 \right) \hat{w}_o + \sum_j (r_{ij,s}^0 - r_{ij}^0) \hat{E}_j$$

and, therefore,

$$\hat{\ell}_{i,s} = (1-\sigma)\mu_{i,s}(\hat{\tau}) - \hat{\eta}_i(\hat{\tau}) + \kappa \sum_o (\chi_{io,s}^0 - \chi_{io}^0) \hat{w}_o + \sum_j (r_{ij,s}^0 - r_{ij}^0) \hat{E}_j$$

where $\chi_{io,s}^0 \equiv \sum_d r_{id,s}^0 x_{od,s}^0$ and $\chi_{io}^0 \equiv \sum_s \ell_{i,s}^0 \chi_{io,s}^0$.

Recall that $E_j = W_j = w_i(n_i)^{\frac{\phi-1}{\phi}} N_i \varrho$ and $\hat{E}_j = \phi_j^0 \hat{w}_j + (1-\phi_j^0) \hat{\Omega}$. Thus,

$$\hat{\ell}_{i,s} = (1-\sigma)\mu_{i,s}(\hat{\tau}) - \hat{\eta}_i(\hat{\tau}) + \sum_j [(\chi_{ij,s}^0 - \chi_{ij}^0) \kappa + (r_{ij,s}^0 - r_{ij}^0) \phi_j^0] \hat{w}_j + \rho_{i,s}^0 \hat{\Omega}$$

where $\rho_{i,s}^0 \equiv -\sum_d (r_{id,s}^0 - r_{id}^0) \phi_d^0$. Since $\hat{\Omega} = \sum_j \omega_j^0 \hat{w}_j$, this expression becomes

$$\hat{\ell}_{i,s} = (1-\sigma)\mu_{i,s}(\hat{\tau}) - \hat{\eta}_i(\hat{\tau}) + \sum_j \pi_{ij,s}^0 \hat{w}_j \quad (13)$$

where $\pi_{ij,s}^0 \equiv (\chi_{ij,s}^0 - \chi_{ij}^0) \kappa + (r_{ij,s}^0 - r_{ij}^0) \phi_j^0 + \rho_{i,s}^0 \omega_j^0$. Notice that, up to a first-order approximation, the change in the share of employment in sector s is $\Delta \ell_{i,s} = \ell_{i,s}^0 \hat{\ell}_{i,s}$ and share of population employed in sector s is $\Delta n_{i,s} = \ell_{i,s}^0 n_i^0 (\hat{\ell}_{i,s} + \hat{n}_i)$.

A.2 Proofs for Section 4

The equilibrium requires the gross revenue $\{R_{i,s}\}_{i,s}$ to solve

$$R_{i,s} = \sum_j \frac{(\tau_{ij,s} p_{i,s})^{1-\sigma}}{\sum_o (\tau_{oj,s} p_{o,s})^{1-\sigma}} \left(\xi_{j,s} E_j + \sum_k \xi_{j,sk}^M a_{j,k}^M R_{j,k} \right) \quad \text{for all } (i,s), \quad (14)$$

where $E_i = W_i = \varrho w_i^\phi (w_i^\phi + b_i^\phi)^{\frac{1-\phi}{\phi}} N_i$.

The equilibrium can be expressed as a wage vector in terms of an excess labor demand system: $D_i(\mathbf{w}|\boldsymbol{\tau}) = 0$ for all i , such that

$$D_i(\mathbf{w}|\boldsymbol{\tau}) \equiv \sum_s a_{i,s}^L R_{i,s}(\mathbf{w}|\boldsymbol{\tau}) - W_i(\mathbf{w}|\boldsymbol{\tau}). \quad (15)$$

A.2.1 Proof of Excess Labor Demand in (15)

Step 1. We now implicitly characterize $P_{j,s}(\mathbf{w})$ from the combination of $P_{j,s}$ in (7), $p_{i,s}$ in (31), n_i in (8), N_i in (30) with $b_i = \bar{b}_i(P_i)^\lambda (\Omega(\mathbf{w}))^{1-\lambda}$. Thus, any $\{P_{j,s}\}_{j,s} \in \{P_{j,s}(\mathbf{w})\}_{j,s}$ solves

$$(P_{j,s})^{1-\sigma} = \sum_i (\tau_{ij,s})^{1-\sigma} \left(\frac{w_i}{\bar{b}_i (\Pi_k(P_{i,k})^{\xi_{i,k}})^\lambda (\Omega(\mathbf{w}))^{1-\lambda}} \right)^{\phi\psi(\sigma-1)} \left[(w_i)^{a_{i,s}^L} \left(\Pi_k(P_{i,k})^{\xi_{i,ks}^M} \right)^{a_{i,s}^M} \right]^{1-\sigma}.$$

Step 2. For any $\mathbf{w}$, take $\{P_{j,s}\}_{j,s} \in \{P_{j,s}(\mathbf{w})\}_{j,s}$ to compute $P_i(\mathbf{w}|\boldsymbol{\tau}) = \Pi_k(P_{i,k})^{\xi_{i,k}}$, and $P_{i,s}^M(\mathbf{w}|\boldsymbol{\tau}) = \Pi_k(P_{i,k})^{\xi_{i,ks}^M}$. We then obtain $p_{i,s}(\mathbf{w}|\boldsymbol{\tau})$ from (31), $n_i(\mathbf{w}|\boldsymbol{\tau})$ from (8), $N_i(\mathbf{w}|\boldsymbol{\tau})$ from (30), and

$$E_j(\mathbf{w}|\boldsymbol{\tau}) = W_j(\mathbf{w}|\boldsymbol{\tau}) = w_j^\phi \left(w_j^\phi + \left(\bar{b}_j (\Pi_k(P_{j,k})^{\xi_{j,k}})^\lambda (\Omega(\mathbf{w}))^{1-\lambda} \right)^\phi \right)^{\frac{1-\phi}{\phi}} N_j(\mathbf{w}|\boldsymbol{\tau}) \varrho.$$

Step 3. To define the revenue function, we solve for

$$R_{i,s} - \sum_k \sum_j \frac{(\tau_{ij,s} p_{i,s}(\mathbf{w}|\boldsymbol{\tau}))^{1-\sigma}}{\sum_o (\tau_{oj,s} p_{o,s}(\mathbf{w}|\boldsymbol{\tau}))^{1-\sigma}} \xi_{j,sk}^M a_{j,k}^M R_{j,k} = \sum_j \frac{(\tau_{ij,s} p_{i,s}(\mathbf{w}|\boldsymbol{\tau}))^{1-\sigma}}{\sum_o (\tau_{oj,s} p_{o,s}(\mathbf{w}|\boldsymbol{\tau}))^{1-\sigma}} \xi_{j,s} E_j(\mathbf{w}|\boldsymbol{\tau})$$

This system has a unique solution because, for every (j,k) ,

$$\sum_s \sum_i \frac{(\tau_{ij,s} p_{i,s}(\mathbf{w}|\boldsymbol{\tau}))^{1-\sigma}}{\sum_o (\tau_{oj,s} p_{o,s}(\mathbf{w}|\boldsymbol{\tau}))^{1-\sigma}} \xi_{j,sk}^M a_{j,k}^M = \sum_s \xi_{j,sk}^M a_{j,k}^M = a_{j,k}^M < 1,$$

so that

$$[R_{i,s}(\mathbf{w}|\boldsymbol{\tau})]_{i,s} = \sum_{d=0}^{\infty} (\bar{A}(\mathbf{w}|\boldsymbol{\tau}))^d \left[\sum_j \frac{(\tau_{ij,s} p_{i,s}(\mathbf{w}|\boldsymbol{\tau}))^{1-\sigma}}{\sum_o (\tau_{oj,s} p_{o,s}(\mathbf{w}|\boldsymbol{\tau}))^{1-\sigma}} \xi_{j,s} E_j(\mathbf{w}|\boldsymbol{\tau}) \right]_{i,s}$$

where

$$\bar{\mathbf{A}}(\mathbf{w}|\boldsymbol{\tau}) \equiv \left[\frac{(\tau_{ij,s} p_{i,s}(\mathbf{w}|\boldsymbol{\tau}))^{1-\sigma}}{\sum_o (\tau_{oj,s} p_{o,s}(\mathbf{w}|\boldsymbol{\tau}))^{1-\sigma}} \xi_{j,sk}^M a_{j,k}^M \right]_{is,jk}.$$

A.2.2 Proof of Equation (32)

In all the remaining proofs of this section, we simplify notation by omitting the superscript 0. The system in (14) implicitly defines $\{R_{i,s}\}_{i,s}$ as a function of $\boldsymbol{\tau}$ for any given $\{p_{i,s}\}_{i,s}$. We can then use the implicit function theorem to write

$$\frac{\partial \ln R_{i,k}}{\partial \ln \tau_{od,s}} = \mathbb{I}_{s=k} \frac{X_{id,s}}{R_{i,s}} (1-\sigma) (\mathbb{I}_{i=o} - x_{od,s}) + \sum_{k'} \sum_j \frac{x_{ij,s} \xi_{j,sk'}^M a_{j,k'}^M R_{j,k'}}{R_{i,k}} \frac{\partial \ln R_{j,k'}}{\partial \ln \tau_{od,s}}$$

which implies that

$$\left[\frac{\partial \ln R_{i,k}}{\partial \ln \tau_{od,s}} \right]_{ik} = (1-\sigma) (\bar{\mathbf{I}} + \bar{\mathbf{b}}^U) \left[\mathbb{I}_{s=k} \frac{X_{id,s}}{R_{i,s}} (\mathbb{I}_{i=o} - x_{od,s}) \right]_{ik}$$

where we have used the definition $\bar{\mathbf{b}}^U \equiv \sum_{d=1}^{\infty} (\bar{\mathbf{r}}^U)^d$, where $\bar{\mathbf{r}}^U \equiv [r_{is,jk}^U]_{is,jk}$ with $r_{is,jk}^U \equiv X_{ij,sk}^M / R_{i,s}$.

Thus,

$$\begin{aligned} \left[\sum_{s,o,d} \frac{\partial \ln R_{i,k}}{\partial \ln \tau_{od,s}} \hat{\tau}_{od,s} \right]_{ik} &= (1-\sigma) (\bar{\mathbf{I}} + \bar{\mathbf{b}}^U) \left[\sum_{o,d} \frac{X_{id,k}}{R_{i,k}} (\mathbb{I}_{i=o} - x_{od,k}) \hat{\tau}_{od,k} \right]_{ik} \\ &= (1-\sigma) (\bar{\mathbf{I}} + \bar{\mathbf{b}}^U) [\mu_{i,k}(\hat{\boldsymbol{\tau}})]_{ik} \\ &= (1-\sigma) \left[\mu_{i,k}(\hat{\boldsymbol{\tau}}) + \sum_{j,k'} b_{ik,jk'}^U \mu_{j,k'}(\hat{\boldsymbol{\tau}}) \right]_{ik} \\ &= (1-\sigma) \left[\mu_{i,k}(\hat{\boldsymbol{\tau}}) + \mu_{i,k}^U(\hat{\boldsymbol{\tau}}) \right]_{ik} \end{aligned}$$

This immediately implies (32) because $\hat{\eta}_i^R(\hat{\boldsymbol{\tau}}) \equiv \sum_{s,o,d} \frac{\partial \ln R_i}{\partial \ln \tau_{od,s}} \hat{\tau}_{od,s} = \sum_k \ell_{i,k} \sum_{s,o,d} \frac{\partial \ln R_{i,k}}{\partial \ln \tau_{od,s}} \hat{\tau}_{od,s}$.

A.2.3 Proof of Equation (33)

Equations (7) and (31) define $\{P_{i,s}^M\}_{i,s}$ as a function of $\boldsymbol{\tau}$ for any given $\{w_i, L_i\}_i$ from the solution of

$$P_{i,s}^M = \Pi_k \left(\sum_o (\tau_{oi,k})^{1-\sigma} \left(\frac{w_o}{b_o} \right)^{\phi\psi(\sigma-1)} w_o^{(1-\sigma)a_{o,s}^L} (P_{o,k}^M)^{(1-\sigma)a_{o,s}^M} \right)^{\frac{\xi_{i,ks}^M}{1-\sigma}}.$$

Using the implicit function theorem, we have that

$$\frac{\partial \ln P_{i,s}^M}{\partial \ln \tau_{od,k}} = \mathbb{I}_{i=d} \xi_{i,ks}^M x_{oi,k} + \sum_{k'} \sum_j \xi_{i,k's}^M x_{ji,k'} a_{j,k'}^M \frac{\partial \ln P_{j,k'}^M}{\partial \ln \tau_{od,k}}$$

which, by defining $\bar{\mathbf{g}}^D \equiv (\bar{\mathbf{I}} + \bar{\mathbf{b}}^D)$ implies that

$$\left[\frac{\partial \ln P_{i,s}^M}{\partial \ln \tau_{od,k}} \right]_{is} = \bar{\mathbf{g}}^D [\mathbb{I}_{i=d} \xi_{i,ks}^M x_{oi,k}]_{is}.$$

Thus, $[\hat{\eta}_{i,s}^M(\hat{\boldsymbol{\tau}})]_{i,s} = \bar{\mathbf{g}}^D [\mu_{i,s}^M(\hat{\boldsymbol{\tau}})]_{i,s} = \mu_{i,s}^M(\hat{\boldsymbol{\tau}}) + \sum_{j,k} b_{is,jk}^D \mu_{j,k}^M(\hat{\boldsymbol{\tau}})$.

A.2.4 Proof of Equations (34)–(35)

There are four parts to the derivation of equations (34)–(35). The first part is to characterize the matrix of spatial links, $\bar{\gamma}$, and the vector of shifts in excess labor demand, $\hat{\eta}$ (as in Sections 3.2.1 and 3.2.2). The second part is to characterize the reduced-form response of wages to shifts in excess labor demand (as in Section 3.2.3). The third is the derivation of the reduced-form responses in labor market outcomes to shocks in bilateral productivity. The last part is the derivation of (34) for observed and unobserved shocks, and the associated moment conditions in (35) (as in Section 3.3).

Part A: Derivation of the matrix of spatial links, $\bar{\gamma}$, and the vector of shifts in excess labor demand, $\hat{\eta}$. We first establish that, by totally differentiating the equilibrium conditions, we can write them as

$$\bar{\gamma}\hat{w} = \hat{\eta}. \quad (16)$$

Step 1. We first derive changes in labor market outcomes as a function of $\hat{w}$ and $\hat{P}$. Expression (8) with $b_i = \bar{b}_i P_i^\lambda (\Omega(\mathbf{w}))^{1-\lambda}$ implies that $\hat{n}_i = \phi(1-n_i)(\hat{w}_i - \lambda \hat{P}_i - (1-\lambda)\sum_j \omega_j \hat{w}_j)$. So,

$$\hat{n} = \phi \bar{\phi}^{n,w} \hat{w} + \phi \lambda \bar{\phi}^{n,P} \hat{P} \quad (17)$$

such that $\phi_{ij}^{n,w} \equiv (1-n_i)(\mathbb{I}_{i=j} - (1-\lambda)\omega_j)$ and $\phi_{ij}^{n,p} \equiv -(1-n_i)\mathbb{I}_{i=j}$.

Expression (30) with $b_i = \bar{b}_i P_i^\lambda (\Omega(\mathbf{w}))^{1-\lambda}$ implies

$$\begin{aligned} \hat{N}_i &= \vartheta \sum_j \left(\mathbb{I}_{i=j} - \frac{N_j}{N_{c(i)}} \mathbb{I}_{c(i)=c(j)} \right) \left(\hat{w}_j - \hat{P}_j + (\phi-1)(1-n_j) \left(\hat{w}_j - \lambda \hat{P}_j \right) \right) \\ &+ (\phi-1)(1-\lambda) \left(n_i - \sum_{j \in c(i)} \frac{N_j}{N_{c(j)}} n_j \right) \sum_d \omega_d \hat{w}_d. \end{aligned}$$

Thus,

$$\hat{N} = \vartheta \bar{\phi}^{N,w} \hat{w} + \vartheta \bar{\phi}^{N,P} \hat{P}, \quad (18)$$

where $\tilde{N} \equiv [\mathbb{I}_{i=j} - \mathbb{I}_{c(i)=c(j)} N_j / N_{c(j)}]_{i,j}$, $\bar{n} \equiv [n_i \mathbb{I}_{i=j}]_{i,j}$,

$$\bar{\phi}^{N,w} \equiv \tilde{N}(\bar{I} + (\phi-1)(\bar{I} - \bar{n})) + (\phi-1)(1-\lambda)\tilde{N}\mathbf{n}\omega', \quad \text{and} \quad \bar{\phi}^{N,P} \equiv -\tilde{N}(\bar{I} + \lambda(\phi-1)(\bar{I} - \bar{n})).$$

Expressions (17) and (18) imply that $\hat{W}_i = \hat{w}_i + \hat{n}_i(\phi-1)/\phi + \hat{N}_i$ is given by

$$\hat{E} = \hat{W} = \bar{\phi}^{W,w} \hat{w} + \left((\phi-1)\lambda \bar{\phi}^{n,P} + \vartheta \bar{\phi}^{N,P} \right) \hat{P}, \quad (19)$$

where $\bar{\phi}^{W,w} \equiv \bar{I} + (\phi-1)\bar{\phi}^{n,w} + \vartheta \bar{\phi}^{N,w}$.

Step 2. We now derive expressions for changes in price indices as a function of wages, and exogenous shocks. From (31),

$$\hat{P}_{i,s}^M - \sum_{j,k} \xi_{i,ks}^M x_{ji,k} a_{j,k}^M \hat{P}_{j,k}^M = \mu_{i,s}^M(\hat{\tau}) + \sum_{j,k} \xi_{i,ks}^M x_{ji,k} \left(a_{j,k}^L \hat{w}_j - \frac{\psi}{1-n_j} \hat{n}_j \right)$$

which, by defining $\bar{\mathbf{x}}_{NS \times NS}^D \equiv [a_{j,k}^M \xi_{i,ks}^M x_{ji,k}]_{is,jk}$, $\bar{\mathbf{x}}_{NS \times NS}^M \equiv [\xi_{i,ks}^M x_{ji,k}]_{is,jk}$, $\bar{\mathbf{a}}_{NS \times N}^L \equiv [a_{i,k}^L \mathbb{I}_{i=j}]_{ik,j}$, $\bar{\mathbf{v}}_{NS \times N}^n \equiv [\mathbb{I}_{i=j}(1-n_i)^{-1}]_{ik,j}$, implies

$$(\bar{I} - \bar{\mathbf{x}}^D) \hat{P}^M = \mu^M(\hat{\tau}) + \bar{\mathbf{x}}^M (\bar{\mathbf{a}}^L \hat{w} - \psi \bar{\mathbf{v}}^n \hat{n})$$

and, therefore,

$$\hat{\mathbf{P}}^M = \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) + \bar{\mathbf{g}}^D \bar{\mathbf{x}}^M (\bar{\mathbf{a}}^L \hat{\mathbf{w}} - \psi \bar{\mathbf{v}}^n \hat{\mathbf{n}}). \quad (20)$$

From Shepard's lemma, $\hat{P}_i = \sum_s \xi_{i,s} \sum_j x_{ji,s} (\hat{\tau}_{ji,s} + \hat{p}_{j,s})$, which implies that

$$\hat{P}_i = \sum_s \xi_{i,s} \sum_j x_{ji,s} \left(\hat{\tau}_{ji,s} + a_{j,s}^L \hat{w}_j + a_{j,s}^M \hat{P}_{j,s}^M - \frac{\psi}{1-n_j} \hat{n}_j \right)$$

which, by defining $\bar{\mathbf{x}}_{N \times NS}^C \equiv [\xi_{i,s} x_{ji,s}]_{i,j,s}$, $\bar{\mathbf{a}}_{NS \times NS}^M \equiv [a_{j,k}^M \mathbb{I}_{is=jk}]_{i,s,jk}$,

$$\hat{\mathbf{P}} = \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\mathbf{x}}^C (\bar{\mathbf{a}}^L \hat{\mathbf{w}} - \psi \bar{\mathbf{v}}^n \hat{\mathbf{n}}) + \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M \hat{\mathbf{P}}^M.$$

Substituting (20) into this expression,

$$\hat{\mathbf{P}} = \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \right) + \bar{\mathbf{x}}^C \bar{\mathbf{c}}^M (\bar{\mathbf{a}}^L \hat{\mathbf{w}} - \psi \bar{\mathbf{v}}^n \hat{\mathbf{n}}),$$

where $\bar{\mathbf{c}}^M \equiv \bar{\mathbf{I}} + \bar{\mathbf{a}}^M \bar{\mathbf{g}}^D \bar{\mathbf{x}}^M$.

By plugging (17) into this expression,

$$\begin{aligned} \hat{\mathbf{P}} &= \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \right) + \bar{\mathbf{x}}^C \bar{\mathbf{c}}^M \bar{\mathbf{a}}^L \hat{\mathbf{w}} \\ &\quad - \psi \bar{\mathbf{x}}^C \bar{\mathbf{c}}^M \bar{\mathbf{v}}^n \left(\phi \bar{\boldsymbol{\phi}}^{n,w} \hat{\mathbf{w}} + \phi \lambda \bar{\boldsymbol{\phi}}^{n,P} \hat{\mathbf{P}} \right) \end{aligned}$$

and, therefore,

$$\hat{\mathbf{P}} = \bar{\boldsymbol{\alpha}}^{P,w} \hat{\mathbf{w}} + \boldsymbol{\rho}^P \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \right), \quad (21)$$

with $\bar{\boldsymbol{\alpha}}^{P,w} \equiv \boldsymbol{\rho}^P \bar{\mathbf{x}}^C \bar{\mathbf{c}}^M (\bar{\mathbf{a}}^L - \phi \psi \bar{\mathbf{v}}^n \bar{\boldsymbol{\phi}}^{n,w})$ and $\boldsymbol{\rho}^P \equiv \left(\bar{\mathbf{I}} + \phi \psi \lambda \bar{\mathbf{x}}^C \bar{\mathbf{c}}^M \bar{\mathbf{v}}^n \bar{\boldsymbol{\phi}}^{n,P} \right)^{-1}$.

Substituting $\hat{\mathbf{P}}$ in (21) into (17), we have that

$$\hat{\mathbf{n}} = \phi \bar{\boldsymbol{\alpha}}^{n,w} \hat{\mathbf{w}} + \phi \lambda \bar{\boldsymbol{\alpha}}^{n,P} \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \right) \quad (22)$$

with $\bar{\boldsymbol{\alpha}}^{n,w} \equiv \bar{\boldsymbol{\phi}}^{n,w} + \lambda \bar{\boldsymbol{\phi}}^{n,P} \bar{\boldsymbol{\alpha}}^{P,w}$ and $\bar{\boldsymbol{\alpha}}^{n,P} \equiv \bar{\boldsymbol{\phi}}^{n,P} \boldsymbol{\rho}^P$.

Substituting $\hat{\mathbf{P}}$ in (21) into (18), we have that

$$\hat{\mathbf{N}} = \vartheta \bar{\boldsymbol{\alpha}}^{N,w} \hat{\mathbf{w}} + \vartheta \bar{\boldsymbol{\alpha}}^{N,P} \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \right) \quad (23)$$

with $\bar{\boldsymbol{\alpha}}^{N,w} \equiv \bar{\boldsymbol{\phi}}^{N,w} + \bar{\boldsymbol{\phi}}^{N,P} \bar{\boldsymbol{\alpha}}^{P,w}$ and $\bar{\boldsymbol{\alpha}}^{N,P} \equiv \bar{\boldsymbol{\phi}}^{N,P} \boldsymbol{\rho}^P$.

Substituting $\hat{\mathbf{P}}$ in (21) into (19), we have that

$$\hat{\mathbf{W}} = \bar{\boldsymbol{\alpha}}^{W,w} \hat{\mathbf{w}} + \left(\lambda \bar{\boldsymbol{\alpha}}_{\lambda}^{W,P} + \vartheta \bar{\boldsymbol{\alpha}}_{\vartheta}^{W,P} \right) \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \right) \quad (24)$$

with $\bar{\boldsymbol{\alpha}}^{W,w} \equiv \bar{\boldsymbol{\phi}}^{W,w} + ((\phi-1)\lambda \bar{\boldsymbol{\phi}}^{n,P} + \vartheta \bar{\boldsymbol{\phi}}^{N,P}) \bar{\boldsymbol{\alpha}}^{P,w}$, $\bar{\boldsymbol{\alpha}}_{\lambda}^{W,P} \equiv (\phi-1) \bar{\boldsymbol{\phi}}^{n,P} \boldsymbol{\rho}^P$, and $\bar{\boldsymbol{\alpha}}_{\vartheta}^{W,P} \equiv \bar{\boldsymbol{\phi}}^{N,P} \boldsymbol{\rho}^P$.

Finally, we can solve for the change in the input price index using (20):

$$\hat{\mathbf{P}}^M = \bar{\boldsymbol{\alpha}}^{M,w} \hat{\mathbf{w}} + \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) - \psi \phi \lambda \bar{\boldsymbol{\alpha}}_{\lambda}^{M,P} \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \right) \quad (25)$$

where $\bar{\boldsymbol{\alpha}}^{M,w} \equiv \bar{\mathbf{g}}^D \bar{\mathbf{x}}^M (\bar{\mathbf{a}}^L - \phi \psi \bar{\mathbf{v}}^n \bar{\boldsymbol{\alpha}}^{n,w})$, and $\bar{\boldsymbol{\alpha}}_{\lambda}^{M,P} \equiv \bar{\mathbf{g}}^D \bar{\mathbf{x}}^M \bar{\mathbf{v}}^n \bar{\boldsymbol{\alpha}}^{n,P}$.

Step 3. We now solve for the change in revenue of sector-market pairs. From (14), by defining $\bar{\mathbf{r}}_{NS \times N}^C \equiv [x_{ij,s} \xi_{j,s} E_j / R_{i,s}]_{is,j}$,

$$(\bar{\mathbf{I}} - \bar{\mathbf{r}}^U) \hat{\mathbf{R}} = \left[\sum_d r_{id,s} \hat{x}_{id,s} \right]_{is} + \bar{\mathbf{r}}^C \hat{\mathbf{W}},$$

where, from (6),

$$\sum_d r_{id,s} \hat{x}_{id,s} = (1-\sigma) \mu_{i,s}(\hat{\boldsymbol{\tau}}) + (1-\sigma) \sum_d r_{id,s} (\mathbb{I}_{i=j} - x_{jd,s}) \left(a_{j,s}^L \hat{w}_j + a_{j,s}^M \hat{P}_{j,s}^M - \frac{\psi}{1-n_j} \hat{n}_j \right).$$

By defining $\bar{\boldsymbol{\chi}}_{NS \times NS} \equiv [\mathbb{I}_{s=k} \sum_d r_{id,s} (\mathbb{I}_{i=j} - x_{jd,s})]_{is,jk}$ and $\bar{\mathbf{g}}^U \equiv \bar{\mathbf{I}} + \bar{\mathbf{b}}^U$, we can then write

$$\hat{\mathbf{R}} = (1-\sigma) \bar{\mathbf{g}}^U \boldsymbol{\mu}(\hat{\boldsymbol{\tau}}) + (1-\sigma) \bar{\mathbf{g}}^U \bar{\boldsymbol{\chi}} \left(\bar{\mathbf{a}}^L \hat{\mathbf{w}} - \psi \bar{\mathbf{v}}^n \hat{\mathbf{n}} + \bar{\mathbf{a}}^M \hat{\mathbf{P}}^M \right) + \bar{\mathbf{g}}^U \bar{\mathbf{r}}^C \hat{\mathbf{W}}. \quad (26)$$

Step 4. We now characterize the system in (16). The equilibrium definition in (15) implies that $\hat{W}_i = \sum_s \ell_{i,s} \hat{R}_{i,s}$, which, by defining $\bar{\boldsymbol{\ell}}_{N \times NS} \equiv [\ell_{j,s} \mathbb{I}_{[i=j]}]_{i,j,s}$, and $\bar{\mathbf{v}}_{NS \times N} = [\mathbb{I}_{i=j}]_{ik,j}$, can be written as

$$\hat{\mathbf{W}} = \bar{\boldsymbol{\ell}} \hat{\mathbf{R}}.$$

Plugging (26) into this expression, we have that

$$(\bar{\mathbf{I}} - \bar{\boldsymbol{\ell}} \bar{\mathbf{g}}^U \bar{\mathbf{r}}^C) \hat{\mathbf{W}} = (1-\sigma) \bar{\boldsymbol{\ell}} \bar{\mathbf{g}}^U \boldsymbol{\mu}(\hat{\boldsymbol{\tau}}) + (1-\sigma) \bar{\boldsymbol{\ell}} \bar{\mathbf{g}}^U \bar{\boldsymbol{\chi}} \left(\bar{\mathbf{a}}^L \hat{\mathbf{w}} - \psi \bar{\mathbf{v}}^n \hat{\mathbf{n}} + \bar{\mathbf{a}}^M \hat{\mathbf{P}}^M \right).$$

Using (22)–(24),

$$\bar{\boldsymbol{\gamma}}^w \hat{\mathbf{w}} = (1-\sigma) \bar{\boldsymbol{\ell}} \bar{\mathbf{g}}^U \boldsymbol{\mu}(\hat{\boldsymbol{\tau}}) + (1-\sigma) \bar{\boldsymbol{\alpha}}_\sigma^M \hat{\mathbf{P}}^M + (\lambda \bar{\boldsymbol{\gamma}}_\lambda^P + \vartheta \bar{\boldsymbol{\gamma}}_\vartheta^P) \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \right),$$

where

$$\bar{\boldsymbol{\gamma}}^w \equiv (\bar{\mathbf{I}} - \bar{\boldsymbol{\ell}} \bar{\mathbf{g}}^U \bar{\mathbf{y}}^C) \bar{\boldsymbol{\alpha}}^{W,w} - (1-\sigma) \bar{\boldsymbol{\ell}} \bar{\mathbf{g}}^U \bar{\boldsymbol{\chi}} (\bar{\mathbf{a}}^L - \psi \phi \bar{\mathbf{v}}^n \bar{\boldsymbol{\alpha}}^{n,w}),$$

$$\bar{\boldsymbol{\gamma}}_\lambda^P \equiv -(\bar{\mathbf{I}} - \bar{\boldsymbol{\ell}} \bar{\mathbf{g}}^U \bar{\mathbf{y}}^C) \bar{\boldsymbol{\alpha}}_\lambda^{W,P} - (1-\sigma) \psi \phi \bar{\boldsymbol{\ell}} \bar{\mathbf{g}}^U \bar{\boldsymbol{\chi}} \bar{\mathbf{v}}^n \bar{\boldsymbol{\alpha}}^{n,P},$$

$$\bar{\boldsymbol{\gamma}}_\vartheta^P \equiv -(\bar{\mathbf{I}} - \bar{\boldsymbol{\ell}} \bar{\mathbf{g}}^U \bar{\mathbf{y}}^C) \bar{\boldsymbol{\alpha}}_\vartheta^{W,P},$$

$$\bar{\boldsymbol{\alpha}}_\sigma^M \equiv \bar{\boldsymbol{\ell}} \bar{\mathbf{g}}^U \bar{\boldsymbol{\chi}} \bar{\mathbf{a}}^M.$$

Substituting $\hat{\mathbf{P}}^M$ with (25),

$$\bar{\boldsymbol{\gamma}}^w \hat{\mathbf{w}} = \hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + (1-\sigma) \bar{\boldsymbol{\alpha}}_\sigma^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) + (\lambda \bar{\boldsymbol{\alpha}}_\lambda^P + \vartheta \bar{\boldsymbol{\alpha}}_\vartheta^P) \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \right)$$

where

$$\bar{\boldsymbol{\gamma}} \equiv \bar{\boldsymbol{\gamma}}^w - (1-\sigma) \bar{\boldsymbol{\alpha}}_\sigma^M \bar{\boldsymbol{\alpha}}^{M,w} \quad (27)$$

$$\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) \equiv (1-\sigma) \bar{\boldsymbol{\ell}} \bar{\mathbf{g}}^U \boldsymbol{\mu}(\hat{\boldsymbol{\tau}}), \quad \bar{\boldsymbol{\alpha}}_\lambda^P \equiv \bar{\boldsymbol{\gamma}}_\lambda^P - (1-\sigma) \psi \phi \bar{\boldsymbol{\alpha}}_\sigma^M \bar{\boldsymbol{\alpha}}_\lambda^{M,P}, \quad \bar{\boldsymbol{\alpha}}_\vartheta^P \equiv \bar{\boldsymbol{\gamma}}_\vartheta^P.$$

Thus,

$$\bar{\boldsymbol{\gamma}} \hat{\mathbf{w}} = \hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}})$$

with

$$\hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}}) \equiv \hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + (\lambda \bar{\boldsymbol{\alpha}}_\lambda^P + \vartheta \bar{\boldsymbol{\alpha}}_\vartheta^P) \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + ((1-\sigma) \bar{\boldsymbol{\alpha}}_\sigma^M + \lambda \bar{\boldsymbol{\alpha}}_\lambda^M + \vartheta \bar{\boldsymbol{\alpha}}_\vartheta^M) \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \quad (28)$$

$$\bar{\boldsymbol{\alpha}}_\lambda^M \equiv \bar{\boldsymbol{\alpha}}_\lambda^P \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M, \quad \text{and} \quad \bar{\boldsymbol{\alpha}}_\vartheta^M \equiv \bar{\boldsymbol{\alpha}}_\vartheta^P \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M.$$

Part B: Derivation of the reduced-form response of wages to shifts in excess labor demand. We now establish that the system above, $\bar{\gamma}\hat{w} = \eta(\hat{\tau})$, yields a reduced-form representation for wage changes, $\hat{w} = \bar{\beta}\eta(\hat{\tau})$, where $\bar{\beta}$ has a series expansion representation. We impose the same diagonal dominance condition: In any equilibrium, there is a vector $\{h_i\}_{i \neq m} \gg 0$ such that, for all $i \neq m$,

$$h_i \gamma_{ii} > \sum_{j \neq i, m} |\gamma_{ij}| h_j. \quad (29)$$

The proof now proceeds in the same way as the proof in Section A.1.2. We start by redefining the system to set the change in the wage of market m to zero. Using the same matrix $\bar{M}$ defined in Section A.1.2, we show that, if $\bar{M}\bar{\gamma}\bar{M}'$ is non-singular, then $\hat{w} = \bar{\beta}\eta(\hat{\tau})$ for $\bar{\beta} \equiv \bar{M}'(\bar{M}\bar{\gamma}\bar{M}')^{-1}\bar{M}$. In the rest of the proof, we first show that $\bar{\beta}$ exists and then that it admits a series representation. To simplify exposition, we again abuse notation by defining

$$\bar{\gamma} \equiv \bar{M}\bar{\gamma}\bar{M}', \quad \hat{w} \equiv \bar{M}\hat{w} \quad \text{and} \quad \hat{\eta} \equiv \bar{M}\hat{\eta}(\hat{\tau}).$$

This modified system does not include the row associated with the market clearing condition of market m and imposes that $\hat{w}_m = 0$. To obtain a characterization for the solution of this system, let $\bar{\lambda}$ be the diagonal matrix with the diagonal elements of $\bar{\gamma}$: $\bar{\lambda}$ s.t. $\lambda_{ii} = \gamma_{ii}$ and $\lambda_{ij} = 0$ for $i \neq j$. Thus, we can write the system as

$$\bar{\gamma} = \bar{\lambda}(\bar{I} - \bar{\gamma}) \quad \text{st} \quad \bar{\gamma} \equiv \bar{I} - \bar{\lambda}^{-1}\bar{\gamma},$$

which implies that $\tilde{\gamma}_{ii} = 0$ and $\tilde{\gamma}_{ij} = -\gamma_{ij}/\gamma_{ii}$. Let $\bar{h}$ be the diagonal matrix such that h_i is the diagonal entry in row i . Thus, $\bar{\gamma}\hat{w} = \eta(\hat{\tau})$ is equivalent to

$$\begin{aligned} \bar{\lambda}(\bar{I} - \bar{\gamma})(\bar{h}\bar{h}^{-1})\hat{w} &= \hat{\eta} \\ \bar{\lambda}(\bar{h} - \bar{\gamma}\bar{h})\bar{h}^{-1}\hat{w} &= \hat{\eta} \\ (\bar{\lambda}\bar{h})(\bar{I} - (\bar{h}^{-1}\bar{\gamma}\bar{h}))\bar{h}^{-1}\hat{w} &= \hat{\eta}, \end{aligned}$$

which implies that

$$\hat{w} = \bar{h}(\bar{I} - \bar{\gamma})^{-1}(\bar{\lambda}\bar{h})^{-1}\hat{\eta}, \quad \bar{\gamma} \equiv \bar{h}^{-1}\bar{\gamma}\bar{h}. \quad (30)$$

Notice that, for all i , $\tilde{\gamma}_{ii} = 0$ and $\tilde{\gamma}_{ij} = -\gamma_{ij}h_j/\gamma_{ii}h_i$.

First, we show that $(\bar{I} - \bar{\gamma})$ is non-singular, so that we can write the expression in (30). We proceed by contradiction. Suppose that $(\bar{I} - \bar{\gamma})$ is singular, so $\mu = 0$ is an eigenvalue of $(\bar{I} - \bar{\gamma})$. Take the eigenvector $\mathbf{x}$ associated with the zero eigenvalue and normalize it such that $x_i = 1$ and $|x_j| \leq 1$. Notice that $(\bar{I} - \bar{\gamma})\mathbf{x} = 0$, so that the i -row of this system is

$$1 + \sum_{j \neq i, m} -\tilde{\gamma}_{ij}x_j = 0 \implies 1 + \sum_{j \neq i, m} \frac{\gamma_{ij}}{\gamma_{ii}} \frac{h_j}{h_i} x_j = 0.$$

Thus,

$$\gamma_{ii}h_i = - \sum_{j \neq i, m} \gamma_{ij}h_jx_j \leq \left| \sum_{j \neq i, m} \gamma_{ij}h_jx_j \right| \leq \sum_{j \neq i, m} |\gamma_{ij}|h_j|x_j| \leq \sum_{j \neq i, m} |\gamma_{ij}|h_j$$

where the last inequality holds because $|x_j| \leq 1$ and $h_j > 0$. Thus, $\gamma_{ii}h_i \leq \sum_{j \neq i, m} |\gamma_{ij}|h_j$, which contradicts (29).

Second, we show that $(\bar{I} - \bar{\gamma})^{-1}$ admits a series representation. This is true whenever the largest eigenvalue of $\bar{\gamma}$ is below one. To show this, we proceed by contradiction. Assume that the largest eigenvalue μ is weakly greater than one. Take the eigenvector $\mathbf{x}$ associated with the largest eigenvalue and normalize it such that

$x_i = 1$ and $|x_j| \leq 1$. Notice that $\mu \mathbf{x} = \bar{\bar{\gamma}} \mathbf{x}$ so that the i -row of this system is

$$1 \leq \mu = \sum_{j \neq i, m} -\frac{\gamma_{ij}}{\gamma_{ii}} \frac{h_j}{h_i} x_j$$

Since γ_{ii} and h_i are positive,

$$\gamma_{ii} h_i \leq - \sum_{j \neq i, m} \gamma_{ij} h_j x_j \leq \sum_{j \neq i, m} \gamma_{ij} h_j x_j \leq \sum_{j \neq i, m} |\gamma_{ij}| |h_j| |x_j|$$

Since $|x_j| \leq 1$ and $h_j > 0$, $\sum_{j \neq i, m} |\gamma_{ij}| |h_j| |x_j| \leq \sum_{j \neq i, m} |\gamma_{ij}| h_j$. Thus, $\gamma_{ii} h_i \leq \sum_{j \neq i, m} |\gamma_{ij}| h_j$, which contradicts (29). Thus, the largest eigenvalue of $\bar{\bar{\gamma}}$ is below one, allowing us to write $(\bar{\mathbf{I}} - \bar{\bar{\gamma}})^{-1} = \sum_{d=0}^{\infty} (\bar{\bar{\gamma}})^d$. Substituting this series expansion into (30) yields

$$\hat{\mathbf{w}} = \sum_{d=0}^{\infty} \left(\bar{\mathbf{h}} (\bar{\bar{\gamma}})^d \bar{\mathbf{h}}^{-1} \right) \bar{\boldsymbol{\lambda}}^{-1} \hat{\boldsymbol{\eta}}.$$

Finally, to establish the result, we can follow the same steps in Appendix A.1.2 to show that $\bar{\mathbf{h}} (\bar{\bar{\gamma}})^d \bar{\mathbf{h}}^{-1} = (\bar{\bar{\gamma}})^d$. Thus,

$$\hat{\mathbf{w}} = \bar{\boldsymbol{\beta}} \hat{\boldsymbol{\eta}} = \sum_{d=0}^{\infty} (\bar{\bar{\gamma}})^d \bar{\boldsymbol{\lambda}}^{-1} \hat{\boldsymbol{\eta}},$$

which immediately implies that

$$\hat{w}_i = \sum_j \beta_{ij} \hat{\eta}_j(\hat{\boldsymbol{\tau}}) \quad \text{where} \quad \beta_{ij} = \frac{1}{\gamma_{ii}} \left(\mathbb{I}_{[i=j]} - \frac{\gamma_{ij}}{\gamma_{jj}} \mathbb{I}_{[i \neq j]} \right) + \sum_{d=2}^{\infty} \frac{\tilde{\gamma}_{ij}^{(d)}}{\gamma_{jj}} \quad (31)$$

with $\tilde{\gamma}_{ij}^{(d)}$ denoting the i - j entry of $(\bar{\bar{\gamma}})^d$ defined as $\tilde{\gamma}_{ij} \equiv -\frac{\gamma_{ij}}{\gamma_{ii}} \mathbb{I}_{[i \neq j; i, j \neq m]}$.

Part C: Reduced-form representation for changes in labor market outcomes. We start by writing the reduced-form response in wages:

$$\hat{\mathbf{w}} = \bar{\boldsymbol{\beta}}^R \hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^C \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \quad (32)$$

with

$$\bar{\boldsymbol{\beta}}^R = \bar{\boldsymbol{\beta}}, \quad \bar{\boldsymbol{\beta}}^C \equiv \bar{\boldsymbol{\beta}} (\lambda \bar{\boldsymbol{\alpha}}_{\lambda}^P + \vartheta \bar{\boldsymbol{\alpha}}_{\vartheta}^P), \quad \bar{\boldsymbol{\beta}}^M \equiv \bar{\boldsymbol{\beta}} ((1-\sigma) \bar{\boldsymbol{\alpha}}_{\sigma}^M + \lambda \bar{\boldsymbol{\alpha}}_{\lambda}^M + \vartheta \bar{\boldsymbol{\alpha}}_{\vartheta}^M).$$

The combination of (32) and (22)–(23) implies

$$\hat{\mathbf{n}} = \bar{\boldsymbol{\beta}}^{n,R} \hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{n,C} \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{n,M} \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \quad (33)$$

$$\hat{\mathbf{N}} = \bar{\boldsymbol{\beta}}^{N,R} \hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{N,C} \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{N,M} \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \quad (34)$$

where

$$\begin{aligned} \bar{\boldsymbol{\beta}}^{n,R} &\equiv \phi \bar{\boldsymbol{\alpha}}^{n,w} \bar{\boldsymbol{\beta}}^R, & \bar{\boldsymbol{\beta}}^{n,C} &\equiv \phi \left(\bar{\boldsymbol{\alpha}}^{n,w} \bar{\boldsymbol{\beta}}^C + \lambda \bar{\boldsymbol{\alpha}}^{n,P} \right), & \bar{\boldsymbol{\beta}}^{n,M} &\equiv \phi \left(\bar{\boldsymbol{\alpha}}^{n,w} \bar{\boldsymbol{\beta}}^M + \lambda \bar{\boldsymbol{\alpha}}^{n,P} \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M \right) \\ \bar{\boldsymbol{\beta}}^{N,R} &\equiv \vartheta \bar{\boldsymbol{\alpha}}^{N,w} \bar{\boldsymbol{\beta}}^R, & \bar{\boldsymbol{\beta}}^{N,C} &\equiv \vartheta \left(\bar{\boldsymbol{\alpha}}^{N,w} \bar{\boldsymbol{\beta}}^C + \bar{\boldsymbol{\alpha}}^{N,P} \right), & \bar{\boldsymbol{\beta}}^{N,M} &\equiv \vartheta \left(\bar{\boldsymbol{\alpha}}^{N,w} \bar{\boldsymbol{\beta}}^M + \bar{\boldsymbol{\alpha}}^{N,P} \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M \right) \end{aligned}$$

Finally, from the expression for $\Delta \overline{\ln w_i}$ in (9), $\Delta \overline{\ln w} = \hat{w} - (1/\phi)\hat{n}$. By substituting (31) and (33),

$$\Delta \overline{\ln w} = \bar{\beta}^{w,R} \hat{\eta}^R(\hat{\tau}) + \bar{\beta}^{w,C} \hat{\eta}^C(\hat{\tau}) + \bar{\beta}^{w,M} \hat{\eta}^M(\hat{\tau}) \quad (35)$$

where

$$\bar{\beta}^{w,R} \equiv \bar{\beta}^R - (1/\phi)\bar{\beta}^{n,R}, \quad \bar{\beta}^{w,C} \equiv \bar{\beta}^C - (1/\phi)\bar{\beta}^{n,C}, \quad \bar{\beta}^{w,M} \equiv \bar{\beta}^M - (1/\phi)\bar{\beta}^{n,M}.$$

Part D: Empirical specification in (34)–(35). For outcomes $\Delta \ln Y_i \in \{\Delta \ln n_i, \Delta \ln N_i, \Delta \overline{\ln w}\}$, equations (33)–(35) imply that, up to a first-order approximation,

$$\Delta \ln Y = \bar{\beta}^{Y,R} \hat{\eta}^R(\hat{\tau}) + \bar{\beta}^{Y,C} \hat{\eta}^C(\hat{\tau}) + \bar{\beta}^{Y,M} \hat{\eta}^M(\hat{\tau}).$$

By definition, $\hat{\eta}^R(\hat{\tau})$, $\hat{\eta}^C(\hat{\tau})$ and $\hat{\eta}^M(\hat{\tau})$ are linear combinations of $\hat{\tau}_{ij,s}$. Thus, $\hat{\tau} = \hat{\tau}^{\text{obs}} + \hat{\tau}^{\text{unbs}}$ implies that

$$\Delta \ln Y = \alpha^Y + \bar{\beta}^{Y,R} \hat{\eta}^R(\hat{\tau}^{\text{obs}}) + \bar{\beta}^{Y,C} \hat{\eta}^C(\hat{\tau}^{\text{obs}}) + \bar{\beta}^{Y,M} \hat{\eta}^M(\hat{\tau}^{\text{obs}}) + \nu^Y$$

where $\nu \equiv \bar{\beta}^{Y,R} \hat{\eta}^R(\hat{\tau}^{\text{unbs}}) + \bar{\beta}^{Y,C} \hat{\eta}^C(\hat{\tau}^{\text{unbs}}) + \bar{\beta}^{Y,M} \hat{\eta}^M(\hat{\tau}^{\text{unbs}})$, $\alpha^Y \equiv I^{-1} \sum_i E[\nu_i | \mathcal{W}^0]$, and $\nu_i^Y \equiv \nu_i - \alpha^Y$.

We now establish that $E[\nu_i^Y \hat{\eta}_j^E(\hat{\zeta})] = 0$ for $E \in \{R, C, M\}$. The proof is analogous to that in Appendix A.1.9. First notice that the definitions of $\bar{\beta}^{Y,E}$ and $\hat{\eta}^E(\cdot)$ imply that we can write

$$\nu_i^Y = \sum_{s,d,o} \beta_{i,sdo}^{Y,\tau}(\theta | \mathcal{W}^0) \hat{\tau}_{od,s}^{\text{unbs}} - \alpha^Y, \quad \text{and} \quad \hat{\eta}_j^E(\hat{\zeta}) = \sum_{s,d,o} H_{j,sdo}^{E,\tau}(\mathcal{W}^0) \hat{\zeta}_{od,s}.$$

Thus,

$$\begin{aligned} E[\nu_i^Y \hat{\eta}_j^E(\hat{\zeta}) | \mathcal{W}^0] &= \sum_{s,d,o} \sum_{s',d',o'} \beta_{i,sdo}^{Y,\tau}(\theta | \mathcal{W}^0) H_{j,s'd'o'}^{\tau}(\mathcal{W}^0) E[\hat{\tau}_{od,s}^{\text{unbs}} \hat{\zeta}_{o'd',s'} | \mathcal{W}^0] \\ &\quad - \alpha^Y \sum_{s,d,o} H_{j,sdo}^{\tau}(\mathcal{W}^0) E[\hat{\zeta}_{o'd',s'} | \mathcal{W}^0] \end{aligned}$$

Assumption 1 implies that $E[\hat{\zeta}_{o'd',s'} | \mathcal{W}^0] = E[\hat{\tau}_{od,s}^{\text{unbs}} \hat{\zeta}_{o'd',s'} | \mathcal{W}^0] = 0$ and thus $E[\nu_i^Y \hat{\eta}_j^E(\hat{\zeta})] = E[E[\nu_i^Y \hat{\eta}_j^E(\hat{\zeta}) | \mathcal{W}^0]] = 0$ for any i and j . This immediately implies that (35) holds for any real matrix h_{ij}^E .

A.2.5 Extension of Reduced-Form Response in (22) for a Symmetric Economy with Intermediate Inputs in Production

There are no trade costs, $\tau_{ij,s} = \tau_{i,s}$ for all j , the input spending shares are the same in all sectors and markets, $a_{i,s}^M = a^M$ and $\xi_{i,ks}^M = \xi_k^M$. We consider the same labor supply structure in Section 3 where $\lambda = \vartheta = 0$. Thus,

$$P_{i,s}^M = P^M = \Pi_k \left[\sum_o (\tau_{o,k} p_o)^{1-\sigma} \right]^{\frac{\xi_k}{1-\sigma}},$$

$$p_i = (w_i)^{1-\psi\phi-a^M} (P^M)^{a^M} \bar{b}_i^{\psi\phi} (\Omega(\mathbf{w}))^{\psi\phi}.$$

By defining $\kappa \equiv (\sigma-1)(1-\psi\phi-a^M)$, these expressions imply that

$$x_{ij,s} = x_{i,s} = \frac{(\tau_{i,s} p_i)^{1-\sigma}}{\sum_o (\tau_{o,s} p_o)^{1-\sigma}} = \frac{\tau_{i,s}^{1-\sigma} w_i^{-\kappa} \bar{b}_i^{\psi\phi(1-\sigma)}}{\sum_o \tau_{o,s}^{1-\sigma} w_o^{-\kappa} \bar{b}_o^{\psi\phi(1-\sigma)}}.$$

In this case, labor market clearing requires that

$$\begin{aligned} W_i &= (1-a^M) \sum_s \sum_j x_{i,s} (\xi_s W_j + \sum_k \xi_s^M a^M R_{j,k}) \\ W_i &= \sum_s \sum_j x_{i,s} ((1-a^M) \xi_s W_j + a^M \xi_s^M \sum_k W_{j,k}) \\ W_i &= \sum_s \sum_j x_{i,s} ((1-a^M) \xi_s + a^M \xi_s^M) W_j \\ W_i &= \sum_s \sum_j x_{i,s} e_s W_j \end{aligned}$$

where $e_s \equiv (1-a^M) \xi_s + a^M \xi_s^M$ is the share of gross spending on sector s (common to all markets i).

Finally, the supply of labor efficiency units is

$$W_j = w_j^\phi \left(w_j^\phi + \bar{b}_j^\phi (\Omega(\mathbf{w}))^\phi \right)^{\frac{1-\phi}{\phi}} \bar{N}_j \varrho.$$

The combination of the equilibrium conditions above implies that the equilibrium wage vector solves $D_i(\mathbf{w}|\boldsymbol{\tau})=0$ for all i such that

$$D_i(\mathbf{w}|\boldsymbol{\tau}) \equiv \sum_s \sum_j \left[\frac{\tau_{i,s}^{1-\sigma} w_i^{-\kappa} \bar{b}_i^{\psi\phi(1-\sigma)}}{\sum_o \tau_{o,s}^{1-\sigma} w_o^{-\kappa} \bar{b}_o^{(1-\sigma)\psi\phi}} e_s - \mathbb{I}_{i=j} \right] w_j^\phi \left(w_j^\phi + \bar{b}_j^\phi (\Omega(\mathbf{w}))^\phi \right)^{\frac{1-\phi}{\phi}} \bar{N}_j \varrho.$$

Given the alternative definition of the labor demand parameter κ , this excess labor demand system is isomorphic to that in (12) for the model of Section 3 in the special case of $\tau_{ij,s} = \tau_{i,s}$ and $\xi_{j,s} = e_s$ for all j . This implies that Corollary 3 holds for this economy, and wage responses are given by (22).

A.2.6 Adding Endogenous Transfers to the Model

We now extend our model to allow for endogenous transfers across markets to finance the non-employment benefits. We maintain the same assumptions of Section 4 for production and labor supply. In this case, it is useful to write the location choice in terms of per-capita spending in each market, so that

$$\frac{N_i}{N_j} = \frac{P_i^{-\vartheta} (E_i/N_i)^\vartheta}{P_j^{-\vartheta} (E_j/N_j)^\vartheta}$$

and, therefore,

$$N_i = \frac{(E_i/P_i)^{\frac{\vartheta}{1+\vartheta}}}{\sum_{j \in \mathcal{I}_{c(i)}} (E_j/P_j)^{\frac{\vartheta}{1+\vartheta}}}. \quad (36)$$

We further assume that a fraction ϖ of non-employment benefits is financed with local taxes and that the remaining balance is financed with a common national income tax. Hence,

$$E_i = W_i + B_i - v_i(W_i + B_i) - v_c(W_i + B_i)$$

such that, in equilibrium,

$$v_i(W_i + B_i) = \varpi B_i, \quad \text{and} \quad v_c \sum_{i \in \mathcal{I}_c} (W_i + B_i) = (1-\varpi) \sum_{i \in \mathcal{I}_c} B_i.$$

Using the fact that $\frac{W_i}{B_i} = \frac{n_i}{1-n_i}$, this expression is equivalent to

$$E_i = W_i + (1-\varpi) \left[1 - n_i - \frac{\sum_{j \in \mathcal{I}_{c(i)}} (1-n_j)(W_j/n_j)}{\sum_{j \in \mathcal{I}_{c(i)}} (W_j/n_j)} \right] \frac{W_i}{n_i}. \quad (37)$$

Notice that, as in our baseline specification, $E_i = W_i$ if $\varpi = 1$. For any $\varpi \in [0, 1]$, equations (8), (36), and (37) constitute a system of equations that can be locally solved as a function of $\mathbf{w}$ and $\mathbf{P}$. Conditional on these expressions, we can follow exactly the same steps in Appendix A.2.4 to characterize the reduced-form responses in the model.

A.3 Proofs of General Version of the Model in Section 4

A.3.1 Environment

Suppose that each country c is populated by a continuum of individuals divided into multiple groups indexed by $g = 1, \dots, G$. Workers of group g in market i receive a wage of w_{gi} for each efficiency unit supplied. As in the baseline, each market has a competitive representative firm that produces a differentiated tradable intermediate good in each sector s , whose endogenous production cost is $p_{i,s}$ and iceberg trade cost of selling in j is $\tau_{ij,s}$.¹ There is a representative firm that produces a single non-tradable final consumption good in each market, and sells it at price of P_i .

We now present the three central parts of the model that, up to a first-order approximation, yield log-linear equilibrium relationships that are sufficient to derive the reduced-form representation for the model's predictions.

Labor Supply. Assume that worker preferences and efficiency implicitly define all labor outcomes as a (local) function of the wage rate and the price of the consumption good across markets. That is, for any group g in market i , we have the following local representation for one outcome Y_{gi} out of the employment rate n_{gi} , population N_{gi} , employment L_{gi} , spending E_{gi} , wage bill W_{gi} , or log average wage $\Delta \ln w_{gi}$:

$$Y_{gi} = \Phi_{gi}^Y(\{w_{fj}\}_{f,j}, \{P_j\}_j).$$

Notice that the models in Sections 3 and 4 satisfy this general restriction. It allows for (endogenous or exogenous) transfers across markets (for example, as in Appendix A.2.6). The general representation also covers a generalized Roy model with arbitrary individual heterogeneity in market-specific efficiency and preferences (for example, as in [Adão \(2016\)](#)). In addition, it allows for a rich structure of preferences for leisure and home production (as specified in Appendix B of the old version of our paper, [Adao et al. \(2019a\)](#)), as well as competitive search environments (such as the one described in Appendix A.4).

The first-order approximation for changes in any outcome Y_{gi} is

$$\hat{Y}_{gi} = \sum_{f,j} \phi_{gi,fj}^{Y,w} \hat{w}_{fj} + \sum_j \phi_{gi,j}^{Y,P} \hat{P}_j, \quad (38)$$

where $\phi_{gi,fj}^{Y,w} \equiv \frac{\partial \ln \Phi_{gi}^Y(\mathbf{w}, \mathbf{P})}{\partial \ln w_{fj}}$ and $\phi_{gi,j}^{Y,P} \equiv \frac{\partial \ln \Phi_{gi}^Y(\mathbf{w}, \mathbf{P})}{\partial \ln P_j}$ are the labor supply elasticities with respect to wages and prices, respectively.

Final Consumption Good. Assume that the production function for the final good combines the differentiated good from all origins: $C_j = F_j^C(\{c_{ij,s}\}_{i,s})$ where $c_{ij,s}$ is the quantity of the differentiated good of sector s from i used to produce the final good in market j . Assume that F_j^C is continuous, twice differentiable, increasing in all arguments, strictly quasi-concave, and homogeneous of degree one. Thus, cost minimization and zero profit imply that

$$P_j(\{\tau_{oj,k} p_{o,k}\}_{o,k}) \equiv \min_{\{c_{ij,s}\}_{i,s}} \sum_{o,k} \tau_{oj,k} p_{o,k} \quad \text{such that} \quad F_j^C(\{c_{ij,s}\}_{i,s}) = 1.$$

¹It is straightforward to define markets as groups of sectors by assuming that, for a subset of sectors, $\tau_{ij,s} = \infty$ for all j (including i).

The first-order approximation for changes in the final good price and final spending shares are given by

$$\hat{P}_j = \sum_{i,s} x_{ij,s}^C (\hat{\tau}_{ij,s} + \hat{p}_{i,s}), \quad \text{and} \quad \hat{x}_{ij,s}^C = \sum_{o,k} \chi_{ij,s,ok}^C (\hat{\tau}_{oj,k} + \hat{p}_{o,k}), \quad (39)$$

where $x_{ij,s}^C \equiv \frac{\partial \ln P_j(\{\tau_{oj,k} p_{o,k}\}_{o,k})}{\partial \ln(\tau_{ij,s} p_{i,s})}$ is the share of good s from i in final spending of market j , and $\chi_{ij,s,ok}^C \equiv \frac{\partial^2 \ln P_j(\{\tau_{oj,k} p_{o,k}\}_{o,k})}{\partial \ln(\tau_{ij,s} p_{i,s}) \partial \ln(\tau_{oj,k} p_{o,k})}$ is the elasticity of $x_{ij,s}^C$ to changes in the cost of good k from o . This final good production structure allows for arbitrary final spending shares and cross-price elasticities in the initial equilibrium. It is equivalent to allowing individuals to have arbitrary homothetic preferences for the differentiated products of different sectors and origins.

Differentiated Good. Assume that the production function for the differentiated good of sector s from market i is subject to external economies of scale and combines labor of different groups and inputs from different sectors and origins. That is, $Q_{i,s} = \Psi_{i,s}(\{n_{gj}, N_{gj}\}_{g,j}) F_{i,s}(\{L_{gi,s}\}_g, F_{i,s}^M(\{M_{oi,ks}\}_{o,k}))$, where $L_{gi,s}$ is the number of efficiency units employed in sector s of market i , $M_{oi,ks}$ is the quantity of the differentiated good of sector k from o used to produce good s in market i , and $\Psi_{i,s}(\{n_{gj}, N_{gj}\}_{g,j})$ is the endogenous productivity term (but external to the firm) in sector s of market i that depends on employment outcomes across groups and markets. Assume that $F_{i,s}$ and $F_{i,s}^M$ are continuous, twice differentiable, increasing in all arguments, strictly quasi-concave, and homogeneous of degree one. This production function allows us to solve the firm's cost minimization problem in two stages.

Consider first the cost minimization problem of selecting intermediate inputs:

$$P_{i,s}^M(\{\tau_{oi,k} p_{o,k}\}_{o,k}) \equiv \min_{\{M_{oi,ks}\}_{o,k}} \sum_{o,k} \tau_{oi,k} p_{o,k} M_{oi,ks} \quad \text{s.t.} \quad F_{i,s}^M(\{M_{oi,ks}\}_{o,k}) = 1,$$

which implies that

$$\hat{P}_{i,s}^M = \sum_{j,k} x_{ji,ks}^M (\hat{\tau}_{ji,k} + \hat{p}_{j,k}), \quad \text{and} \quad \hat{x}_{ji,ks}^M = \sum_{o,h} \chi_{ji,ks,oh}^M (\hat{\tau}_{oi,h} + \hat{p}_{o,h}), \quad (40)$$

where $x_{ji,ks}^M = \frac{\partial \ln P_{i,s}^M(\{\tau_{oi,k} p_{o,k}\}_{o,k})}{\partial \ln(\tau_{ji,k} p_{j,k})}$ is the share of spending on sector k from j in the total input spending of sector s in market i , and $\chi_{ji,ks,oh}^M \equiv \frac{\partial^2 \ln P_{i,s}^M(\{\tau_{oi,k} p_{o,k}\}_{o,k})}{\partial \ln(\tau_{ji,k} p_{j,k}) \partial \ln(\tau_{oi,h} p_{o,h})}$ is the elasticity of $x_{ji,ks}^M$ to changes in the cost of the good from sector h of market o .

We then solve the firm's optimal spending on labor and inputs:

$$c_{i,s}(\{w_{gi}\}_g, P_{i,s}^M) = \min_{\{L_{gi,s}\}_g, M_{i,s}} \sum_g w_{gi} L_{gi,s} + P_{i,s}^M M_{i,s} \quad \text{s.t.} \quad F_{i,s}(\{L_{gi,s}\}_g, M_{i,s}) = 1,$$

which implies that

$$\hat{c}_{i,s} = \sum_g a_{gi,s}^L \hat{w}_{gi} + a_{i,s}^M \hat{P}_{i,s}^M, \quad \hat{a}_{gi,s}^L = \sum_{g'} \epsilon_{gi,s,g'}^L \hat{w}_{g'i} + \epsilon_{gi,s}^{LM} \hat{P}_{i,s}^M, \quad \text{and} \quad \hat{a}_{i,s}^M = \sum_g \epsilon_{gi,s}^{LM} \hat{w}_{gi} + \epsilon_{i,s}^M \hat{P}_{i,s}^M \quad (41)$$

where $a_{gi,s}^L \equiv \frac{\partial \ln c_{i,s}(\{w_{gi}\}_g, P_{i,s}^M)}{\partial \ln w_{gi}}$ and $a_{i,s}^M \equiv \frac{\partial \ln c_{i,s}(\{w_{gi}\}_g, P_{i,s}^M)}{\partial \ln P_{i,s}^M}$ are the shares of labor and inputs on the total cost of sector s from i , and $\epsilon_{gi,s,g'}^L \equiv \frac{\partial^2 \ln c_{i,s}(\{w_{gi}\}_g, P_{i,s}^M)}{\partial \ln w_{gi} \partial \ln w_{g'i}}$, $\epsilon_{gi,s}^{LM} \equiv \frac{\partial^2 \ln c_{i,s}(\{w_{gi}\}_g, P_{i,s}^M)}{\partial \ln w_{gi} \partial \ln P_{i,s}^M}$ and $\epsilon_{i,s}^M \equiv \frac{\partial^2 \ln c_{i,s}(\{w_{gi}\}_g, P_{i,s}^M)}{\partial \ln P_{i,s}^M \partial \ln P_{i,s}^M}$ are the elasticities of labor and input cost shares with respect to changes in wages and input prices.

Expressions in (40)–(41) allow for a flexible structure of production. In the initial equilibrium, each

sector-market pair can have arbitrary spending shares on labor of different groups and intermediate goods from different sectors and origins. In addition, we flexibly allow for a nested elasticity structure in the labor and input demand functions. We do not impose parametric restrictions on the cross-price elasticity matrix for intermediate spending, allowing for different substitution patterns across goods from different sectors and markets. This structure also yields an arbitrary demand substitution pattern for labor of different groups. Importantly, while changes in the unit input cost can differentially affect demand for different labor groups, the nested production function imposes that the cost of inputs of distinct sectors and origins only affect factor demand through a single unit input cost index.

Finally, production cost of s in i is $p_{i,s} = c_{i,s}(\{w_{gi}\}_g, P_{i,s}^M) / \Psi_{i,s}(\{n_{gj}, N_{gj}\}_{g,j})$, which implies that

$$\hat{p}_{i,s} = \sum_g a_{gi,s}^L \hat{w}_{gi} + a_{i,s}^M \hat{P}_{i,s}^M - \sum_{g,j} \psi_{is,gj}^n \hat{n}_{gj} - \sum_{g,j} \psi_{is,gj}^N \hat{N}_{gj}, \quad (42)$$

where $\psi_{is,gj}^n \equiv \frac{\partial \ln \Psi_{i,s}(\{n_{gj}, N_{gj}\}_{g,j})}{\partial \ln n_{gj}}$ and $\psi_{is,gj}^N \equiv \frac{\partial \ln \Psi_{i,s}(\{n_{gj}, N_{gj}\}_{g,j})}{\partial \ln N_{gj}}$. This general agglomeration elasticity matrix allows for technology diffusion between regions, as in ? and ?, and differences across sectors in economies of scale – e.g., ?, ?, Kucheryavyy et al. (2023). In addition, the cross-market elasticity of labor productivity may also incorporate congestion forces implied by the re-allocation of other factors of production, like land and capital (see Appendix B of the old version of our paper, Adao et al. (2019a)).

Equilibrium. The equilibrium requires both goods and labor markets to clear. For every sector s and market i , the vector of gross revenues $\{R_{i,s}\}_{i,s}$ must solve

$$R_{i,s} = \sum_j x_{ij,s}^C E_j + \sum_j x_{ij,sk}^M a_{j,k}^M R_{j,k}. \quad (43)$$

For every group g and market i , the vector of wages must guarantee that

$$W_{gi} = \sum_s a_{gi,s}^L R_{i,s}. \quad (44)$$

A.3.2 Reduced-form Representation

The following proposition characterizes the reduced-form representation of the wage change for each group and market as a function of measures of market-level shock exposure.

Proposition 1. *For an arbitrary $\hat{\tau} \equiv \{\hat{\tau}_{ij,s}\}_{ij,s}$, the vector of wage change, $\hat{\mathbf{w}} \equiv \{\hat{w}_{gi}\}_{gi}$, solves $\bar{\gamma} \hat{\mathbf{w}} = \boldsymbol{\eta}(\hat{\tau})$. Under the diagonal dominance condition in (29), $\hat{\mathbf{w}}$ has a representation of the form:*

$$\hat{\mathbf{w}} = \bar{\beta} \hat{\boldsymbol{\eta}}(\hat{\tau}) \quad \text{such that} \quad \hat{\boldsymbol{\eta}}(\hat{\tau}) = \hat{\boldsymbol{\eta}}^R(\hat{\tau}) + \bar{\alpha}^C \hat{\boldsymbol{\eta}}^C(\hat{\tau}) + \bar{\alpha}^M \hat{\boldsymbol{\eta}}^M(\hat{\tau}), \quad (45)$$

where

$$\eta_{gi}^R(\hat{\tau}) = \sum_s \ell_{gi,s}^0 (\mu_{i,s}(\hat{\tau}) + \sum_{j,k} b_{is,jk}^U \mu_{j,k}(\hat{\tau})), \quad \eta_i^C(\hat{\tau}) = \sum_{s,o} x_{oi,s}^C \hat{\tau}_{oi,s}, \quad \eta_{i,s}^M(\hat{\tau}) = \mu_{i,s}^M(\hat{\tau}) + \sum_{j,k} b_{is,jk}^D \mu_{j,k}^M(\hat{\tau}) \quad (46)$$

such that

$$\mu_{i,s}(\hat{\tau}) \equiv \sum_{o,h} \sum_j (r_{ij,s}^C \chi_{ij,s,oh}^C + \sum_k r_{ij,sk}^U \chi_{ij,sk,oh}^M) \hat{\tau}_{oj,h} \quad \text{and} \quad \mu_{i,s}^M(\hat{\tau}) \equiv \sum_{j,k} x_{ji,ks}^M \hat{\tau}_{ji,k}, \quad (47)$$

$$\bar{\mathbf{b}}^U \equiv \sum_{d=1}^{\infty} (\bar{\mathbf{r}}^U)^d \quad \text{and} \quad \bar{\mathbf{b}}^D \equiv \sum_{d=1}^{\infty} (\bar{\mathbf{x}}^D)^d, \quad (48)$$

with $\bar{\mathbf{r}}_{NS \times NS}^U \equiv [x_{ij,sk}^M a_{j,k}^M R_{j,k}/R_{i,s}]_{is,jk}$, $\bar{\mathbf{x}}_{NS \times NS}^D \equiv [x_{ji,ks}^M a_{j,k}^M]_{is,jk}$, $\mathbf{W}^0 \equiv \{\{x_{ij,s}^C\}_j, \{x_{ij,sk}^M\}_{j,k}, \{a_{gi,s}^L\}_g\}_j$ and $\boldsymbol{\theta} \equiv \{\chi_{ji,ks,oh}^M, \chi_{ji,s,oh}^C, \psi_{is,gj}^n, \psi_{is,gj}^N, \phi_{gi,fj}^{Y,w}, \phi_{gi,j}^{Y,P}\}$.

Appendix A.3.3 contains the proof of this proposition. It generalizes the reduced-form representation in Section 4 for the model with the non-parametric links in production and labor supply described in Appendix A.3.1. It maps wage changes to measures of shock exposure that depend on the initial bilateral trade shares for both final and intermediate consumption, the initial factor spending shares in production, and the elasticity matrices governing cross-market links in labor supply ($\phi_{gi,fj}^{Y,w}$ and $\phi_{gi,j}^{Y,P}$), productivity ($\psi_{is,gj}^n$ and $\psi_{is,gj}^N$) and trade demand ($\chi_{ji,ks,oh}^M$ and $\chi_{ji,s,oh}^C$). The definitions of consumption and input exposure are identical to those in Section 4 for arbitrary bilateral (final and intermediate) spending shares, $x_{ji,ks}^M$ and $x_{ji,s}^C$. However, the definition of revenue exposure needs to be extended to account for the more general demand for goods: the shock's impact on the sales of s in i , $\mu_{i,s}(\hat{\tau})$, is now a function of the cross-price demand elasticities, $\chi_{ij,s,oh}^C$ and $\chi_{ij,sk,oh}^M$. With the nested CES demand in Section 4, these elasticities take the simple form of $\chi_{ij,s,oh}^C = \chi_{ij,sk,oh}^M = (1-\sigma)(\mathbb{I}_{i=o} - x_{oj,s})\mathbb{I}_{s=k}$.

In Appendix A.3.3, we also characterize the reduced-form representation for labor market and price outcomes. For any $\hat{Y}_i \in \{\{\hat{w}_{gi}, \hat{n}_{gi}, \hat{N}_{gi}, \hat{E}_{gi}, \hat{\ln w}_{gi}\}_g, \hat{P}_i, \{\hat{P}_{i,s}^M\}_s\}$, we show that $\hat{Y}_i$ can be written as

$$\hat{Y}_i = \sum_{g,j} \beta_{ijg}^{Y,R}(\boldsymbol{\theta}|\mathbf{W}^0) \hat{\eta}_{gj}^R(\hat{\tau}|\boldsymbol{\theta}) + \sum_j \beta_{ij}^{Y,C}(\boldsymbol{\theta}|\mathbf{W}^0) \hat{\eta}_j^C(\hat{\tau}) + \sum_{j,s} \beta_{ij,s}^{Y,M}(\boldsymbol{\theta}|\mathbf{W}^0) \hat{\eta}_{j,s}^M(\hat{\tau}). \quad (49)$$

The steps in Appendix A.2.4 imply that, under Assumption 1,

$$\Delta \ln Y_i = \alpha^Y + \sum_{g,j} \beta_{ijg}^{Y,R}(\boldsymbol{\theta}|\mathbf{W}^0) \hat{\eta}_{gj}^R(\hat{\tau}^{\text{obs}}) + \sum_j \beta_{ij}^{Y,C}(\boldsymbol{\theta}|\mathbf{W}^0) \hat{\eta}_j^C(\hat{\tau}^{\text{obs}}) + \sum_{j,s} \beta_{ij,s}^{Y,M}(\boldsymbol{\theta}|\mathbf{W}^0) \hat{\eta}_{j,s}^M(\hat{\tau}^{\text{obs}}) + \nu_i^Y \quad (50)$$

such that

$$E\left[\nu_i^Y \hat{\eta}_{gj}^R(\hat{\zeta}|\boldsymbol{\theta})\right] = E\left[\nu_i^Y \hat{\eta}_j^C(\hat{\zeta})\right] = E\left[\nu_i^Y \hat{\eta}_{j,s}^M(\hat{\zeta})\right] = 0 \quad \text{for any } i, j, g, s. \quad (51)$$

A.3.3 Proof of Proposition 1

Step 1. We first derive expressions for changes in price indices as a function of wages, and exogenous shocks. From (40) and (42),

$$\hat{P}_{i,s}^M = \sum_{j,k} x_{ji,ks}^M \left(\hat{\tau}_{ji,k} + \sum_g a_{gj,k}^L \hat{w}_{gj} + a_{j,k}^M \hat{P}_{j,k}^M - \sum_{g,o} \psi_{jk,go}^n \hat{n}_{go} - \sum_{g,o} \psi_{jk,go}^N \hat{N}_{go} \right)$$

which, by defining $\bar{\mathbf{x}}_{NS \times NS}^D \equiv [x_{ji,ks}^M a_{j,k}^M]_{is,jk}$, $\bar{\mathbf{x}}_{NS \times NS}^M \equiv [x_{ji,ks}^M]_{is,jk}$, $\bar{\mathbf{a}}_{NS \times NG}^L \equiv [a_{gj,k}^L \mathbb{I}_{i=j}]_{ik,gj}$, $\bar{\boldsymbol{\psi}}_{NS \times NG}^n = [\psi_{is,gj}^n]_{is,gj}$, $\bar{\boldsymbol{\psi}}_{NS \times NG}^N = [\psi_{is,gj}^N]_{is,gj}$ and $\boldsymbol{\mu}_{NS \times 1}^M \equiv [\hat{\mu}_{i,s}(\hat{\tau})]_{i,s}$ with $\hat{\mu}_{i,s}(\hat{\tau}) \equiv \sum_{j,k} x_{ji,ks}^M \hat{\tau}_{ji,k}$, implies

$$(\bar{\mathbf{I}} - \bar{\mathbf{x}}^D) \hat{\mathbf{P}}^M = \boldsymbol{\mu}^M(\hat{\tau}) + \bar{\mathbf{x}}^M (\bar{\mathbf{a}}^L \hat{\mathbf{w}} - \bar{\boldsymbol{\psi}}^n \hat{\mathbf{n}} - \bar{\boldsymbol{\psi}}^N \hat{\mathbf{N}})$$

and, therefore,

$$\hat{\mathbf{P}}^M = \hat{\boldsymbol{\eta}}^M(\hat{\tau}) + \bar{\mathbf{g}}^D \bar{\mathbf{x}}^M (\bar{\mathbf{a}}^L \hat{\mathbf{w}} - \bar{\boldsymbol{\psi}}^n \hat{\mathbf{n}} - \bar{\boldsymbol{\psi}}^N \hat{\mathbf{N}}) \quad (52)$$

for $\bar{\mathbf{g}}^D \equiv (\bar{\mathbf{I}} - \bar{\mathbf{x}}^D)^{-1} = \bar{\mathbf{I}} + \sum_{d=1}^{\infty} (\bar{\mathbf{x}}^D)^d$ and $\boldsymbol{\eta}^M(\hat{\tau})$ in (46).

From (39),

$$\hat{P}_i = \sum_{j,s} x_{ji,s}^C \left(\hat{\tau}_{ji,s} + \sum_g a_{gj,s}^L \hat{w}_{gj} + a_{j,s}^M \hat{P}_{j,s}^M - \sum_{g,o} \psi_{jk,go}^n \hat{n}_{go} - \sum_{g,o} \psi_{jk,go}^N \hat{N}_{go} \right)$$

which, by defining $\bar{\mathbf{x}}_{N \times NS}^C \equiv [x_{ji,s}^C]_{i,j,s}$ and $\bar{\mathbf{a}}_{NS \times NS}^M \equiv [a_{j,k}^M \mathbb{I}_{is=jk}]_{is,jk}$,

$$\hat{\mathbf{P}} = \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\mathbf{x}}^C \left(\bar{\mathbf{a}}^L \hat{\mathbf{w}} - \bar{\boldsymbol{\psi}}^n \hat{\mathbf{n}} - \bar{\boldsymbol{\psi}}^N \hat{\mathbf{N}} \right) + \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M \hat{\mathbf{P}}^M$$

with $\boldsymbol{\eta}^C(\hat{\boldsymbol{\tau}})$ in (46).

Substituting (52) into this expression,

$$\hat{\mathbf{P}} = \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \right) + \bar{\mathbf{x}}^C \bar{\mathbf{c}}^M \left(\bar{\mathbf{a}}^L \hat{\mathbf{w}} - \bar{\boldsymbol{\psi}}^n \hat{\mathbf{n}} - \bar{\boldsymbol{\psi}}^N \hat{\mathbf{N}} \right), \quad (53)$$

where $\bar{\mathbf{c}}^M \equiv \bar{\mathbf{I}} + \bar{\mathbf{a}}^M \bar{\mathbf{g}}^D \bar{\mathbf{x}}^M$.

Using (38) for outcome Y ,

$$\hat{\mathbf{Y}} = \bar{\boldsymbol{\phi}}^{Y,w} \hat{\mathbf{w}} + \bar{\boldsymbol{\phi}}^{Y,P} \hat{\mathbf{P}}$$

where we define $\bar{\boldsymbol{\phi}}_{GN \times GN}^{Y,w} \equiv [\phi_{gi,fj}^{Y,w}]_{gi,fj}$ and $\bar{\boldsymbol{\phi}}_{GN \times N}^{Y,P} \equiv [\phi_{gi,j}^{Y,P}]_{gi,j}$.

To obtain an expression for the price index, we now combine this expression with (53) to derive

$$\hat{\mathbf{P}} = \bar{\boldsymbol{\rho}}^P \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \right) + \bar{\boldsymbol{\alpha}}^{P,w} \hat{\mathbf{w}} \quad (54)$$

where $\bar{\boldsymbol{\rho}}^P \equiv \left[\bar{\mathbf{I}} + \bar{\mathbf{x}}^C \bar{\mathbf{c}}^M \left(\bar{\boldsymbol{\psi}}^n \bar{\boldsymbol{\phi}}^{n,P} + \bar{\boldsymbol{\psi}}^N \bar{\boldsymbol{\phi}}^{N,P} \right) \right]^{-1}$ and $\bar{\boldsymbol{\alpha}}^{P,w} \equiv \bar{\boldsymbol{\rho}}^P \bar{\mathbf{x}}^C \bar{\mathbf{c}}^M \left(\bar{\mathbf{a}}^L - \bar{\boldsymbol{\psi}}^n \bar{\boldsymbol{\phi}}^{n,w} - \bar{\boldsymbol{\psi}}^N \bar{\boldsymbol{\phi}}^{N,w} \right)$.

Using (38) for outcome Y ,

$$\hat{\mathbf{Y}} = \bar{\boldsymbol{\alpha}}^{Y,w} \hat{\mathbf{w}} + \bar{\boldsymbol{\alpha}}^{Y,C} \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \right) \quad (55)$$

where $\bar{\boldsymbol{\alpha}}^{Y,w} \equiv \bar{\boldsymbol{\phi}}^{Y,w} + \bar{\boldsymbol{\phi}}^{Y,P} \bar{\boldsymbol{\alpha}}^{P,w}$ and $\bar{\boldsymbol{\alpha}}^{Y,C} \equiv \bar{\boldsymbol{\phi}}^{Y,P} \bar{\boldsymbol{\rho}}^P$.

Together with (52) and (53), equation (55) implies

$$\hat{\mathbf{P}}^M = \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) - \bar{\boldsymbol{\alpha}}^{M,C} \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \right) + \bar{\boldsymbol{\alpha}}^{M,w} \hat{\mathbf{w}}, \quad (56)$$

where $\bar{\boldsymbol{\alpha}}^{M,C} \equiv \bar{\mathbf{g}}^D \bar{\mathbf{x}}^M \left(\bar{\boldsymbol{\psi}}^n \bar{\boldsymbol{\alpha}}^{n,C} + \bar{\boldsymbol{\psi}}^N \bar{\boldsymbol{\alpha}}^{N,C} \right)$ and $\bar{\boldsymbol{\alpha}}^{M,w} \equiv \bar{\mathbf{g}}^D \bar{\mathbf{x}}^M \left(\bar{\mathbf{a}}^L - \bar{\boldsymbol{\psi}}^n \bar{\boldsymbol{\alpha}}^{n,w} - \bar{\boldsymbol{\psi}}^N \bar{\boldsymbol{\alpha}}^{N,w} \right)$.

Step 2. We now solve for the change in revenue of sector-market pairs. From (43), by defining $\bar{\mathbf{r}}_{NS \times N}^C \equiv [x_{ij,s}^C E_j / R_{i,s}]_{is,j}$ and $\bar{\mathbf{r}}_{NS \times NS}^U \equiv [x_{ij,sk}^M a_{j,k}^M R_{j,k} / R_{i,s}]_{is,jk}$,

$$(\bar{\mathbf{I}} - \bar{\mathbf{r}}^U) \hat{\mathbf{R}} = \left[\sum_j r_{ij,s}^C \hat{x}_{ij,s}^C + \sum_{j,k} r_{ij,sk}^U \hat{x}_{ij,sk}^M \right]_{is} + \bar{\mathbf{r}}^U \hat{\mathbf{a}}^M + \bar{\mathbf{r}}^C \hat{\mathbf{E}}.$$

Using (39) and (40), this expression becomes

$$(\bar{\mathbf{I}} - \bar{\mathbf{r}}^U) \hat{\mathbf{R}} = \boldsymbol{\mu}(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\chi}} \hat{\mathbf{P}} + \bar{\mathbf{r}}^U \hat{\mathbf{a}}^M + \bar{\mathbf{r}}^C \hat{\mathbf{E}}.$$

where $\bar{\boldsymbol{\chi}}_{NS \times NS} \equiv [\sum_j (r_{ij,s}^C \chi_{ij,s,oh}^C + \sum_k r_{ij,sk}^U \chi_{ij,ks,oh}^M)]_{is,oh}$ and $\boldsymbol{\mu}(\hat{\boldsymbol{\tau}})$ defined in (47).

From (41) and (42), we have that

$$\begin{aligned}\hat{\mathbf{p}} &= \bar{\mathbf{a}}^L \hat{\mathbf{w}} + \bar{\mathbf{a}}^M \hat{\mathbf{P}}^M - \bar{\psi}^n \hat{\mathbf{n}} - \bar{\psi}^N \hat{\mathbf{N}} \\ \hat{\mathbf{a}}^M &= \bar{\epsilon}^{LM} \hat{\mathbf{w}} + \bar{\epsilon}^M \hat{\mathbf{P}}^M\end{aligned}$$

where $\bar{\epsilon}_{NS \times NG}^{LM} \equiv [\epsilon_{gi,s}^{LM} \mathbb{I}_{i=j}]_{is,gj}$ and $\bar{\epsilon}^M \equiv [\epsilon_{i,s}^M \mathbb{I}_{is=jk}]_{is,jk}$.

By defining $\bar{\mathbf{g}}^U \equiv \bar{\mathbf{I}} + \bar{\mathbf{b}}^U$, the expressions above imply that

$$\hat{\mathbf{R}} = \bar{\mathbf{g}}^U \boldsymbol{\mu}(\hat{\boldsymbol{\tau}}) + \bar{\mathbf{g}}^U \bar{\boldsymbol{\chi}} \left(\bar{\mathbf{a}}^L \hat{\mathbf{w}} + \bar{\mathbf{a}}^M \hat{\mathbf{P}}^M - \bar{\psi}^n \hat{\mathbf{n}} - \bar{\psi}^N \hat{\mathbf{N}} \right) + \bar{\mathbf{g}}^U \bar{\mathbf{r}}^U \left(\bar{\epsilon}^{LM} \hat{\mathbf{w}} + \bar{\epsilon}^M \hat{\mathbf{P}}^M \right) + \bar{\mathbf{g}}^U \bar{\mathbf{r}}^C \hat{\mathbf{E}}.$$

Using the expressions for $\hat{\mathbf{n}}$ and $\hat{\mathbf{N}}$ in (55) and $\hat{\mathbf{E}}$ in (55),

$$\hat{\mathbf{R}} = \bar{\mathbf{g}}^U \boldsymbol{\mu}(\hat{\boldsymbol{\tau}}) + \bar{\alpha}^{R,w} \hat{\mathbf{w}} + \bar{\alpha}^{R,M} \hat{\mathbf{P}}^M + \bar{\alpha}^{R,C} \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \right) \quad (57)$$

where

$$\begin{aligned}\bar{\alpha}^{R,w} &\equiv \bar{\mathbf{g}}^U \left(\bar{\boldsymbol{\chi}} \bar{\mathbf{a}}^L + \bar{\mathbf{r}}^U \bar{\epsilon}^{LM} - \bar{\boldsymbol{\chi}} (\bar{\psi}^n \bar{\alpha}^{n,w} + \bar{\psi}^N \bar{\alpha}^{N,w}) + \bar{\mathbf{r}}^C \bar{\alpha}^{E,w} \right) \\ \bar{\alpha}^{R,M} &\equiv \bar{\mathbf{g}}^U \left(\bar{\boldsymbol{\chi}} \bar{\mathbf{a}}^M + \bar{\mathbf{r}}^U \bar{\epsilon}^M \right) \\ \bar{\alpha}^{R,C} &\equiv \bar{\mathbf{g}}^U \left(\bar{\mathbf{r}}^C \bar{\alpha}^{E,C} - \bar{\boldsymbol{\chi}} (\bar{\psi}^n \bar{\alpha}^{n,C} + \bar{\psi}^N \bar{\alpha}^{N,C}) \right)\end{aligned}$$

Step 3. The final step is characterizing the system in (16). From the labor market clearing condition in (44), $\hat{W}_{gi} = \sum_s \ell_{gi,s} (\hat{a}_{gi,s}^L + \hat{R}_{i,s})$. When combined with (41), we get that

$$\hat{W}_{gi} = \sum_s \ell_{gi,s} \left(\sum_{g'} \epsilon_{gi,s,g'}^L \hat{w}_{g'i} + \epsilon_{gi,s}^{LM} \hat{P}_{i,s}^M + \hat{R}_{i,s} \right)$$

which, in matrix notation, yields

$$\hat{\mathbf{W}} = \bar{\alpha}^{\epsilon,L} \hat{\mathbf{w}} + \bar{\alpha}^{\epsilon,M} \hat{\mathbf{P}}^M + \bar{\ell} \hat{\mathbf{R}},$$

where $\bar{\alpha}_{NG \times NG}^{\epsilon,L} \equiv \{\mathbb{I}_{i=j} \sum_s \ell_{gi,s} \epsilon_{gi,s,g'}^L\}_{gi,g'j}$ is the matrix of cross-group elasticities of labor demand with respect to wages, $\bar{\alpha}_{NG \times NS}^{\epsilon,M} \equiv \{\mathbb{I}_{i=j} \ell_{gi,s} \epsilon_{gi,s}^{LM}\}_{gi,js}$ is the matrix of group elasticities of labor demand with respect to input cost, and $\bar{\ell}_{GN \times NS} \equiv [\ell_{gi,s} \mathbb{I}_{i=j}]_{gi,js}$ is the matrix of the share of sector s in employment of group g in market i .

By applying (55) and (57) into the expression above, we get

$$(\bar{\alpha}^{W,w} - \bar{\alpha}^{\epsilon,L} - \bar{\ell} \bar{\alpha}^{R,w}) \hat{\mathbf{w}} = \hat{\boldsymbol{\eta}}^R + (\bar{\alpha}^{\epsilon,M} + \bar{\ell} \bar{\alpha}^{R,M}) \hat{\mathbf{P}}^M + (\bar{\ell} \bar{\alpha}^{R,C} - \bar{\alpha}^{W,C}) \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \right),$$

where, by definition, $\hat{\boldsymbol{\eta}}^R \equiv \bar{\ell} \bar{\mathbf{g}}^U \boldsymbol{\mu}(\hat{\boldsymbol{\tau}})$.

By applying (56) into this expression, we get that

$$\bar{\boldsymbol{\gamma}} \hat{\mathbf{w}} = \hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + \bar{\alpha}^C \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\alpha}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}), \quad (58)$$

where

$$\begin{aligned}\bar{\boldsymbol{\gamma}} &\equiv \bar{\alpha}^{W,w} - \bar{\alpha}^{\epsilon,L} - \bar{\ell} \bar{\alpha}^{R,w} - (\bar{\alpha}^{\epsilon,M} + \bar{\ell} \bar{\alpha}^{R,M}) \bar{\alpha}^{M,w}, \\ \bar{\alpha}^C &\equiv \bar{\ell} \bar{\alpha}^{R,C} - \bar{\alpha}^{W,C} - (\bar{\alpha}^{\epsilon,M} + \bar{\ell} \bar{\alpha}^{R,M}) \bar{\alpha}^{M,C}, \\ \bar{\alpha}^M &\equiv \bar{\alpha}^{\epsilon,M} + \bar{\ell} \bar{\alpha}^{R,M} + \bar{\alpha}^C \bar{\mathbf{x}}^C \bar{\mathbf{a}}^M.\end{aligned}$$

The representation in (45) follows from the same steps in Part B of Appendix A.2.4 under the diagonal dominance condition in (29).

Step 4. We derive reduced-form expressions for all labor market outcomes $\hat{Y}_{gi} \in \{\hat{n}_{gi}, \hat{N}_{gi}, \hat{E}_{gi}, \overline{\ln w_{gi}}\}$ using (55):

$$\hat{\mathbf{Y}} = \bar{\beta}^{Y,R} \hat{\eta}^R(\hat{\tau}) + \bar{\beta}^{Y,C} \hat{\eta}^C(\hat{\tau}) + \bar{\beta}^{Y,M} \hat{\eta}^M(\hat{\tau}) \quad (59)$$

where

$$\bar{\beta}^{Y,R} \equiv \bar{\alpha}^{Y,w} \bar{\beta}, \quad \bar{\beta}^{Y,C} \equiv \bar{\alpha}^{Y,w} \bar{\beta} \bar{\alpha}^C + \bar{\alpha}^{Y,C}, \quad \bar{\beta}^{Y,M} \equiv \bar{\alpha}^{Y,w} \bar{\beta} \bar{\alpha}^M + \bar{\alpha}^{Y,C} \bar{x}^C \bar{a}^M.$$

Using (54),

$$\hat{\mathbf{P}} = \bar{\beta}^{C,R} \hat{\eta}^R(\hat{\tau}) + \bar{\beta}^{C,C} \hat{\eta}^C(\hat{\tau}) + \bar{\beta}^{C,M} \hat{\eta}^M(\hat{\tau}), \quad (60)$$

where

$$\bar{\beta}^{C,R} \equiv \bar{\alpha}^{P,w} \bar{\beta}, \quad \bar{\beta}^{C,C} \equiv \bar{\alpha}^{P,w} \bar{\beta} \bar{\alpha}^C + \bar{\rho}^P, \quad \bar{\beta}^{C,M} \equiv \bar{\alpha}^{P,w} \bar{\beta} \bar{\alpha}^M + \bar{\rho}^P \bar{x}^C \bar{a}^M.$$

A.4 Adding Frictional Unemployment to Model of Section 3

We now outline an extension of the model in Section 3 featuring frictional unemployment. It yields an expression for the change in the employment rate in terms of changes in w_i/b_i with an elasticity that combines the parameters controlling responses in both the labor force participation and the unemployment rate.

Environment. We consider the same preferences as in our baseline model, with $l(\iota)$ and $u(\iota)$ denoting ι 's efficiency units and non-employment income. As in the baseline, individuals draw $(l(\iota), u(\iota))$ independently from a Frechet distribution with shape parameter $\phi > 1$ and scale 1. Given the uncertainty in the job search process, we assume that individuals are risk neutral.

As in our baseline, each sector s of market i has a representative firm that produces a differentiated good subject to iceberg trade costs. We now assume that production depends on a CES aggregator of the continuum of non-traded inputs available in the market, $\nu \in \mathcal{V}_i$:

$$Q_{i,s} = \left[\int_{\nu \in \mathcal{V}_i} (q_{i,s}(\nu))^{\frac{\mu-1}{\mu}} d\nu \right]^{\frac{\mu}{\mu-1}}, \quad (61)$$

where $\mu > 1$ is the elasticity of substitution between non-traded varieties.

We assume that the economy has a fixed pool of potential producers of the non-traded inputs that operate in monopolistic competition. In order to produce, firms need to get matched with a worker. If the owner of the firm does not post a vacancy, she gets an outside option payoff of $\bar{v}_i$. We consider a competitive search environment in which firm ν posts a wage offer $w_i(\nu)$. We analyze a symmetric equilibrium in which all firms post the same wage (i.e., $w_i(\nu) = w_i$), and then are randomly matched with a worker in the economy. Conditional on being matched to individual ι , intermediate producers have a linear production function such that $y_i(\nu) = l(\iota)$. The matching technology is such that, if V_i vacancies are posted and N_i^P workers search for a job, the number of matches is

$$M_i = (V_i)^\alpha (N_i^P)^{1-\alpha}. \quad (62)$$

Labor Force Participation. We first solve for the share of individuals in market i that look for a job given an offered wage rate of w_i . Consider the case in which individual ι searches for a job. With probability M_i/N_i^P , she finds a job and has a payoff of $(1-v_i)w_i l(\iota)/P_i$; with probability $1-M_i/N_i^P$, she does not find a job and has a payoff of $(1-v_i)b_i u(\iota)/P_i$. If the same individual ι does not search for a job, she gets a payoff of $(1-v_i)b_i u(\iota)/P_i$.

Thus, the maximization of expected utility implies that the market's labor force participation is

$$n_i^p = \Pr \left[\frac{M_i}{N_i^p} w_i l(\iota) + \left(1 - \frac{M_i}{N_i^p} \right) b_i u(\iota) > b_i u(\iota) \right] = \Pr[w_i l(\iota) > b_i u(\iota)] \Rightarrow n_i^p = \frac{w_i^\phi}{w_i^\phi + b_i^\phi}. \quad (63)$$

As in our baseline, the mean efficiency of those searching for jobs is $\bar{l}_i = \varrho(n_i^p)^{-\frac{1}{\phi}}$.

Unemployment Rate. Given the cost minimization problem of the representative firm in market i , the demand for the output of the intermediate producer ν is

$$q_i(\nu) = \frac{(p_i(\nu))^{-\mu}}{\int_{\tilde{\nu} \in \mathcal{V}_i} (p_i(\tilde{\nu}))^{1-\mu} d\tilde{\nu}} R_i.$$

Thus, the profit maximization problem of firm ν yields the typical constant markup expression for the price of the intermediate good:

$$\tilde{p}_i(\nu) = \frac{\mu}{\mu-1} w_i \quad \forall \nu \in \mathcal{V}_i.$$

This implies that the production cost of firms in market i is $p_{i,s} \equiv \left[\int_{\nu \in \mathcal{V}_i} (\tilde{p}_i(\nu))^{1-\mu} d\nu \right]^{\frac{1}{1-\mu}} = \frac{\mu}{\mu-1} w_i (M_i)^{\frac{1}{1-\mu}}$. In equilibrium, the number of successful matches must be equal to the number of employed individuals ($M_i = L_i$), so

$$p_{i,s} = \frac{\mu}{\mu-1} w_i (L_i)^{-\psi} \quad \text{such that} \quad \psi \equiv \frac{1}{\mu-1}. \quad (64)$$

Finally, the free entry condition implies that the expected profit of posting a vacancy must be equal to the outside option of not posting it. Given that the probability of filling a vacancy is M_i/V_i and that the expected efficiency of a match is $\bar{l}_i$, we have that

$$\bar{\nu}_i = (\tilde{p}_i(\nu) - w_i) \bar{l}_i \frac{M_i}{V_i} = \frac{1}{\mu-1} w_i \bar{l}_i \left(\frac{N_i^p}{V_i} \right)^{1-\alpha} \Rightarrow \frac{N_i^p}{V_i} = \left(\frac{(\mu-1)\bar{\nu}_i}{w_i \bar{l}_i} \right)^{\frac{1}{1-\alpha}}$$

This expression determines the share of individuals searching for a job that get matched to a producer:

$$n_i^m = \frac{M_i}{N_i^p} = \left(\frac{V_i}{N_i^p} \right)^\alpha = \left(\frac{w_i \bar{l}_i}{(\mu-1)\bar{\nu}_i} \right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{w_i \varrho(n_i^p)^{-\frac{1}{\phi}}}{(\mu-1)\bar{\nu}_i} \right)^{\frac{\alpha}{1-\alpha}}.$$

Assuming that the outside option of producers is proportional to the non-employment transfer ($\bar{\nu}_i = \nu_i b_i$), we derive our main expression for the share of individuals in market i that are employed:

$$n_i = n_i^m n_i^p = \left(\frac{\varrho}{(\mu-1)\nu_i} \frac{w_i}{b_i} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{(w_i/b_i)^\phi}{1 + (w_i/b_i)^\phi} \right)^{\left(1 - \frac{\alpha}{1-\alpha} \frac{1}{\phi} \right)}. \quad (65)$$

Up to a first order approximation, this expression implies that

$$\hat{n}_i = \hat{n}_i^m + \hat{n}_i^p = \left(\frac{\alpha}{1-\alpha} n_i^p + \phi(1-n_i^p) \right) (\hat{w}_i - \hat{b}_i).$$

The elasticity of the employment rate to the wage rate has two components. As before, it entails the elasticity of the labor force participation margin, $\phi(1-n_i^p)$. But now it also encompasses the elasticity of the matching rate,

$\frac{\alpha}{1-\alpha}n_i^p$, which depends on the matching technology parameter α . Whenever $\alpha=0$, all individuals searching for a job get a match and this term disappears.

A.5 Model Integration

A.5.1 Algorithm

The reduced-form representation in (49) is a first-order approximation for changes in the market-level outcomes: $\hat{Y}_i \in \{\{\hat{w}_{gi}, \hat{n}_{gi}, \hat{N}_{gi}, \hat{E}_{gi}, \widehat{\ln w_{gi}}\}_g, \hat{P}_i, \{\hat{P}_{i,s}^M\}_s\}$. It can be integrated to compute exact (non-linear) changes in these outcomes using the following algorithm.

1. Consider $r=1, \dots, R$ repetitions. Let the initial conditions be $\mathcal{W}^0 \equiv \{\{x_{ij,s}^C\}_j, \{x_{ij,sk}^M\}_{s,j,k}, \{a_{gi,s}^L\}_g\}_j$, and the initial elasticities be $\theta^0 \equiv \{\chi_{ji,ks,oh}^M, \chi_{ji,s,oh}^C, \psi_{is,gj}^n, \psi_{is,gj}^N, \phi_{gi,fj}^{Y,w}, \phi_{gi,j}^{Y,P}\}$.
2. Given θ^{r-1} and $\mathcal{W}^{r-1}$, for $\hat{\tau}^r = \hat{\tau}/R$:
 - (a) Compute $Y_i^r = Y_i^{r-1} \exp(\hat{Y}_i^r)$ where $\hat{Y}_i^r$ is given by (49);
 - (b) Compute $p_{i,s}^r = p_{i,s}^{r-1} \exp(\hat{p}_{i,s}^r)$ where $\hat{p}_{i,s}^r$ is given by (42);
 - (c) Compute $\mathcal{W}^r \equiv \{\{x_{ij,s}^{C,r-1} \exp(\hat{x}_{ij,s}^{C,r})\}_j, \{x_{ij,sk}^{M,r-1} \exp(\hat{x}_{ij,sk}^{M,r})\}_{j,k}, \{a_{gi,s}^L \exp(\hat{a}_{gi,s}^{L,r})\}_g\}_j$ where $\hat{x}_{ij,s}^{C,r}$, $\hat{x}_{ij,sk}^{M,r}$, and $\hat{a}_{gi,s}^{L,r}$ are respectively given by (39), (40) and (41);
 - (d) Compute $\theta^r \equiv \{\chi_{ji,ks,oh}^{M,r}, \chi_{ji,s,oh}^{C,r}, \psi_{is,gj}^{n,r}, \psi_{is,gj}^{N,r}, \phi_{gi,fj}^{Y,w,r}, \phi_{gi,j}^{Y,P,r}\}$ using the definitions above and outcomes in iteration r .
3. Repeat step 2 for each r . The overall change in any outcome is $Y_i'/Y_i^0 = Y_i^R/Y_i^0$.

A.5.2 Illustration

To illustrate the integration algorithm, we compare the impact of shocks obtained with the linear approximation and the integration algorithm. We consider the predictions implied by the calibration of the model in Appendix A.6 and one draw of the i.i.d. sector-level shocks $\hat{\zeta}_s$.

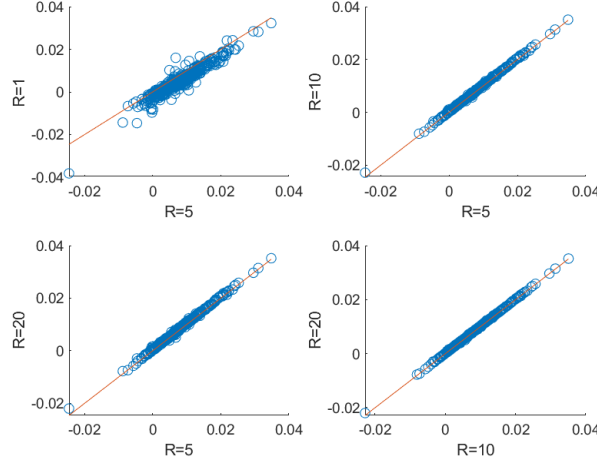
Let us define the integral of our reduced-form formulas with R partitions of the shock as

$$\hat{Y}_i^M(\hat{\zeta}|\theta, \mathcal{W}^0, R) \equiv \sum_{r=1}^R \sum_j \beta_{ij}^Y(\theta|\mathcal{W}^{r-1}) \sum_s \ell_{j,s}^{r-1} \left(\hat{\zeta}_s/R \right),$$

where $\mathcal{W}^r = \mathbf{H}(\hat{\zeta}/R|\theta, \mathcal{W}^{r-1})$ defines the law of motion for the endogenous variables in $\mathcal{W}^0$ following the shifts in sectoral demand caused by a trade shock, $\hat{\zeta}_s/R$. When R is large enough, this algorithm converges to the exact solution for the model's predicted impact of the shock, as we show below.

Figure A.1 reports the predicted changes in the employment rate that we obtain with different partitions R . The top left panel shows that the first-order approximation yields predictions that are similar to the ones implied by the integration algorithm with $R=5$. The correlation between them is close to one. The remaining panels show that there is very little gain in accuracy from using more than five partitions of the shock. For this reason, our simulations use five partitions when we compute the non-linear solution of the model.

Figure A.1: Impact of a Trade Shock on the Employment Rate, Integral of First-Order Approximation



Notes: Each dot is the predicted change in the employment rate of each of the 722 CZs that we compute using the integration algorithm in Appendix A.5.1 for R partitions of the simulated shift in sectoral demand. Predictions computed using the model calibration of Appendix A.6.

A.6 Monte Carlo Simulations

We present Monte Carlo simulations to illustrate the properties of our empirical strategy in the context of the impact of the China shock on U.S. CZs, as in Section 2. Specifically, we consider $\varsigma = 1, \dots, 3000$ simulated economies subject to random observed and unobserved shocks. For each simulation, the reduced-form expressions in (25) yield changes in wages and employment for each U.S. CZ, $\Delta \ln w_i(\varsigma)$ and $\Delta \ln n_i(\varsigma)$. Motivated by the strategy in Section 2, we consider a foreign shock with an identical impact on the sectoral demand of all U.S. CZs such that $Z_i(\varsigma) = \sum_s \ell_{j,s}^0 \hat{z}_{F,s}^{\text{obs}}(\varsigma)$ where $\hat{z}_{F,s}^{\text{obs}}(\varsigma)$ is i.i.d. across sectors and normally distributed with a mean of zero and a standard deviation of σ_ς . We further assume that $\nu_i^n(\varsigma)$ and $\nu_i^w(\varsigma)$ are i.i.d. across i and normally distributed with a mean of zero and a standard deviation of σ_ν .

We use the estimates in Section 2 to calibrate our simulations. First, we consider the stacked sample with 722 CZs in two periods, 1990-2000 and 2000-2007. To specify the variables in $\mathbf{W}^0$, we use a sector-level trade matrix between U.S. CZs and foreign countries.² For illustrative purposes, we calibrate the parameters of the model such that the trade elasticity is five ($\sigma=6$), agglomeration or congestion forces are zero ($\psi=0$), and the labor supply elasticity matches the ratio of employment-to-wage response in Table 1 ($\phi=2.5$).³ Finally, we set σ_ς and σ_ν to match the standard deviation of outcomes and shock exposure across regions and periods, as reported in Table B.1.

For each ς , we consider a researcher who uses the reduced-form specification in (25) to estimate the aggregate and differential effects of the observed sectoral shocks $\{\hat{z}_{F,s}^{\text{obs}}(\varsigma)\}_s$. To simplify our analysis, we assume that the researcher only estimates the labor supply elasticity ϕ , and considers two sets of moment conditions based on different specifications of $\{h_{ij}^n, h_{ij}^w\}$ in (28): the efficient moments, computed with the procedure in Appendix A.1.10 (Optimal IV), and the region's own shock exposure, computed with $h_{ij}^n = h_{ij}^w = \mathbb{I}_{[i=j]}$ (Direct IV).

²Section 5.2 and Appendix C provide a detailed description of the data construction and the specification of spatial links.

³In our model, it can be shown that the impact of a trade cost shock on employment divided by its impact on wages is given by $\phi(1 - n_i^0)/n_i^0$. We obtain $\phi=2.5$ by taking the ratio of estimates in columns (3) and (1) of Table 1 and setting $n_i^0=0.7$ (the median across U.S. CZs).

Panel A of Table A.1 presents moments of the labor supply elasticity estimates, $\hat{\phi}(\varsigma)$, across 3,000 simulated economies. Columns (1) and (2) show that both sets of moments yield estimates that are centered at the true value of 2.5. Note that the standard deviation of estimates obtained from the Optimal IV is 10% lower than that of estimates from the Direct IV.

Table A.1: Monte Carlo Simulations

Data simulated from: Estimation moment:	Linear Model		Non-linear Model	
	Optimal IV	Direct IV	Optimal IV	Direct IV
	(1)	(2)	(3)	(4)
<i>Panel A: Estimates of $\phi(\varsigma)$</i>				
Median	2.51	2.52	2.25	2.19
St. Dev.	0.71	0.78	0.71	0.77
<i>Panel B: Regression of $\Delta^{\text{true}}\ln n_i(\varsigma)$ on $\Delta^{\text{pred}}\ln n_i(\varsigma)$</i>				
Median Constant	0.00	0.00	0.00	0.00
Median Slope	1.00	0.99	0.96	0.98
Median R^2	1.00	1.00	0.97	0.97

Notes: Panel A reports moments of the estimates of $\hat{\phi}(\varsigma)$ obtained from the $\varsigma = 1, \dots, 3000$ simulated economies. Panel B reports moments across simulated economies of the regression of the true impact of the shock, $\Delta^{\text{true}}\ln n_i(\varsigma)$, on the researcher's predicted impact of the shock, $\Delta^{\text{pred}}\ln n_i(\varsigma)$, in the pooled sample of 1,444 CZs. Simulated data generated according to the linear specification in (25) in columns (1)-(2) and the non-linear integral specification in Appendix A.5 in columns (3)-(4). Moment conditions built using the Optimal IV in columns (1) and (3) and the Direct IV in columns (2) and (4).

We also compare the researcher's estimated impact with the true impact of the shock on the employment rate of U.S. CZs. Specifically, for each ς , we compute the “true” impact of the shock, $\Delta^{\text{true}}\ln n_i(\varsigma) \equiv \sum_j \beta_{ij}^n(\phi|\mathbf{W}^0)Z_i(\varsigma)$ with $\phi = 2.5$, and the researcher's estimated impact of the shock, $\Delta^{\text{pred}}\ln n_i(\varsigma) \equiv \sum_j \beta_{ij}^n(\hat{\phi}(\varsigma)|\mathbf{W}^0)Z_i(\varsigma)$ with $\hat{\phi}(\varsigma)$ obtained from (25) and (28). We then run a regression of $\Delta^{\text{true}}\ln n_i(\varsigma)$ on $\Delta^{\text{pred}}\ln n_i(\varsigma)$ in the pooled sample of 1,444 CZs in 1990-2000 and 2000-2007. Across our simulated economies, Panel B shows that this regression yields a median constant of zero, a median slope of one, and a median R^2 of one. In line with the discussion above, these results confirm that the reduced-form specification in (25) provides consistent estimates of both the differential and the aggregate impact of trade shocks across regions.

The last two columns of Table A.1 implement simulations to evaluate the performance of our linear specification when the impact of the shock is non-linear. To this end, we repeat the simulation described above, but we assume instead that $\Delta\ln n_i(\varsigma) = \Delta^{\text{true}}\ln n_i(\varsigma) + \nu_i^n$, with $\Delta^{\text{true}}\ln n_i(\varsigma)$ implied by the integration algorithm in Appendix A.5. We define $\Delta\ln w_i(\varsigma)$ and $\Delta^{\text{true}}\ln w_i(\varsigma)$ analogously.⁴

The estimates of the labor supply elasticity are centered at around 2.2, which is 12% lower than the true parameter value. However, Panel B shows that the moments of the regression of the true impact on the researcher's predicted impact in columns (3)-(4) are similar to those in columns (1)-(2). This indicates that non-linearities do not introduce a large bias in the researcher's prediction for the shock's impact on U.S. CZs. Intuitively, our strategy adjusts the estimate of the labor supply elasticity in order to better capture the impact of the shock on regional outcomes. Although this is not a general result,

⁴For large shocks, our specification follows ? who provide a general decomposition of quantitative spatial and trade models into two additive components, one related to the impact of the shock of interest (i.e., $\Delta^{\text{true}}\ln n_i(\varsigma)$) and another related to the impact of other shocks (i.e., ν_i^n), without loss of generality.

these simulations suggest that, in our application, the linear approximation performs very well.

A.7 Proofs and Additional Results for Section 5

A.7.1 Additional derivations for Section 5.1

Proof of Expression (36). The gravity trade demand $X_{ij,s}$ in (6) implies that

$$\Delta \ln X_{ij,s}^t = (1-\sigma)\hat{\tau}_{ij,s}^t - \kappa \hat{p}_{i,s} + \hat{E}_j^t - \underbrace{\left(\sum_o \tau_{oj,s}^{1-\sigma} w_o^{-\kappa} \bar{b}_o^{\kappa-\sigma+1} \right)}_{\equiv \Lambda_{j,s}^t}. \quad (66)$$

Up to a first order approximation, the definition of $\Delta M_{\text{China},s}^t \equiv \sum_j \frac{\Delta X_{\text{China},j,s}^t}{L_{\text{US},s}^{t_0}}$ is equal to

$$\begin{aligned} \Delta M_{\text{China},s}^t &= \sum_j \frac{X_{\text{China},j,s}^{t_0}}{L_{\text{US},s}^{t_0}} \Delta \ln X_{\text{China},j,s}^t \\ &= \sum_j \frac{X_{\text{China},j,s}^{t_0}}{L_{\text{US},s}^{t_0}} \left((1-\sigma)\hat{\tau}_{\text{China},j,s}^t - \kappa \hat{p}_{\text{China},s}^t + \Lambda_{j,s}^t \right). \end{aligned}$$

We set the Chinese wage as the numeraire ($\hat{w}_{\text{China}}^t = 0$) without loss of generality. Together with (10), the expression above implies that

$$\Delta M_{\text{China},s}^t = \sum_j \frac{E_{j,s}^{t_0}}{L_{\text{US},s}^{t_0}} \left((1-\sigma)x_{\text{China},j,s}^{t_0} \hat{\tau}_{\text{China},j,s}^t \right) + \frac{\sum_j X_{\text{China},j,s}^{t_0} \Lambda_{j,s}^t}{L_{\text{US},s}^{t_0}}.$$

The decomposition $(1-\sigma)x_{\text{China},j,s}^{t_0} \hat{\tau}_{\text{China},j,s}^t = \hat{\tau}_{\text{China},s}^{\text{obs},t} + \hat{\varepsilon}_{\text{China},j,s}^t$ implies that

$$\Delta M_{\text{China},s}^t = \left(\frac{\sum_j E_{j,s}^{t_0}}{L_{\text{US},s}^{t_0}} \right) \left(\hat{\tau}_{\text{China},s}^{\text{obs},t} + \sum_j \frac{E_{j,s}^{t_0}}{\sum_{j'} E_{j',s}^{t_0}} \hat{\varepsilon}_{\text{China},j,s}^t \right) + \frac{\sum_j X_{\text{China},j,s}^{t_0} \Lambda_{j,s}^t}{L_{\text{US},s}^{t_0}}.$$

Under the assumption that $\sum_j \frac{E_{j,s}^{t_0}}{\sum_{j'} E_{j',s}^{t_0}} \hat{\varepsilon}_{\text{China},j,s}^t = 0$, this expression immediately yields expression (36):

$$\Delta M_{\text{China},s}^t = \left(\frac{\sum_j E_{j,s}^{t_0}}{L_{\text{US},s}^{t_0}} \right) \hat{\tau}_{\text{China},s}^{\text{obs},t} + \frac{\sum_j X_{\text{China},j,s}^{t_0} \Lambda_{j,s}^t}{L_{\text{US},s}^{t_0}}, \quad (67)$$

Measurement of $\hat{\tau}_{\text{China},s}^{\text{obs},t}$. To obtain an expression to recover $\hat{\tau}_{\text{China},s}^{\text{obs},t}$, we re-write (67) as

$$\hat{\tau}_{\text{China},s}^{\text{obs},t} = \left(\frac{L_{\text{US},s}^{t_0}}{\sum_j E_{j,s}^{t_0}} \right) \Delta M_{\text{China},s}^t - \frac{\sum_j X_{\text{China},j,s}^{t_0} \Lambda_{j,s}^t}{\sum_j E_{j,s}^{t_0}}. \quad (68)$$

We now describe how we measure each of the terms in the right hand side of (68). As in Section 2, we follow ADH by setting $\Delta M_{\text{China},s}^t$ to be the per-worker growth in Chinese imports by eight developed countries between years t_0 and t , and $L_{\text{US},s}^{t_0}$ the U.S. employment in sector s at t_0 . We use the trade matrix for year t_0 described in Appendix C to measure $E_{j,s}^{t_0} = \sum_i X_{ij,s}^{t_0}$ and $X_{\text{China},j,s}^{t_0}$ for each j in the set of eight developed countries used to compute $\Delta M_{\text{China},s}^t$ in ADH. Finally, we obtain estimates of $\Lambda_{j,s}^t$ for the set of eight developed countries in ADH

using the gravity equation in (66). Specifically, we estimate separately for each sector s the following specification:

$$\Delta \ln X_{ij,s}^t = \Psi_{i,s}^t + \Lambda_{j,s}^t + \eta_{ij,s}^t \quad \text{where} \quad \Psi_{\text{China},s} = 0. \quad (69)$$

The restriction of $\Psi_{\text{China},s} = 0$ follows from the facts that, by (66), $\Psi_{i,s} \equiv \kappa \hat{w}_i^t$ and that, without loss of generality, we set the Chinese wage as the numeraire ($\hat{w}_{\text{China}}^t = 0$). In practice, we implement the estimation of (69) without a constant, and with fixed-effects for the eight destinations used in ADH and for all origins except China. To obtain $\hat{\tau}_{\text{China},s}^{\text{obs},t}$, we use equation (68) to recover shifts for each 4-digit SIC sector, which we then winsorize from extreme values and average within each 3-digit sector.

A.7.2 Specification of Transfer Numeraire

The definition $\Omega(\mathbf{w}) = (W_{\text{US}}(\mathbf{w}))^{\bar{\omega}} (W_{\text{W}}(\mathbf{w}))^{1-\bar{\omega}}$ implies that

$$\omega_j^{t_0} \equiv \frac{\partial \ln \Omega(\mathbf{w}^{t_0})}{\partial \ln w_j} = \bar{\omega} \frac{W_j^{t_0}}{\sum_{i \in \mathcal{I}_{\text{US}}} W_i^{t_0}} \mathbb{I}_{j \in \mathcal{I}_{\text{US}}} + (1-\bar{\omega}) \frac{W_j^{t_0}}{\sum_c \sum_{i \in \mathcal{I}_c} W_i^{t_0}} \quad (70)$$

where $W_i^{t_0}$ is the GDP of market i in the initial period t_0 .

To compute $\omega_j^{t_0}$, we need to specify $\bar{\omega}$. We do so using the series of the opportunity cost of not working in Chodorow-Reich and Karabarbounis (2016). In our model, the average change in the payoff of not working in the U.S. is $\hat{z}_{\text{US}}^t \equiv \sum_{i \in \mathcal{I}_{\text{US}}} \frac{N_i}{N_{\text{US}}} (\hat{b}_i - \hat{P}_i)$. By defining the U.S. price index change as $\hat{P}_{\text{US}}^t \equiv \sum_{i \in \mathcal{I}_{\text{US}}} \frac{N_i}{N_{\text{US}}} \hat{P}_i$, the fact that $\hat{b}_i^t = \hat{\Omega}^t$ yields

$$\begin{aligned} \hat{z}_{\text{US}}^t &= \bar{\omega} \hat{W}_{\text{US}}^t + (1-\bar{\omega}) \hat{W}_{\text{W}}^t - \hat{P}_{\text{US}}^t \\ &= \left(\hat{W}_{\text{US}}^t - \hat{P}_{\text{US}}^t \right) - (1-\bar{\omega}) \left(\hat{W}_{\text{US}}^t - \hat{W}_{\text{W}}^t \right) \\ &= \left(\hat{W}_{\text{US}}^t - \hat{P}_{\text{US}}^t \right) - (1-\bar{\omega}) \left(\left(\hat{W}_{\text{US}}^t - \hat{P}_{\text{US}}^t \right) - \left(\hat{W}_{\text{W}}^t - \hat{P}_{\text{W}}^t \right) + \left(\hat{P}_{\text{US}}^t - \hat{P}_{\text{W}}^t \right) \right). \end{aligned}$$

By defining the real income of a country as $\widehat{RW}_c^t = \hat{W}_c^t - \hat{P}_c^t$ and the relative price as $\hat{P}_c^t = \hat{P}_c^t - \hat{P}_{\text{W}}^t$, this expression is equivalent to

$$\hat{z}_{\text{US}}^t = \widehat{RW}_{\text{US}}^t - (1-\bar{\omega}) \left(\widehat{RW}_{\text{US}}^t - \widehat{RW}_{\text{W}}^t + \hat{P}_{\text{US}}^t \right).$$

Thus,

$$\text{Cov} \left(\widehat{RW}_{\text{US}}^t, \hat{z}_{\text{US}}^t \right) = \text{Var} \left(\widehat{RW}_{\text{US}}^t \right) - (1-\bar{\omega}) \text{Cov} \left(\widehat{RW}_{\text{US}}^t, \widehat{RW}_{\text{US}}^t - \widehat{RW}_{\text{W}}^t + \hat{P}_{\text{US}}^t \right)$$

and, therefore,

$$\bar{\omega} = 1 - \frac{\text{Var} \left(\widehat{RW}_{\text{US}}^t \right) - \text{Cov} \left(\widehat{RW}_{\text{US}}^t, \hat{z}_{\text{US}}^t \right)}{\text{Cov} \left(\widehat{RW}_{\text{US}}^t, \widehat{RW}_{\text{US}}^t - \widehat{RW}_{\text{W}}^t + \hat{P}_{\text{US}}^t \right)}. \quad (71)$$

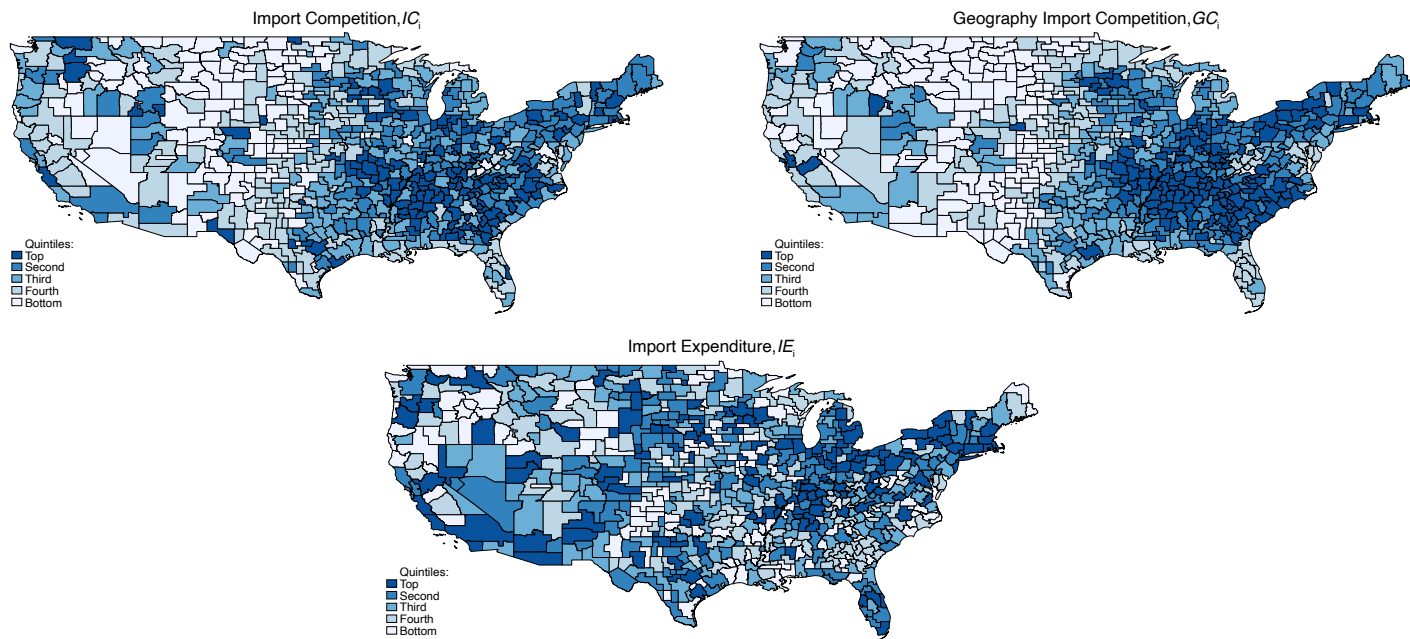
We obtain $\bar{\omega} = 0.62$ using expression (71) computed with year-to-year log-changes in every variable between 1961 and 2012. To measure $\hat{z}_{\text{US}}^t$, we use the series in Chodorow-Reich and Karabarbounis (2016) of the opportunity cost of employment implied by their separable utility specification at the first quarter of each year (available for 1961-2012). We measure all other variables using data from the Penn World Tables produced with the methodology in ?. Specifically, we measure $\widehat{RW}_c^t$ using the annual series of the real domestic absorption at current PPPs (CDA) divided by population (pop), and $\hat{P}_c^t$ using the annual series of the Price level of CDA (PPP/XR). We compute the world average of the log-change in each variable as the average log-change in that variable across all countries, weighted by the country's share in world GDP in the previous year.

B Online Appendix: Additional Empirical Results

B.1 Adjustment of U.S. Regional Markets to Trade Shocks: Three Stylized Facts

This appendix presents additional empirical results that complement those in Section 2.

Figure B.1: Regional Exposure to the China Shock, 1990-2007



Notes: For each CZ, the left panel reports IC_i^t , the right panel reports GC_i^t , and the bottom panel reports IE_i^t .

Table B.1: Summary Statistics of Outcomes for U.S. CZs

	1990-2000		2000-2007		1990-2007		Total	
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
$100 \times$ Change in average weekly log-wage	12.39	4.65	3.84	5.51	16.23	6.47	10.82	7.62
$100 \times$ Change in log of employment rate	1.27	4.23	1.71	5.31	2.98	6.42	1.99	5.44
IC_i^t	1.01	1.06	2.52	2.54	3.52	3.35	2.35	2.71
IE_i^t	2.51	0.58	7.39	1.31	9.90	1.62	6.60	3.31
GC_i^t	1.03	0.88	2.60	1.99	3.64	2.73	2.42	2.28

Notes: Sample of 722 Commuting Zones.

Table B.2: Direct Impact of the China Shock on U.S. CZs – Additional Specifications

	(1)	(2)	(3)	(4)
Panel A: Change in average weekly log-wage				
IC_i^t	-0.47 (0.13)	-0.24 (0.12)	-0.16 (0.10)	-0.26 (0.13)
Panel B: Change in log of employment rate				
IC_i^t	-0.52 (0.09)	-0.24 (0.06)	-0.26 (0.06)	-0.32 (0.14)
Panel C: Change in log of working-age population				
IC_i^t	0.28 (0.18)	0.05 (0.10)	0.33 (0.20)	0.03 (0.30)
Controls:				
Baseline controls in Table 1	Y	Y	Y	Y
Controls of col 9 in Table 2		Y		Y
Sum of exposure shares $\times$ 00-07 dummy		Y	Y	Y
Weights:				
Population weights				Y

Notes: Sample of 1444 Commuting Zones in 1990-2000 and 2000-2007. Controls in each specification described in tables 1 and 2.

Table B.3: Differential Impact of the China Shock on U.S. States – Quantitative Predictions from CDP and Estimates from Alternative Specifications

	Baseline	Alternative specifications			Alternative control sets					Alternative shocks				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Change in log emp.	0.016 (0.005)	0.016 (0.005)	0.012 (0.004)	0.015 (0.005)	0.017 (0.005)	0.016 (0.005)	0.014 (0.004)	0.016 (0.005)	0.014 (0.005)	0.033 (0.011)	0.027 (0.010)	0.033 (0.011)	0.014 (0.005)	0.018 (0.006)
Alternative:	-	Shift-share inference	Pop. weights	Demog. composition	Lagged pop. growth	Own commuting share	2-digit manuf. emp. shares t_0	State fixed effects	Controls in cols (4)-(8)	total sh. ctrls × 00-07 dum.	Controls in cols (9)-(10)	Col (4), 00-07 period	Average sectoral NTR Gap	China gravity origin dummy

Notes: Each column reports the estimated coefficient of a regression where the dependent variable is the impact of the China shock on the log employment rate of each U.S. state (multiplied by 100) between 2000 and 2007 as predicted by the quantitative spatial model in CDP and the regressor is the estimated treatment effect obtained from the specification in the corresponding column of Table 2. All regressions contain a constant.

Table B.4: Differential Impact of the China Shock on U.S. CZs – Alternative Measures of Indirect Spatial Exposure

	(1)	(2)	(3)	(4)	(5)
Panel A: Change in average weekly log-wage					
IC_i^t	-0.38 (0.11)	-0.28 (0.11)	-0.40 (0.12)	-0.37 (0.12)	-0.42 (0.12)
GC_i^t	-0.64 (0.15)	-6.01 (1.65)	-0.45 (0.11)	-0.92 (0.23)	-0.83 (0.27)
IE_i^t	0.08 (0.16)	0.12 (0.16)	0.07 (0.17)	0.07 (0.16)	0.02 (0.17)
Panel B: Change in log of employment rate					
IC_i^t	-0.37 (0.08)	-0.24 (0.07)	-0.40 (0.08)	-0.38 (0.09)	-0.42 (0.09)
GC_i^t	-0.64 (0.14)	-7.11 (0.97)	-0.44 (0.10)	-0.84 (0.17)	-0.72 (0.13)
IE_i^t	-0.16 (0.14)	-0.10 (0.13)	-0.18 (0.15)	-0.18 (0.15)	-0.22 (0.15)
Panel C: Change in log of working-age population					
IC_i^t	0.13 (0.15)	0.08 (0.14)	0.14 (0.16)	0.14 (0.16)	0.15 (0.16)
GC_i^t	0.37 (0.22)	3.02 (2.14)	0.24 (0.16)	0.39 (0.33)	0.54 (0.38)
IE_i^t	0.42 (0.30)	0.40 (0.30)	0.43 (0.29)	0.44 (0.29)	0.45 (0.29)
Spatial Indirect Effect Specification:					
Definition of z_{ij}	$\frac{D_{ij}^{-5}}{\sum_k D_{ik}^{-5}}$	$\frac{D_{ij}^{-1}}{\sum_k D_{ik}^{-1}}$	$\frac{D_{ij}^{-8}}{\sum_k D_{ik}^{-8}}$	$\frac{L_j^0 D_{ij}^{-5}}{\sum_k L_k^0 D_{ik}^{-5}}$	$\frac{1}{2} \left(\frac{M_{ij}^0}{L_i^0} + \frac{M_{ij}^0}{L_j^0} \right)$

Notes: Sample of 1444 Commuting Zones in 1990-2000 and 2000-2007. Spatial indirect effects given by $GC_i^t \equiv \sum_{j \neq i} z_{ij} IC_i^t$ where z_{ij} is specified in each column, D_{ij} is the distance between CZs i and j , L_j^0 is the population of CZ j in 1990, and M_{ij}^0 is the bilateral migration flow between CZs i and j . Robust standard errors in parentheses are clustered by state.

Table B.5: Differential Impact of the China Shock on U.S. CZs – Alternative Measures of Indirect Spatial Exposure II

	(1)	(2)	(3)	(4)	(5)
Panel A: Change in average weekly log-wage					
IC_i^t	-0.22 (0.12)	-0.22 (0.12)	-0.22 (0.12)	-0.20 (0.12)	-0.23 (0.12)
GC_i^t	-0.27 (0.11)	-2.21 (1.32)	-0.20 (0.08)	-0.56 (0.22)	-0.41 (0.24)
IE_i^t	-0.02 (0.15)	-0.01 (0.15)	-0.03 (0.15)	-0.03 (0.15)	-0.05 (0.15)
Panel B: Change in log of employment rate					
IC_i^t	-0.16 (0.06)	-0.13 (0.06)	-0.17 (0.06)	-0.16 (0.06)	-0.17 (0.06)
GC_i^t	-0.24 (0.10)	-4.71 (1.05)	-0.16 (0.08)	-0.34 (0.17)	-0.28 (0.16)
IE_i^t	-0.37 (0.12)	-0.32 (0.11)	-0.38 (0.12)	-0.38 (0.12)	-0.39 (0.12)
Panel C: Change in log of working-age population					
IC_i^t	-0.04 (0.12)	-0.05 (0.12)	-0.04 (0.12)	-0.02 (0.12)	-0.03 (0.12)
GC_i^t	0.05 (0.17)	1.75 (1.95)	0.04 (0.13)	-0.24 (0.27)	-0.21 (0.24)
IE_i^t	0.60 (0.25)	0.57 (0.26)	0.60 (0.25)	0.61 (0.25)	0.60 (0.24)
Spatial Indirect Effect Specification:					
Definition of z_{ij}	$\frac{D_{ij}^{-5}}{\sum_k D_{ik}^{-5}}$	$\frac{D_{ij}^{-1}}{\sum_k D_{ik}^{-1}}$	$\frac{D_{ij}^{-8}}{\sum_k D_{ik}^{-8}}$	$\frac{L_j^0 D_{ij}^{-5}}{\sum_k L_k^0 D_{ik}^{-5}}$	$\frac{1}{2} \left(\frac{M_{ij}^0}{L_i^0} + \frac{M_{ij}^0}{L_j^0} \right)$

Notes: This table is a version of Table B.4 that includes the same controls as in column 11 of Table 2.

Table B.6: Differential Impact of the China Shock on U.S. CZs – Additional Labor Market Outcomes

	Change in the share of working-age population that is				
	(1) Employed	(2) Employed in Manuf.	(3) Employed, Non-Manuf.	(4) Unemployed	(5) Out of labor force
Panel A: Baseline Specification					
IC_i^t	-0.26 (0.06)	-0.16 (0.05)	-0.09 (0.04)	0.09 (0.03)	0.16 (0.04)
GC_i^t	-0.44 (0.10)	-0.23 (0.06)	-0.21 (0.08)	0.18 (0.05)	0.26 (0.07)
IE_i^t	-0.13 (0.10)	-0.10 (0.04)	-0.03 (0.09)	0.03 (0.04)	0.09 (0.07)
Panel B: ADH specification with geography-based indirect effect					
IC_i^t	-0.45 (0.07)	-0.37 (0.04)	-0.08 (0.08)	0.13 (0.03)	0.32 (0.07)
GC_i^t	-0.35 (0.09)	-0.09 (0.06)	-0.27 (0.08)	0.12 (0.04)	0.23 (0.08)
Panel C: Controls in column 11 of Table 2					
IC_i^t	-0.11 (0.04)	-0.07 (0.04)	-0.04 (0.04)	0.04 (0.02)	0.07 (0.03)
GC_i^t	-0.17 (0.07)	-0.09 (0.05)	-0.08 (0.06)	0.06 (0.04)	0.11 (0.06)
IE_i^t	-0.26 (0.08)	-0.20 (0.05)	-0.06 (0.08)	0.08 (0.04)	0.18 (0.07)

Notes: Sample of 1444 Commuting Zones in 1990-2000 and 2000-2007. In Panel A, all specifications include the set of baseline controls in Table 1. In Panel B, all specifications include the set of baseline controls used in ADH and weight observations by the initial population share for Panel B. Panel C includes the same controls as in column 11 of table 2. Robust standard errors in parentheses are clustered by state.

Table B.7: Differential Impact of the China Shock on U.S. CZs – Additional Migration Outcomes

	Change in the log of		
	(1) Population	(2) In-migration	(3) Out-migration
Panel A: Baseline specification			
IC_i^t	0.13 (0.15)	0.30 (0.25)	0.21 (0.25)
GC_i^t	0.37 (0.22)	0.25 (0.35)	0.27 (0.34)
IE_i^t	0.42 (0.30)	1.11 (0.37)	0.96 (0.35)
Panel B: Specification from Borusyak et al. (2023)			
DIC_i^t	0.04 (0.13)	0.14 (0.18)	0.03 (0.21)

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. In Panel A, we report estimates analogous to those in Table 1 for the migration outcomes described at the top of each column. In panel B, the regressor is the difference in the exposure of the CZ and those in its initial migration network, as defined in Borusyak et al. (2023). Robust standard errors in parentheses are clustered by state.

Table B.8: Differential Impact of the China Shock on U.S. CZs – Additional Migration Outcomes II

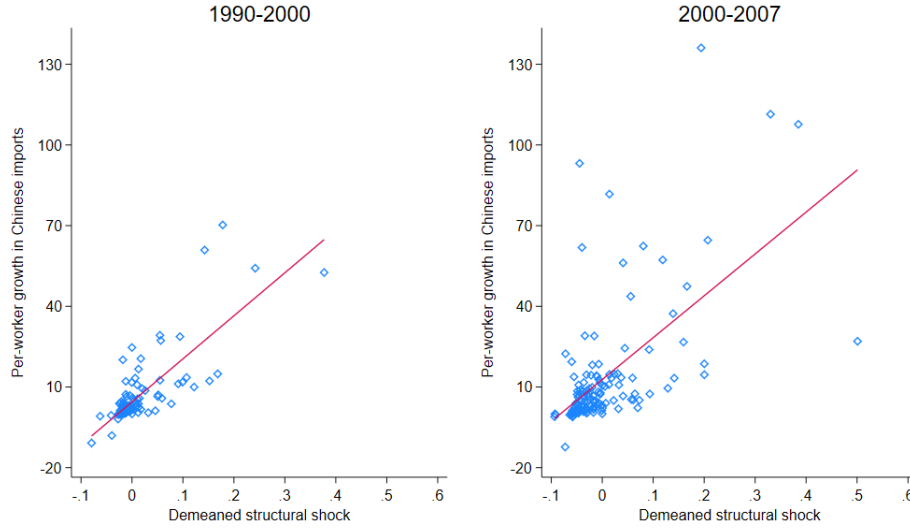
	Change in the log of		
	(1)	(2)	(3)
	Population	In-migration	Out-migration
Panel A: Baseline specification			
IC_i^t	-0.04 (0.12)	-0.04 (0.19)	-0.11 (0.22)
GC_i^t	0.05 (0.17)	-0.55 (0.29)	-0.54 (0.26)
IE_i^t	0.60 (0.25)	1.03 (0.50)	0.69 (0.47)
Panel B: Specification from BDK2023			
DIC_i^t	0.07 (0.08)	-0.03 (0.17)	-0.16 (0.23)

Notes: This table is a version of Table B.7 that includes the same controls as in column 11 of Table 2.

B.2 The General Equilibrium Effect of The China Shock

B.2.1 Measuring the China Shock

Figure B.2: Measures of the China Shock across Sectors



Notes: The graphs plot, across sectors and separately for each period, the per-worker growth in Chinese imports by the eight developed countries used in ADH ($\Delta M_{\text{China},s}^t$) against the demeaned structural shock ($\hat{\zeta}_{\text{China},s}^{\text{obs},t}$). The red line is the best linear fit.

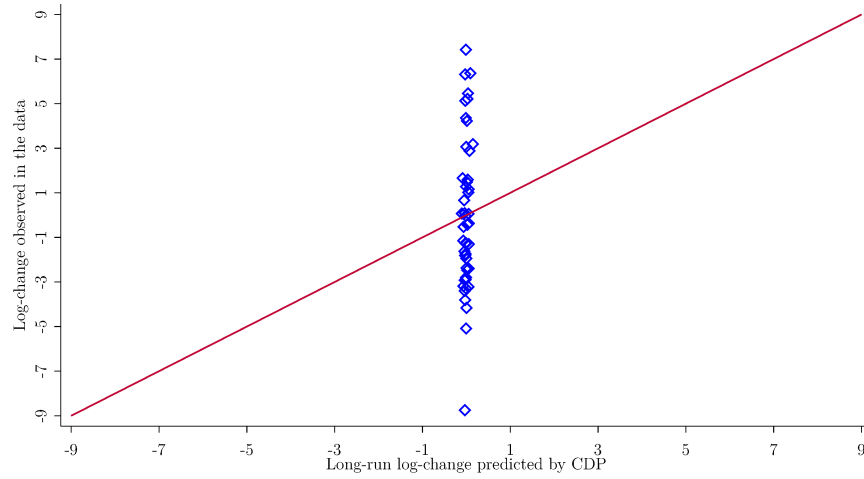
Table B.9: Sectors with the Highest and Lowest Exposure to the China Shock

Sectors most affected by the China shock	Sectors least affected by the China shock
RUBBER AND PLASTIC FOOTWEAR	BROAD WOVEN FABRICS, COTTON
CUT STONE AND STONE PRODUCTS	FATS AND OILS
CEMENT, HYDRAULIC	LOGS, PULPWOOD
CORSETS AND ALLIED GARMENTS	NEWSPAPERS
OUTERWEAR, NSPF, TEXTILE FIBERS	CIGARS
TOYS AND GYMNASTIC GOODS	GUIDED MISSILES AND SPACE VEHICLES
LUGGAGE AND RELATED ITEMS	PAVING AND ROOFING MATERIALS
SHIRTS, NIGHTWEAR	PRINTING BLOCKS
WOMEN'S AND MISSES' OUTERWEAR	INTERLEAVED CARBON SETS
SUITS AND COATS, MEN'S	PERIODICALS

Notes: The table reports the 3-digit SIC sectors with the highest (left column) and lowest (right column) values of the shift in demand caused by China's productivity growth as measured by $\hat{\tau}_{\text{China},s}^{\text{obs},t}$ with equation (36) between 1990 and 2007.

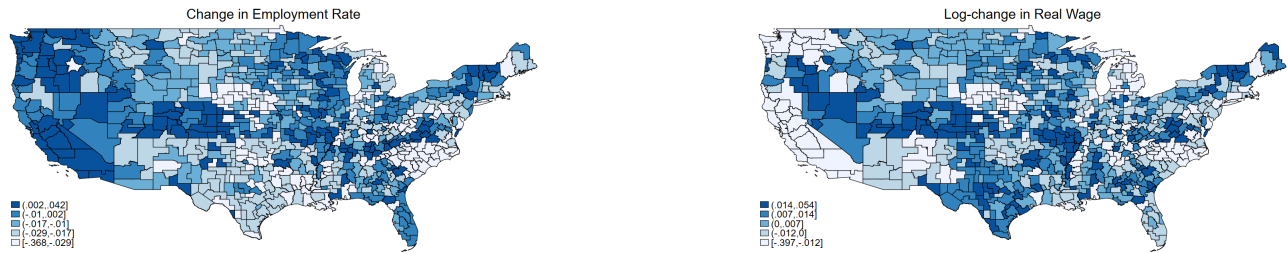
B.2.2 Additional Results

Figure B.3: Log-change in Employment Rate across U.S. States, 2000-2007



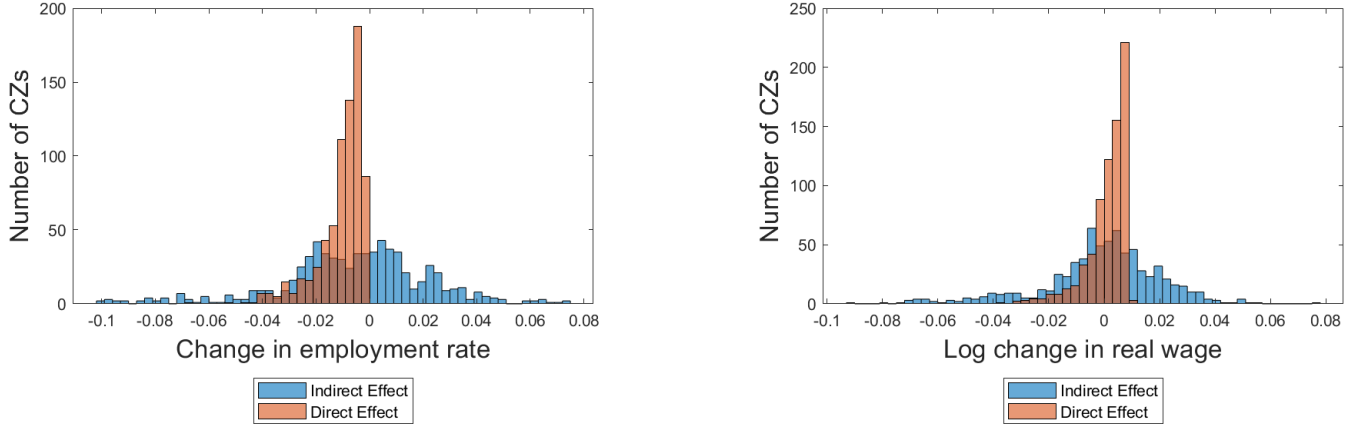
Notes: The figure plots the log-change in the employment rate (multiplied by 100) of U.S. states observed in the data between 2000 and 2007 (vertical axis), against the corresponding log-change predicted by the quantitative spatial model in CDP after the China shock (horizontal axis). The red line is the 45 degree line. We obtain the predicted responses of CDP from their replication files. All variables are normalized to have mean zero.

Figure B.4: Impact of the China Shock on U.S. CZ in General Equilibrium



Notes: The map on the left displays the response of the employment rate to the China shock for each of the 722 CZs that we compute with the sum of the predicted effects for that CZ in 1990-2000 and 2000-2007 implied by the specification in (34) given the parameter estimates in Panel A of Table 3 and the observed shock $\hat{\tau}_{\text{China},s}^{\text{obs},t}$ in Section 5.1. The map on the right displays the analog for the predicted response of the log of the real wage.

Figure B.5: Decomposition of the Impact of China Shock on U.S. CZs



Notes: The histogram on the left separately displays the direct effect and the indirect (domestic) effect of the China shock on the employment rate of each of the 722 CZs. For each CZ, these predicted effects are the sum of the employment rate responses for 1990-2000 and 2000-2007 implied by the specification in (34) given the parameter estimates in Panel A of Table 3 and the observed shock $\hat{\tau}_{China,s}^{obs,t}$ in Section 5.1. The histogram on the right displays the analog for the predicted response of the log of the real wage. For expositional purposes, we trim 9 CZs with change in the employment rate or log of real wage lower than -0.1. Those CZs account for 0.6% of the U.S. population.

C Online Appendix: Data Construction

This appendix describes the procedure to construct the data used in Section 5.

C.1 Bilateral Trade Matrix

C.1.1 Data Construction

Country-to-country bilateral trade matrix. We start by creating a country-to-country matrix of trade flows at the 4-digit SIC classification. We consider the countries listed in Table C.1. We obtain international trade flows at the product-country level from the BACI dataset, assembled by CEPII, which we aggregate at the 4-digit SIC level. Since the starting year of the BACI dataset is 1995, we use the trade flows for 1995 and 2000.⁵ To obtain domestic spending shares for each country, we note first that our gravity model implies that $X_{ij,s}^t = (\tau_{ij,s}^t p_{i,s}^t)^{1-\sigma} (P_{j,s}^t)^{\sigma-1} E_{j,s}^t$. For any sector s within an aggregate sector S , assume that, for $i \neq j$, $\tau_{ij,s}^t = \tilde{\tau}_{i,S}^{O,t} \tilde{\tau}_{j,S}^{D,t} e^{\tilde{\tau}_{ij,s}^t}$. Thus,

$$\ln X_{ij,s}^t = \tilde{\tau}_{ij,s}^t + \alpha_{i,s}^t + \varphi_{j,s}^t, \quad (\text{B.1})$$

where $\alpha_{i,s}^t \equiv \ln \left(\left(\tilde{\tau}_{i,S}^{O,t} p_{i,s}^t \right)^{1-\sigma} \right)$ and $\varphi_{j,s}^t \equiv \ln \left(\left(\tilde{\tau}_{j,S}^{D,t} P_{j,s}^t \right)^{\sigma-1} E_{j,s}^t \right)$.

To get the domestic trade flows, notice that $X_{ii,s}^t = (p_{i,s}^t)^{1-\sigma} (P_{i,s}^t)^{\sigma-1} E_{i,s}^t = \left(e^{\alpha_{i,s}^t} e^{\varphi_{i,s}^t} \right) / \left(\tilde{\tau}_{i,S}^{O,t} \tilde{\tau}_{i,S}^{D,t} \right)$. Since $X_{ii,S}^t = \sum_{k \in S} X_{ii,k}^t$,

$$X_{ii,s}^t = X_{ii,S}^t \frac{e^{\alpha_{i,s}^t} e^{\varphi_{i,s}^t}}{\sum_{k \in S} e^{\alpha_{i,k}^t} e^{\varphi_{i,k}^t}} \quad (\text{B.2})$$

We use (B.2) to compute $X_{ii,s}^t$. In each year t , we obtain $\alpha_{i,s}^t$ and $\varphi_{j,s}^t$ from the estimation of (B.1) with bilateral trade flows by sector, and $X_{ii,S}^t$ from the domestic sales in two aggregate sectors in the Eora MRIO dataset: manufacturing and non-manufacturing.

Table C.1: Sample of Countries

Argentina	Czech Republic	Malaysia	Singapore
Australia	Denmark	Mexico	Slovakia
Austria	Finland	Netherlands	South Africa
Baltic Republics	France	New Zealand	South Korea
Belarus	Germany	Norway	Spain
Benelux	Greece	Pakistan	Sweden
Brazil	Hungary	Philippines	Switzerland
Bulgaria	India	Poland	Taiwan
Canada	Indonesia	Portugal	Thailand
Chile	Ireland	Rest of World	Ukraine
China	Italy	Romania	United Kingdom
Colombia	Japan	Russia	Uruguay
Croatia	Kazakhstan	Saudi Arabia	Venezuela

Notes: Baltic Republics includes Estonia, Lithuania and Latvia.

⁵Although trade data is available for 1990 from UN Comtrade, it is quite sparse across countries and industries.

CZ employment share. We use the same imputation procedure of ADH to compute employment in each 4-digit SIC manufacturing industry for 1980, 1990 and 2000 using the County Business Pattern (CBP). In year t , we use $L_{i,s}^t$ to denote employment in CZ i and 4-digit SIC industry s and $\ell_{i,s}^t = L_{i,s}^t / L_i^t$ to denote the associated employment share.

CZ gross spending shares. We construct gross spending by sector and CZ, $e_{i,s}^t$, using

$$e_{i,s}^t \equiv \frac{E_{i,s}^t}{E_i^t} = \frac{\xi_{i,s}^t + \sum_k \xi_{i,sk}^{M,t} a_k^t \ell_{i,k}^t}{1 + \sum_k a_k^t \ell_{i,k}^t}. \quad (\text{B.3})$$

where, in year t , $\xi_{i,sk}^{M,t}$ is the share of spending on intermediates of sector s by sector k (common to all CZs, $\xi_{i,sk}^{M,t} = \xi_{sk}^{M,t}$), a_k^t is the ratio of intermediate cost to labor cost of sector k (common to all CZs), and $\xi_{i,s}^t$ is consumers' spending share on final goods of sector s (common to all CZs, $\xi_{i,s}^t = \xi_s^t$). We compute $\xi_{sk}^t \equiv \frac{M_{sk}^t}{\sum_{s'} M_{s'k}^t}$ where M_{sk}^t is the spending of industry k on industry s in the BEA 1992 U.S. Input-Output table used in [Acemoglu et al. \(2016a\)](#). For manufacturing SIC-4 industries, we compute a_k^t using total material costs divided by payroll in the NBER manufacturing database for year t . For non-manufacturing industries, we compute a_k^t as average the material to payroll ratio across all U.S. non-manufacturing industries in the WIOD database (?). Finally, we obtain ξ_s^t from the BEA 1992 U.S. Input-Output table.

CZ exports and imports. We follow three steps to create exports and imports for each CZ and industry. First, we compute the CZ spending on sector s as $E_{i,s}^t = e_{i,s}^t L_i^t$ where $e_{i,s}^t$ is the sectoral spending share described above and L_i^t is the total employment in the CZ. Second, for each sector s , we compute the share of CZ i in national spending, $\tilde{e}_{i,s}^t = E_{i,s}^t / \sum_j E_{j,s}^t$, and in national employment, $\tilde{\ell}_{i,s}^t = L_{i,s}^t / \sum_j L_{j,s}^t$. Third, we use the US Census data at the state-sector level for 1997 to compute the share of each state in the exports/imports to/from each foreign country in a SCTG category, which is the 40-sector classification used by the US Census.⁶ This yields $\beta_{state,i,s} = \frac{X_{state,i,s}^{state,i,s}}{X_{US,i,s}^{state,i,s}}$, where i is any of 52 foreign importer, and $\beta_{i,state,s} = \frac{X_{state,i,s}^{state,i,s}}{X_{US,i,s}^{state,i,s}}$, where i is any of 52 foreign exporters. We use the same share $\beta_{state,i,s}$ and $\beta_{i,state,s}$ for all SIC-4 industries within the same SCTG category. Finally, in each year t , we take US imports $X_{i,US,s}^t$ and US exports $X_{US,i,s}^t$ in each sector s and foreign country i , and split them across CZs using the following expressions:

$$X_{ij,s}^t = \frac{\tilde{e}_{j,s}^t}{\sum_{j' \in state} \tilde{e}_{j',s}^t} \beta_{i,state,s} X_{i,US,s}^t \quad \text{and} \quad X_{ji,s}^t = \frac{\tilde{\ell}_{j,s}^t}{\sum_{j' \in state} \tilde{\ell}_{j',s}^t} \beta_{state,i,s} X_{US,i,s}^t.$$

CZ-to-CZ bilateral trade matrix. We follow three steps to impute trade flows across CZs using the gravity trade structure of our model. First, for each SCTG category, we use state-to-state shipment data from the Commodity Flow Survey in 1997 to estimate

$$\ln X_{ij,s} = \delta_s + \beta_1 \ln D_{ij} + \beta_2 \ln E_{j,s} + \beta_3 \ln R_{i,s} + \beta_4 d_{i=j} + \varepsilon_{ij,s} \quad (\text{B.4})$$

where i is the origin state, j is the destination state, s is the SCTG category, D_{ij} is the bilateral distance between the population centroids of states i and j , $E_{j,s}$ are expenditures, $R_{i,s}$ are revenues, $d_{i=j}$ is a dummy that equals 1 when $i=j$.

⁶We construct state-sector exports and imports as follows. First, we use the U.S. Merchandise Trade Data for 1997 released by the U.S. Census to create a mapping from each of the 44 U.S. districts to the 50 U.S. states, in terms of share of imports and exports to each foreign country. Note that this is done at the aggregate level as this information is not available at the industry-level. We then use U.S. Census data to create district-level exports and imports at the HS-6 level for 1997. Finally, we use the mapping previously constructed to obtain state-HS6, and then state-SIC 4 digit, trade flows with our sample of foreign countries.

Second, we use the estimated coefficients to impute trade flows across CZs with the following gravity specification:

$$\ln X_{ij,s}^t \equiv \hat{\beta}_1 \ln D_{ij} + \hat{\beta}_2 \ln \tilde{e}_{j,s}^t + \hat{\beta}_3 \ln \tilde{\ell}_{i,s}^t + \hat{\beta}_4 d_{state(i)=state(j)} \quad (\text{B.5})$$

where D_{ij} is the distance between the population centroids of CZs i and j , and $d_{state(i)=state(j)}$ is a dummy equal 1 if i and j belong to the same state.

Lastly, we re-scale the imputed CZ-to-CZ trade flows so that the sum of the bilateral flows in each SIC sector across all CZs is equal to the total U.S. domestic sales in each SIC sector in the country-to-country trade matrix.

Trade balance. Finally, we impose that trade is balanced at the regional level, as in the baseline model. We use the trade flows obtained above to compute matrix $\bar{\mathbf{x}}^t$ whose entries correspond to the share of spending of each region j on another region i . Under trade balance, the vector of total revenue in the world economy, $\mathbf{R}^t$, must satisfy $\bar{\mathbf{x}}^t \mathbf{R}^t = \mathbf{R}^t$ and, therefore, $(\bar{\mathbf{I}} - \bar{\mathbf{x}}^t) \mathbf{R}^t = 0$. Notice that it is always possible to find a vector $\mathbf{R}^t$ that satisfies this system since $(\bar{\mathbf{I}} - \bar{\mathbf{x}}^t)$ is singular ($\sum_i x_{ij}^t = 1$ for every j). Thus, we find the vector $\mathbf{R}^t$ as the eigenvector of $(\bar{\mathbf{I}} - \bar{\mathbf{x}}^t)$ associated with the eigenvalue of zero. Without loss of generality, we then normalize it such that world GDP is one, $\sum_i R_i^t = 1$.

C.1.2 Validation Tests

We first evaluate the correlation between the expenditure shares $e_{i,s}^t$ constructed in equation (B.3) and the spending shares implied by the shipment data for U.S. states. To this end, for each of the 40 SCTG categories, we compute state-level total shipment inflow in the Commodity Flow Survey (CFS) for 1997. We then construct state-level spending shares at each SCTG category using the expenditure shares $e_{i,s}^t$ in equation (B.3) for the CZs in the state. Specifically, we first aggregate our expenditure shares at the SCTG level using a crosswalk between SIC-4 and SCTG categories, and then compute total spending by SCTG in each state using the total expenditure of the CZs in that state. Table C.2 reports the result of a regression of the expenditure shares computed from the CFS on our constructed gross spending shares in 1990 and 2000. We can see that they are positively and significantly correlated, with an OLS coefficient close to 1 and a R^2 of 0.95.

We then proceed to assess whether our constructed CZ-level trade matrix reproduces the patterns of observed trade flows for U.S. states. We use the CFS to measure bilateral shipments between U.S. states in each SCTG category for 1997, 2002 and 2007. To obtain comparable data, we aggregate the bilateral trade flows for the CZs in the same state and the SIC sectors in the same SCTG category. Table C.3 reports the results of regressing actual shipment data on the corresponding trade flow obtained from our trade matrix. Column (1) considers domestic flows between U.S. states, column (2) considers export flows from U.S. states to foreign countries, and column (3) considers import flows from foreign countries to U.S. states. All specifications include sector fixed-effects. We can see that the predicted trade flows are significantly and positively related to the actual flows, with coefficients close to 1. Notice also that our imputed data captures a large share of the variation in bilateral trade flows. The R^2 is above 0.8 for exports and imports of U.S. states, and around 0.5 for domestic flows between U.S. states.

Table C.2: Validation Test – Gross Expenditure Shares

Dependent variable:	Observed expenditure shares, 1997	
	(1)	(2)
Constructed expenditure shares, 1990	1.18*** (0.02)	
Constructed expenditure shares, 2000		1.17*** (0.02)
Constant	-0.01*** (0.00)	-0.01*** (0.00)
Observations	1,392	1,392
R^2	0.91	0.91

Notes: Sample of 1,392 state-SCTG pairs, where SCTG is the industry classification used in the CFS. Dependent variable is the observed expenditure share in 1997 computed from the CFS. The regressors are the expenditure shares computed in equation (B.3), aggregated at the state-SCTG level. Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table C.3: Validation Test – Bilateral Trade Flows

	(1)	(2)	(3)
Panel A: Log of Actual Flows in 1997			
Log of Predicted Flows in 1997	1.09*** (0.01)	0.97*** (0.00)	1.00*** (0.00)
Observations	64,512	69,888	69,888
R^2	0.55	0.97	1
Panel B: Log of Actual Flows in 2002			
Log of Predicted Flows in 2002	1.01*** (0.01)	0.86*** (0.00)	0.90*** (0.00)
Observations	64,512	69,888	69,888
R^2	0.55	0.84	0.89
Panel C: Log of Actual Flows in 2007			
Log of Predicted Flows in 2007	1.06*** (0.01)	0.80*** (0.00)	0.87*** (0.00)
Observations	64,512	69,888	69,888
R^2	0.52	0.83	0.87
Flow type:			
U.S. state to U.S. state	Yes	No	No
U.S. state to Country	No	Yes	No
Country to U.S. state	No	No	Yes

Notes: The dependent variable in column (1) is the actual shipment flow reported in the CFS for state-state-SCTG triples. The dependent variables in columns (2) and (3) are trade flows constructed from the US Census trade data for state-country-SCTG triples. The regressors are the trade flows constructed using our methodology for the years 1997, 2002 and 2007, aggregated at the state-state-SCTG or state-country-SCTG level. All regressions include sector fixed effects. Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

C.2 Trade in Intermediate and Final Goods

The methodology described in the previous section yields a bilateral matrix of gross trade flows between 722 U.S. CZs and 52 countries. While this is enough to implement the model of Section 3, the estimation of the more general model with input-output links of Section 4 requires bilateral trade flows in intermediate and final goods. We now describe how we proceed to construct such data. Our procedure relies on the fact that, in our model, trade flows in final goods and intermediate inputs between two markets i and j can be written respectively as $X_{ij,s}^C = x_{ij,s} \xi_{j,s} E_j$ and $X_{ij,sk}^M = x_{ij,s} \xi_{j,sk}^M a_{j,k}^M R_{j,k}$, where $x_{ij,s}$ is the matrix of *gross* trade shares within sector s . This property is the by-product of the assumption that the elasticity of substitution between products of different origins is the same for final consumption and intermediate consumption in all sectors, as in [Caliendo and Parro \(2015\)](#) and in the literature reviewed by [Costinot and Rodríguez-Clare \(2014\)](#). Therefore, we only need to complement the bilateral matrix of trade shares described above with data on the sectoral spending shares of final and intermediate expenditures. To reduce the computational burden, we construct the bilateral trade flows in intermediate and final goods at the 3-digit sector level.

C.2.1 Final Spending Shares

Our main data source is the Consumer Expenditure Survey (CEX) Public-use Micro-data from the U.S. Bureau of Labor Statistics for the years of 1996 and 2000. We first combine the individual-level information in the interview and diary databases to generate annual average household expenditure in each U.S. state on the different product categories in the CEX (i.e., the UCC codes). We then construct a crosswalk from the UCC product classification used in the CEX to 3-digit SIC sectors, using the UCC description provided by the BLS. For the states without data in the CEX, we assign the final expenditure shares of the US Census division to which that state belongs. For all foreign countries, we set the share of final spending on each SIC sector to be the same as that reported in the 1992 U.S. IO table from the BEA.

C.2.2 Intermediate Spending Shares

We measure the sectoral intermediate spending shares in each CZ and country by assuming that $a_{j,k}^M = a_j (1 - a_k^L)$ where a_k^L is the share of labor in sector k 's total cost (common to all countries). We first describe how we calibrate a_i and then how we construct each variable.

First, from the good market clearing condition,

$$R_{i,s} = \sum_j x_{ij,s} \left(\xi_{j,s} E_j + \sum_k \xi_{j,sk}^M a_j a_k^M R_{j,k} \right),$$

where E_j is market j 's expenditure on final goods, and $\xi_{j,s}$ and $\xi_{j,sk}^M$ are market j 's final and intermediate spending shares. We can write this expression in matrix form and invert it:

$$\mathbf{R}(\mathbf{a}) = \sum_{d=0}^{\infty} (\bar{\mathbf{A}}(\mathbf{a}))^d \mathbf{F}, \quad (\text{B.6})$$

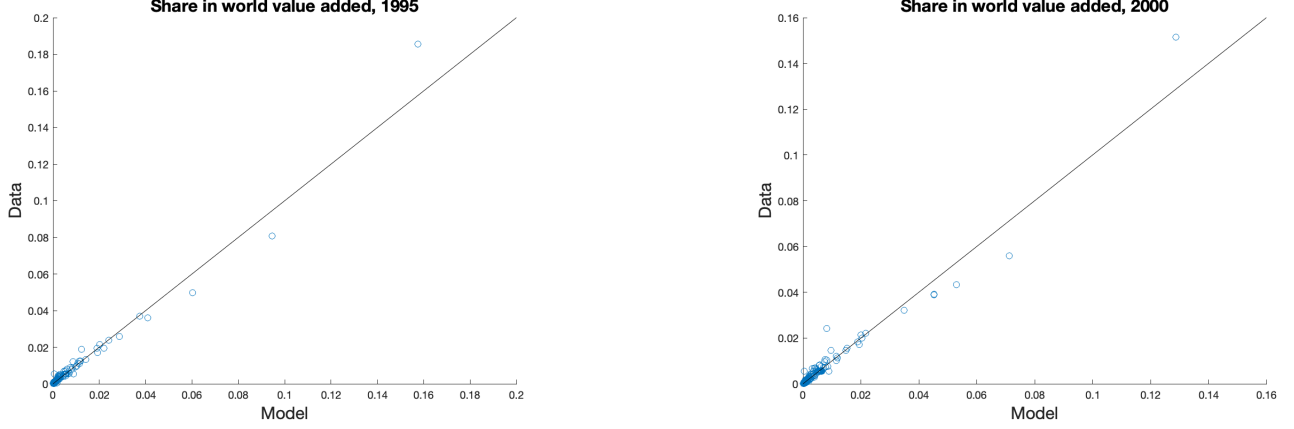
where $\mathbf{F} \equiv \left[\sum_j x_{ij,s} \xi_{j,s} E_j \right]_{is}$, $\bar{\mathbf{A}}(\mathbf{a}) = \bar{\mathbf{x}}^A \text{diag}(a_j)$ and $\bar{\mathbf{x}}^A \equiv [x_{ij,s} \xi_{j,sk}^M a_k^M]_{is,jk}$.

Second, from the labor market clearing condition,

$$W_i(\mathbf{a}) = \sum_s (1 - a_s^M a_j) R_{i,s}(\mathbf{a}), \quad (\text{B.7})$$

where $R_{i,s}(\mathbf{a})$ is given by (B.6).

Figure C.1: Share of World Value Added



Notes: For each CZ and foreign country, each graph plots the share of world value added observed in the data against the corresponding share predicted by our calibration procedure.

Finally, we calibrate $\mathbf{a}$ to minimize the difference between the observed value-added in market i , W_i , and the one predicted by equation (B.7), $W_i(\mathbf{a})$:

$$\mathbf{a}^* = \arg \min_{a_j \in (0, 1 / \max_k \{a_k^M\})} \sqrt{\sum_i (W_i - W_i(\mathbf{a}))^2}.$$

To implement this calibration, we use the within-sector bilateral trade shares, $x_{ij,s}$, that we constructed with the methodology described in Appendix C.1. The labor shares a_k^L are obtained from the NBER Manufacturing database. For all markets, we use the 1992 BEA IO table to measure final and intermediate spending share. We aggregate all the shares at the 3-digit sector level. For all countries, we measure value-added from the WIOT. For the U.S., we split value-added across CZs by setting value-added in CZ i to $W_i = \left(\frac{W_i^P}{\sum_{j \in US} W_j^P} \right) W_{US}$, where W_i^P is the CZ's wage bill in the CBP. Similarly, we use the WIOT to measure aggregate final expenditure in each country, and split total final expenditure in the U.S. across CZs using the same payroll shares, $E_i = \left(\frac{W_i^P}{\sum_{j \in US} W_j^P} \right) E_{US}$. Figure C.1 shows that our calibration procedure almost exactly matches the observed shares of value added across U.S. CZs and foreign countries.